NEUTRON KINETICS

Time dependent diffusion equation (1 group)

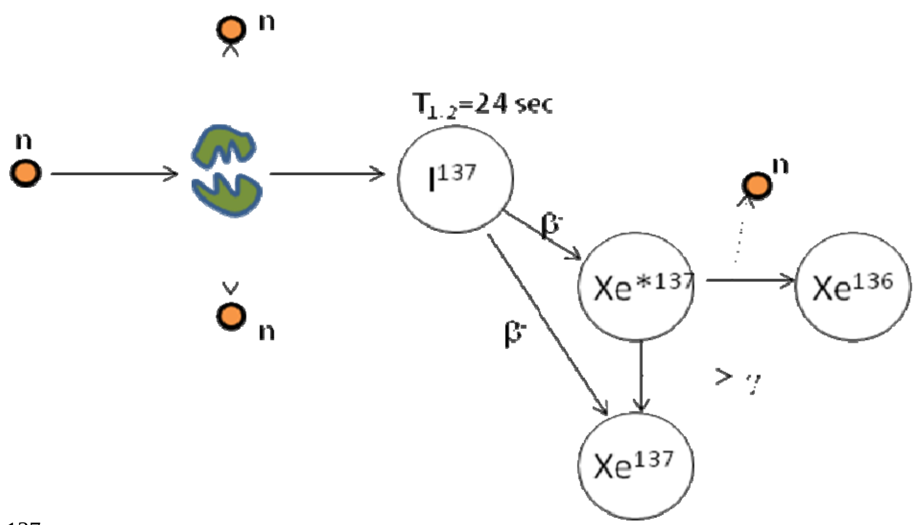
$$D\nabla^2\phi \quad - \quad \Sigma_a\phi(r,t) \quad + \quad S(r,t) \quad = \quad \frac{\partial n}{\partial t} \qquad (1)$$

In-leakage - Abs. + Prod. = change/sec

n - neutron density (n/cm^3)
ϕ = $v \times n$
v_{fast} = 2.10^5 m/s
v_{th} = 4.10^3 m/s

The source has two contributions:

- *Prompt neutrons* – directly from fission
- *Delayed neutrons* – from fission product decay



I^{137} is a so-called precursor.
I^{*137} is in an isomeric state.

Precursors are grouped into 1-8 families, each characterized by its average half life (from 0.2 sec to 1 minute).

If there is only one family: $T_{1/2}$ = 8 sec.

Compare this with the neutron generation time: l = 0.05 ms - $T_{1/2}/l \sim 10^5$.
The fraction of delayed neutrons for U-fission: $\beta = 0.007$.

Equation for precursor concentration C

$$\frac{\partial C}{\partial t} = \beta\nu\Sigma_f\phi - \lambda C \quad , \quad \lambda = \frac{\ln 2}{T_{1/2}} \tag{2}$$

The diffusion equation becomes

$$D\nabla^2\phi - \Sigma_a\phi(r,t) + (1-\beta)\nu\Sigma_f\phi + \lambda C = \frac{1}{v}\frac{\partial\phi}{\partial t} \tag{3}$$

1. Point Kinetics

If the migration of neutrons is neglected in the diffusion equation, i.e. if $\nabla^2\phi=0$, then an equation without space dependence is obtained. The core is treated as one point.

In one-group theory $k_\infty = \nu\Sigma_f/\Sigma_a$. Further, $\phi = vn \Rightarrow$

$$\left[(1-\beta)k_\infty - 1\right]\Sigma_a vn + \lambda C = \frac{dn}{dt} \tag{4}$$

The neutron mean free path because of absorption = $1/\Sigma_a$ (cm). Define

$$l_0 = \frac{1}{v\cdot\Sigma_a} \qquad \text{Neutron 'life time'}$$

$$l = \frac{l_0}{k_\infty} \qquad \text{'Effective life time'}$$

$$\rho = \frac{k_{eff}-1}{k_{eff}} = \frac{k_\infty - 1}{k_\infty} \qquad \text{'Reactivity'}$$

$\Rightarrow$

$$\left.\begin{aligned} \frac{dn}{dt} &= \frac{\rho-\beta}{l}n + \lambda C \\ \frac{dC}{dt} &= \frac{\beta}{l}n - \lambda C \end{aligned}\right\} \tag{5}$$

These are the point kinetics equations (with only one family of delayed neutrons).

2. The In-Hour Equation

Solve the point kinetics equations assuming that

- Up to time *t* the reactor is just critical (ρ=0)
- At time *t* a constant reactivity insertion ρ is applied.

By assuming the solution $n(t) = \mathrm{A}e^{\omega t}$, $C(t) = \mathrm{B}e^{\omega t}$ in the point kinetics equation one arrives at the *inhour* equation

$$\rho = l\omega + \frac{\beta\omega}{\omega+\lambda} \quad , \quad T_{1/2} \approx \frac{\ln 2}{\omega} \tag{6}$$

$\Rightarrow$

$$n(t) = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t}$$

$$\left.\begin{aligned} \omega_1 &\approx \frac{\lambda\rho}{\beta-\rho} \\ \omega_2 &\approx -\frac{\beta-\rho}{l} \end{aligned}\right\} \tag{7}$$

If small reactivity insertion: $\rho << \beta$

$$n(t) \approx A_1 e^{\lambda/\beta \cdot \rho \cdot t} = A_1 e^{\rho/l' \cdot t} \tag{8}$$

l = 0.05 ms	Life time of prompt neutrons
l' = β/λ = 80 ms	Prompt + Delayed neutrons

If large reactivity insertion: $\rho > \beta$

$$n(t) \approx A_2 e^{\rho/l \cdot t} \tag{9}$$

The solution explodes.