“I always dream of a pen that would be a syringe.” — Jacques Derrida

BEHAVIORAL NETWORK SCIENCE

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Hohenheim 2023

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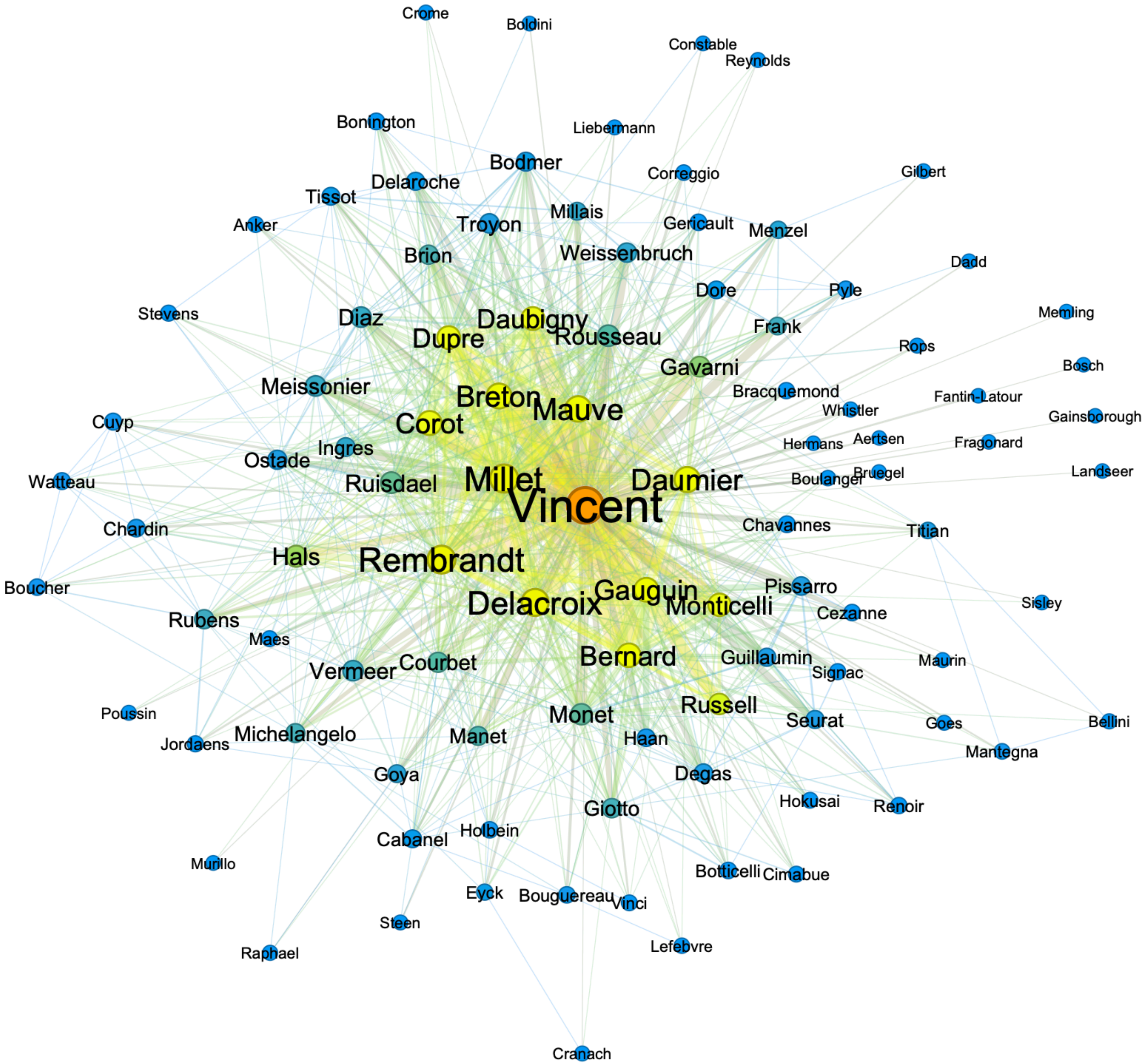
Today

Goal: Getting our hands dirty with R and your own data (or some of my data on Van Gogh, if you prefer)

Small worlds networks and other comparisons.


Crome
Boldini
Constable
Reynolds
Bonington
Liebermann
Bodmer
Correggio
Delaroche
Tissot
Gilbert
Millais
Anker
Troyon
Gericault
Menzel
Brion
Weissenbruch
Dadd
Dore
Pyle
Stevens
Diaz
Daubigny
Memling
Dupre
Rousseau
Frank
Rops
Gavarni
Bosch
Meissonier
Bracquemond
Fantin-Latour
Breton
Whistler
Mauve
Gainsborough
Cuyp
Corot
Aertsen
Hermans
Fragonard
Ingres
Ostade
Boulanger
Bruegel
Landseer
Watteau
Ruisdael
Millet
Daumier
Vincent
Chavannes
Titian
Chardin
Hals
Rembrandt
Boucher
Pissarro
Sisley
Gauguin
Delacroix
Monticelli
Cezanne
Rubens
Maes
Bernard
Guillaumin
Maurin
Vermeer
Courbet
Signac
Russell
Poussin
Monet
Seurat
Goes
Bellini
Michelangelo
Jordaens
Manet
Haan
Mantegna
Goya
Degas
Hokusai
Renoir
Giotto
Holbein
Cabanel
Murillo
Botticelli
Cimabue
Eyck
Bouguereau
Vinci
Steen
Raphael
Lefebvre
Cranach

What’s the control network?

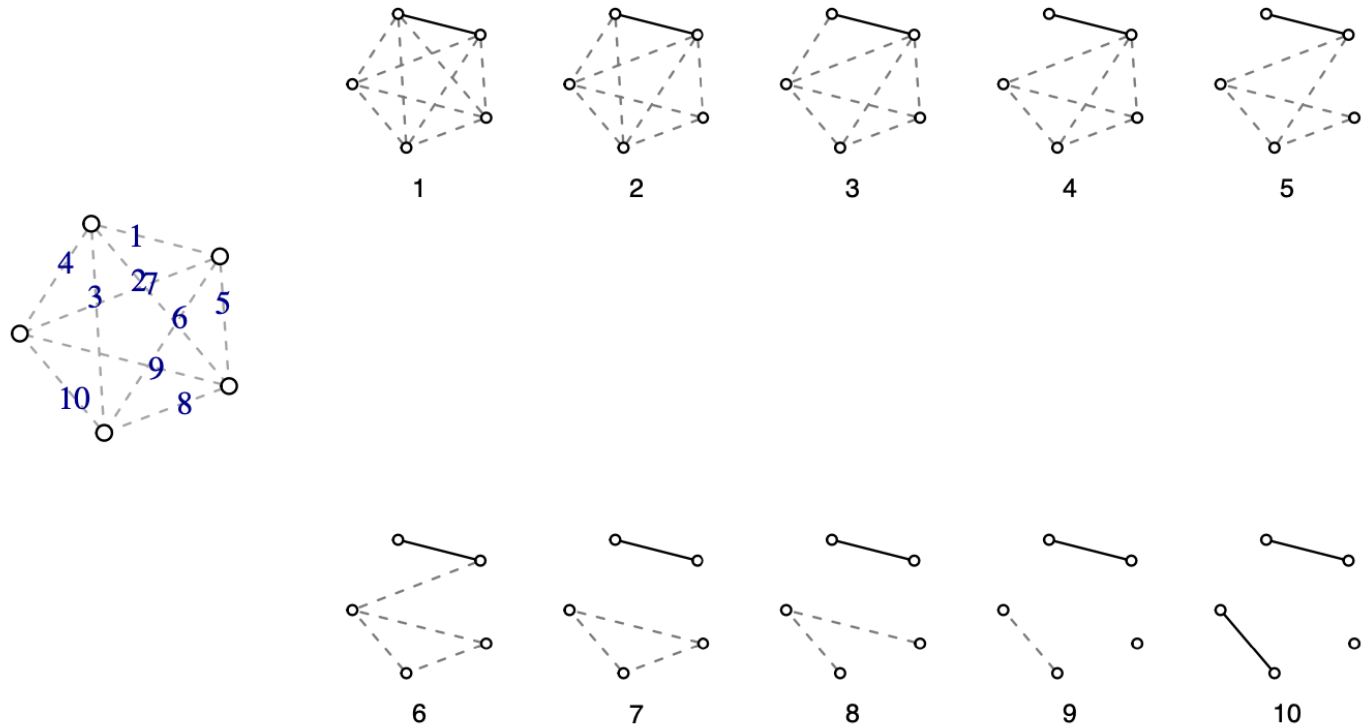
- I can measure structure on networks.
- But this structure needs comparators to be meaningful.
- There are many potential models.

Erdös-Renyi random graphs

G(N,E) — Define N nodes and probability of edge (or density)

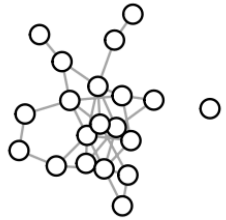
This is *the* standard model for a ‘null’ graph.

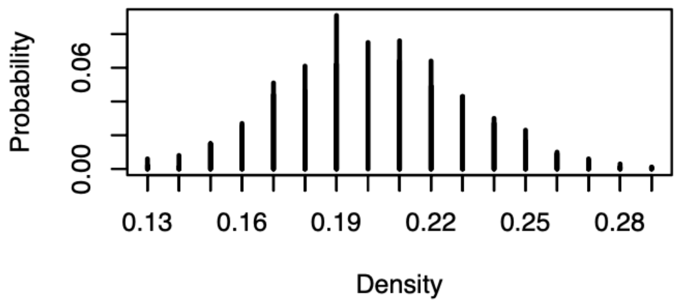
Erdös-Renyi random graphs



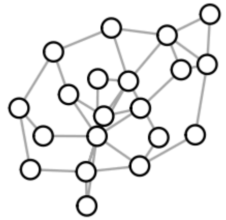
G(N,E) — Define N nodes and probability of edge (or density)

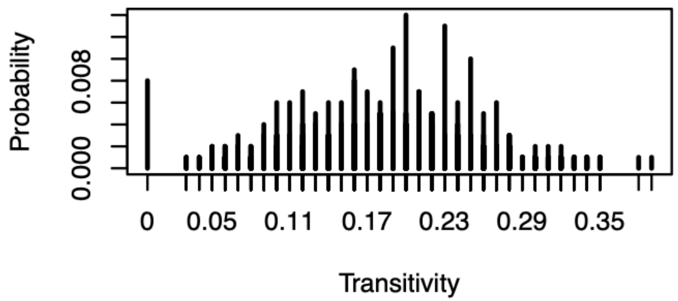
Erdös-Renyi random graphs are distributions

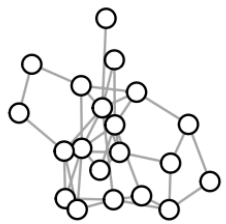


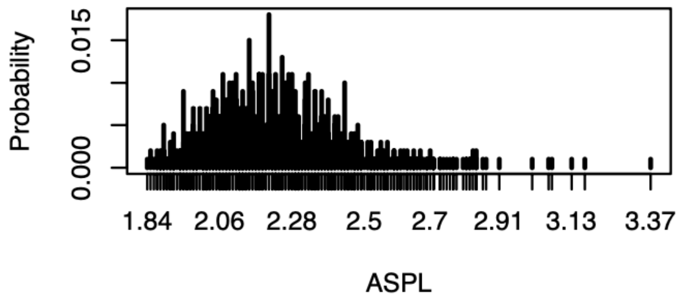


Fixed probability

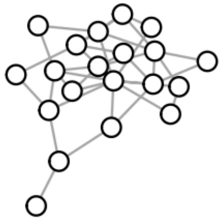


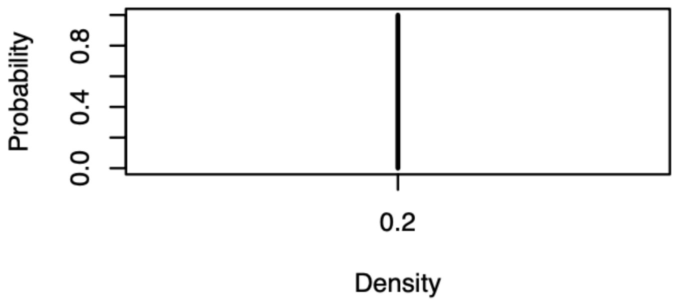




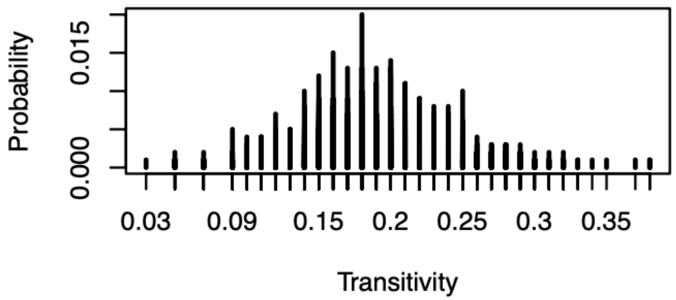


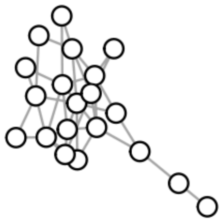
Erdös-Renyi random graphs are distributions

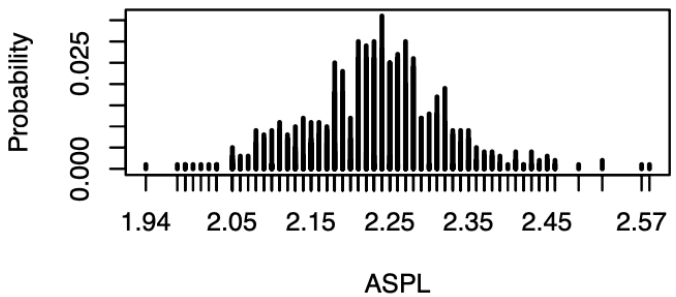




Fixed Edge number







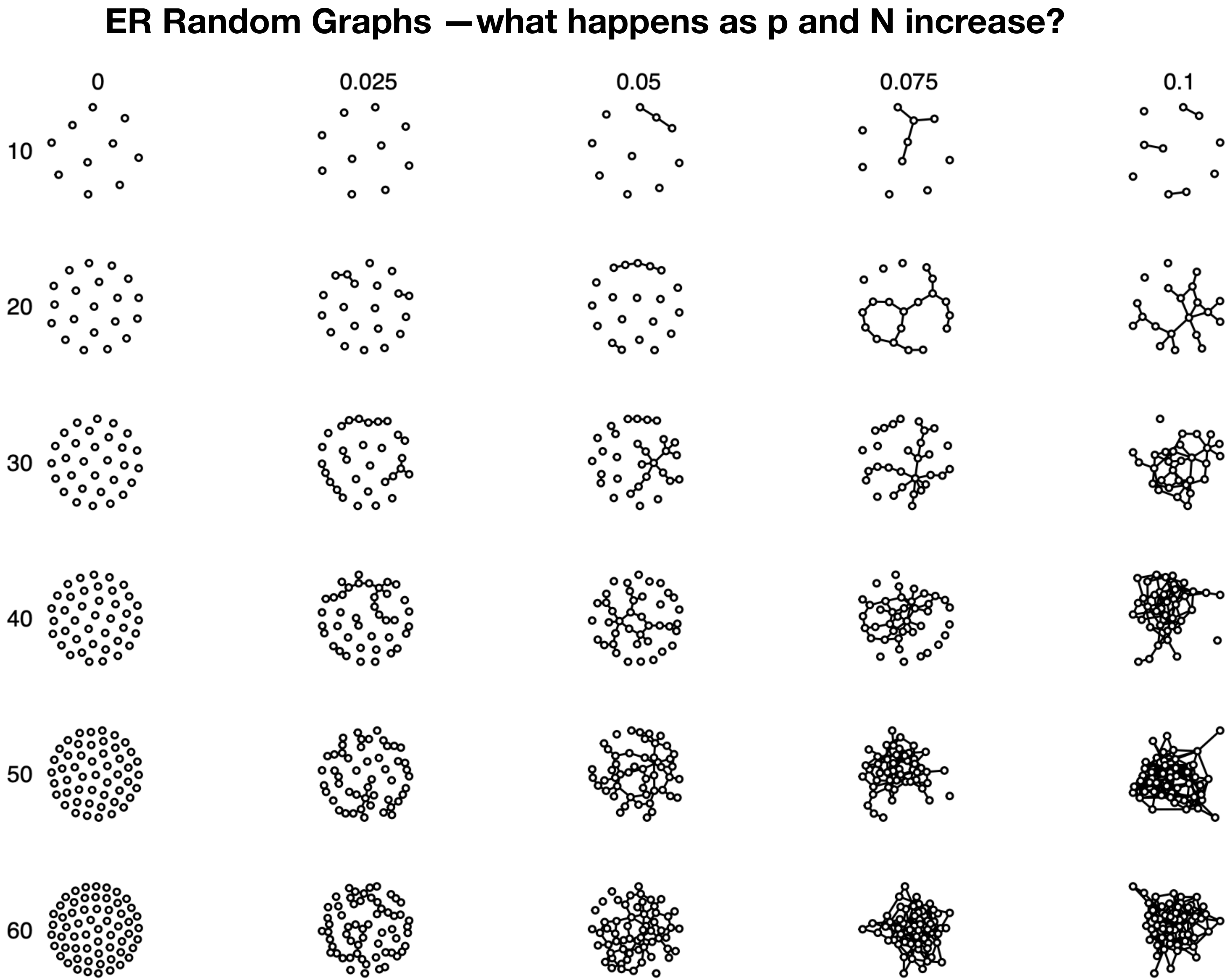
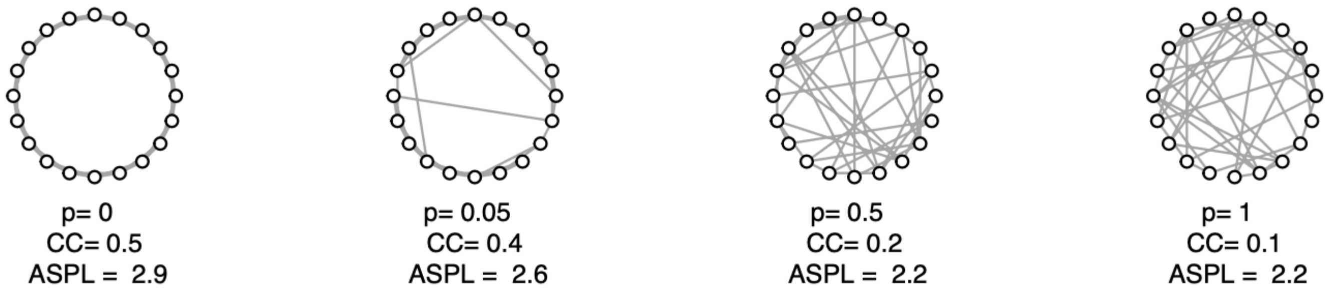


Figure 2: The influence of density and number of nodes on components and connectivity.

Small world networks

(Watts Strogatz)

Small world networks: nodes are closer together than they would be in a random network.



Start with structured lattice (ring lattice) and rewire edges with probability p

Small world networks

(Watts Strogatz)

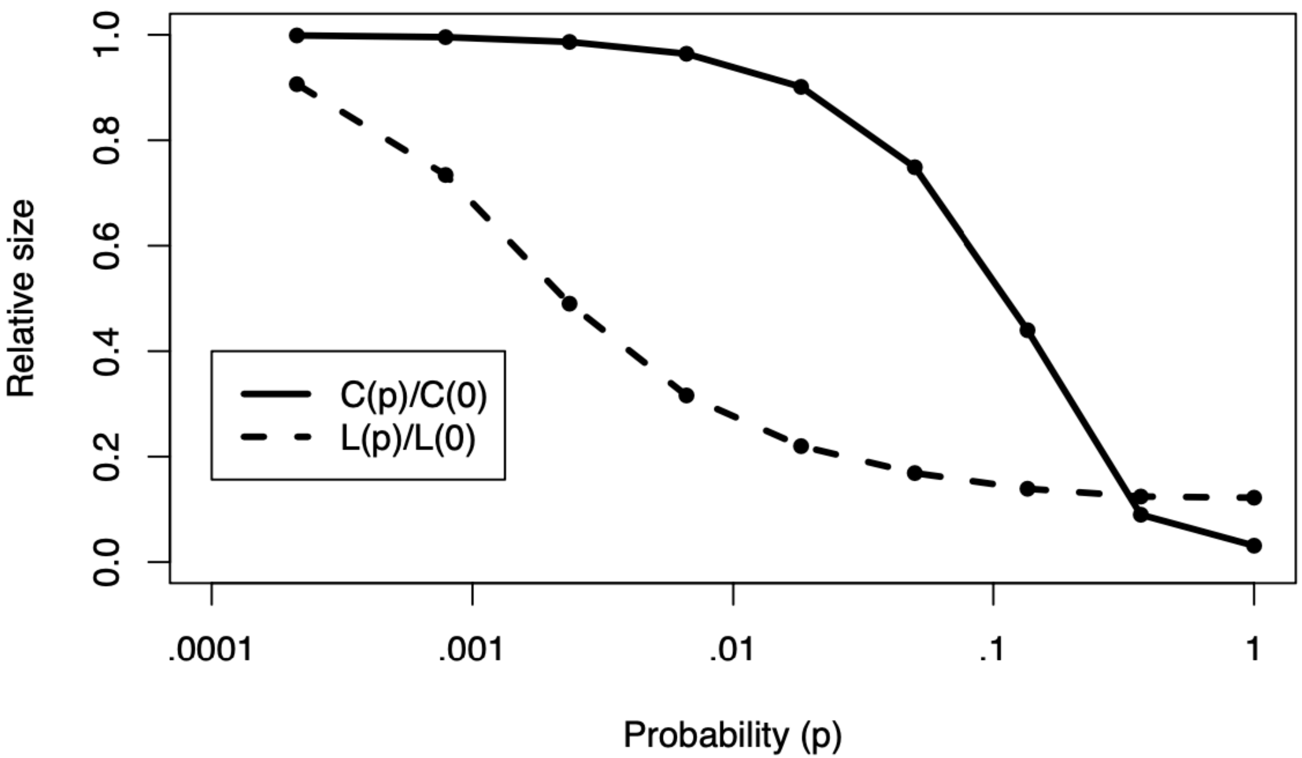
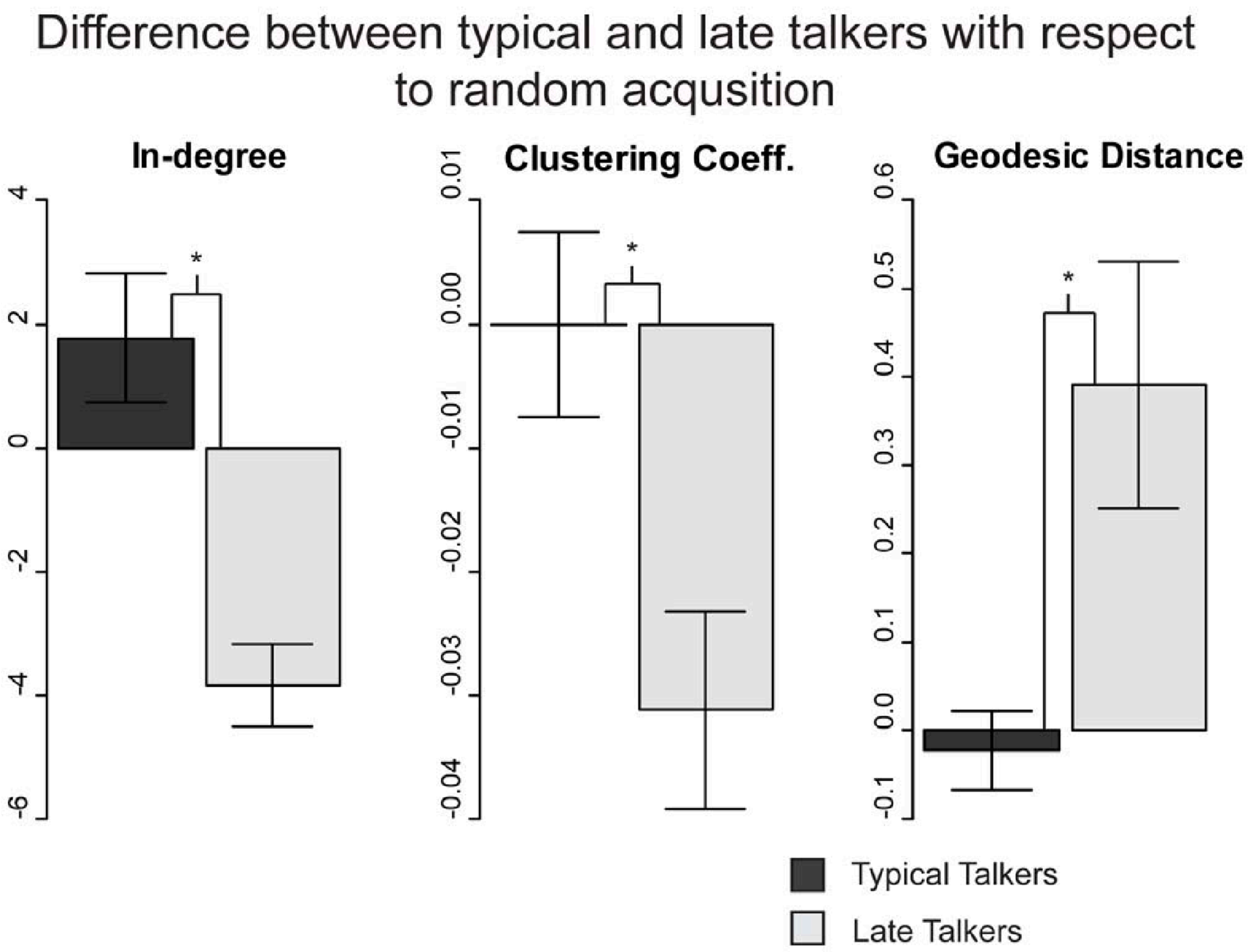


Figure 2: Relative clustering coefficient and average shortest path length for the Watts-Strogatz small world model across a range of rewiring probabilities, p.

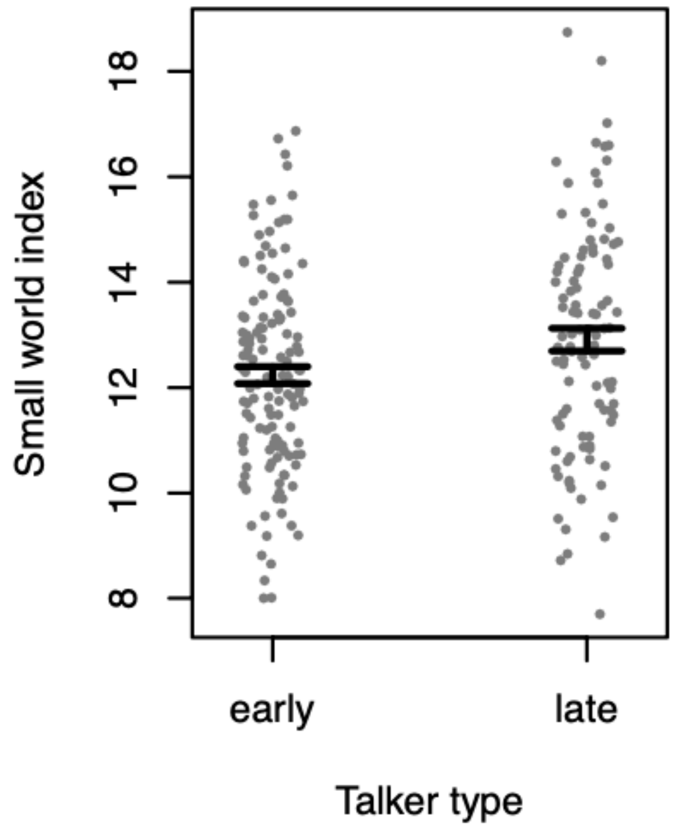
Clustering coefficient and the words children learn



Late talkers have lower degree and lower clustering coefficient and have average shortest path length (ASPL = geodesic distance)

Small world Index

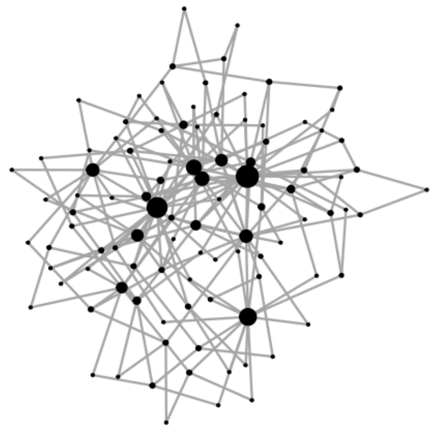
$$SWI = \frac{\frac{C_{observed}}{C_{random}}}{\frac{L_{observed}}{L_{random}}} = \frac{\gamma}{\lambda}$$



Preferential Attachment

(Barabasi)

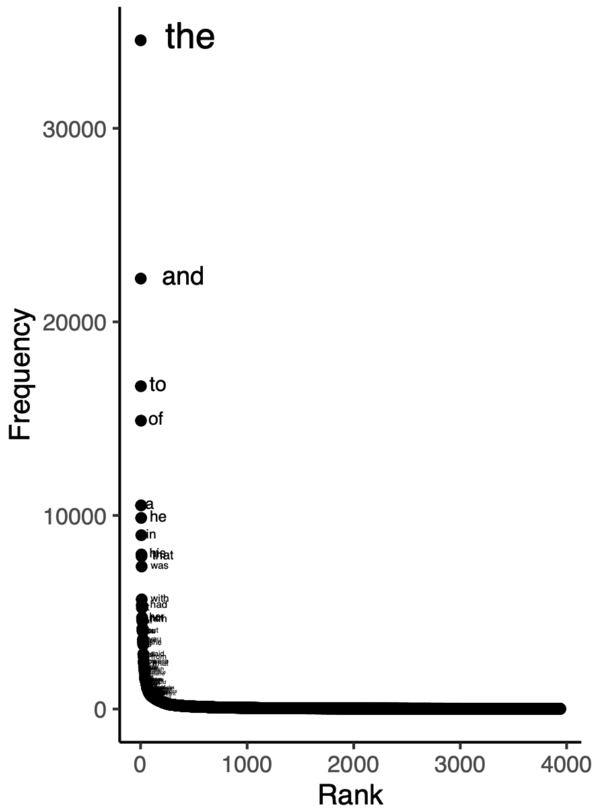
Nodes are added one at a time and preferentially attach to existing nodes ~ f(degree)

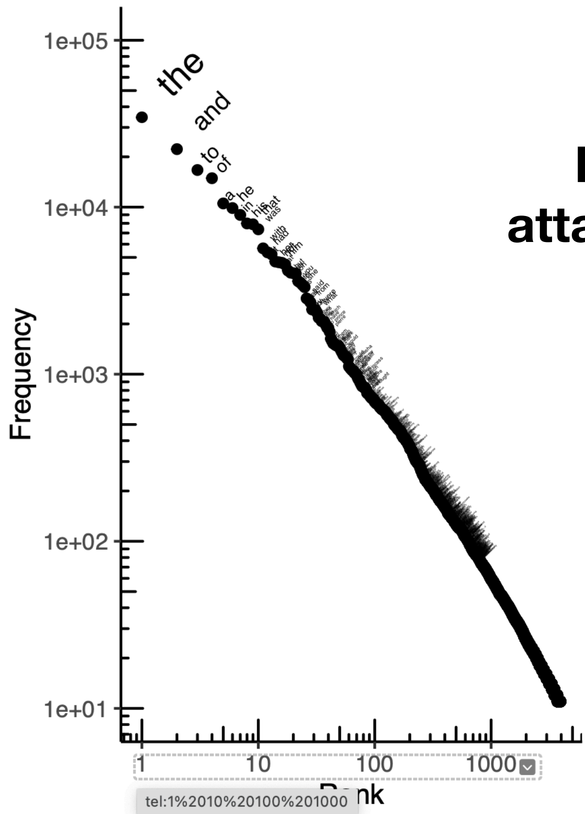


$$P(i) = \frac{k_i^\alpha + a}{\sum_{j \in N}^{N} k_j^\alpha + a}$$

Scale-free Networks

Scale-free networks are linear on a log-log plot





Preferential attachment gives us this

Scale-free Networks

Fractals are scale-free

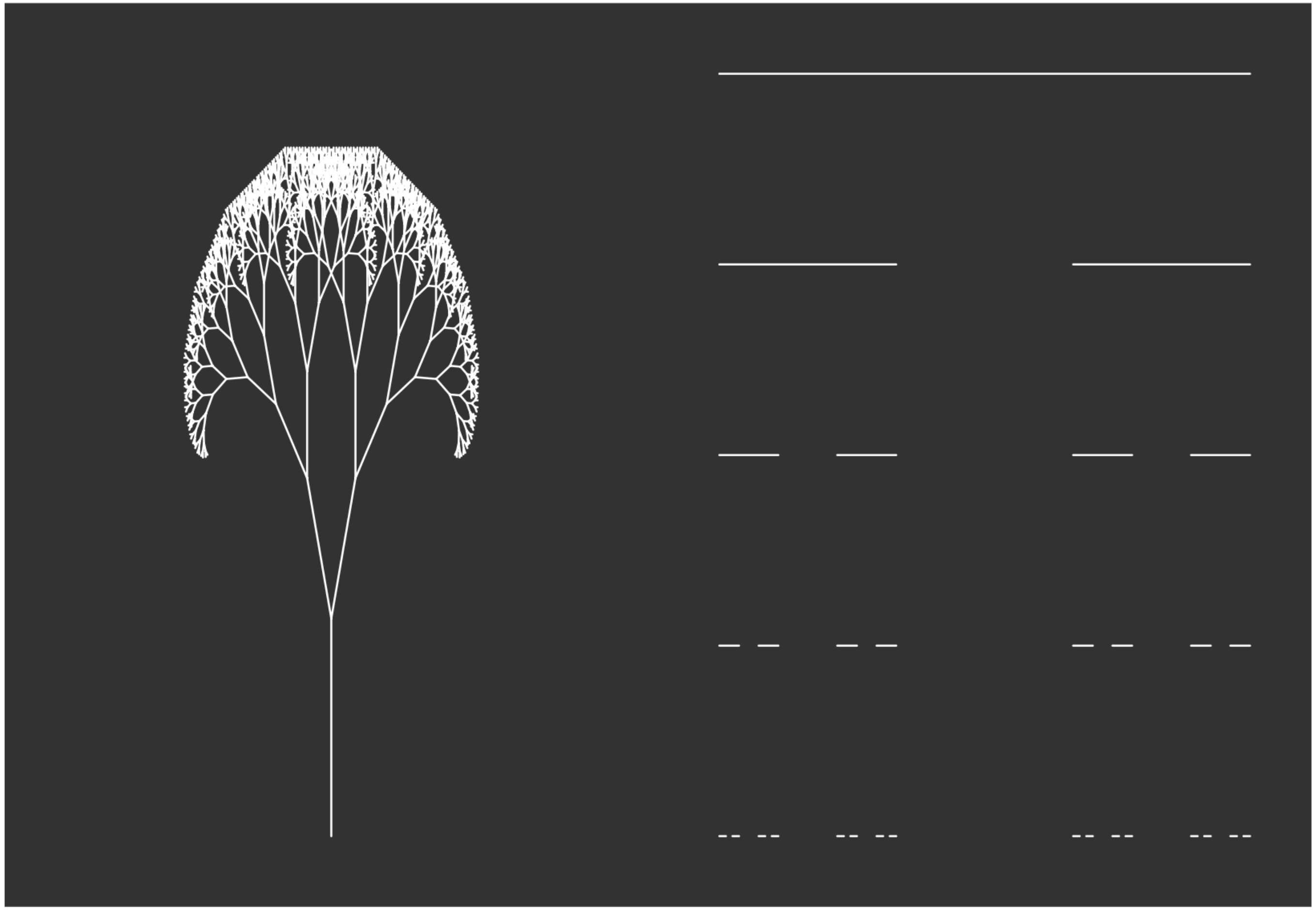
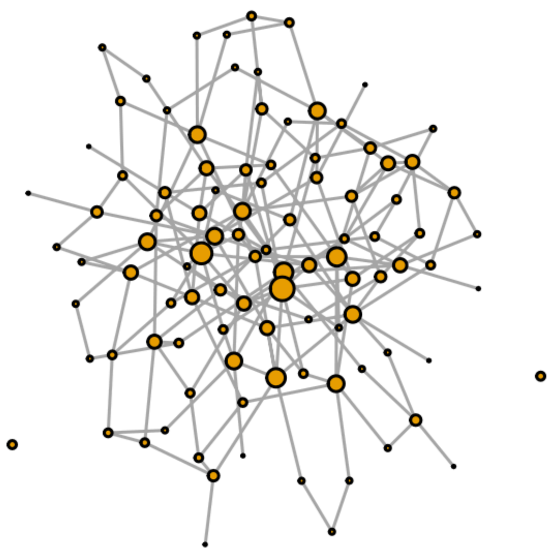


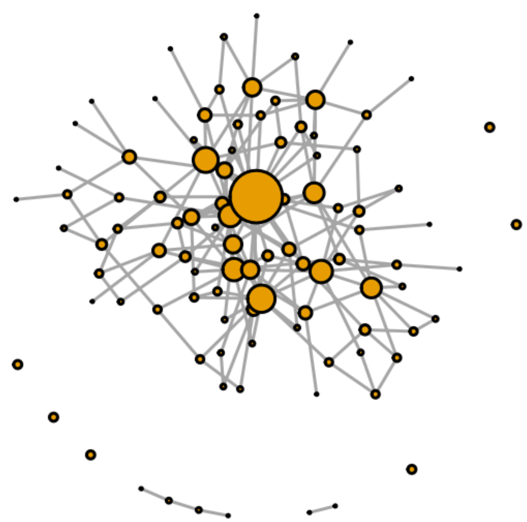
Figure 2: Fractals exhibiting scale-free and power-law distributions of edge lengths. On the left is a recursive tree: each branch leads to two new lines of reduced size. On the right is a Cantor Set: each line produces two new lines of 1/3 the size as we move downwards. Note that as we move up in size, the number of larger lines is always half the number of lines one-size smaller.

Scale-free Networks

ER random network



Scale–free network



Attack Tolerance

Scale-free networks are resilient to random attacks, but not to targeted attacks

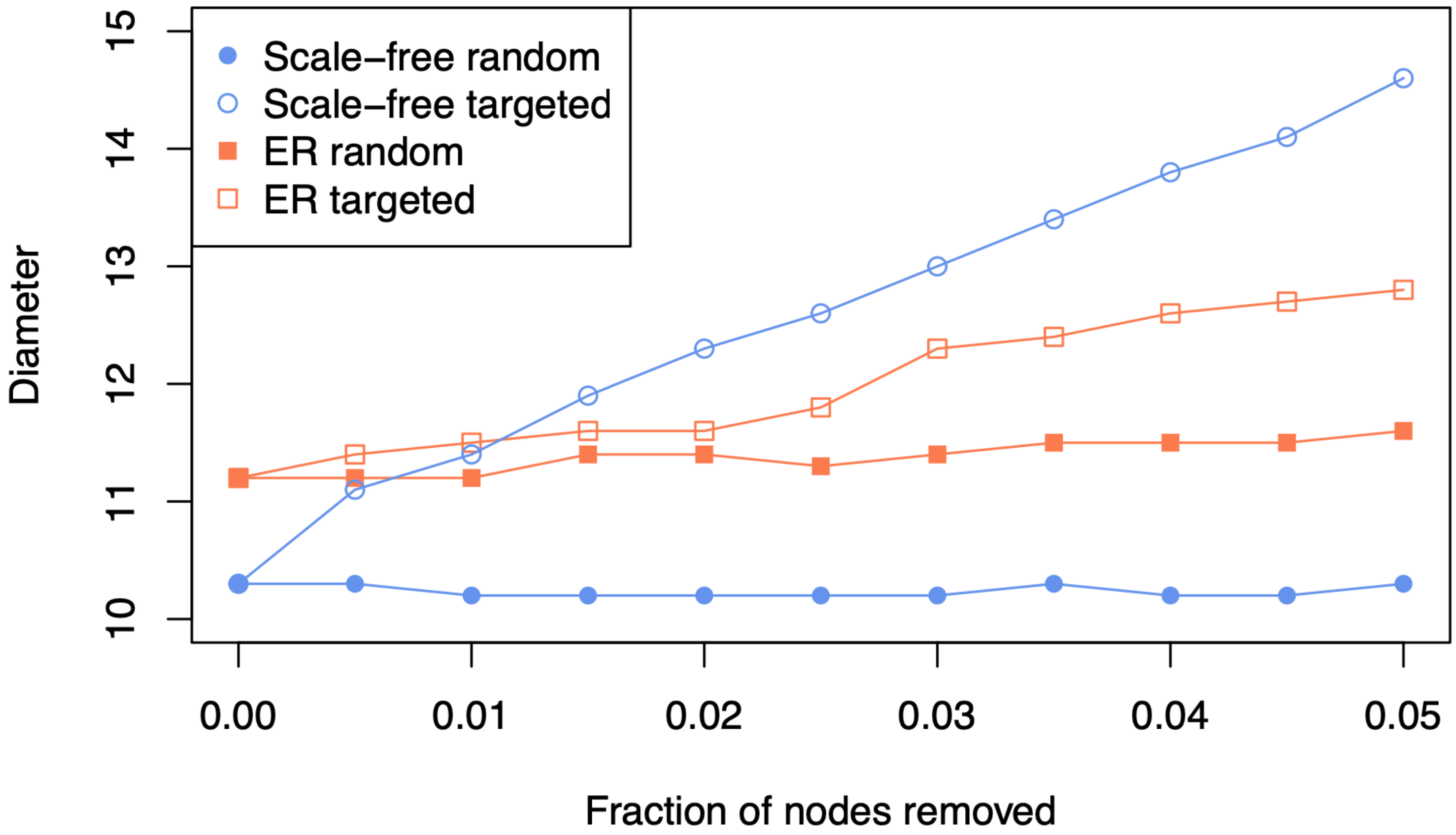


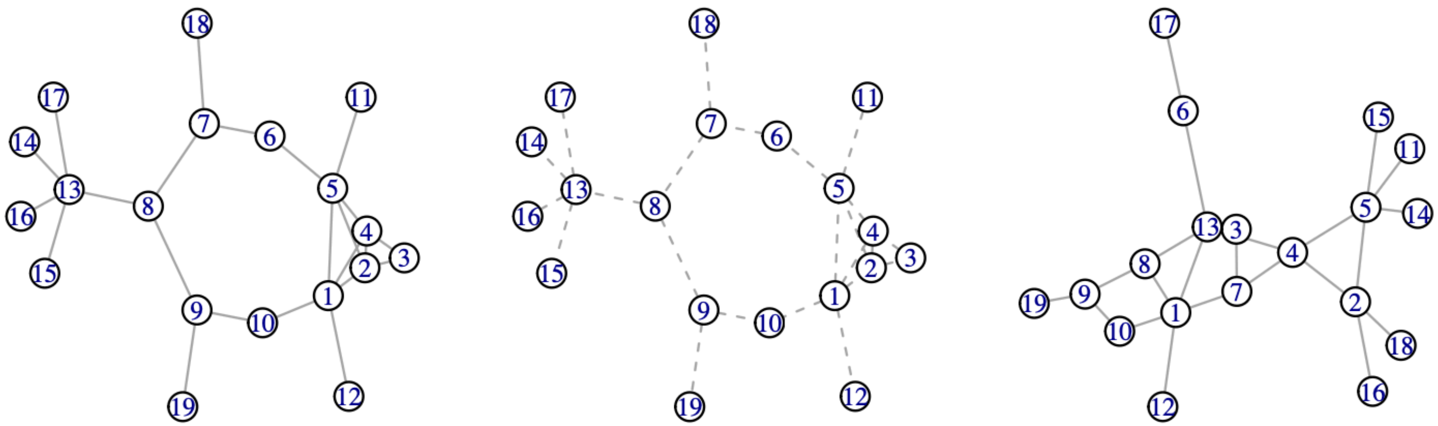
Figure 5: Targeted and random attacks on random and scale-free networks. All networks have N=5000 nodes and E=10000 edges. Random error attacks remove a fraction, f, of nodes with a uniform probability across all nodes. Targeted attacks remove the fraction, f, of nodes with the highest degree. Scale-free networks are more resilient to random attacks, but more susceptible to targeted attacks.

Null models

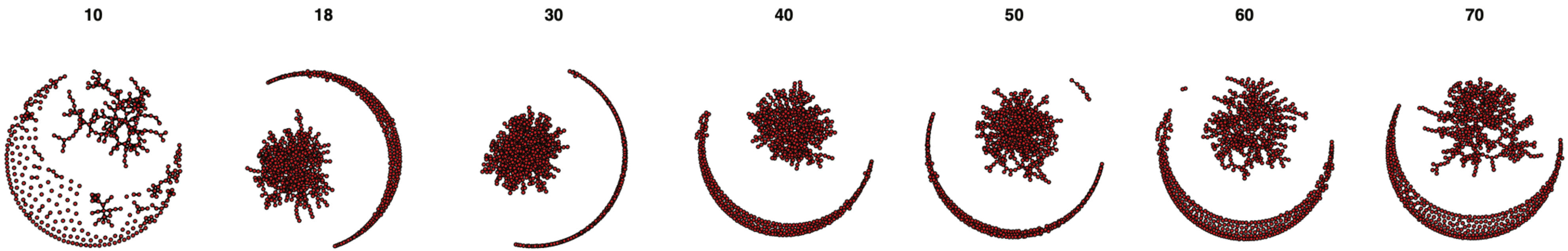
- Other kinds of ‘null’ models are variations of the data we start with.

Configuration Models

Preserve degree distribution, but rewire edges

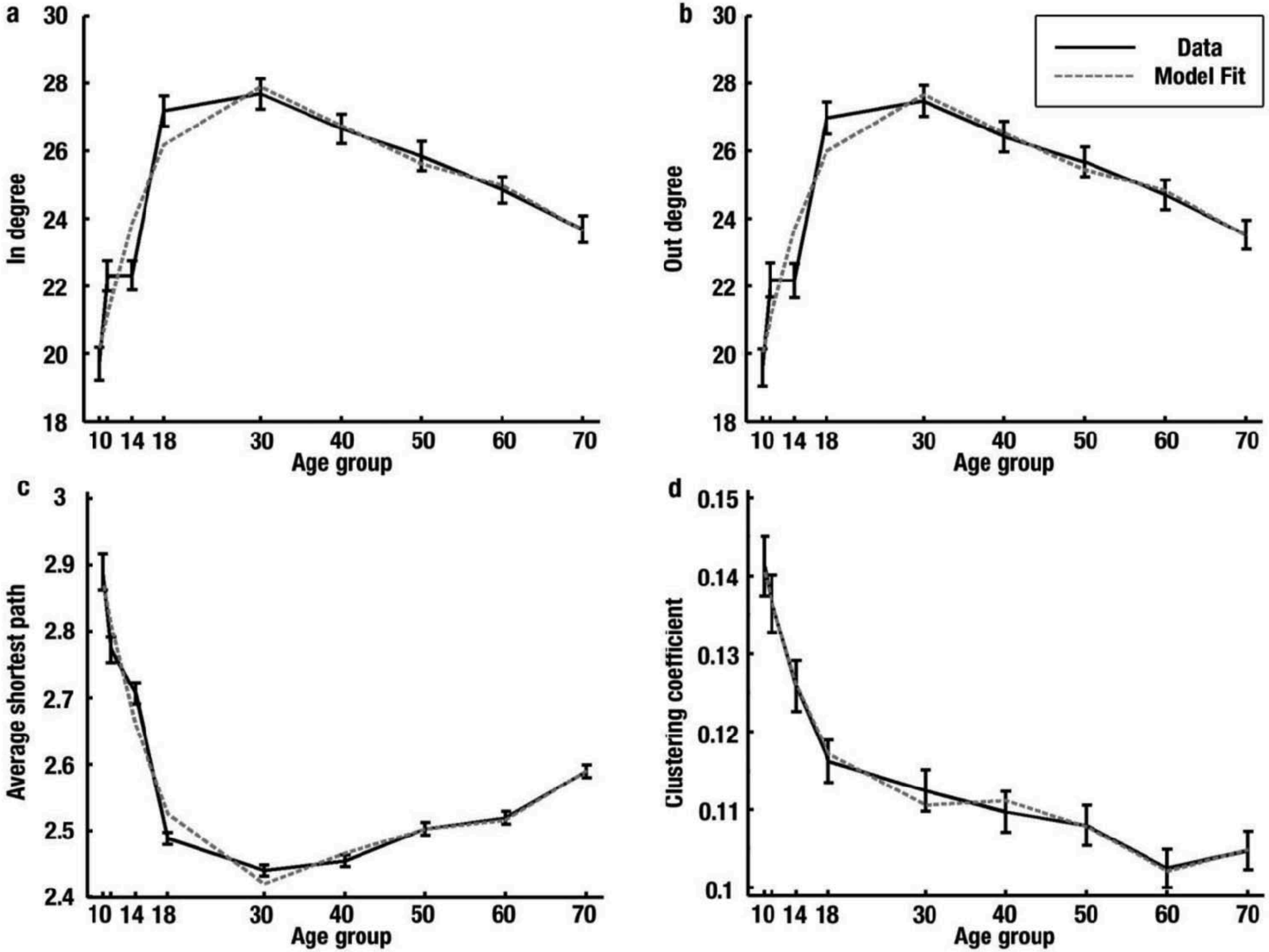


Comparisons between networks



- 420 nodes, 8000 participants (~1000 per age group)

Network comparisons



Network comparisons

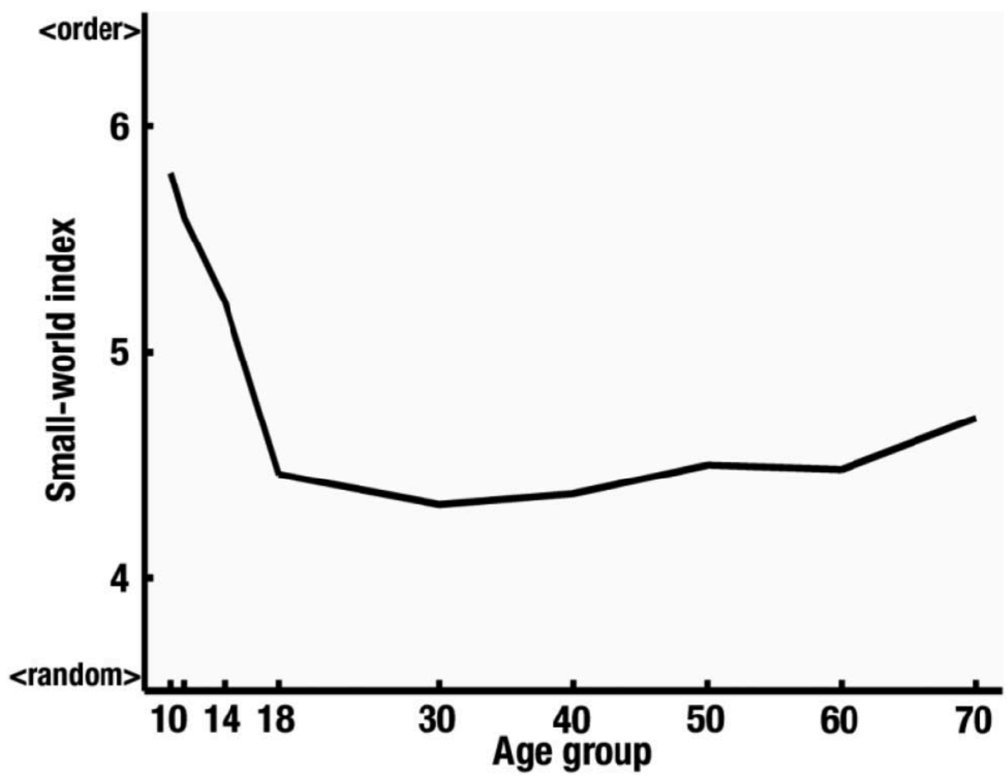
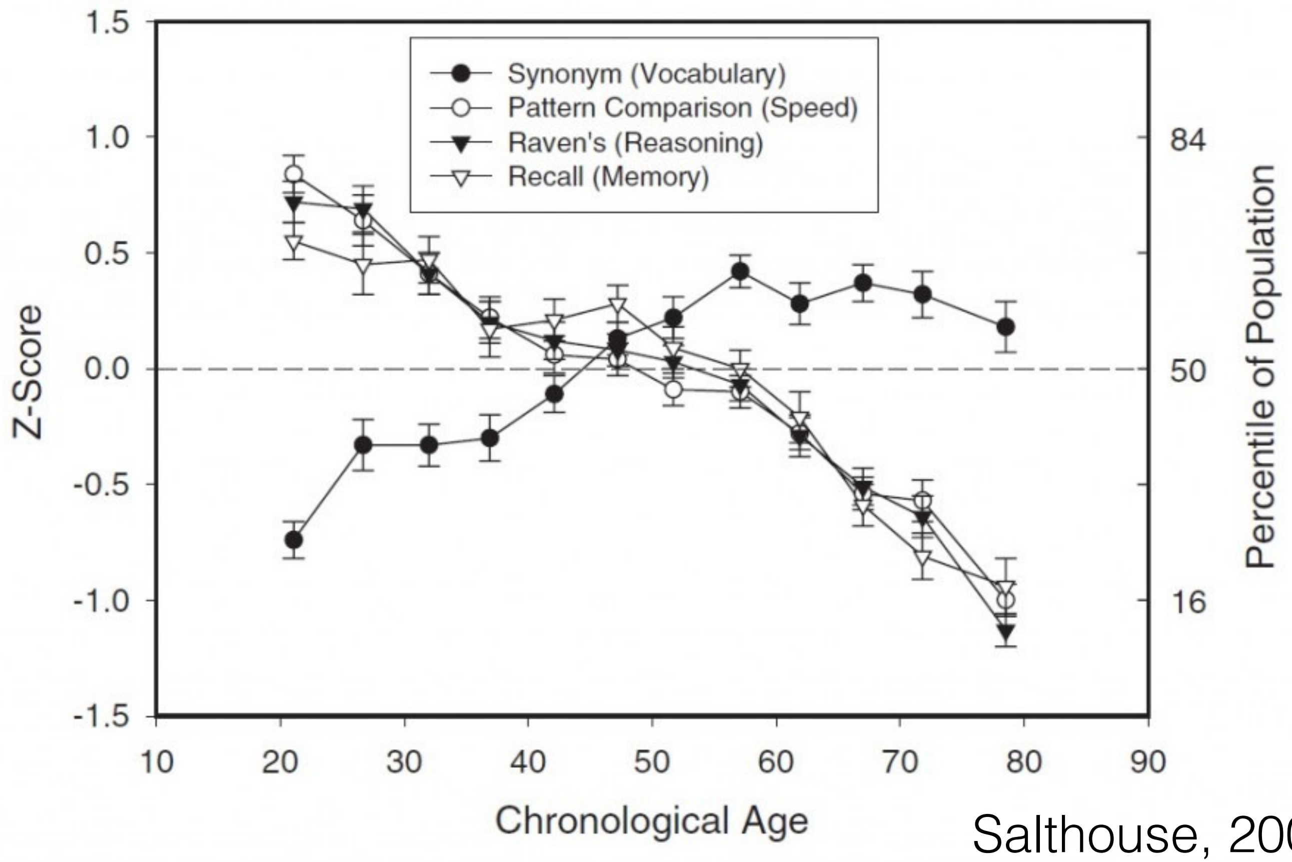


Figure 5. The small-world index across the life span.

Modeling Growth

Vocabulary grows across the lifespan



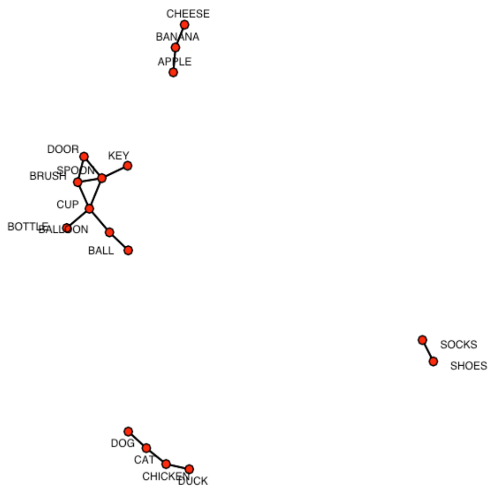
Salthouse, 2004

Fig. 1. Means (and standard errors) of performance in four cognitive tests as a function of age. Each data point is based on between 52 and 156 adults.

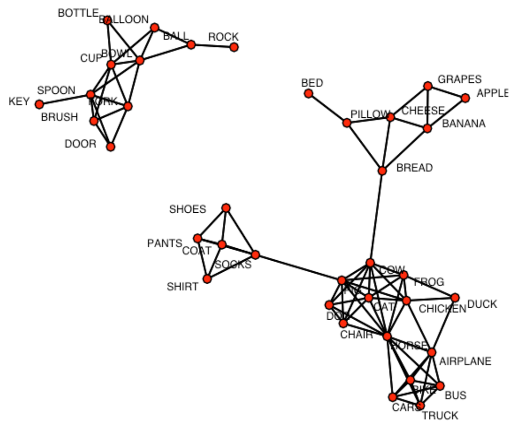
Growth vocabulary

relationships
(edges)
features
associates
semantic
taxonomic
phonology

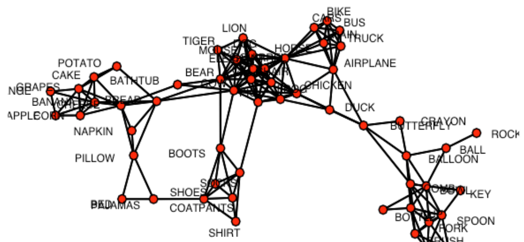
15 months



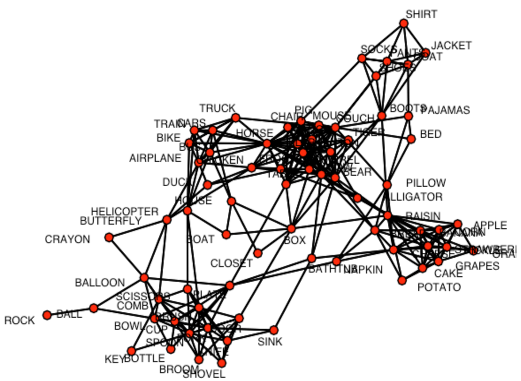
16 months



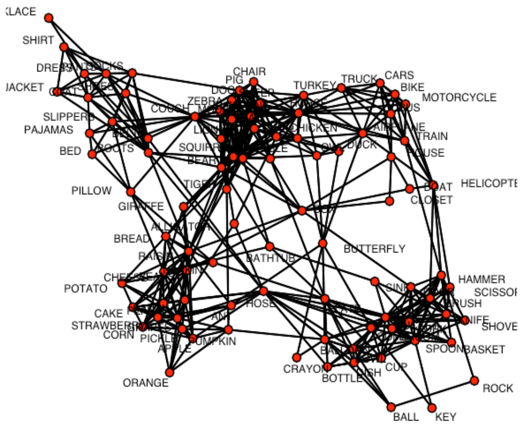
17 months



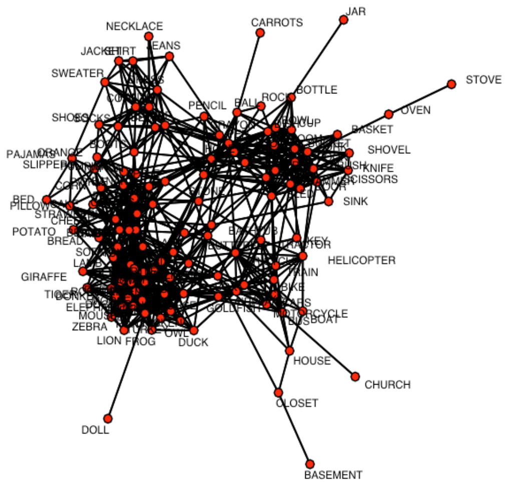
18 months



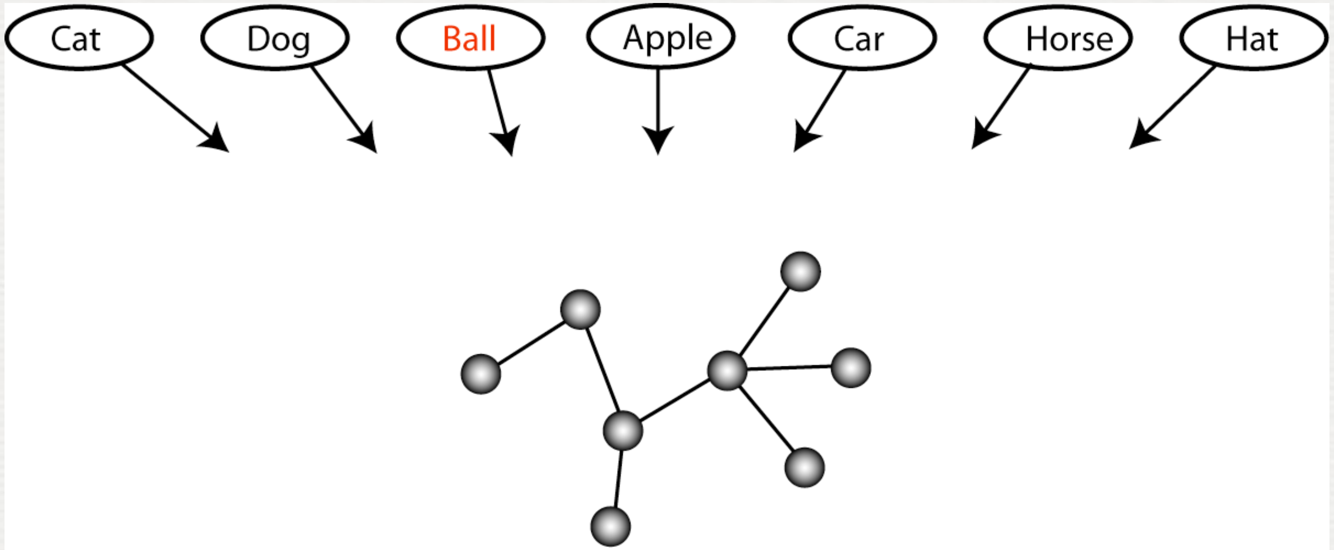
19 months



20 months …

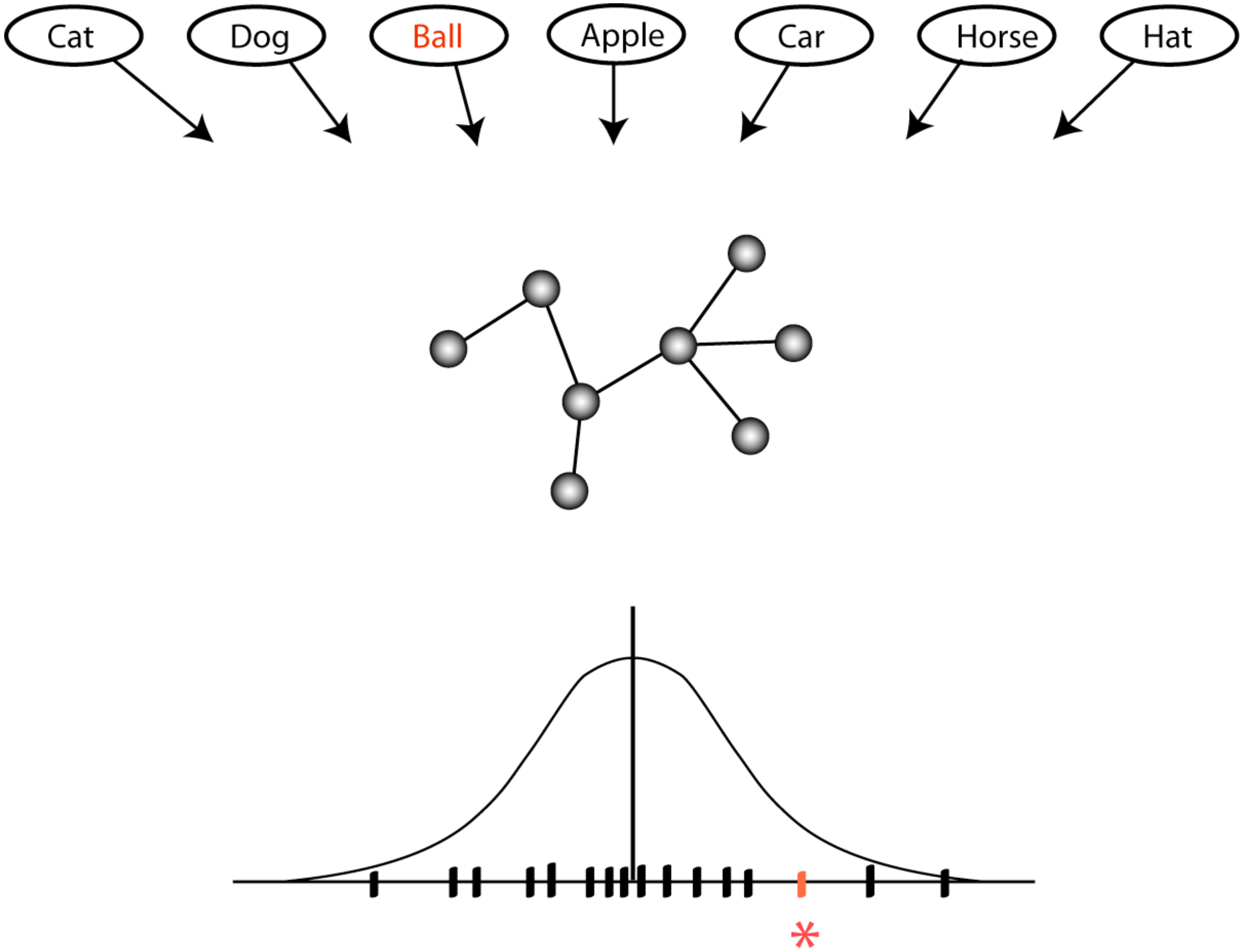


MODELING WORD ACQUISITION: WHICH WORD SHOULD BE LEARNED NEXT?



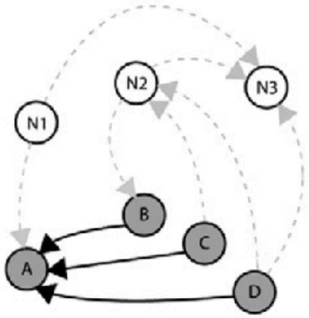
$$P(w_i) = \frac{e^{\beta d_i}}{\sum_j e^{\beta d_j}}$$

Modeled for each month from 15-25 months


Cat
Dog
Ball
Apple
Car
Horse
Hat
*

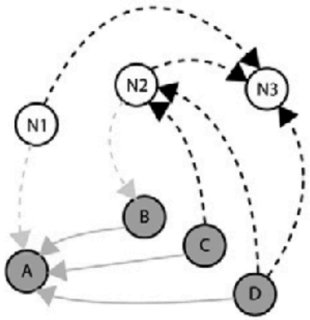
Three growth models

Preferential Attachment Model



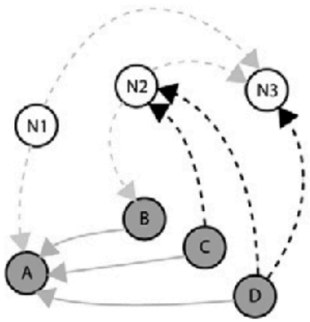
Node	Value	Add
N1	3	+
N2	0	-
N3	0	-

Preferential Acquisition Model



Node	Value	Add
N1	0	-
N2	2	-
N3	3	+

Lure of the Associates Model



Node	Value	Add
N1	0	-
N2	2	+
N3	1	-

$$P(w_i) = \frac{e^{\beta d_i}}{\sum_j e^{\beta d_j}}$$

Fit Beta to data and see which model has best fit