5   ADVERSARIAL SEARCH

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function MINIMAX-DECISION(*state*) **returns** *an action*
   return $\arg\max_{a\ \in\ \text{ACTIONS}(s)}$ MIN-VALUE(RESULT(*state*, *a*))

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function MAX-VALUE(*state*) **returns** *a utility value*
   if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
   $v \leftarrow -\infty$
   for each *a* **in** ACTIONS(*state*) **do**
     $v \leftarrow$ MAX(*v*, MIN-VALUE(RESULT(*s*, *a*)))
   return *v*

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function MIN-VALUE(*state*) **returns** *a utility value*
   if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
   $v \leftarrow \infty$
   for each *a* **in** ACTIONS(*state*) **do**
     $v \leftarrow$ MIN(*v*, MAX-VALUE(RESULT(*s*, *a*)))
   return *v*

Figure 5.3   An algorithm for calculating minimax decisions. It returns the action corresponding to the best possible move, that is, the move that leads to the outcome with the best utility, under the assumption that the opponent plays to minimize utility. The functions MAX-VALUE and MIN-VALUE go through the whole game tree, all the way to the leaves, to determine the backed-up value of a state. The notation $\text{argmax}_{a \in S} f(a)$ computes the element *a* of set S that has the maximum value of $f(a)$.

function ALPHA-BETA-SEARCH(*state*) **returns** an action
  $v \leftarrow$ MAX-VALUE(*state*, $-\infty$, $+\infty$)
  return the *action* in ACTIONS(*state*) with value v

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function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

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function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Figure 5.7   The alpha–beta search algorithm.   Notice that these routines are the same as the MINIMAX functions in Figure **??**, except for the two lines in each of MIN-VALUE and MAX-VALUE that maintain α and β (and the bookkeeping to pass these parameters along).