Solutions / key

Quiz 1

Calculus - Mr. Davis

September 29, 2020

Section 1: Instructions - (algebraically) Compute the following limits:

1. $\lim_{h\to 3} \frac{2h^2-18}{h-3} = \lim_{h\to 3} \frac{2(h^2-9)}{h-3} = \lim_{h\to 3} \frac{2(h-3)(h+3)}{h-3} = \lim_{h\to 3} 2(h+3) = 12$

2. $\lim_{t\to 4} \frac{1-\sqrt{3t+4}}{4-t} = \lim_{t\to 4} \frac{t^2-3t-4}{(4-t)(t^2+3t+4)} = \lim_{t\to 4} \frac{(t-4)(t+1)}{(4-t)(t^2+3t+4)} = \lim_{t\to 4} \frac{-(t+1)}{(t^2+\sqrt{3t+4})} = -\frac{5}{8} = -\frac{5}{8}$

3. $\lim_{\theta\to 0} \frac{\sin\theta}{\sqrt{\theta}}$ $\quad \frac{\sin\theta}{\sqrt{\theta}} \cdot \frac{\sqrt{\theta}}{\sqrt{\theta}} = \sqrt{\theta}\,\frac{\sin\theta}{\theta} \quad \therefore \lim_{\theta\to 0} \sqrt{\theta}\,\frac{\sin\theta}{\theta} = 1\cdot\lim_{\theta\to 0}\sqrt{\theta} = 0$

4. $\lim_{\psi\to \pi/2} \frac{\cot\psi}{\cos\psi} = \lim_{\psi\to \pi/2} \frac{\cos\psi}{\sin\psi}\cdot\frac{1}{\cos\psi} = \lim_{\psi\to \pi/2} \frac{1}{\sin\psi} = 1$

5. $\lim_{h\to 0} \frac{\frac{1}{7+h}-\frac{1}{7}}{h} = -\frac{1}{49}$ $\qquad\longrightarrow\quad \frac{1}{h}\left(\frac{1}{7+h}-\frac{1}{7}\right)$

$= \frac{1}{h}\left(\frac{7}{7(7+h)} - \frac{(7+h)}{7(7+h)}\right)$

6. Given the function: $K(y) = \begin{cases} y^2+c, & y < -2 \\ 1-3y & y \ge -2 \end{cases}$

Find the value(s) of c such that:

$$\lim_{x\to -2^-} K(y) = \lim_{x\to -2^+} K(y)$$

or state that no values exist.

$\lim_{y\to -2^-} K(y) = \lim_{y\to -2^-} 1-3y = 1-3(-2) = 7$

$\lim_{y\to -2^+} K(y) = \lim_{y\to -2^+} y^2+c = 4+c$

$\Rightarrow c = 3$