Holiday Homework: Ch. 1 318-336

318. $f(x) = 2x^2 + 9x - 5$

degree = 2, y-intercept = -5

zeros:

$$x = \frac{-9 \pm \sqrt{81 - 4(2)(-5)}}{2(2)}$$

$$= \frac{-9 \pm \sqrt{81 + 40}}{4} = \frac{-9 \pm 11}{4}$$

$$= -\frac{1}{2}, -5$$

319. $f(x) = x^3 + 2x^2 - 2x = x(x^2 + 2x - 2)$

degree = 3, y-intercept = 0

zeros:

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-2)}}{2} =$$

$$= \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$= \sqrt{3} - 2, -\sqrt{3} - 2$$

320. $\dfrac{\tan^2 x}{\sec^2 x} + \cos^2 x$

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\frac{1}{\cos^2 x}} + \cos^2 x = 1$$

321. $\cos^2\theta - \sin^2\theta = \cos(2\theta)$

322. $6\cos^2 x - 3 = 0$

$\cos^2 x = \frac{1}{2} \Rightarrow \cos x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \pm\frac{\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}, \pm\frac{3\pi}{4}}.$

323. $\sec^2 x - 2\sec x + 1 = 0$

set $\sec x = y$:

$y^2 - 2y + 1 = 0$

$(y-1)(y-1) = 0 \quad y = 1$

$\sec x = 1 \Rightarrow \frac{1}{\cos x} = 1$

$\cos x = 1 \Rightarrow \boxed{x = 0, \pm\pi, \pm 2\pi,}$

324. $5^x = 16$

$\log(5^x) = \log(16)$

$x = \frac{\log 16}{\log 5} \approx 1.723$

(doesn't matter which base)

325. $\log_2(x+4) = 3$

$2^{\log_2(x+4)} = 2^3$

$x + 4 = 8$

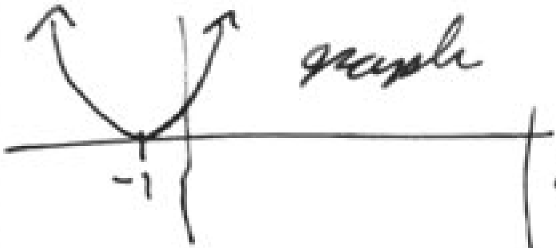
$x = 4$

checking answer

$\log_2(4+4) = \log_2(8) = 3\ \checkmark$

326. $f(x) = x^2 + 2x + 1 = (x+1)^2$

inverse $f^{-1}(x) = \pm\sqrt{x} - 1$



graph $\therefore$ one-to-one over $[-1, \infty)$ $(-\infty, -1]$

other way:

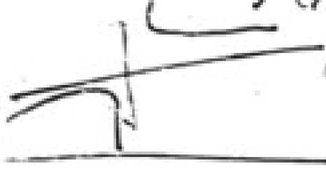
vertex (minimum) of $f(x)$ at $\left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$\left(\frac{-2}{2\cdot 1}, f\left(\frac{-2}{2\cdot 1}\right)\right)$

$= (-1, 0)$. Since $a > 0$,

$(-1, 0)$ is a minimum.

327. $f(x) = \frac{1}{x}$ $\boxed{(-\infty, 0) \quad (0, \infty)}$



328. $f(x) = \sqrt{9-x}$

one-to-one on $(-\infty, 9)$

inverse:

$y = \sqrt{9-x}$

$y^2 = 9 - x$

$x = 9 - y^2$

$f(x) = 9 - x^2$ on $(-\infty, 0)$

3 29. $f(x) = x^2 + 3x + 4$

$= (x+4)(x-1)$

minimum $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$= \left(\frac{-3}{2}, f\left(\frac{-3}{2}\right)\right) = \left(\frac{-3}{2}, \left(\frac{-3}{2}\right)^2 + 3\left(\frac{-3}{2}\right) + 4\right)$

$= \left(\frac{-3}{2}, \frac{9}{4} - \frac{18}{4} + \frac{16}{4}\right) = \left(\frac{-3}{2}, \frac{7}{4}\right)$

completing the square:

$f(x) = \left(x + \frac{3}{2}\right)^2 + \frac{7}{4}$

$y = \left(x + \frac{3}{2}\right)^2 + \frac{7}{4}$

$\pm\sqrt{\left(y - \frac{7}{4}\right)} = x + \frac{3}{2}$ $\quad x = -\frac{3}{2} \pm \sqrt{y - \frac{7}{4}}$

inverse $f^{-1}(x) = -\frac{3}{2} \pm \sqrt{x - \frac{7}{4}}$

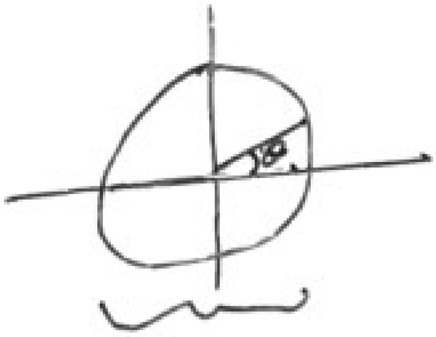
$x > \frac{7}{4}$

one to one on

$\left[-\frac{3}{2}, \infty\right)$ (inverse - take "−")

$\left(-\infty, -\frac{3}{2}\right]$ (inverse - take "+")

330.



1 mi

$\theta = 55° = 55° \cdot \frac{\pi}{180°}$
$= \frac{11\pi}{36}$

$r = 0.5$ mi

$\omega = \frac{\Delta\theta}{\Delta t}$

$v = \omega r$

$v = \left(\frac{11\pi}{36}\right)\left(\frac{1}{2}\text{ mi}\right) \quad = \frac{11\pi}{36} \cdot \frac{1}{1\text{ sec}}$

$\boxed{v = \frac{11\pi}{72}\text{ mi/sec}} \quad = \frac{11\pi}{36}\,\frac{\text{rad}}{\text{sec}}$

331.

# of shirts	cost (dollars)
20	\$440
100	\$1000

fixed cost
- price no matter how many shirts purchase

variable cost
- cost/shirt

$y = m \cdot x + b$

$m = \frac{1000 - 440}{100 - 20} = \frac{560}{80} = 7$

$y = 7x + b$

$1000 = 7(100) + b$

$b = 300$

(a) $y = 7x + 300$

(b) $7x + 300 = 10x$
$300 = 3x \Rightarrow \boxed{x = 100}$

$\boxed{\text{The man needs to sell 100 shirts to break even.}}$

332.

(a) $y = 7x + 300 = c = f(x)$

inverse: $f^{-1}(x) = \dfrac{x - 300}{7}$

input a cost (x) and get a number of shirts.

(b) \$8000 to spend:

$$\frac{(8000) - 300}{7} = \frac{7700}{7} = \boxed{1100 \text{ shirts}}$$

335. $y = e^{rt}$

$r = -0.0001210$

$e^{(-0.0001210)\cdot 2000} \approx 78.5\%$

336. $t = \frac{1}{r}\ln y = \ln y^{\frac{1}{r}}$

$\approx 11{,}457$ years old.

given a percentage of carbon in the sample, the inverse gives you the number of years.