Multiple choice

20 min no calc

5 min calc

10 min Break

Free response

15 min calc

20 min no calc

Corrections

§ I A

5. θ should be x

7. $\sqrt[2]{}$ should be $\sqrt[3]{}$

~~1.(A) $0 < x < 0.633$~~

Choice

Please complete the following questions in the time allotted. [...]nswer on the sheet.

: Section I, Part A

[...] minutes

[...] be used for this section of the exam.

1. $$\lim_{x\to\infty} \frac{\sqrt{9x^4+1}}{x^2-3x+5} \text{ is}$$ $= \lim_{x\to\infty} \dfrac{\frac{1}{x^2}\sqrt{9x^4+1}}{\frac{1}{x^2}(x^2-3x+5)}$

$= \lim_{x\to\infty} \dfrac{\sqrt{9+\frac{1}{x^4}}}{1-\frac{3}{x}+\frac{5}{x^2}} = \text{B}$

(A) 1

(B) 3

(C) 9

(D) nonexistent

2. A particle moving along a line has position function $s(t)$ given below.

$$s(t) = -4\sec t + 2\csc t$$

Compute the velocity, $v(t)$, of the particle.

$v(t) = \dot{s}(t)$

$= -4\sec t \tan t - 2\csc t \cot t$

$= \text{A}$

(A) $v(t) = -4\sec t\tan t - 2\csc t\cot t$

(B) $v(t) = -4\csc t - 2\sec t$

(C) $v(t) = -4\sec^2 t - 2\csc^2 t$

(D) $v(t) = -4\sec t\tan t + 2\csc t\cot t$

3. Give the value of c such that slope of the tangent line of the following implicit function at $x = 0$ is 4.

$$4x^2 + cx - 2e^y = -2$$

at $x=0$ $-2e^y = -2$

(A) -2

(B) 4

(C) 8

(D) -4

(E) -8

$8x + c - 2e^y \dfrac{dy}{dx} = 0$

$-2e^y \dfrac{dy}{dx} = -8x - c$

$\dfrac{dy}{dx} = e^{-y}\left(\dfrac{8x+c}{2}\right)$

at $x=0$: $\left.\dfrac{dy}{dx}\right|_{x=0} = e^{-y}\left(\dfrac{0+c}{2}\right) = 4$

$\Rightarrow c = 8$

C

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

apply chain rule to g(x)

4. Find $g'(x)$ given the following function:

$$g(x) = \ln(\ln(2 - \cos x))$$

$$\frac{\sin x}{\ln(2-\cos x)(2-\cos x)} = D$$

(A) $\dfrac{\cos x}{(2-\cos x)\ln(2-\cos x)}$

(B) $\dfrac{\sin x}{\ln(2-\cos x)}$

(C) $-\dfrac{\cos x}{\ln(2-\cos x)}$

(D) $\dfrac{\sin x}{(2-\cos x)\ln(2-\cos x)}$

5. Evaluate the following limit:

$$\lim_{x\to 0} \frac{\sin 3x}{\sin 8x} = \lim_{x\to 0} 3\,\frac{\sin 3x}{3x}\cdot\frac{8x}{8\sin 8x}$$

$$= \frac{3}{8} \quad C$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

(A) $\frac{8}{3}$

(B) 1

(C) $\frac{3}{8}$

(D) The limit does not exist.

$$f(x) = \begin{cases} 2x - 4 & \text{for } x \le 0 \\ 2x + 5 & \text{for } x > 0 \end{cases}$$

6. Let f be the piecewise-linear function defined above. Which of the following statements are true?

I $\lim_{h\to 0^-} \frac{f(3+h)-f(3)}{h} = 2$

II $\lim_{h\to 0^+} \frac{f(3+h)-f(3)}{h} = 2$

$$\Rightarrow \lim_{h\to 0} \frac{5(3+h)-5(3)}{h} = 2$$

III $f'(3) = 2$

$= 2$

(A) None

(B) II only

(C) I and II only

(D) I, II, and III

same as $\lim_{h \to 0} \frac{f(8+h) - f(8)}{h} = f'(8)$

for $f(x) = \sqrt[3]{x}$

7.

$$\lim_{h \to 0} \frac{\sqrt[3]{8+h} - 2}{h} \text{ is:}$$

(A) 0

(B) $\frac{1}{12}$

(C) 1

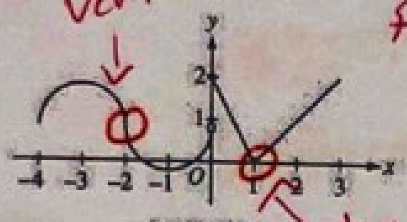
(D) 192

$f(x) = \sqrt[3]{x}$

$f'(8) = \frac{1}{3} x^{-2/3} \Big|_8 = \frac{1}{3 \cdot 4} = \frac{1}{12}$

vertical tangent

$f'(-2) \to \pm\infty$



Graph of f

$\lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} \neq \lim_{h \to 0^+}$ ~

8. The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all the values of x, $-4 < x < 3$, at which f is continuous but not differentiable?

at $x = 0$ $f(x)$ is not continuous

(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

9. The best linear approximation for $f(x) = \tan(x^2)$ at $x = \frac{\sqrt{\pi}}{2}$ is

(A) $y - 1 = 2\sqrt{\pi}(x - \frac{\sqrt{\pi}}{2})$

A

(B) $y + 1 = 2\sqrt{\pi}(x - \frac{\sqrt{\pi}}{2})$

(C) $y + 1 = \frac{2}{\sqrt{\pi}}(x - \frac{\sqrt{\pi}}{2})$

(D) $y - 1 = \frac{2}{\sqrt{\pi}}(x - \frac{\sqrt{\pi}}{2})$

$f'(x) = 2x \sec^2(x^2)$

$f'\left(\frac{\sqrt{\pi}}{2}\right) = \sqrt{\pi} \sec^2\left(\frac{\pi}{4}\right) = 2\sqrt{\pi}$

$f\left(\frac{\sqrt{\pi}}{2}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

10. If $y = e^{-x^3}$, then $y''(1)$ equals

(A) $-\frac{1}{e}$

(B) $-\frac{3}{e}$

(C) $\frac{1}{e}$

(D) $\frac{3}{e}$

D

$\cancel{y'(x) = -x^3 e^{-x^3}}$

$\cancel{y''(x) = -3x^2 e^{-x^3} + (-x^3)(3x^3 e^{-x^3})}$

STOP. End of section.

$-3x^2 e^{-x^3}$

$y''(x) = -6x e^{-x^3} + (-3x^2)(-3x^2 e^{-x^3})$

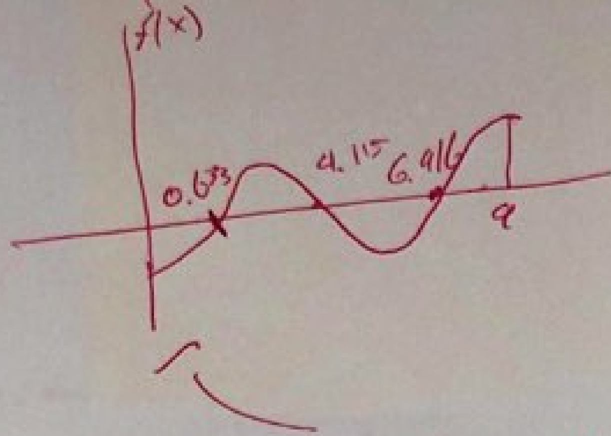
$= 9x^4 e^{-x^3} - 6x e^{-x^3}$

$y''(1) = \frac{9-6}{e} = \frac{3}{e}$

I B

11. (A) should have $0 < x < 0.633$

13. $f(1.6) = 22$



Choice: Section I, Part B

ime: 15 minutes

A calculator may be required for some questions on this section of the exam.

Need x s.t. $f'(x) < 0$ or

11. The derivative of a function f is given by $f'(x) = e^{\sin x} - \cos x - 1$ for $0 < x < 9$. On what intervals is f decreasing?

$f'(x) < 0$ for $0 < x < 0.633$

(A) $0 < x < 0.633$ and $4.115 < x < 6.916$

(B) $0 < x < 1.947$ and $5.744 < x < 8.230$

(C) $0.633 < x < 4.115$ and $6.916 < x < 9$

(D) $1.947 < x < 5.744$ and $8.230 < x < 9$

12. The temperature of a room, in degrees Fahrenheit, is modeled by H, a differentiable function of the number of minutes after the thermostat is adjusted. Of the following, which is the best interpretation of $H'(5) = 2$?

(A) The temperature of the room is 2 degrees Fahrenheit, 5 minutes after the thermostat is adjusted.

(B) The temperature of the room increases by 2 degrees Fahrenheit during the first 5 minutes after the thermostat is adjusted.

(C) The temperature of the room is increasing at a constant rate of $\frac{2}{5}$ degrees Fahrenheit per minute.

(D) The temperature of the room is increasing at a rate of 2 degrees Fahrenheit per minute, 5 minutes after the thermostat is adjusted.

H is °F, t is [minutes]

$$\left[\frac{H}{t}\right] = \frac{[°F]}{[\text{minutes}]} = \left[\frac{dH}{dt}\right]$$

$$= \lim_{x \to 1.5} \frac{f(x) - f(1.5)}{x - 1.5}$$

$$f'(1.5) = \lim_{h \to 0} \frac{f(1.5+h) - f(1.5)}{h}$$

$$\approx \frac{f(1.6) - f(1.4)}{1.6 - 1.4}$$

$$= \frac{22 - 14}{.2} = 40$$

13. From the values of the differentiable function f show
$f'(1.5)$:

x	1.0	1.3	1.4	1.6
$f(x)$	8	10	14	=22

$$f'(1.5) \approx \frac{f(1.6) - f(1.4)}{.2}$$

[redacted]

(A) 8

(C) (B) 18

(C) 40

(D) 80

14. Let f, g, h be differentiable functions with $\frac{d}{dx} f(x) = g(x)$ and $h(x) =$ [redacted]
$\sin x$.
Which of the following expressions correctly gives the value of $\frac{d}{dx} f(h(1))$?

(A) $g(0.841)$

(B) $0.540 \cdot g(1)$

(C) $0.540 \cdot g(0.841)$

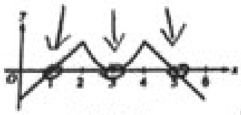
(D) $0.841 \cdot g(0.841)$

$$\frac{d}{dx}\left[f(h(x))\right] = f'(h(x)) \cdot h'(x)$$

$$\boxed{x=1} \rightarrow = f'(h(1)) \cdot h'(1)$$

$$= \text{(C)}$$

zero slope at
$x = 1, 3, 5$



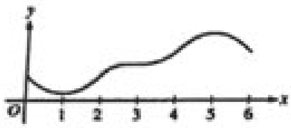
Graph of f'

decreasing on
$(0,1) \cup (5,6)$

15. The graph of f', the derivative of the function f, is shown above. Which of the following could be the graph of f?

increasing
otherwise

(A)

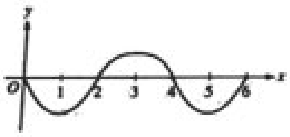


A

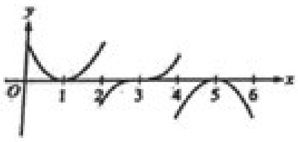
concave down
$(2, \cancel{4}\,3) \cup (4, 6)$

concave up
$(0,1) \cup (\cancel{4,6})$
$(3,4)$

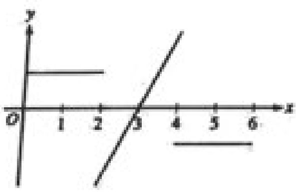
(B)



(C)



(D)



STOP. End of section.

2. Free Response

SECTION II, Part A

Section time: 15 minutes

A graphing calculator is required for this problem.

1.For $-\infty < t \leq 6$, a particle is moving along the x-axis. The particle's postion is given by $x(t) = 2\sin(e^{t/4}) + 1$. Answer the following questions:

a Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

b Find the velocity $\nu(t)$ of the particle as a position of time.

c For $0 \leq t \leq 6$, the particle changes direction exactly once. Find the position of the particle at that time.

d Using $\nu(t)$ from part (b), evaluate the limit $\lim_{t\to-\infty} \nu(t)$.

End of section.

(a) From a graph, x(t) increases from [0, 1.806] and decreases from [1.806, 6].

So $D = x(1.806) - x(0)$

$+ \; x(1.806) - x(6)$

$= [3 - 2.683] + [3 - (-.947)]$

$= [3 - 2.683] + 3.947 = 4.264$ [units]

(b) $v(t) = 2\cos(e^{t/4}) \cdot e^{t/4} \cdot \frac{1}{4}$

$= \frac{1}{2} e^{t/4} \cos(e^{t/4})$

(c) when $v(t) = 0$

$\Rightarrow \cos(e^{t/4}) = 0$

i.e. $e^{t/4} = \pi/2$

$t = 4\ln\left(\frac{\pi}{2}\right) \approx 1.806$

(d) $\lim_{t\to-\infty} v(t) = \lim_{t\to-\infty} \frac{1}{2}\cos(e^{t/4})\, e^{t/4} = \frac{1}{2}\lim_{t\to-\infty} (0)\cos(0) = 0.$

SECTION II, Part B

Section time: 20 minutes **No graphing calculator is allowed for these problems.**

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	3
6	4	5	3	-1

2. The functions f and g have continuous second derivatives. The table above gives the values of the functions and their derivatives at selected values of x.

a Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

b Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$

c Evaluate $l'(2)$ if $l(x) = g(x+1)f(\frac{3}{2}x)$

(a) $k'(x) = f'(g(x)) \cdot g'(x)$

$k'(3) = f'(g(3)) \cdot g'(3)$

$= f'(6) \cdot g'(3)$

$= 5 \cdot 3 = 15$

$k(3) = f(g(3)) = f(6) = 4$

$y - 4 = 15(x - 3)$

(b) $h'(x) = \dfrac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2}$

$h'(1) = \dfrac{f(1)g'(1) - g(1)f'(1)}{[f(1)]^2} = \dfrac{(-6)(8) - (2)(3)}{[-6]^2}$

$= \dfrac{-48 - 6}{36} = \dfrac{-54}{36} = \dfrac{-6 \cdot 9}{9 \cdot 4} = \boxed{-\frac{3}{2}}$

(c) $l'(x) = \frac{3}{2}g(x+1)f'(\frac{3}{2}x)$

$+ g'(x+1)f(\frac{3}{2}x)$

$l'(2) = \frac{3}{2}g(3)f'(3) + g'(3)f(3)$

$= \frac{3}{2}(6)(7) + (3)(8) = 86.$

3. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the following differential equation for the next 20 years.

$$\frac{dW}{dt} = \frac{1}{25}(W - 300)$$

W is measured in tons, and t is measured in years from the start of 2010.

a Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste ~~that~~ the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$)

b Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part A is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

c Let C be any constant non-zero value. Verify that $W(t) = 300 + Ce^{t/25}$ is a function that satisfies the differential equation described above.

END OF EXAM

(a) $\frac{dw}{dt} = \frac{1}{25}(1400 - 300) = \frac{1100}{25} = \cancel{400}\ 44$

$25 \times 4 = 100$

$y - 1400 = \cancel{4}\ 44(t)$

$y = 1400 + 44(\frac{1}{4}) = 1411$ tons $\approx w$.

(b) $\frac{d^2w}{dt^2} = \frac{1}{25}\frac{dw}{dt} = \frac{1}{25}\left(\frac{1}{25}(w - 300)\right)$

$= \frac{1}{25^2}(1400 - 300) > 0$

Since $\frac{d^2w}{dt^2} > 0$, our estimate of $w(\frac{1}{4}) = 1411$ tons

is an underestimate

(c) $w(t) = 300 + Ce^{t/25} \Rightarrow$

$w'(t) = \frac{C}{25}e^{t/25}$

$\frac{w - 300}{25} = \frac{Ce^{t/25}}{25}$

$\Rightarrow \frac{1}{25}(w - 300) = \frac{dw}{dt}$