A Timeless 4D Spatial Manifold with Monotonic Kinematic Foliation:
Geometric Origins of Time, Fermions, Dark Energy, and Cosmic Birefringence

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19 November 2025

Abstract

We present a speculative but mathematically structured framework in which physical reality is modeled as a timeless, orientable Riemannian 4-manifold M with metric g_{ab}. A global, monotonic kinematic foliation by 3-manifolds $\{\Sigma_t\}$ is generated by a smooth vector field v^a. We demonstrate that while the bulk geometry is Riemannian, the effective field theory confined to the foliation leaves inherits a Lorentzian signature due to the sign difference between extrinsic (kinetic) and intrinsic (gradient) curvature terms in the projected action. The accumulated twist $\phi(t)$ of the foliation behaves as a homogeneous pseudoscalar field. We derive its effective potential from geometric monodromy and instanton effects, and identify a parity-violating coupling $\phi F\tilde{F}/(4f_a)$ that predicts isotropic cosmic birefringence $\beta = \Delta\phi/(2f_a)$. Finally, we propose a geometric ansatz for fermion abundance: the intersection of the moving leaves with fixed 4D topological solitons. Using index-theorem arguments, we show how these intersections support localized zero modes that behave as fermionic operators. The model yields falsifiable predictions compatible with recent detections of cosmic birefringence and dark energy evolution.

Contents

1 Introduction

The observed universe raises four interconnected puzzles: the origin and arrow of cosmic time, the perfect identity of elementary particles, the nature of dark energy, and the 7σ detection of isotropic cosmic birefringence reported in 2025 [2,4]. We explore a unified geometric framework that addresses all four.

While 20th-century physics successfully unified space and time into a Lorentzian manifold, we propose that the Lorentzian signature is emergent rather than fundamental. We posit that the fundamental geometry is a Riemannian 4-manifold M, and that "time" is the kinematic parameter of a preferred monotonic foliation. This approach aims to explain the arrow of time not as a thermodynamic accident, but as a topological constraint on the foliation.

Furthermore, we address the "identity of indiscernibles" problem for fermions—why every electron is identical—by proposing they are intersection points of global 4D solitons with the spatial leaf [7,8].

2 Geometry and The Emergence of Time

2.1 Notation and Induced Structure

We use indices a, b for the 4D Riemannian bulk and i, j for spatial indices on the leaf Σ_t. Let n^a be the unit normal to Σ_t. The evolution vector is $\partial_t = N n^a + N^i e_i{}^a$. The extrinsic curvature is defined as:

$$K_{ij} \equiv \frac{1}{2N} \left(\dot{h}_{ij} - D_i N_j - D_j N_i \right),$$

where D_i is the covariant derivative on the leaf. We define the scalar twist-rate invariant $\omega^2 \equiv K_{ij} K^{ij} + \lambda K^2$, and the accumulated twist $\phi(t)$ as:

$$\phi(t) \equiv Z \int_{t_0}^{t} \omega(t') \, \mathrm{d}t'.$$

2.2 Effective Lorentzian Signature

Standard General Relativity assumes a Lorentzian bulk $(-+++)$ to generate Lorentzian dynamics. Here, our bulk is Riemannian $(++++)$. However, the dynamics observed on Σ_t are governed by the *effective action* derived from the 4D curvature.

In the Hamiltonian formulation of the bulk geometry, the extrinsic curvature K_{ij} plays the role of generalized momentum (kinetic energy), while the intrinsic curvature $R^{(3)}$ acts as the potential energy. In the projected action (derived in Sec. 3), these terms appear with opposite signs: $\mathcal{L} \sim K^2 + R^{(3)}$ in Euclidean space. Upon identifying $K \sim \dot{\phi}$, we obtain a structure $\dot{\phi}^2 - (\nabla\phi)^2$ (after appropriate renormalization of the time parameter). Thus, the Lorentzian signature is a property of the effective equations of motion for the confined fields, consistent with emergent gravity proposals [5].

3 Effective Action from Projection

3.1 Gauss–Codazzi Projection

The Gauss–Codazzi relation decomposes the 4D Ricci scalar. Neglecting boundary terms, the bulk action projects to:

$$S \supset \int \mathrm{d}^4 x \sqrt{h} \, \frac{M_{\mathrm{Pl}}^2}{2} \left(R^{(3)} + K_{ij} K^{ij} - K^2 \right).$$
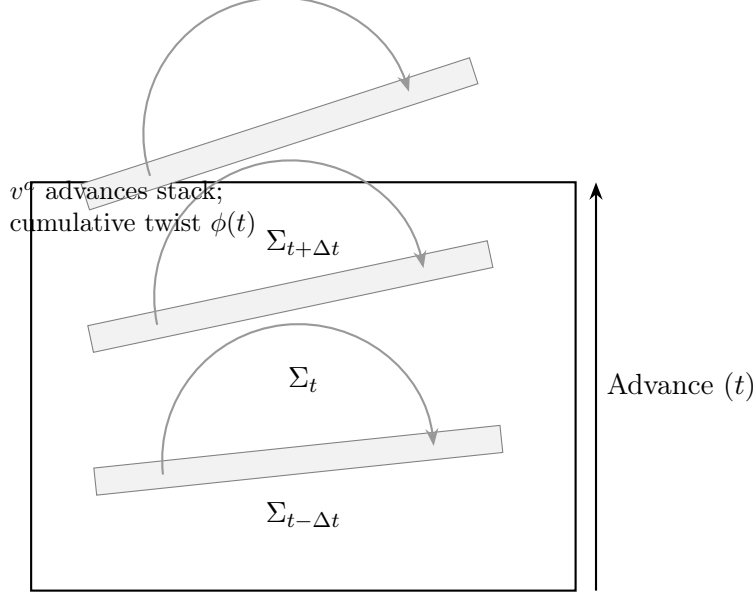
Figure 1: Schematic representation of the twisting foliation. The scalar field $\phi(t)$ is a geometric record of the foliation's cumulative twist.

We identify the extrinsic curvature terms $K_{ij}K^{ij} - K^2$ as the kinetic energy density of the foliation geometry. By defining the field ϕ proportional to the scalar mode of K, and imposing canonical normalization, we obtain the kinetic term:

$$\mathcal{L}_{\phi,\mathrm{kin}} = -\frac{1}{2}(\partial_t \phi)^2.$$

Combined with spatial gradients from $R^{(3)}$, the effective equation of motion is $\Box\phi - V'(\phi) = 0$, which is hyperbolic.

3.2 Potential and Chern-Simons Coupling

We motivate the effective potential by two geometric effects:

- **Monodromy:** As the foliation cycles through non-trivial flux sectors of M, the shift symmetry is softly broken, producing $V_{\mathrm{mono}}(\phi) = \mu^3 \phi$.

- **Residual Periodicity:** Discrete symmetries generate $V_{\mathrm{res}}(\phi) = \Lambda_4^4 \left(1 - \cos\frac{\phi}{f_{\mathrm{res}}}\right)$.

Additionally, the global twist of the spatial frame drags the polarization basis of any gauge fields confined to the slice. This results in a Chern-Simons interaction [1]:

$$\mathcal{L}_{\mathrm{CS}} = \frac{\phi}{4f_a} F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

4 Cosmological Dynamics and Birefringence

4.1 Parameter Pipeline

The model connects Dark Energy and Birefringence via the field ϕ.

1. **Equation of State:** w_ϕ is determined by the slow roll of ϕ down $V(\phi)$. Small deviations from $w = -1$ are controlled by μ and Λ_4, consistent with DESI 2025 results [3].

2. **Birefringence:** The accumulated twist $\Delta\phi$ between recombination and today drives a polarization rotation $\beta = \Delta\phi/(2f_a)$ [attachment_0](*attachment*).

4.2 Fiducial Model

Using fiducial parameters $\mu \approx 7$ meV and $f_a \approx 0.41 M_{Pl}$, the model predicts:

$$\beta \approx \frac{2.4}{2 \times 0.41 \, M_{\mathrm{Pl}}} \approx 0.34°$$

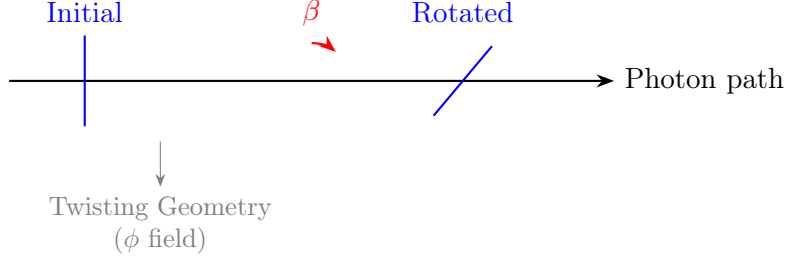This matches the joint analysis of Planck, ACT, SPT, and SPIDER data [2, 4].

Figure 2: Cosmic birefringence arises geometrically from the relative rotation of the spatial leaves between the time of emission and detection.

5 Fermions from Global 4D Solitons

5.1 The Intersection Ansatz

We propose that fermions are not fundamental point particles, but the intersection loci of the 3D spatial leaf Σ_t with static, codimension-two topological solitons $\mathcal{S}$ extending through the 4D bulk. This extends the topological preon models of Bilson-Thompson [7] and the algebraic spinor approaches of Furey [6, 8] into a geometric foliation context.
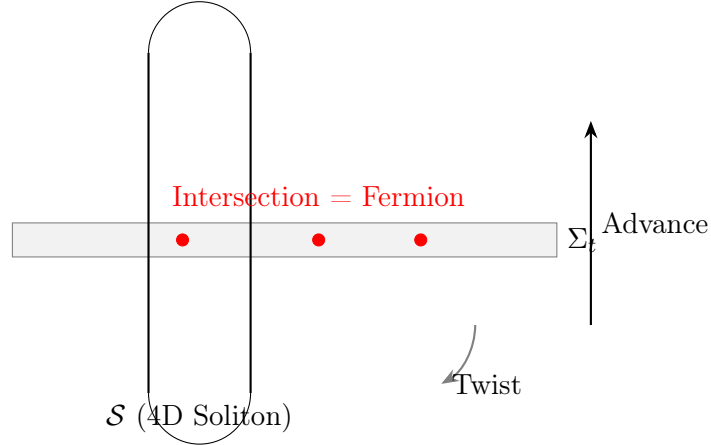

Figure 3: Geometric origin of fermions. A static 4D soliton $\mathcal{S}$ is sliced by the advancing leaf. The intersection behaves as a particle.

5.2 Index Theorem and Quantization

The identification of geometric intersections with quantum operators is supported by the Atiyah-Singer Index Theorem. For a Dirac operator coupled to the background of a 4D vortex, the theorem guarantees the existence of chiral zero modes localized at the core (the intersection point).

Quantization of the moduli space of these zero modes yields fermionic statistics. This mechanism allows a single global geometric object $\mathcal{S}$ to manifest as $N \sim 10^{80}$ identical particles when sliced by the foliation, effectively solving the identity problem.

6 Conclusion

This framework offers a geometric unification where time is a foliation parameter, dark energy is the kinetic residue of that foliation, and fermions are topological signatures of bulk solitons. The model is falsifiable via the predicted correlation between the dark energy equation of state and cosmic birefringence [1, 2].

References

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