Rocket Rex Design and Simulation

Riley Martin[1] and Thomas Tran[2] and Karen Flores[3]
San Diego State University, San Diego, CA, 92182

and

Gilbert Rodriguez[4] and Josh Wootten[5]
San Diego State University, San Diego, CA, 92182

The purpose of this technical report was for five students to successfully design, simulate, and analyze a rocket capable of reaching the Von Karmen Line at 100 kilometers, which is the recognized boundary between Earth's atmosphere into space. We design our project as if we were competing in the Base 11 Space Challenge.

Nomenclature

a	= Acceleration magnitude
a_x	= Horizontal Velocity
a_y	= Vertical Acceleration
A_e	= Exit Area
A^*	= Throat Area
A_c	= Chamber Area
$\frac{A_e}{A^*}$	= Area Ratio
A_{ref}	= Reference Area
c	= Speed of sound
c^*	= Specific Impulse
C_D	= Coefficient of Drag
C_T	= Thrust coefficient
d	= Body tube diameter
F_T	= Thrust force
g	= Gravity
h	= Height
I_{sp}	= Specific Impulse
I_T	= Total Impulse
L	= Length of rocket
m_0	= Initial Total Mass
m_L	= Payload Mass
m_S	= Structural Mass
m_P	= Propellant Mass
$\dot{m}$	= Mass flow rate
M	= Mach number
$\bar{M}$	= Molecular Mass
P_0	= Chamber pressure
P_e	= Exit pressure
q	= Dynamic pressure
$\bar{R}$	= Universal gas constant
R	= Specific gas constant
t	= Time
t_b	= Burn time
T	= Temperature
T_0	= Chamber temperature
u	= Velocity magnitude
V_x	= Horizontal Velocity
V_y	= Vertical Velocity
γ	= Specific heat ratio
ρ	= Density
θ	= Angle of rocket from horizontal axis

[1] Lab Assistant, Teachers Assistant, Student, Aerospace Engineering.
[2] Femineer Student Assistant, SDSU AIAA Vice-Chair, Aerospace Engineering.
[3] Student, Aerospace Engineering.
[4] Student, Aerospace Engineering.
[5] Astrodynamics Tutor, Student, Aerospace Engineering.

Figure 1 : Rocket Rex Design.

I. Introduction

THIS technical report will show the design process and final outcomes for Rocket Rex. Our rocket was designed around the Base 11 Space Challenge design requirements and given parameters. The Base 11 Space Challenge was a prize-winning competition that instructed student-led university teams to design, build, and launch a single-stage, liquid-propelled rocket that would pass the Von Karman line. The Von Karman line is the Fédération Aéronautique Internationale recognized boundary of space at the altitude of 100 km. This report will show the design process of the rocket's geometry, then going into the systems chosen to reach the Von Karmen line with the other requirements accounted for, and finally show the simulation results of our rocket.

II. Design Parameters

We were given numerous parameters to abide by, and in order to begin the design of our rocket, we must first decide which propellant we will use. For both simplicity, power, efficiency, and cost reasons, we chose the fuel oxidizer combination of liquid oxygen and kerosene. Once the propellent combination was found, we calculated the thrust to be 12000 N in order to reach the minimum apogee requirement of 100 km and would be applied over a burn time of 60 seconds. We then computed that our specific impulse was 720000 Ns, found by multiplying our chosen thrust by the burn time. The maximum impulse given for our rocket was given to be 889600 Ns and we well below that maximum. The following parameter chosen was the chamber pressure that was computed to be 4.5 MPa and the chamber combustion temperature was found on an online site to be 3670 K for RP-1. Using these parameters, we found that the oxidizer to fuel ratio is 2.3. Using the parameters all found above, we can now utilize the and run CEA in order to compute the specific impulse of 280.1 seconds. This specific impulse for our chosen fuel and oxidizer compound was within the reasonable range. After finding and computing these above values, we are now able to design the remainder of the rocket.

Parameter	Requirement	Notes:
Minimum Apogee	100 km	When launched vertically from sea level.
Maximum Apogee	150 km	When launched vertically from sea level.
Allowable Fuel	RP-1, LCH_4, Alcohol	No solids, hybrids, or monopropellants. No toxic or hypergolic propellants are permitted.
Allowable Oxidizer	LOx, H_2O_2, NO_2	
Allowable stages	1	Single stage only; no jettison of boosters or other hardware.
Maximum Total Impulse	889,600 Newton-sec (200,000 lbf.-sec)	
Propellant Feed System	Pump-fed	The pump feed system may be either an electric centrifugal pump or a pistonless pump.
Payload	5-kg, 100mm x 100mm x 300mm	Assume mass is uniformly distributed in this rectangular payload.
Maximum GLOM	1200-kg	
Maximum Motor Chamber Pressure	1000 psia	Must be regeneratively cooled.
Motor Cooling	Regeneratively cooled with 5% BLC	The 5% BLC cooling is by fuel mass.
Recovery System	2-stage parachute recovery	Drogue parachute deployed at apogee. Main parachute deployed at 2500 ft AGL.
Maximum Parachute Descent Rate	3 m/s (9.8 ft/s)	
Factor of Safety	1.5	Apply to tanks (hoop stress calculations), etc.
Fin Thickness	0.75 inch	It is assumed that fins will be used to aid in aerodynamic stability. This is to avoid having to do a fin flutter analysis.

Table 1 : Initial Rocket Design Requirements.

Parameter	Design Choice
Fuel	RP-1
Oxidizer	LOX
Payload Mass	5 kg
Chamber Pressure	4.5 MPa
Chamber Temperature	3670 K
Mixture Ratio (O:F)	2.3
Burn Time	60 seconds
Total Impulse	720000 Newton-second
Thrust	12000 N
Specific Impulse	280 seconds
Density of RP-1	900 km/m^3
Density of LOx	1141 km/m^3

Table 2 : Rocket Design Parameters.

III. Rocket Systems

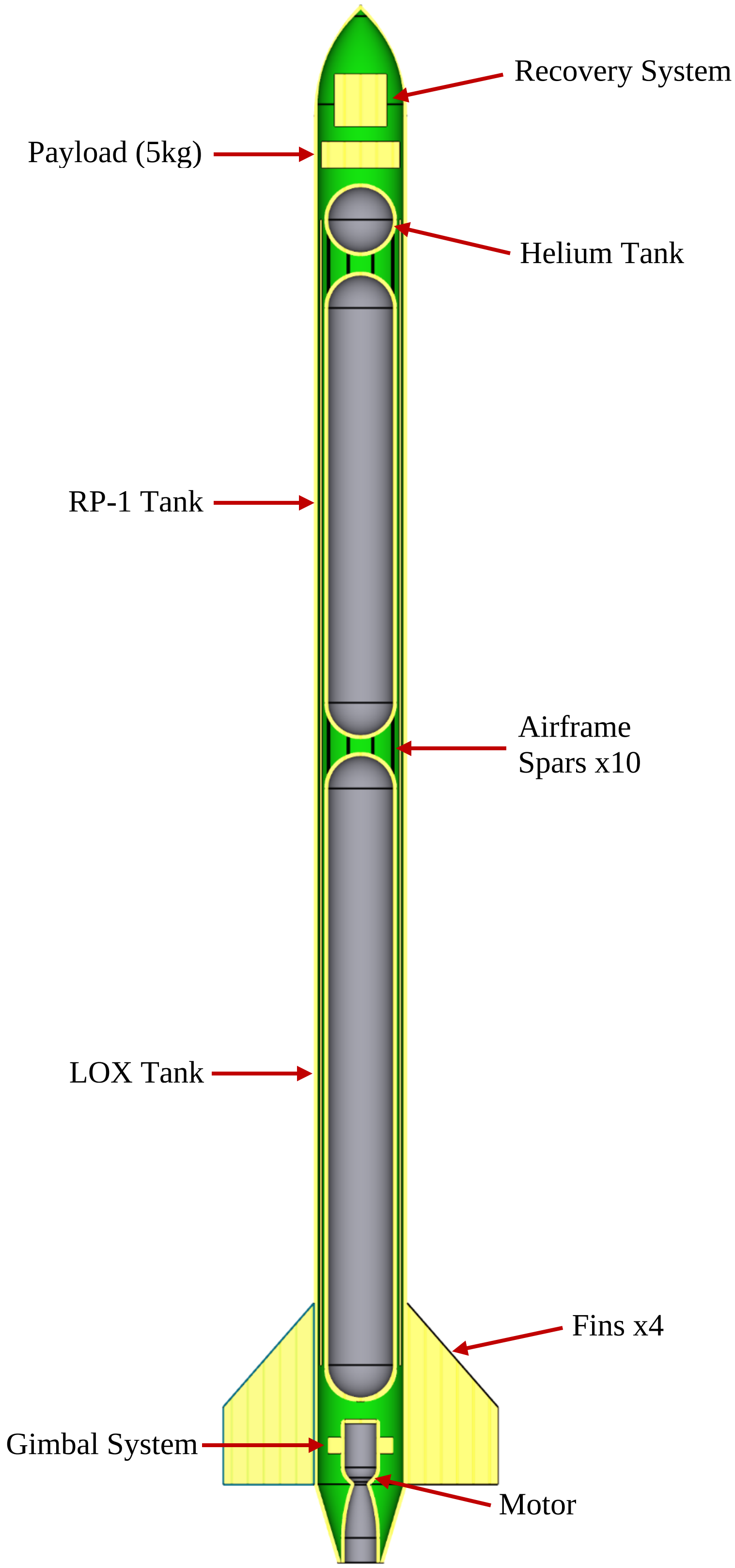

Figure 2 : Components within Rocket Rex.

A. Rocket Geometry

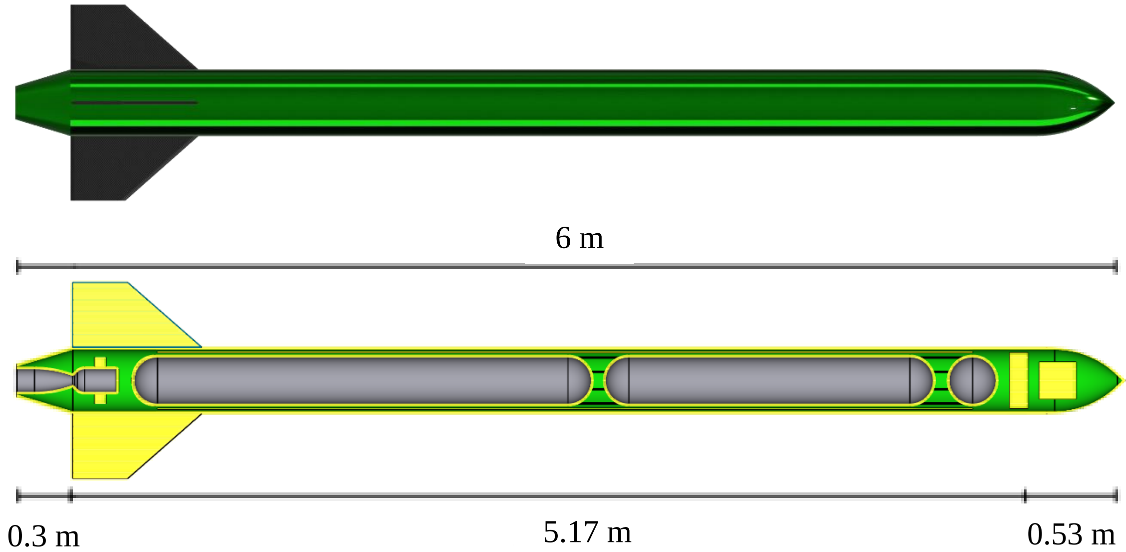

Figure 3 : External Dimensions of Rocket Rex.

The geometry of Rocket Rex was determined using the mission requirements. The majority of the contents consisted of the nosecone, gas tanks, and the motor. The sizes of the fuel, oxidizer, and pressurant tanks were determined using the mass and pressure of each fluid. The length of our motor was based on the required thrust for the competition to reach a height of 100-150 km. There was plenty of space left in the tip of the rocket for the payload as well as the recovery system. The extra room will allow for a smooth deployment of the parachutes. The total length of Rocket Rex came out to be 6 meters.

The diameter of our rocket needed to be big enough to fit all our components, but also needed to be small to achieve an aerodynamic form factor that would be beneficial for the competition. The outer diameter of the rocket came out to be 36 cm with the diameter of the tanks inside being 28 cm. The extra room inside the rocket allows for us to include our airframe spars for structural stability. The spars consist of 5 mm thick 6061 aluminum that run from the helium tank all the way to the bottom of the LOX tank. They have a total length of 4.41 m. The empty space inside the rocket would also allow for the necessary connection valves for each tank. The LOX, RP-1, and helium tanks have lengths of 250 cm, 180 cm, and 28 cm respectively. The payload within the nosecone has the required dimensions of 100 cm x 100cm x 300cm. The fins were also designed based on the requirements for the mission.

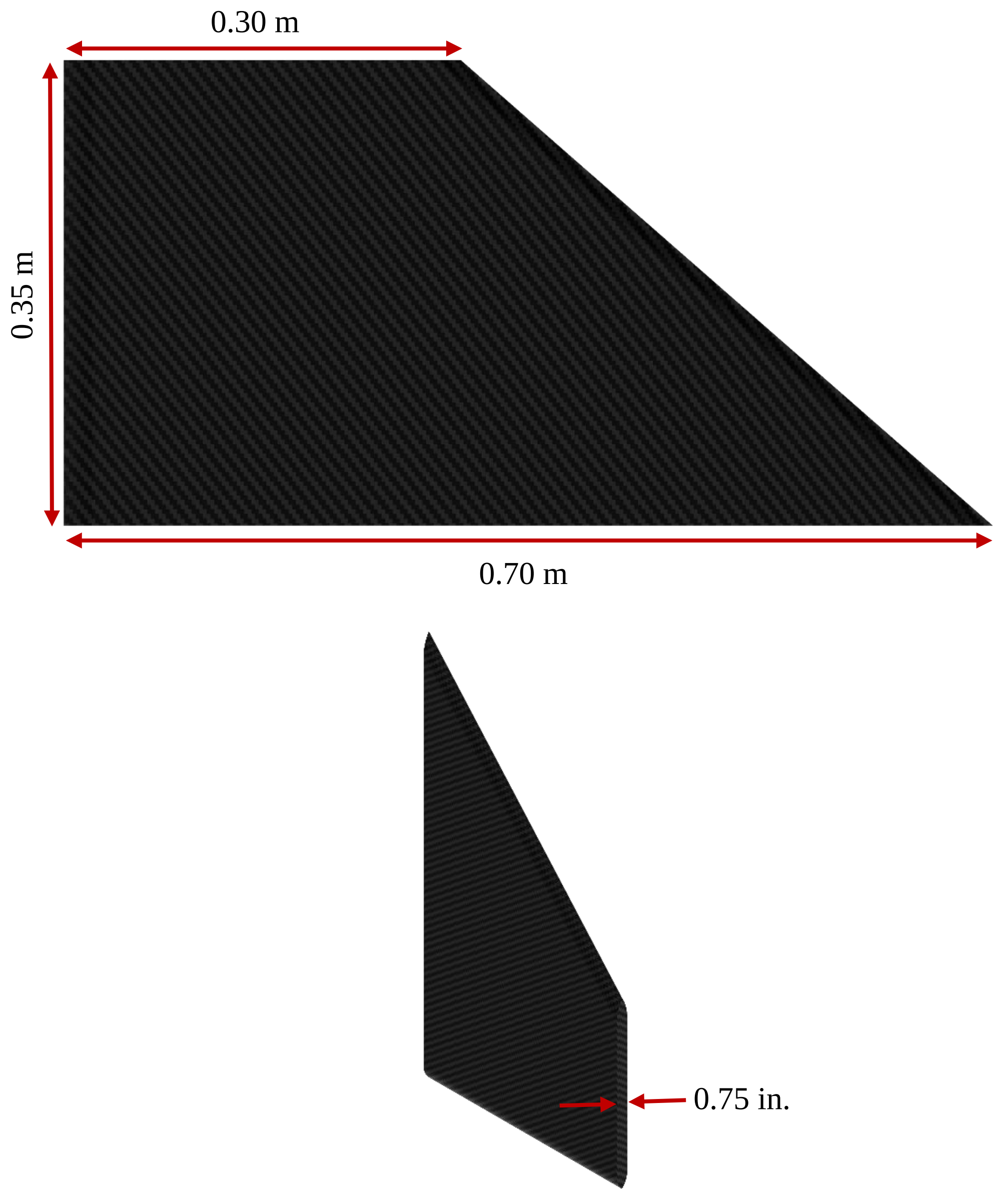

Figure 4 : Fin Dimensions.

Rocket Rex has 4 trapezoidal fins made from carbon fiber to keep the rocket lighter. The leading and trailing edges of the fins have been chamfered and filleted to be more aerodynamic. Earlier in the design process, we came across a scaling problem with our fins, but after further renditions of our rocket we were able to thicken the fins without scaling up the entire rocket. We were initially at the size of much bigger rockets such as the Atlas V rocket. We chose this trapezoidal shape for our fins because we wanted to keep the geometry simple yet stable at high altitudes. The only notable disadvantage of trapezoidal fins is that they have more drag when compared to elliptical fins. We kept the thickness of the fins at 0.75 in for aerodynamic stability and to avoid fin flutter.

B. Mass Overview

Mass Components	Mass (kg)
Nosecone	2.9958
Nosecone Divider	0.7278
Body	32.7052
Airframe	12.966
Fins (x4)	2.9428
Boat Tail	1.5163
LOx	182.62
RP-1	79.4
Helium	0.5
LOx Tank	40.7375
RP-1 Tank	25.7759
Helium Tank	5.2658
Plumbing	3.726
Avionics & Wiring	5
Thrust Vector Control Hardware	2.1206
Valves	1.1491
Payload	5
Recovery System	15.32
Motor	28.4046
Injector	1.06002

Table 3: Mass Components and Subcomponents of Rocket Rex

All mass components were generated by Fusion 360 ™ while including the densities of the wet mass components. The avionics and wiring components were conservatively estimated. The total GLOM is below the maximum 1200 kg. We meet our payload requirement of 5 kg. Our total wet weight was calculated to be 449.93 kg and its dry weight to be 187.4 kg.

C. Motor Design

One of the most important parameters of a rocket is the design of the motor. There are many ways to produce thrust and for this rocket liquid propellants were chosen which means that a motor must be designed that can accommodate that. Shown below are many of the important motor parameters.

Motor Parameters	Values
Heat Capacity Ratio	1.24
Gamma	0.656
Thrust Coefficient	1.4655
Characteristic Velocity	1875.626 m/s
Throat Area	18.2 cm^2
Exit Area	114 cm^2
Area Ratio	6.26
Mass Flow Rate	4.37 kg/s
Throat Diameter	4.8 cm
Exit Diameter	12 cm
Nozzle Length	28.88 cm
Characteristic Length	1.27 m
Chamber Area	91 cm^2
Chamber Diameter	12.2 cm
Chamber Length	23.1 cm
Chamber Volume	2540 cm^3

Table 4 : Calculated Motor Parameters.

The first step in the motor design process is to choose a thrust and a chamber pressure value that are adequate for the given application. In the case of the Rocket Rex a thrust of 12kN was chosen because this gives a total impulse within the given parameters and a chamber pressure of 4.5MPa because this is also within the given requirements. As shown in the charts above and in the propellant section that is to come, liquid oxygen and RP-1 were chosen as the propellant/oxidizer mixture which means that the motor must be designed with that in mind. Important properties of this mix is that it has a specific heat capacity ratio of 1.24 and also ignites at 3670K. A specific impulse of 280s was determined and the specific gas constant was also found which was 413. With all of the given values the coefficient of thrust can now be found. This is a very easy value to find using the formula shown below.

$$C_T = \frac{I_{SP} * g * \Gamma}{\sqrt{R * T_0}}$$

The next value that needs to be determined is the characteristic velocity of Rocket Rex. This can be found using the equation shown below.

$$c^* = \frac{I_{SP} * g}{C_T}$$

Another value that can easily be found at this point is the throat area. This is an important design point when it comes to the motor of a rocket and can help with modeling the final result as well. The throat area equation can be found below.

$$A^* = \frac{F_T}{C_T * P_0}$$

The next value that can be found is the exit area. A few assumptions must be made. The first assumption that must be made is that the exit pressure is the same as the ambient pressure at sea level which is 101.325kPa. The next step that must be taken is to take the area ratio equation for exit area over throat area and multiply it by the throat area. This results in the following equation below and allows for the exit area to be found.

$$A_e = (\frac{P_0}{P_e})^{\frac{1}{\gamma}} * \frac{\Gamma^2 * A^*}{C_T}$$

The next most logical value to find for the Rocket Rex's motor is the mass flow rate. This is extremely important because the combination of mass flow rate and burnout time will allow for the discover of the mass of propellant and oxidizer needed. The mass flow rate of this motor can be found using the following equation.

$$\dot{m} = \frac{A^* * P_0}{c^*}$$

Now that these parameters have been found it is time to determine the geometry of the motor. Already the exit and throat area have been found and it was decided that the best shape for this motor to perform as desired would be to have a bell shape. This would be harder to build than a cone shape however, it has better performance and seems feasible that a college rocket project group could fabricate a bell-shaped nozzle. A very important feature of a nozzle is determining the length and in order to do that the graph shown below was used.

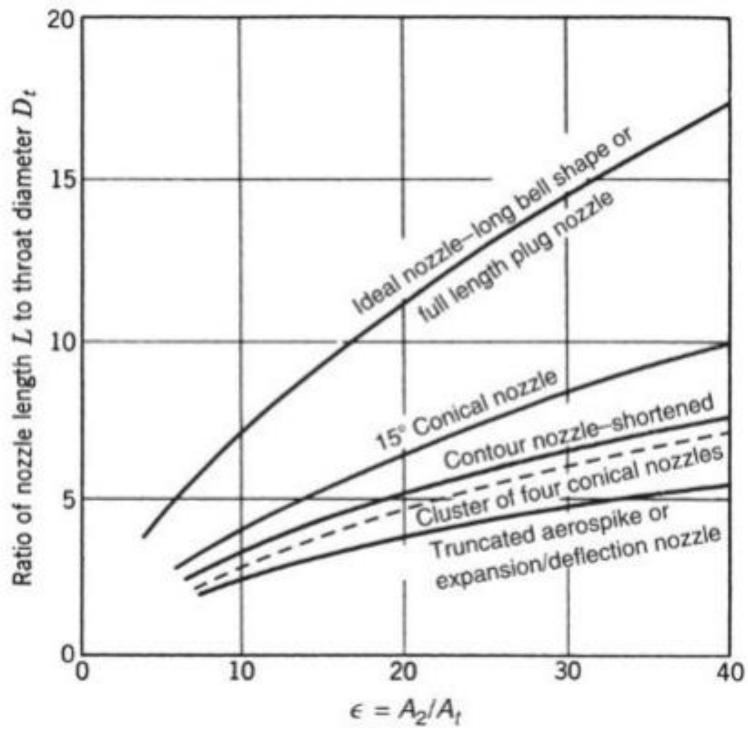

Figure 5 : Area ratio vs. Length to Diameter Ratio Chart.

In order to use this chart the ratio of the combustion chamber and throat area must be found. Then after that value has been found, locate where on the top line the y value or nozzle length ratio falls and multiply that by the throat diameter to find the length of the throat since the throat diameter is already known. The ratio can be found using the equation below.

$$\frac{A_2}{A_t} = \frac{A_2}{A^*} = \frac{8}{D^{*0.6}} + 1.25$$

This equation also can easily give chamber area if the ratio is multiplied by the throat area. The next value that can be found is the chamber length. This is different from the nozzle length as it is found by using the chamber volume. The chamber volume is found using the equation shown below and can be rearranged to find the length.

$$V_c = L^* \dot{A}^* \rightarrow L_c = \frac{V_c}{A_c}$$

With all of the values that were determined above about the geometry of the motor a 3D model can be shown below.

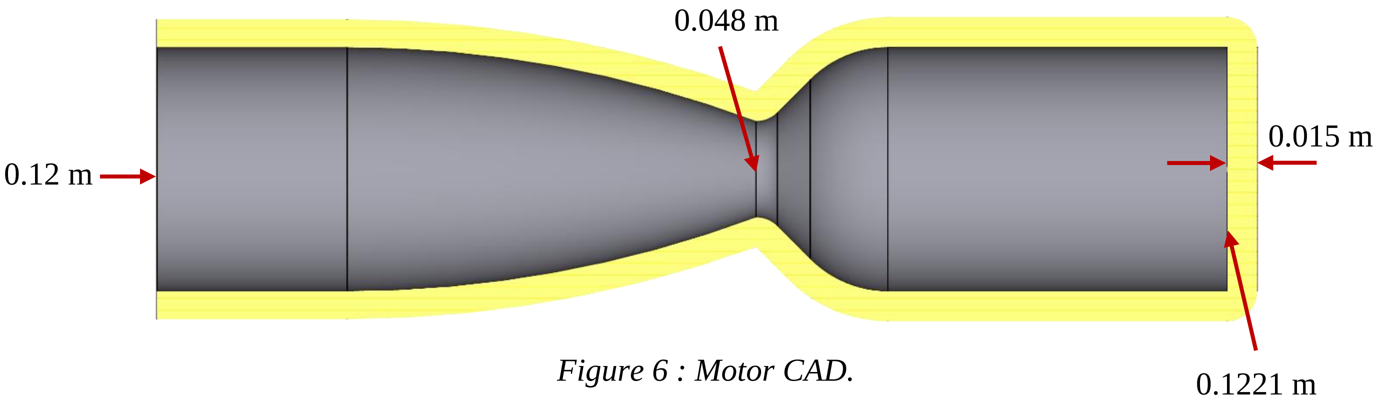

Figure 6 : Motor CAD.

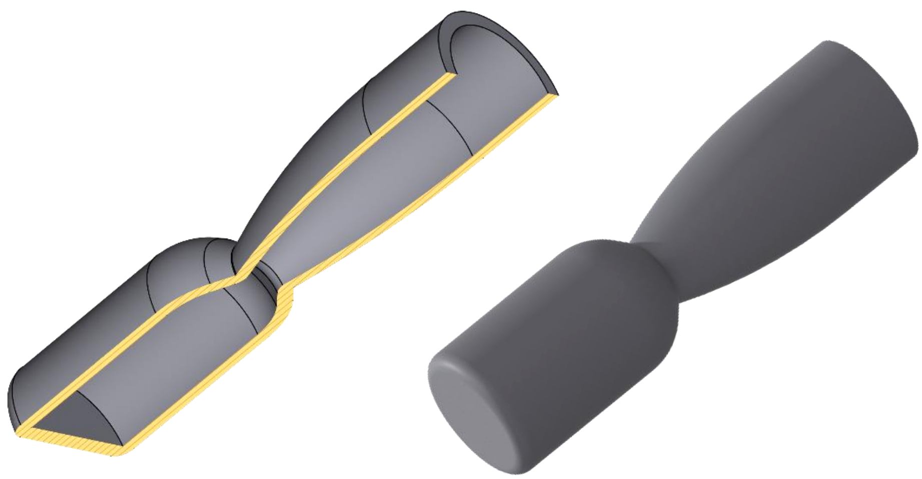

Figure 7 : 3D Motor Shell.

D. Thermal Analysis of the Motor

The motor is made from Inconel-625. Inconel is a very high strength, nickel-chromium based, superalloy developed by Special Metals Corporation for its use in gas turbine blades and combustors. Inconel has a variable service temperature(s) range from cryogenic to 1800°F (982°C) before losing strength or functionality. The thermal conductivity of Rocket Rex was found to be $9.8\,\frac{W}{mK}$. For this type of rocket design, it is best to use regenerative cooling as the primary method of temperature regulation for the motor itself. *Figure 8* below illustrates the stress-strain relationship of Inconel at various temperatures.

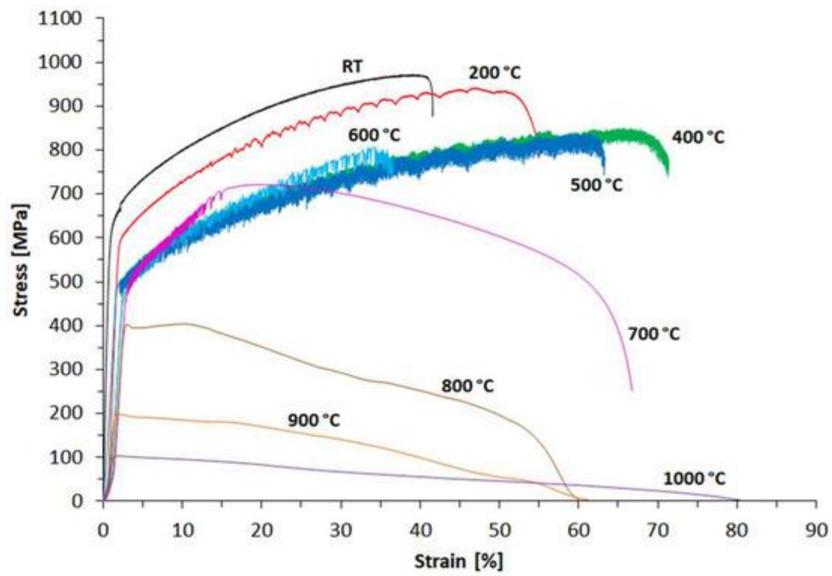

Figure 8 : Inconel Stress-Strain Relationship Curve(s).

This figure allows for the illustration of high temperature resistance and permeability of Inconel. The alloy itself performs well enough under temperature extremes, making it suitable for increased motor capability. The table below is representative of initial motor design constraints, and the chosen motor material.

Heat Transfer Design Values		Units
P_0	4.50E+06	Pa
T_w	973.15	K
rc	0.054	m
O:F	2.3	-
D^*	0.048	m
D_e	0.120	m
M^*	1	-
P_e	101325	Pa

Table 5 : Initial Motor Design Condition(s).

NASA CEA Values		Units
$\boldsymbol{T_0}$	3142.6	K
$\boldsymbol{C_p}$	2010	J/kg*K
$\boldsymbol{\gamma}$	1.12	-
$\boldsymbol{\mu}$	1.004	-
$\boldsymbol{k_g}$	3150	W/cm*K
$\boldsymbol{c}*$	1738.6	m/s
M	1	-

Table 6 : NASA CEA Values.

Table # above illustrates the values obtained from NASA's Chemical Equilibrium with Applications program. These results allow for the complete thermal analysis of the specified motor under the calculated constraints. These values provide: the throat area, A_{throat}, the temperature at the throat, T_{throat}, the Prandtl Number, Pr, the correction factor, σ, the gas coefficient, h_g, the heat flux at the throat, $Q_{flux,throat}$, the change in temperature, ΔT, and the temperature at the cold side of the wall, $T_{w,cold}$.

Area at throat calculation:

$$A_{throat} = \frac{\pi}{4} * D_{throat}^2$$

Similarly, the temperature at the throat:

$$T_{throat} = \frac{2}{\gamma + 1} * T_0$$

The Prandtl number is equated to approximate the ratio of momentum diffusivity to thermal diffusivity:

$$Pr = \frac{c_p * \mu}{k_g}$$

The inclusion of the correction factor is calculated as:

$$\sigma = \frac{1}{\left[\frac{1}{2} * \frac{T_w}{T_0} * \left(1 + \frac{\gamma - 1}{2}\right) * M^2\right]^{0.68} * \left(1 + \frac{\gamma - 1}{2} * M^2\right)^{0.12}}$$

The gas convection at the throat:

$$h_g = \left[\frac{0.026}{D^2} * \left(\frac{\mu * c_p}{Pr^{0.6}}\right) * \left(\frac{P_0^{0.8}}{c^*}\right) * \left(\frac{D^{*0.1}}{rc}\right)\right] * \left(\frac{A^*}{A}\right)^{0.9} * \sigma$$

The heat flux at the throat is calculated as follows:

$$Q_{flux,throat} = h_g * (T_{throat} - T_w)$$

The change in temperature as follows:

$$\Delta T = \frac{Q_{flux,throat} * \delta}{c}$$

The thickness was estimated to be around 0.4mm to equate the most accurate result(s):

$$\delta = 0.4mm$$

Thermal conductivity of Inconel-625 from ASM material sheet:

$$k_{Inconel-625} = 9.8\frac{W}{m * K}$$

The values for each were then calculated. The results are tabulated below.

Heat Transfer Calculated Values		Units
$\boldsymbol{T_{throat}}$	2965.70	K
$\boldsymbol{A_{throat}}$	0.0018	m^2
$\boldsymbol{Pr}$	0.64	-
$\boldsymbol{\sigma}$	3.40	-
$\boldsymbol{h_g}$	226126.14	$\frac{W}{m^2 * K}$
$\boldsymbol{Q_{flux,throat}}$	450.57	$\frac{MW}{m * K}$
$\boldsymbol{k_{Inconel-625}}$	9.8	$\frac{W}{m * K}$
$\boldsymbol{\delta}$	0.4	mm
$\boldsymbol{\Delta T}$	563.21	K
$\boldsymbol{T_{w,cold}}$.	972.59	K

Table 7 : Initial Motor Design Condition(s).

The highest yield temperature of Inconel-625, according to the stress-strain curve, is around 1250K. This is far above the calculated ΔT, 563.21K, and thus the motor should withstand these conditions. The $T_{w,cold}$, value is lower than the $T_w$value. Therefore, the engine is still withstanding its regenerative cooling process without the need for additional coolants. This could be expanded, and the regenerative cooling method should become more efficient with further research/testing.

E. Injector Design

For purposes of having better combustion stability and easy manufacturing, a pintle type injector was chosen to be pursued. Oxidizer flow will be through straight drilled radial orifices while the fuel will flow axially. A chamber pressure of 4.5 MPa was chosen along with an oxidizer to fuel ratio of 2.3. Recirculation of the propellant in the chamber was expected, so to account for it, a thickness of four times the diameter was chosen. Each orifice has a diameter of 0.5 mm, yielding a discharge coefficient of 0.81. Bernoulli's equation, Eq 19, was used to determine the flow through the orifice, considering pressure drop across the injector, density of the propellant, and exit velocity of the orifice.

$$u_2 = \sqrt{\frac{2\Delta P}{\rho}}$$

High injection velocities are dependent on the pressure drop across the injector and were needed to ensure fine atomization of the propellant. That is why the pressure drop was assumed to be 25 percent of the chamber pressure, yielding a 1.125 MPa pressure drop. This then gives a pressure in the manifold of 5.63 MPa. Using Eq 20, the mass flow rate could now be determined.

$$\dot{m} = C_d A_0 \sqrt{2\Delta P \rho} = \rho Q$$

The volumetric flow, Q, was also calculated with the equation above. Having this mass flow rate calculated, along with the diameter of the orifice, the area of the orifice could now be calculated and is listed in Table 7. The NASA CEA was used to estimate values of the design such as the specific impulse, thrust coefficient, and characteristic velocity. All critical values for the injector design can be seen below in Table 8. Following that, a sketch with the pintle injector dimensions is shown for visualization of the design.

Parameters			
Oxidizer to Fuel Ratio	O/F	2.3	–
Chamber Pressure	P2 = P0	4.5 E +6	Pa
Diameter of Orifice	d	5.08 E -4	m
Area of Orifice	Ao	2.03 E -7	m^2
Orifice Thickness	T	1.52 E -3	m
Discharge Coefficient	Cd	0.81	–
Pressure Difference Across Injector	ΔP	1.125 E +6	Pa
Pressure of Manifold	P1	4.5 E +6	Pa

Fuel Orifice			
RP-1 Density	ρ_f	806	kg/m^3
Exit velocity	u_f	52.84	m/s
Mass Flow Rate	$\dot{m}_f$	6.99 E -3	kg/s
Volumetric Flow Rate	Q_f	1.07 E -5	m^3/s
Oxidizer Orifice			
LOx Density	ρ_{ox}	1141	kg/m^3
Exit velocity	u_{ox}	44.41	m/s
Mass Flow Rate	$\dot{m}_{ox}$	8.32 E -3	kg/s
Volumetric Flow Rate	Q_{ox}	9.00 E -5	m^3/s

Table 8 : Injector Design Parameters.

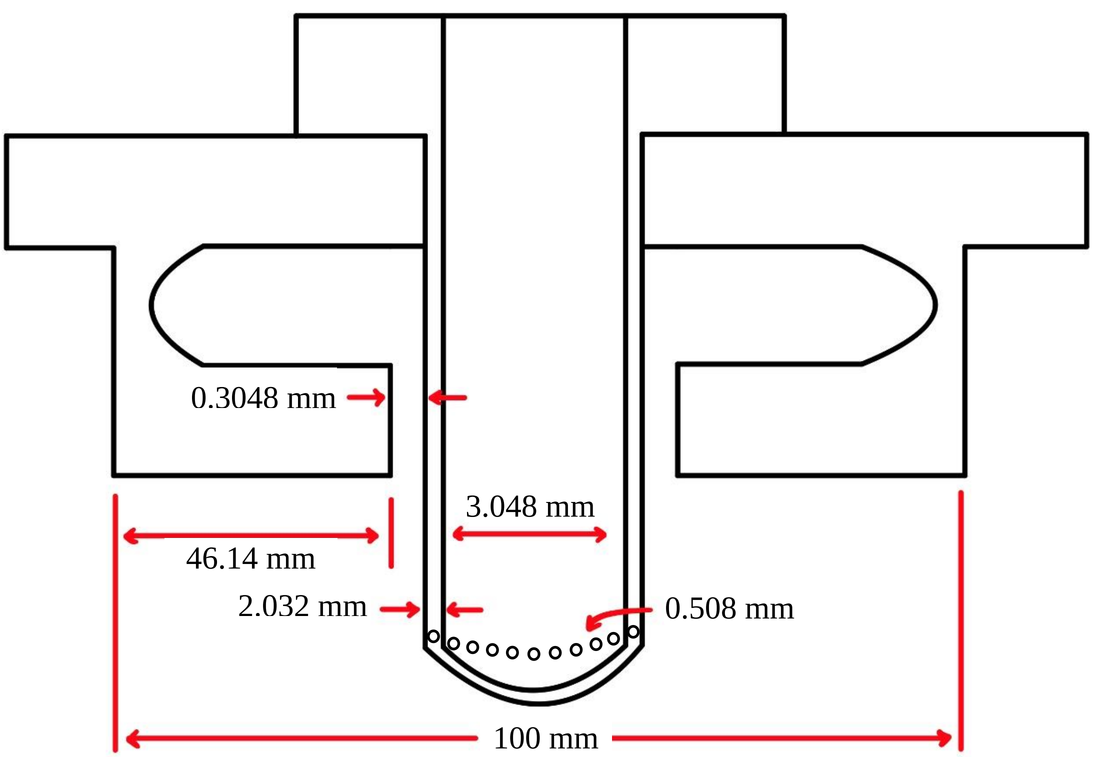

Figure 9 : Dimensions of the Pintle Injector.

F. Propellant

The chosen propellant for Rocket Rex was RP-1 and the chosen oxidizer was liquid oxygen also known as LOX. The context of this design is that it needs to be not only easy to make but also affordable so that a college team could theoretically build it and kerosene and liquid oxygen are obtainable and relatively affordable for a college team. The fuel to oxidizer ratio was chosen to be 2.3 and through the calculations shown in the mass and motor design sections it was determined that the mass of the fuel needed for Rocket Rex was about 262kg. This leaves about 183kg of liquid oxygen and 79kg of kerosene. The density of both liquid oxygen and kerosene are both known at $1141\frac{kg}{m^3}$ and $806\frac{kg}{m^3}$ respectively. This allows for the volume of each needed to be calculated by dividing the mass of each by the density. This results in a liquid oxygen volume of $0.16m^3$ and a kerosine volume of about $0.1m^3$. This leads to a total volume of fuel/oxidizer of about $0.26m^3$.

These volumes are important to determine because this allows for the determination of how large the tanks of each must be. Each tank was chosen to be a cylinder with round ends as this appears to be one of the easiest to build and is extremely structurally sound. Since the volume of the tanks was found above the only other parameter that needs to be found is the thickness of the tanks. The material that was chosen for the tanks was the Aluminum T6 2014 alloy because of its extremely high yield strength of 414 MPa.

To determine if this is strong enough the factor of safety and pressure drop in the tank must be considered. Assuming a pressure drop of 30% and a factor of safety of 1.5 the actual maximum designed pressure in the tank is 1.95 times greater than the chosen pressure of 4.5MPa. This would give an actual maximum pressure of 8.775MPa. The yield point of Aluminum T6 2014 is much greater than that so it is safe to assume the tank would be able to withstand the pressure.

The diameter of the tanks was decided to be 0.3m as this allows for them to easily fit inside the Rocket Rex. Using the equations below the thickness of the tank can be found, which was determined to be 0.0064m.

$$t = \frac{P_{tank} * D_i * (Factor\ of\ Safety)}{\sigma_y}$$

Now with the given diameter of 0.3m, the thickness of 0.0064m, and the chosen tank geometry, the tanks can be properly fabricated and implemented into the design.

G. Stability Analysis

Rocket Rex sets out to have a stable rocket configuration. This entails that the center of pressure should habitually always lie behind the center of gravity. It is concluded that the rocket is stable, both wet and dry. The figure(s) below illustrate the method(s) for discovering the center of pressure. The center of gravity and center of pressure are 0.475 m apart in the wet configuration. The center of gravity and center of pressure are 0.408 m apart in the dry configuration. There is a total of 0.057 m of CG movement throughout the flight.

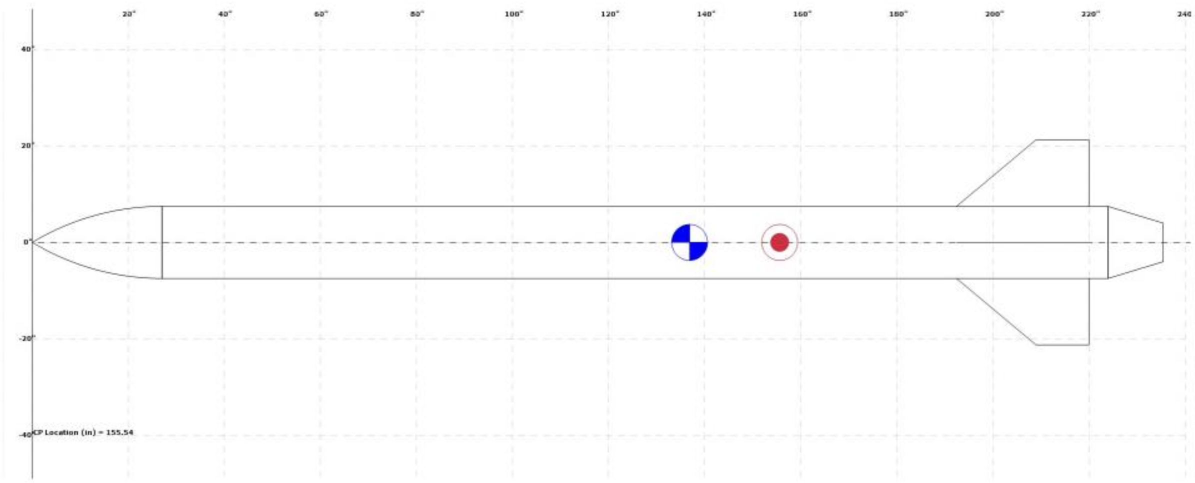

Figure 10 : Depiction of Center of Pressure for Wet Mass.

Figure 11 : Depiction of Center of Pressure for Dry Mass.

Thrust Vectoring aboard Rocket Rex is the next essential part to ensure rocket stability. To adhere to the most effective thrust vectoring method, it is best to create a pre-determined flight plan. This way, there is already a pre-determined state vector for the rocket to follow as its main orientation. Data must be fed in real-time in order to interpret the rocket's true orientation and make corrections if needed. The thrust vectoring correction(s) will be made with actuators and control moment gyroscope(s) to execute desired orientation(s).

H. Recovery

The recovery system for Rocket Rex will consist of two parachutes: the drogue chute that will deploy at 100 km and the main chute that will deploy at 2500 ft. Both parachutes will use a barometric altimeter and black powder to trigger deployment. The mass of the barometric altimeter was found to be 0.4 kg and other deployment necessities were found to be about 0.5 kg.

The drogue chute is a small-scale chute that will deploy at apogee of the flight to aid in the control of the main chute descent rate of the Rocket Rex. Our drogue chute will aim to keep the rocket at a descent rate of 9.03 ft/s. While using the online calculator site *Model Rocket Parachute Descent Rate Calculator*, we found the parachute size of 300 in diameter round parachute that will support Rocket Rex at its dry mass of about 187.4 kg. The drogue chute mass was determined to be 3.6 kg.

The main chute had to be quite a bit larger in size in order to slow the rocket to the required, safe final descent rate of 2.99 ft/s. This descent rate was reached with a round parachute of diameter 905 in. The main chute will deploy at the required height of 2500 ft, and the descent time was calculated to be 68 seconds. The main parachute's mass is 10.8 kg. making the total recovery system mass 15.32 kg.

IV. Simulation Results

Utilizing our MATLAB code from the numerical simulation homework, we were able to create a simulation of the rocket's fully detailed flight characteristics and path. We were found the values for the expected apogee, velocity at burnout, Mach number at burnout, thrust profile, and the time of each key event. Using the results, we have computed throughout the report including drag coefficients, mass, and thrust; we can depict the flight path of Rocket Rex. The first figure shown is the Drag Coefficient vs Mach in order to find the next following values.

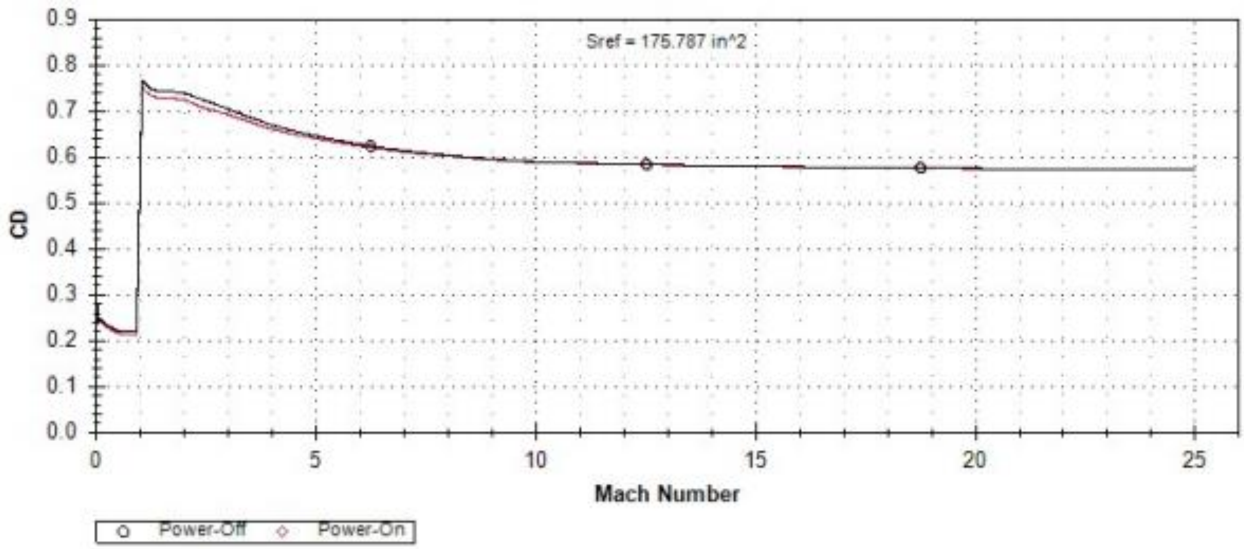

Figure 12 : Drag Coefficient vs Mach.

The thrust profile of Rocket Rex is shown below. As shown in the first 60 seconds of flight until burnout is achieved, the thrust is constant at 12000 N.

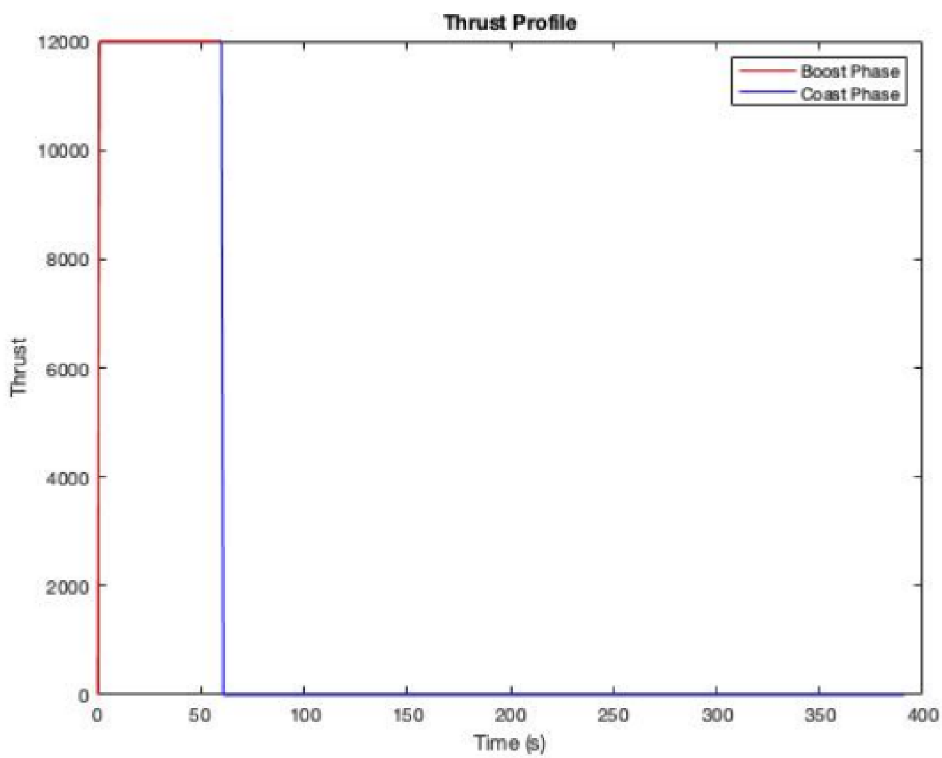

Figure 13 : Thrust Profile.

In order to find the altitude, we used the equation shown below to calculate the height,

$$Y(n) = Y(n-1) + V_y * dt$$

where n represents the time step in our code. Using the horizontal velocity, we can find the horizontal location, which gives us the altitude of the rocket simulation and we can plot that across the time for Rocket Rex, depicted below.

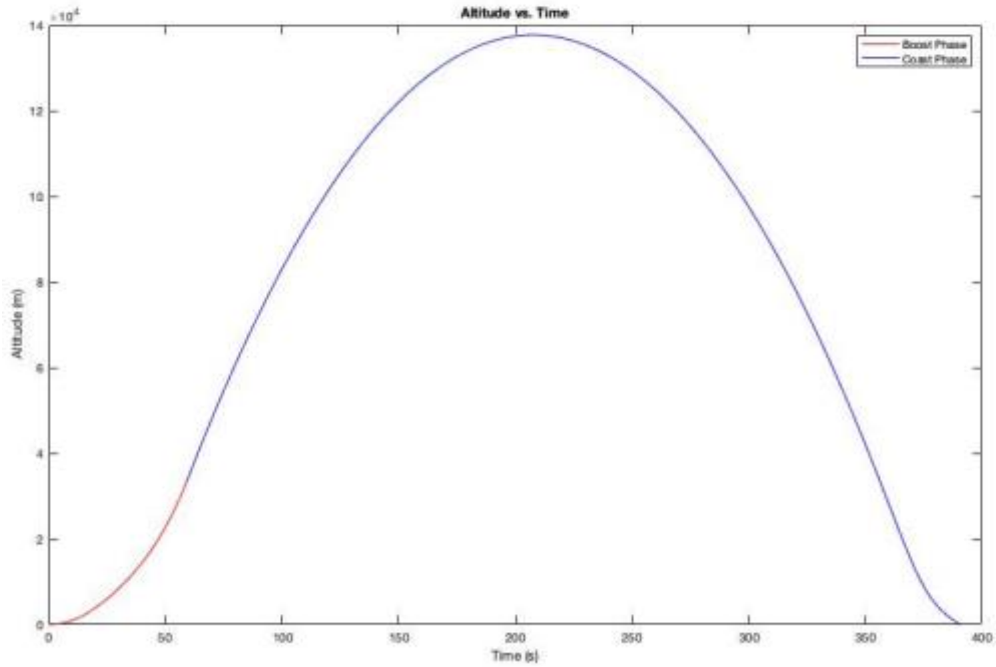

Figure 14 : Altitude vs Time.

Referencing the figure above, we can see that the maximum apogee reaches the height of 137682 meters in 206 seconds. In order to compute for the velocity for both the x and y components, we use the following equations,

$$V_y(n) = V_y(n-1) + \frac{1}{2}(A_y(n) + A_y(n-1)) * dt$$

$$V_x(n) = V_x(n-1) + \frac{1}{2}(A_x(n) + A_x(n-1)) * dt$$

with both components computed, the velocity along path can be found using the formula below,

$$V(n) = \sqrt{(V_x(n))^2 + (V_y(n))^2}$$

which shows us the Velocity Along the Flight Path vs Time graph and the Vertical Velocity vs Time graph of Rocket Rex's simulation.

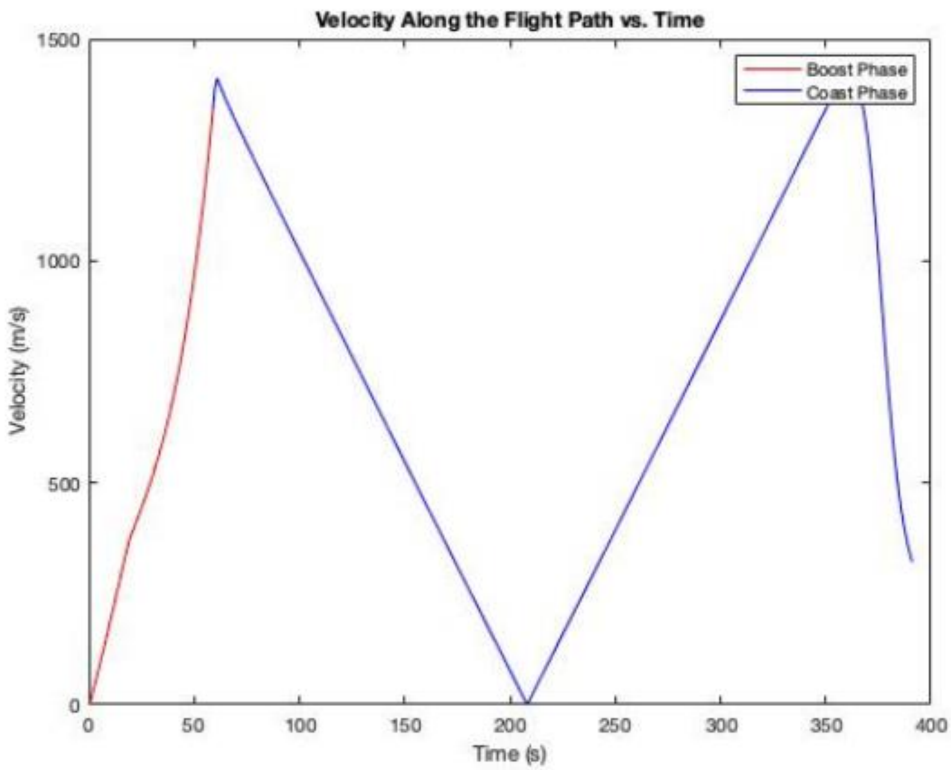

Figure 15 : Velocity Along the Flightpath vs Time.

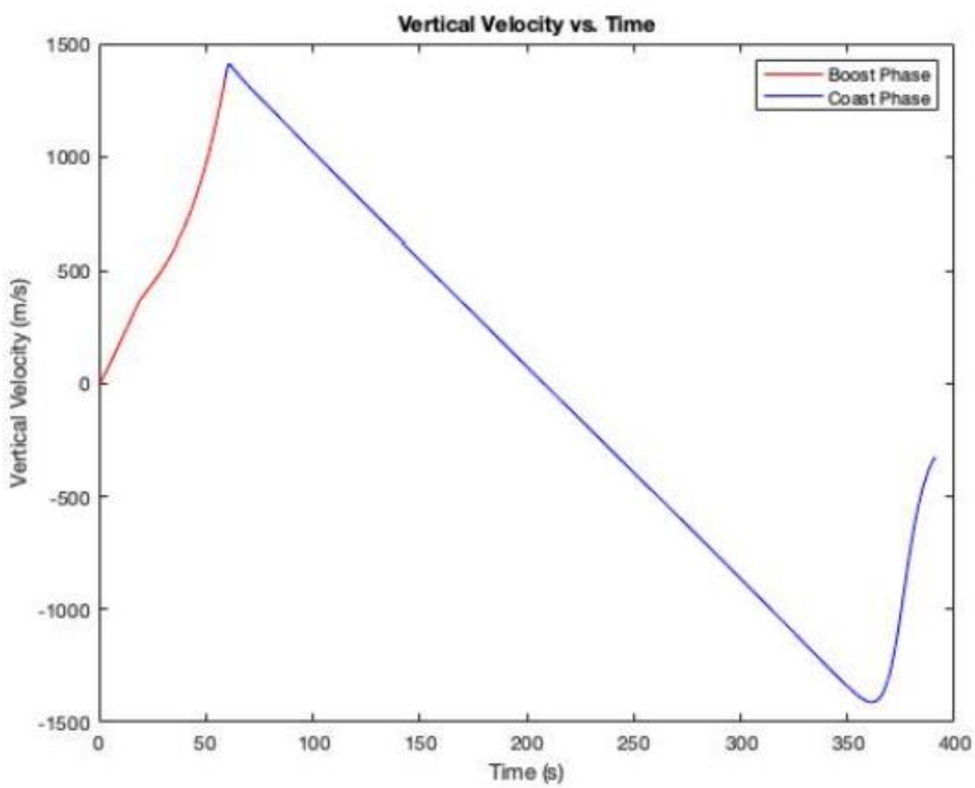

Figure 16 : Vertical Velocity vs Time.

Referencing the figures above, we can find that the velocity at burnout is 1341 m/s and a Mach number is 4.26 at burnout.

The next step is to find the dynamic pressure using the formula for dynamic pressure including the time step is, as follows

$$q(n) = \frac{1}{2}\rho(n)V(n)^2$$

Once plugging this equation into our simulation code, we calculated that the maximum dynamic pressure is 65400 Pa for Rocket Rex, depicted in the following figure.

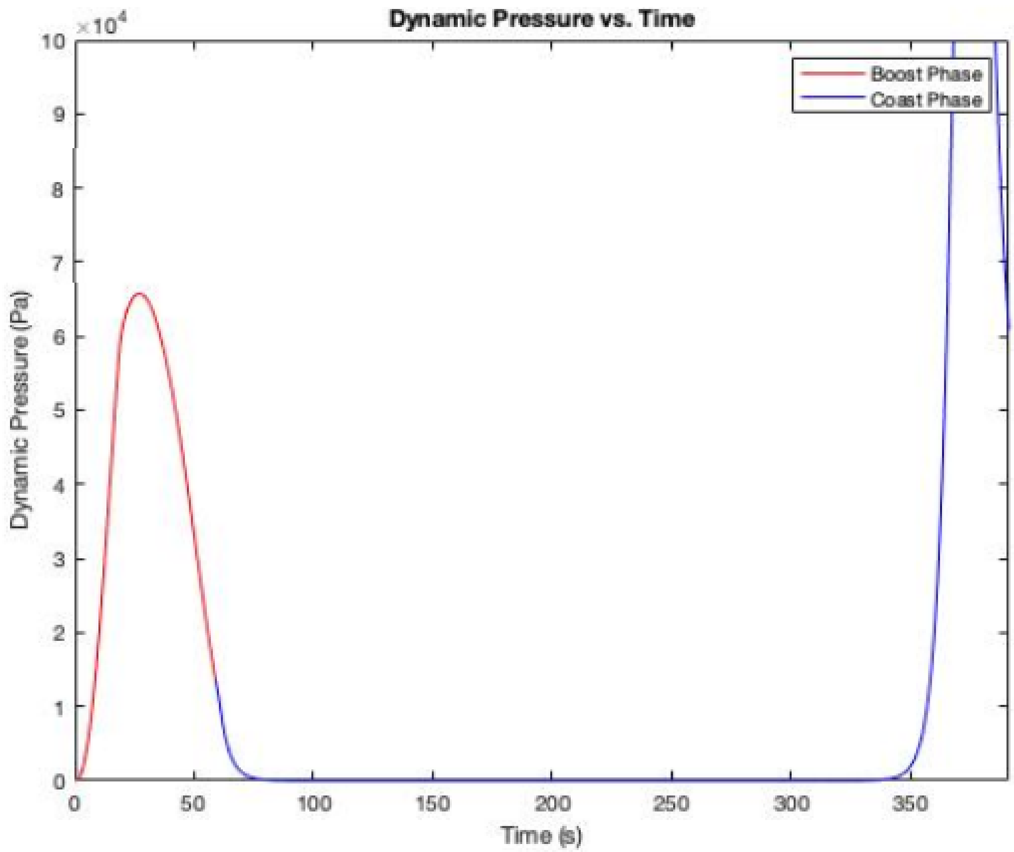

Figure 17 : Dynamic Pressure vs Time.

Lastly, we must plot the Acceleration Along the Flight Path vs Time of Rocket Rex's simulation shown below.

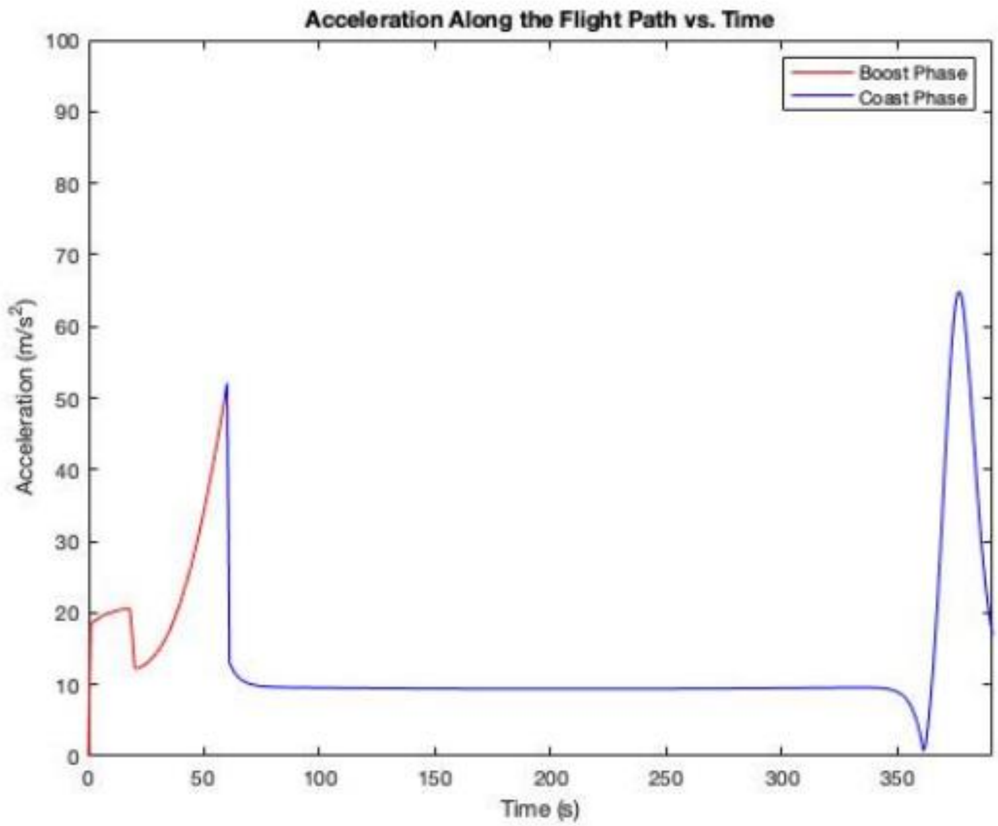

Figure 18 : Acceleration vs Time.

As shown above, we can see that Rocket Rex's acceleration is 50.11 $\frac{m}{s^2}$ at burnout.

V. Optimization

To optimize the design, the thrust was decreased because the apogee maximum of 150 kilometers was being surpassed. The second thrust used in calculations resulted in not reaching the minimum 100 kilometers apogee, so the thrust was once again changed, this time to 12,000 Newtons, achieving the distance of an apogee within the required range. A different way that this could have been achieved, was by altering the burn time, but the thrust modification was ultimately used.

Trapezoidal fins were used in this design for increased stability but moving forward an elliptical shape basis for the fins would be used. This change would be done since elliptical fins would have a lower drag coefficient, and research would be extended into figuring out if this change would be optimizing and efficient. Concerning the heat transfer analysis, the engine was only cooled 1 K even with the materials used that are well suited for rocket nozzles and engines. NASA has developed and used GRCop-84, so a better result was expected. If an alloy with a lowered $Q_{flux\ throat}$ could be used, the amount of heat transfer would be decreased. The thickness of 0.4 mm yielded a low ΔT if it was to be lower a risk of damage to the motor. If a larger thickness was chosen, there would be a significant larger ΔT, so further research would be needed to optimize this.

VI. Conclusion

In conclusion, we were able design a rocket that reached and exceeded the Von Karman line, while meeting the minimum apogee requirement of 100 km and maximum apogee requirement of 150 km, with the Rocket Rex's maximum apogee of 138 km. This altitude was achieved by finding all the necessary design parameters involving the rocket geometry, mass components, motor design, thermal analysis, injector design, propellant decision, stability analysis, and recovery design. We then verified all of our computation in the simulation results. As a competitor in the Base 11 Mock Challenge, we kept cost in mind and made a relatively realistic budget for a university student team-built rocket. Rocket Rex successfully fits all parameters given including reaching the Von Karmen line with a final thrust of 12000 N with a burn time of 60 seconds and a GLOM of 449 kg.

Acknowledgements

We would like to acknowledge and dedicate this project to Professor Tedesco. He allowed us time to complete this project with very helpful feedback throughout the course and after our presentations.

References

[1] Tedesco, Carl. Homework 2 Rocket Simulation May 4, 2021
[2] Van Milligan, T. What is The Best Fin Shape for a Model Rocket? Apogee Components, inc. May 2, 2017.
[3] Hennin, B. Why Should You Airfoil Your Rocket Fins? Apogee Components, inc. January 31, 2012.
[4] Ellis, David. GRCop-84: A High-Temperature Copper Alloy for High-Heat Flux Applications.
[5] Ellis, David. Thermophysical Properties of GRCop-84.
[6] Ellis, David. Aerospace Structural Materials Handbook Supplement GRCop-84.
[7] Fruity Chutes Inc. Professional Aerospace Recovery Systems. https://fruitychutes.com/
[8] *Model Rocket Parachute Descent Rate Calculator*, https://descentratecalculator.onlinetesting.net/

Appendix

MATLAB Code

```
A = 0 . 0 9 6 ; % Rocket projected area (m^ 2 )
Initial_Mass = 428.94 ;
ms = 180.5;
mL = 5 ;
Mass_Burnout = ms + mL;
mp = Initial_Mass - Mass_Burnout ;
tb = 60;
a = 1.2 ;
b = 2.9 * 10 ^ (-5 ) ;
Mach(1) = 0 ;
Theta(1) = 90; % Initial angle (deg)


clear;clc;
tic
% Parameters
dt = 1 ; % Time se p
FINAL = 392;

% Preallocate memory for arrays
Mach = zeros( 1 , FINAL ) ;
t = zeros( 1 , FINAL ) ;
Thrust = zeros( 1 , FINAL ) ;
Mass = zeros( 1 , FINAL ) ;
Theta = zeros( 1 , FINAL ) ;
Fn = zeros( 1 , FINAL ) ;
Drag = zeros( 1 , FINAL ) ;
Dragx = zeros( 1 , FINAL ) ;
Dragy = zeros( 1 , FINAL ) ;
Fx = zeros( 1 , FINAL ) ;
Fy = zeros( 1 , FINAL ) ;
kx1 = zeros( 1 , FINAL ) ;
kx2 = zeros( 1 , FINAL ) ;
kx3 = zeros( 1 , FINAL ) ;
kx4 = zeros( 1 , FINAL ) ;
ky1 = zeros( 1 , FINAL ) ;
ky2 = zeros( 1 , FINAL ) ;
ky3 = zeros( 1 , FINAL ) ;
ky4 = zeros( 1 , FINAL ) ;
Ax = zeros( 1 , FINAL ) ;
Ay = zeros( 1 , FINAL ) ;
Aa = zeros( 1 , FINAL ) ;
Vx = zeros( 1 , FINAL ) ;
```

```
Vy = zeros( 1 , FINAL ) ;
V = zeros( 1 , FINAL ) ;
x = zeros( 1 , FINAL ) ;
y = zeros( 1 , FINAL ) ;
Distance_x = zeros( 1 , FINAL ) ;
Distance_y = zeros( 1 , FINAL ) ;
Distance = zeros( 1 , FINAL ) ;
T = zeros( 1 , FINAL ) ;
c = zeros( 1 , FINAL ) ;
dynamicPressure = zeros( 1 , FINAL ) ;
Gravity = zeros( 1 , FINAL ) ;
density = zeros( 1 , FINAL ) ;
C = zeros( 1 , FINAL ) ;
F = zeros( 1 , FINAL ) ;

%% Start
clc,clear
%% Givens
t = 0; %seconds
a0 = 0; %m/s^2
u0 = 0; %m/s
h0 = 0; %m
mL = 5; %kg
mS = 60; %kg
mB = mS + mL; %kg
m0 = 449; %kg
mP = m0 - mL - mS; %kg
rho = 1.226; %kg/m^3
gE = 9.81;
T = 12000; %Newtons
area = 0.070; %m^2
theta = 0; %degrees
tB = 60; %seconds
mdot = mP / tB;
Cp = 3.8565; %m
Re = 6371 * 10^3; %m
a_density = 1.226; %kg/m^3
b_density = 2.9*10^-5; %kg/m^3
R = m0 / mB;
uEQ = T / mdot; %m/s
gamma = 1.4;
R_c = 287;
%% Initializing Machtrices
timeIterations = 313; %guess and check
t = zeros(1, timeIterations);
T = zeros(1, timeIterations);
m = zeros(1, timeIterations);
```

```
theta = zeros(1, timeIterations);
Fn = zeros(1, timeIterations);
Fx = zeros(1, timeIterations);
h = zeros(1, timeIterations);
g = zeros(1,timeIterations);
Fy = zeros(1, timeIterations);
Fd = zeros(1, timeIterations);
Fdx = zeros(1, timeIterations);
Fdy = zeros(1, timeIterations);
Ax = zeros(1, timeIterations);
Ay = zeros(1, timeIterations);
Vx = zeros(1, timeIterations);
Vy = zeros(1, timeIterations);
V = zeros(1, timeIterations);
x = zeros(1, timeIterations);
y = zeros(1, timeIterations);
Dx = zeros(1, timeIterations);
Dy = zeros(1, timeIterations);
D = zeros(1, timeIterations);
a1 = zeros(1, timeIterations);
a2 = zeros(1, timeIterations);
Sx = zeros(1, timeIterations);
Sy = zeros(1, timeIterations);
SMAG = zeros(1, timeIterations);
Kx1 = zeros(1, timeIterations);
Kx2 = zeros(1, timeIterations);
Kx3 = zeros(1, timeIterations);
Kx4 = zeros(1, timeIterations);
Ky1 = zeros(1, timeIterations);
Ky2 = zeros(1, timeIterations);
Ky3 = zeros(1, timeIterations);
Ky4 = zeros(1, timeIterations);
rhoh = zeros(1, timeIterations);
%% Initial Conditions
dt = 1;
t(1) = 0;
theta(1) = 2;
Vx(1) = 0;
Vy(1) = 0;
V(1) = 0;
x(1) = 0;
y(1) = 0.1;
Dx(1) = 0;
Dy(1) = 0;
rhoh(1) = 0;
m(1) = 130;
h(1) = 0;
```

```
g(1) = 9.81;
rho(1) = 1.226;
Sx(1) = 0;
Sy(1) = 0;
SMAG(1) = 0;
%% Iteration Loops
for j = 1:timeIterations
  if j <= 30
    m(j + 1) = m(j) - mdot * dt;
    T = 8800;
    g = gE * (Re / (Re + h(j))) ^ 2;
    rho = a_density * exp((-b_density) * h(j) ^ 1.15);
    SoS = SoSfxn(h(j));
    mach(j) = V(j) / SoS;
    drag(j) = dragfxn(mach(j),8800);
    rhoh(j + 1) = (1/2 * rho * V(j).^2);

    Kx1 = (T .* sind(theta(j)) - drag(j) .* 0.5 .* rho .*
(Vx(j)).^2 .* area) ./ m(j);
    Kx2 = (T .* sind(theta(j)) - drag(j) .* 0.5 .* rho .* (Vx(j)
+ 0.5 .* Kx1 .* dt).^2 * area) ./ m(j);
    Kx3 = (T .* sind(theta(j)) - drag(j) .* 0.5 .* rho .* (Vx(j)
+ 0.5 .* Kx2 .* dt).^2 * area) ./ m(j);
    Kx4 = (T .* sind(theta(j)) - drag(j) .* 0.5 .* rho .* (Vx(j)
+ Kx3 .* dt).^2 .* area) ./ m(j);
    Ky1 = (T .* cosd(theta(j)) - drag(j) .* 0.5 .* rho .*
(Vy(j)).^2 .* area - (m(j) .* g)) ./ m(j);
    Ky2 = (T .* cosd(theta(j)) - drag(j) .* 0.5 .* rho .* (Vy(j)
+ 0.5 .* Ky1 .* dt).^2 .* area - (m(j) .* g)) ./ m(j);
    Ky3 = (T .* cosd(theta(j)) - drag(j) .* 0.5 .* rho .* (Vy(j)
+ 0.5 .* Ky2 .* dt).^2 .* area - (m(j) .* g)) ./ m(j);
    Ky4 = (T .* cosd(theta(j)) - drag(j) .* 0.5 .* rho .* (Vy(j)
+ Ky3 .* dt).^2 .* area - (m(j) .* g)) ./ m(j);

    Sx(j + 1) = (1 / 6) .* ((Kx1 + (2 * Kx2) + (2 * Kx3) +
Kx4));
    Sy(j + 1) = (1 / 6) .* ((Ky1 + (2 * Ky2) + (2 * Ky3) +
Ky4));
    SMAG(j + 1) = sqrt((Sx(j + 1)).^2 + (Sy(j + 1)).^2);
    Vx(j + 1) = (Vx(j) + (((Sx(j + 1)) + (Sx(j))) / 2)) * dt;
    Vy(j + 1) = Vy(j) + ((Sy(j + 1) + Sy(j)) / 2) * dt;
    V(j + 1) = sqrt((Vx(j + 1)).^2 + (Vy(j + 1)).^2);
    theta(j + 1) = atand(Vx(j + 1) / Vy(j + 1));
    h(j + 1) = h(j) + V(j + 1) * cosd(theta(j + 1)) * dt;
    x(j + 1) = x(j) + Vx(j + 1) * dt;
    t(j + 1) = t(j) + dt;
```

```
  elseif t(j) > tB && h(j) < 0
    h(j) = 0;

  else
    T = 0;
    m(j + 1) = mB;
    g = gE * (Re / (Re + h(j)))^2;
    rho = a_density * exp(( - b_density) * h(j)^(1.15));
    SoS = SoSfxn(h(j));
    mach(j) = V(j) / SoS;
    drag(j) = dragfxn(mach(j),0);
    rhoh(j)= (1/2 * rho * V(j).^2);

    Kx1 = (T .* sind(theta(j)) - drag(j) .* 0.5 .* rho .*
(Vx(j)).^2 .* area) ./ m(j);
    Kx2 = (T .* sind(theta(j)) - drag(j) .* 0.5 .* rho .* (Vx(j)
+ 0.5 .* Kx1 .* dt).^2 * area) ./ m(j);
    Kx3 = (T .* sind(theta(j)) - drag(j) .* 0.5 .* rho .* (Vx(j)
+ 0.5 .* Kx2 .* dt).^2 * area) ./ m(j);
    Kx4 = (T .* sind(theta(j)) - drag(j) .* 0.5 .* rho .* (Vx(j)
+ Kx3 .* dt).^2 .* area) ./ m(j);
    Ky1 = (T .* cosd(theta(j)) - drag(j) .* 0.5 .* rho .*
(Vy(j)).^2 .* area - (m(j) .* g)) ./ m(j);
    Ky2 = (T .* cosd(theta(j)) - drag(j) .* 0.5 .* rho .* (Vy(j)
+ 0.5 .* Ky1 .* dt).^2 .* area - (m(j) .* g)) ./ m(j);
    Ky3 = (T .* cosd(theta(j)) - drag(j) .* 0.5 .* rho .* (Vy(j)
+ 0.5 .* Ky2 .* dt).^2 .* area - (m(j) .* g)) ./ m(j);
    Ky4 = (T .* cosd(theta(j)) - drag(j) .* 0.5 .* rho .* (Vy(j)
+ Ky3 .* dt).^2 .* area - (m(j) .* g)) ./ m(j);

    Sx(j + 1) = (1 / 6) .* ((Kx1 + (2 * Kx2) + (2 * Kx3) +
Kx4));
    Sy(j + 1) = (1 / 6) .* ((Ky1 + (2 * Ky2) + (2 * Ky3) +
Ky4));
    SMAG(j + 1) = sqrt((Sx(j + 1)).^2 + (Sy(j + 1)).^2);
    Vx(j + 1) = (Vx(j) + (((Sx(j + 1)) + (Sx(j))) / 2)) * dt;
    Vy(j + 1) = Vy(j) + ((Sy(j + 1) + Sy(j)) / 2) * dt;
    V(j + 1) = sqrt((Vx(j + 1)).^2 + (Vy(j + 1)).^2);
    theta(j + 1) = atan2d(Vx(j + 1),Vy(j + 1));
    h(j + 1) = h(j) + V(j + 1) * cosd(theta(j + 1)) * dt;
    x(j + 1) = x(j) + Vx(j + 1) * dt;
    t(j + 1) = t(j) + dt;
  end
end
%% Plots
figure(1)
plot(t(1:tB+1), h(1:tB+1),'r', t(tB+1:313), h(tB+1:313), 'b')
```

```
xlabel('Time (seconds)')
ylabel('Height (meters)')
title('Altitude vs. Time')
legend('Boost','Coast')
grid on

figure(2)
plot(t(1:tB+1), V(1:tB+1),'r', t(tB+1:300), V(tB+1:300), 'b')
xlabel('Time (seconds)')
ylabel('Velocity (meters/seconds)')
title('Velocity vs. Time')
legend('Boost','Coast')
grid on

figure(3)
plot(t(1:tB+1), Vx(1:tB+1),'r', t(tB+1:313), Vx(tB+1:313), 'b')
xlabel('Time (seconds)')
ylabel('Velocity in Horizontal (meters/seconds)')
title('Velocity in Horizontal vs. Time')
legend('Boost','Coast')
grid on

figure(4)
plot(t(1:tB+1), Vy(1:tB+1),'r', t(tB+1:313), Vy(tB+1:313), 'b')
xlabel('Time (seconds)')
ylabel('Velocity in Vertical (meters/seconds)')
title('Velocity in Vertical vs. Time')
legend('Boost','Coast')
grid on

figure(5)
plot(t(1:tB+1), SMAG(1:tB+1),'r', t(tB+1:300), SMAG(tB+1:300),
'b')
xlabel('Time (seconds)')
ylabel('Acceleration (meters/seconds^2)')
title('Acceleration vs. Time')
legend('Boost','Coast')
grid on

figure(6)
plot(t(1:tB+1), rhoh(1:tB+1),'r', t(tB+1:300), rhoh(tB+1:300),
'b')
xlabel('Time (seconds)')
ylabel('Dynamic Pressure (Pascals)')
title('Dynamic Pressure vs. Time')
legend('Boost','Coast')
grid on
```

```
figure(7)
plot(t(1:tB+1), mach(1:tB+1),'r', t(tB+1:300), mach(tB+1:300),
'b')
xlabel('Time (seconds)')
ylabel('Mach')
title('Mach vs. Time')
legend('Boost','Coast')
grid on
```