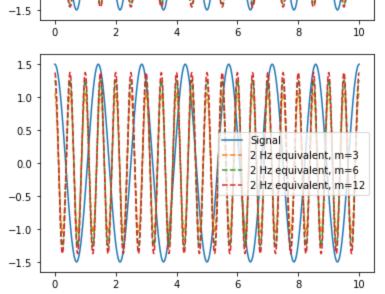
Wind Energy Toolbox wetb()

```
Fatigue load calculation
In [3]: # Signal parameters
        nr_periods = 7
        amplitude = 1.5
        offset = 0
        # Fatigue load parameters
        m = [3,6,12] # Wöhler exponents
        eq_freq =[1,2] # Frequency of equivalent signal
        # Sinosodial signal
        time = np.linspace(0, 10, 1000) #
        neq = time[-1] * np.array(eq_freq)
        signal = amplitude * np.cos(time/10*nr_periods*2*np.pi) + offset
        # Equivalent Load
        eq_load_neq_m = np.array(eq_load(signal, m=m, neq=neq))
        print((eq_load_neq_m.shape))
        for eq_freq_, eq_load_m in zip(eq_freq, eq_load_neq_m):
          plt.figure()
          plt.plot(time, signal, label='Signal')
          for m_, eq_load in zip(m, eq_load_m):
            plt.plot(time, (eq_load/2) * np.cos(time*2*np.pi*eq_freq_), '--', label='%d Hz equivalent, m=%d'%(eq_freq_, m_))
            n_i = nr_periods # No cycles
            n_eq = 10*eq_freq_ # cycles in equivalent signal
            S_i = amplitude*2 # peak to peak amplitude
            expected_eq = np.round(((n_i * S_i**m_) / n_eq)**(1 / m_),4)
            print ("Neq: %d, m: %d, Equivalent load: %s, Expected equivalent load: %s"%(n_eq, m_, np.round(eq_load,4), expected_eq))
          plt.legend()
       Neq: 10, m: 3, Equivalent load: 2.6637, Expected equivalent load: 2.6637
       Neq: 10, m: 6, Equivalent load: 2.8269, Expected equivalent load: 2.8269
       Neq: 10, m: 12, Equivalent load: 2.9121, Expected equivalent load: 2.9121
       Neq: 20, m: 3, Equivalent load: 2.1142, Expected equivalent load: 2.1142
       Neq: 20, m: 6, Equivalent load: 2.5184, Expected equivalent load: 2.5184
       Neq: 20, m: 12, Equivalent load: 2.7487, Expected equivalent load: 2.7487
        1.5
        1.0
        0.5
                                         1 Hz equivalent, m=3
        0.0
                                         1 Hz equivalent, m=6
       -0.5
       -1.0
```



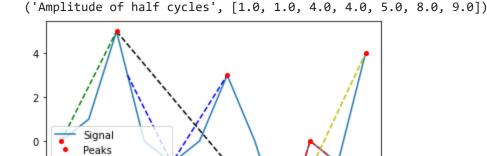
Theory - Signals to Cycles

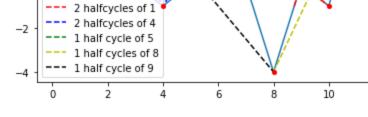
First the signal is reduced to only peak values (local minima and maxima) and then the number of cycles and their amplitude is found using rainflow counting. The plot below illustrates the principle and prints the amplitudes returned by the rainflow counting function.

```
In [4]:
    signal = np.array([-0, 1, 5, 0, -1, 0, 3, 0, -4, 0, -1,4])
    plt.plot(signal, label='Signal')
    plt.plot([0,2,4,6,8,9,10,11],[0,5,-1,3,-4,0,-1,4],'.r', ms=8, label='Peaks', zorder=32)

plt.plot([8.75,9,10],[-1,0,-1],'--r', label="2 halfcycles of 1")
    plt.plot([2,4,4,6],[3,-1,3],'--b', label="2 halfcycles of 4")
    plt.plot([0,2],[0,5],'--g', label="1 half cycle of 5")
    plt.plot([8,11],[-4,4],'--y', label="1 half cycles of 8")
    plt.plot([2,8],[5,-4],'--k', label="1 half cycle of 9")
    plt.legend(loc='lower left')

from wetb.fatigue_tools.fatigue import rainflow_astm
    ampl, mean = rainflow_astm(signal)
    print(("Amplitude of half cycles", sorted(ampl)))
```



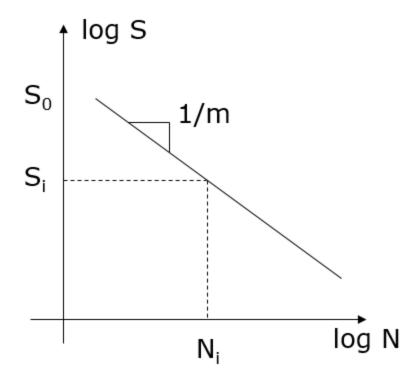


Note, that this is the number of half cycles, i.e. the number of full cycles (n_i below) are:

Cycles	Amplitude
1	1
1	4
0.5	5
0.5	8
0.5	9

Cycles to equivalent load

The SN-curve specifies the relation between the stress, S, and the number to failure, N. In other words, failure will occur after N_i cycles with amplitude, S_i .



The damage factor, d_i , of n_i cycles with amplitude, S_i is: $d_i = \frac{n_i}{N_i}$ where N_i is the number of cycles to failure for the amplitude, S_i . According to the Palmgren-Miner linear damage hypothesis, this relation between stress and number to failure is linear in a log-log coodinate system with the slope: $\frac{1}{m}$ where m is the Wöhler exponent This relation can be formulated as: $N_i = \left(\frac{S_0}{S_i}\right)^m$ Inserting this into the equation above gives us an expression for the damage factor for a number of cycles, n_i with amplitude S_i : $d_i = \frac{n_i}{N_i} = \frac{n_i S_i^m}{S_0^m}$ The total damage, D, can be summed for cycles of differnce amplitudes: $D = \frac{1}{S_0^m} \sum n_i S_i^m$ For a given number of cycles, n_{eq} , the amplitude, S_{eq} , that gives the same total damage can now be found: $\frac{n_{eq} S_{eq}^m}{S_0^m} = D = \frac{1}{S_0^m} \sum n_i S_i^m \Rightarrow S_{eq} = \left(\frac{\sum n_i S_i^m}{S_0^m}\right)^{\frac{1}{m}}$ The 1Hz equivalent stress is found by setting n_{eq} to the signal length in second, i.e. the total

 $rac{n_{eq}S_{eq}^m}{S_0^m} = D = rac{1}{S_0^m} \sum n_i S_i^m \Rightarrow S_{eq} = \left(rac{\sum n_i S_i^m}{n_{eq}}
ight)^{rac{1}{m}}$ The 1Hz equivalent stress is found by setting n_{eq} to the signal length in second, i.e. the total damage of the signal is equal to the damage of a 1Hz sinododial signal of the same length with amplitude S_{eq} .