3^{rd} Statistical Report on Breathometer
Nonlinear Mixed-Effect model
Hierarchical Regression Modelling

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I. DATA PREPROCESSING AND EXPLORATORY ANALYSIS

In this analysis, we focus on the 379 sensor devices that occur at least 15 times in the dataset 'CalibrationData.csv', since more data is better.

Among these device, there are substantially more $CP1$ and $CP4$ observations than $CP2$ and $CP3$. We also filter out all $TP1$ and $TP4$ observations.

For simplicity, we choose to base our analysis on a particular sensor device (PID $=$ 101758).

First, we explore the behaviour of this particular sensor within 25 secs interval by plotting the observations in the 'Data' column in the dataset . See below plot I.1.

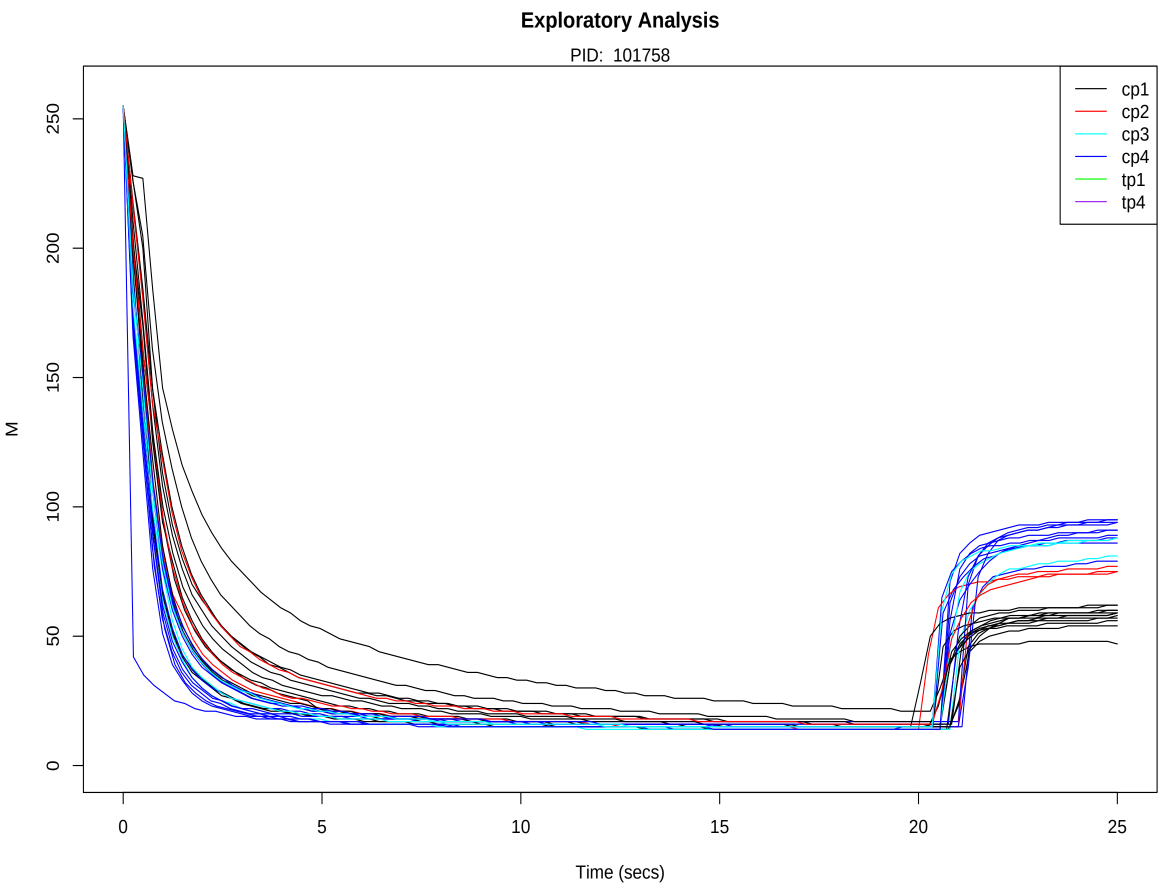

Figure I.1: Exploratory Data Analysis

Each curve corresponds to the sensor behaviour under a particular CP level. The behaviour

is made up of 2 processes: (1) decay process (20 secs), (2) blow-in process (5 secs).
There are several issues with the data:

1. Each data vector has different length.

2. The cut-off point from the decay process is not being reported accurately and hence the time / length for the decay process and blow-in process differs across all CP levels. And this is related to the first problem.

3. If the device is left unused for a while, the device tends to give inaccurate estimates of alcohol concentration level in the 1^{st} attempt.

Table I captures all the timestamp, CP levels and *After.WarmUp* condition observations for sensor device 101758. And there are 28 observations in total for this particular PID. Note that each observation contains a *data vector* , which is too long to be presented in the table I.

A. Getting around the issues

To address the problem that the decay process is not stopping exactly at the 20^{th} second as shown in the first plot I.1, our solution was to use the *After.WarmUp* condition provided in the dataset as the *cut-off* point that stops at the 20^{th} second. Note that this is not necessarily the minimum point of the data vector.

Alternatively, a different approach would be to append the *After.WarmUp* condition to each data vector such that they all have equal length in the first 20 seconds. This is based on the assumption that the sensor sometimes plateau before the 20^{th} second is reached. (Computationally, this is slightly more difficult to achieve but we have written a code on this as well.) For simplicity, we will use the 1^{st} approach in our analysis.

Using the *After.WarmUp* data as *cut-off* point, we break down the entire data vector into 2 processes: decay process (first 20 secs) and blow-in process ($20 - 25$ secs). Note that the data vectors are **still in different lengths** but we ensure the decay process

	Date.Time	PID	TestGroup	Factory	testName	Before.WarmUp	After.WarmUp	Condition
1	0014-05-12 10:36:00	101758	1	HP	qa3_1	255	21	cp1
2	0014-05-12 14:57:00	101758	1	HP	qa3_1	255	15	cp4
3	0014-05-13 11:39:00	101758	1	HP	qa3_1	255	15	cp1
4	0014-05-13 16:49:00	101758	1	HP	qa3_1	255	14	cp4
5	0014-05-14 09:16:00	101758	1	HP	qa3_1	255	14	cp1
6	0014-05-14 14:42:00	101758	1	HP	qa3_1	255	15	cp4
7	0014-05-15 09:34:00	101758	1	HP	qa3_1	255	14	cp1
8	0014-05-15 14:39:00	101758	1	HP	qa3_1	255	15	cp4
9	0014-05-16 09:22:00	101758	1	HP	qa3_1	255	14	cp2
10	0014-05-16 14:35:00	101758	1	HP	qa3_1	255	14	cp3
11	0014-05-19 09:45:00	101758	1	HP	qa3_1	255	16	cp1
12	0014-05-19 14:40:00	101758	1	HP	qa3_1	255	14	cp4
13	0014-05-20 09:20:00	101758	1	HP	qa3_1	255	15	cp1
14	0014-05-20 14:42:00	101758	1	HP	qa3_1	255	17	cp4
15	0014-05-21 09:19:00	101758	1	HP	qa3_1	255	15	cp1
16	0014-05-22 09:11:00	101758	1	HP	qa3_1	255	15	cp1
17	0014-05-22 13:44:00	101758	1	HP	qa3_1	255	15	cp4
18	0014-05-23 09:03:00	101758	1	HP	qa3_1	255	15	cp2
19	0014-05-23 14:30:00	101758	1	HP	qa3_1	255	15	cp3
20	0014-05-26 09:08:00	101758	1	HP	qa3_1	255	17	cp1
21	0014-05-26 14:43:00	101758	1	HP	qa3_1	255	15	cp4
22	0014-05-27 09:11:00	101758	1	HP	qa3_1	255	15	cp1
23	0014-05-27 15:11:00	101758	1	HP	qa3_1	255	15	cp4
24	0014-05-28 09:24:00	101758	1	HP	qa3_1	255	15	cp1
25	0014-05-28 14:45:00	101758	1	HP	qa3_1	255	16	cp4
26	0014-05-29 09:25:00	101758	1	HP	qa3_1	255	16	cp1
27	0014-05-30 09:23:00	101758	1	HP	qa3_1	255	16	cp2
28	0014-05-30 14:45:00	101758	1	HP	qa3_1	255	15	cp3

Table I: Table 1

ends at the 20^{th} second and blow-in process starts after 20^{th} second and ends at the 25^{th} second. And we end up working with two separate data frames, which we can fit *2 separate nonlinear mixed-effects models* on. This will be shown in later sections. Nonetheless, the decomposition leads us to the plot I.2 below.

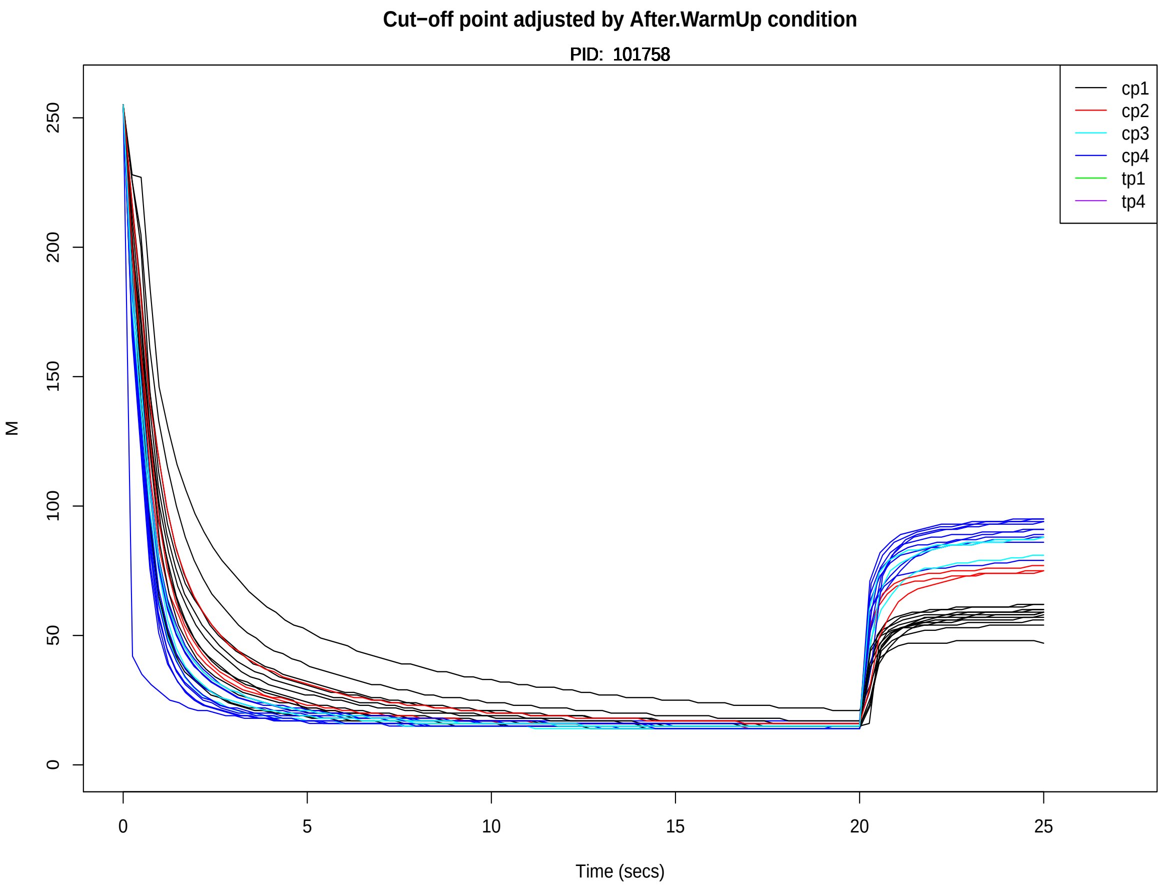

Figure I.2: Adjusted by After.WarmUp

II. DECAY PROCESS

The data frame for the decay process looks like the following:

	df1_datetime	df1_pid	df1_testgroup	df1_factory	df1_testName	df1_condition	df1_cp	df1_subject	decay_time	df1_measures
1	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	0.00	255.00
2	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	0.24	228.00
3	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	0.49	227.00
4	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	0.73	184.00
5	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	0.98	146.00
6	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	1.22	130.00
7	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	1.46	116.00
8	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	1.71	106.00
9	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	1.95	97.00
10	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	2.20	90.00
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
73	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	17.56	23.00
74	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	17.80	22.00
75	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	18.05	22.00
76	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	18.29	22.00
77	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	18.54	22.00
78	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	18.78	22.00
79	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	19.02	22.00
80	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	19.27	21.00
81	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	19.51	21.00
82	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	19.76	21.00
83	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	20.00	21.00
84	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	0.00	255.00
85	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	0.25	171.00
86	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	0.49	123.00
87	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	0.74	77.00
88	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	0.99	53.00
89	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	1.23	39.00
90	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	1.48	32.00
91	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	1.73	28.00
92	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	1.98	25.00
93	0014-05-12 14:57:00	101758	1	HP	qa3_1	cp4	0.80	2	2.22	23.00
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table II: Decay Process Data Frame

Note that a subject refers to an observation under a particular CP level at a particular time point. Essentially, a subject corresponds to a line number in table I and there are 28 lines in table I and hence, 28 subjects in the decay process data frame. This will allow us to fit a nonlinear mixed-effects model to these subjects individually later on.

It is assumed that it takes exactly 20 seconds to reach the $After.WarmUp$ condition.

A. The nonlinear mixed-effect model - Hierarchical Regression Models

Let k denotes one of $(CP1, CP2, CP3, CP4)$.

Let i denotes the i^{th} subject in table I and $i = 1, \cdots, n_i$. n_i refers to the total number of observations for each device. For device 101758, $n_i = 28$.

Let $\vec{t_i} = (t_{i1}, \cdots, t_{i20})^T$ denotes time series vector (decay_time in Table II). And $\vec{t_i}$ and the length of $\vec{t_i}$ varies across different subjects. The same goes for $\vec{y_i}$ as well.

Let $\beta_{i0} = exp(\tilde{\beta}_{i0}), \beta_{i1} = exp(\tilde{\beta}_{i1}), \beta_{i2} = exp(\tilde{\beta}_{i2})$ be the individual (subject)-specific parameters for subject i. And $\vec{y_i} = (y_{i1}, \cdots, y_{i20})^T$ be the vector of measurements (df1_measures in Table II) under the influence of a particular CP level.

The hierarchical model:

- Within Individual Model

$$\vec{y_i} = \beta_{i0} + \beta_{i1}\exp(-\beta_{i2} \times \vec{t_i}) \quad \forall \quad i = 1, \cdots, n_i$$

The decay process is independent of the alcohol levels , since the first 20 seconds of the process involves no alcohol at all and hence, independent of the CP covariates.

- Population Model Level I and II

$$\vec{\beta_i} = A_i\vec{\beta} + \vec{b_k} + \vec{b_{ki}}$$

$\vec{\beta_i}$ is a (3×1) vector of individual-specific parameters.
$\vec{\beta}$ is a $(p \times 1)$ vector of **fixed-effect parameters**, also known as population parameters .

A_i is the $(3 \times p)$ matrix that depends on the individual-level (among individual) covariates, the CP levels. It can also be an identity matrix, which means $A_i = I$ is independent of the individual-level covariates.

From the plot I.2, we can see that clearly the higher the CP level, the quicker the decay rate of the process. So, we suggest that for the decay process:

$$A_i = \begin{bmatrix} 1 & k & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & k \end{bmatrix} \qquad \vec{\beta} = \begin{bmatrix} \beta_{00} \\ \beta_{0k} \\ \beta_{10} \\ \beta_{20} \\ \beta_{2k} \end{bmatrix}$$

, where k denotes the CP level of a particular observation.

$\vec{b_k}$ is the (3×1) random effect vector depending on the CP level, which is dictated by *df1_condition* from Table II. Note that there are multiple subjects in each CP levels. This is essentially the random effects of **the group of subjects under a particular CP level**.

$\vec{b_{ki}}$ is the (3×1) random effect vector of each subject i, given that the subject is under the influence of $k = CP1$, $CP2$, $CP3$ or $CP4$.

Level I of the population model involves only the random effect $\vec{b_k}$, i.e.

$$\vec{\beta_i} = A_i\vec{\beta} + \vec{b_k}$$

This allows us to understand the random effect of different CP levels and fit the nonlinear regression model based on CP levels.

Level II of the popukation model involves both random effects $\vec{b_k}, \vec{b_{ki}}$. This allows us to fit the nonlinear regression model on an individual / subject level.

The population model is assumed to be a *linear function* of individual-level covariates, fixed effects $\vec{\beta}$ and random effects $\vec{b_k}, \vec{b_{ki}}$.

III. BLOW-IN PROCESS

The data frame for the blow-in process looks like the following:

	df2_datetime	df2_pid	df2_testgroup	df2_factory	df2_testName	df2_condition	df2_cp	df2_subject	blow_time	df2_measures
1	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	20.26	30.00
2	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	20.53	41.00
3	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	20.79	44.00
4	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	21.05	46.00
5	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	21.32	47.00
6	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	21.58	47.00
7	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	21.84	47.00
8	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	22.11	47.00
9	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	22.37	47.00
10	0014-05-12 10:36:00	101758	1	HP	qa3_1	cp1	0.20	1	22.63	48.00
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
46	0014-05-13 11:39:00	101758	1	HP	qa3_1	cp1	0.20	3	23.53	54.00
47	0014-05-13 11:39:00	101758	1	HP	qa3_1	cp1	0.20	3	23.82	54.00
48	0014-05-13 11:39:00	101758	1	HP	qa3_1	cp1	0.20	3	24.12	54.00
49	0014-05-13 11:39:00	101758	1	HP	qa3_1	cp1	0.20	3	24.41	54.00
50	0014-05-13 11:39:00	101758	1	HP	qa3_1	cp1	0.20	3	24.71	54.00
51	0014-05-13 11:39:00	101758	1	HP	qa3_1	cp1	0.20	3	25.00	54.00
52	0014-05-13 16:49:00	101758	1	HP	qa3_1	cp4	0.80	4	20.28	63.00
53	0014-05-13 16:49:00	101758	1	HP	qa3_1	cp4	0.80	4	20.56	73.00
54	0014-05-13 16:49:00	101758	1	HP	qa3_1	cp4	0.80	4	20.83	78.00
55	0014-05-13 16:49:00	101758	1	HP	qa3_1	cp4	0.80	4	21.11	81.00
56	0014-05-13 16:49:00	101758	1	HP	qa3_1	cp4	0.80	4	21.39	82.00
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table III: Blow-In process Data Frame

It is assumed that the *blow-in process* takes exactly 5 seconds to plateau.

A. The nonlinear mixed-effect model

Let k denotes one of $(CP1, CP2, CP3, CP4)$.

Let i denotes the i^{th} subject in table I and $i = 1, \cdots, n_i$. n_i refers to the total number of observations for each device. For device 101758, $n_i = 28$.

Let $\vec{t_i} = (t_{i20.1}, \cdots, t_{i25})^T$ denotes time series vector (blow_time in Table III). And $\vec{t_i}$ and the length of $\vec{t_i}$ varies across different subjects. The same goes for $\vec{y_i}$ as well.

Let $d = exp(CP)$ denotes the exponential of the CP levels.

Let $\beta_{i0} = exp(\tilde{\beta}_{i0}), \beta_{i1} = exp(\tilde{\beta}_{i1}), \beta_{i2} = exp(\tilde{\beta}_{i2})$ be the individual (subject)-specific parameters for subject i. And $\vec{y_i} = (y_{i20.1}, \cdots, y_{i25})^T$ be the vector of measurements (df2_measures in Table III) under the influence of a particular CP level.

The hierarchical model:

- Within Individual Model

$$\vec{y_i} = d \times \left(\beta_{i0} - \beta_{i1} \exp\left(-\beta_{i2} \times (\vec{t_i} - 20)\right)\right) \qquad \forall \quad i = 1, \cdots, n_i$$

The blow-in process is dependent of the alcohol levels. Note that the data point at the 20^{th} second belongs to the decay process model and thus, not included in the blow-in process. We care only about everything after the 20^{th} second.

- Population Model Level I and II

$$\vec{\beta_i} = A_i\vec{\beta} + \vec{b_k} + \vec{b_{ki}}$$

$\vec{\beta_i}$ is a (3×1) vector of individual-specific parameters.
$\vec{\beta}$ is a $(p \times 1)$ vector of **fixed-effect parameters**, also known as population parameters .

A_i is the $(3 \times p)$ matrix that depends on the individual-level (among individual) covariates, the CP levels. It can also be an identity matrix, which means $A_i = I$ is independent of the individual-level covariates.

From the plot I.2, we can see that the entire blow-in process is dependent on the CP levels, so we propose:

$$A_i = \begin{bmatrix} 1 & k & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & k & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & k \end{bmatrix} \qquad \vec{\beta} = \begin{bmatrix} \beta_{00} \\ \beta_{0k} \\ \beta_{10} \\ \beta_{1k} \\ \beta_{20} \\ \beta_{2k} \end{bmatrix}$$

, where k denotes the CP level influencing the sensor at a particular time in Table I.

$\vec{b_k}$ is the (3×1) random effect vector depending on the CP level, which is dictated by *df1_condition* from Table II. Note that there are multiple subjects in each CP levels. This is essentially the random effects of **the group of subjects under a particular** CP **level**.

$\vec{b_{ki}}$ is the (3×1) random effect vector of each subject i, given that the subject is under the influence of $k = CP1$, $CP2$, $CP3$ or $CP4$.

Level I of the population model involves only the random effect $\vec{b_k}$, i.e.

$$\vec{\beta_i} = A_i\vec{\beta} + \vec{b_k}$$

This allows us to understand the random effect of different CP levels and fit the nonlinear regression model based on CP levels.

Level II of the popukation model involves both random effects $\vec{b_k}, \vec{b_{ki}}$. This allows us to fit the nonlinear regression model on an individual / subject level.

The population model is assumed to be a *linear function* of individual-level covariates, fixed effects $\vec{\beta}$ and random effects $\vec{b_k}, \vec{b_{ki}}$.

B. R output

From the plot I.2, we can see that clearly the higher the CP level, the quicker the decay rate of the process. So, we suggest that for the decay process:

$$A_i = \begin{bmatrix} 1 & k & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & k \end{bmatrix}$$

The decay process model written in R is as follow:

```
lhs.nlme7 <- nlme(df1_measures ~ meanfuncLHS(df1_cp, decay_time, b0,b1,b2) ,
fixed = list(b0 ~ df1_cp, b1 ~ 1, b2 ~ df1_cp),
random = b0 + b1 + b2 ~ 1| df1_condition /df1_subject,
#groups = ~ df1_subject ,
data = df1,
start = list(fixed = c(b0 = 1.98, 0,b1 = 5.5,b2 = 0.25,0)),
verbose = T
)
summary(lhs.nlme7)

Nonlinear mixed-effects model fit by maximum likelihood
  Model: df1_measures ~ meanfuncLHS(df1_cp, decay_time, b0, b1, b2)
 Data: df1
       AIC      BIC    logLik
  12697.74 12801.37 -6330.868


Random effects:
 Formula: list(b0 ~ 1, b1 ~ 1, b2 ~ 1)
 Level: df1_condition
 Structure: General positive-definite, Log-Cholesky parametrization
               StdDev       Corr
b0.(Intercept) 2.406597e-07 b0.(I) b1
```

```
b1              1.646086e-08 0
b2.(Intercept) 3.061919e-07 0      0

 Formula: list(b0 ~ 1, b1 ~ 1, b2 ~ 1)
 Level: df1_subject %in% df1_condition
 Structure: General positive-definite, Log-Cholesky parametrization
               StdDev    Corr
b0.(Intercept) 0.1136739 b0.(I) b1
b1             0.0150603 -0.980
b2.(Intercept) 0.3639857 -0.537  0.688
Residual       3.4321348

Fixed effects: list(b0 ~ df1_cp, b1 ~ 1, b2 ~ df1_cp)
                   Value  Std.Error   DF   t-value p-value
b0.(Intercept)  2.985340 0.03365430   23   88.7061  0.0000
b0.df1_cp      -0.243927 0.05341907   23   -4.5663  0.0001
b1              5.466212 0.00364132 2309 1501.1606  0.0000
b2.(Intercept) -0.254826 0.12332732 2309   -2.0663  0.0389
b2.df1_cp       1.031356 0.21362588 2309    4.8279  0.0000
 Correlation:
               b0.(I) b0.d1_ b1     b2.(I)
b0.df1_cp      -0.758
b1             -0.538  0.055
b2.(Intercept) -0.311  0.158  0.336
b2.df1_cp       0.144 -0.189 -0.031 -0.829

Standardized Within-Group Residuals:
       Min         Q1        Med         Q3        Max
-4.5431690 -0.5311574 -0.1689055  0.4893373  7.4751148

Number of Observations: 2339
Number of Groups:
```

```
                   df1_condition df1_subject %in% df1_condition
                               4                             28
```

We suggest that for the blow-in process:

$$A_i = \begin{bmatrix} 1 & k & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & k & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & k \end{bmatrix}$$

The blow-in process model written in R is as follow:

```
rhs.nlmetest8 <- nlme(df2_measures ~ meanfuncRHStest(df2_cp, blow_time,b0, b1,b2),
fixed = list(b0~ df2_cp, b1 ~df2_cp, b2~ df2_cp),
random = b0 + b1 + b2 ~ 1 | df2_condition /df2_subject,
data = df2,
start = list(fixed = c(b0 = 3.979,0.0467, b1 = 3.618,0.77,b2 = 0.73,-0.1)),
verbose = T)

summary(rhs.nlmetest8)
```

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: df2_measures ~ meanfuncRHStest(df2_cp, blow_time, b0, b1, b2)
 Data: df2
       AIC      BIC    logLik
  1800.648 1880.725 -881.3239

Random effects:
 Formula: list(b0 ~ 1, b1 ~ 1, b2 ~ 1)
 Level: df2_condition
 Structure: General positive-definite, Log-Cholesky parametrization
```

```
               StdDev       Corr
b0.(Intercept) 4.303546e-02 b0.(I) b1.(I)
b1.(Intercept) 1.768306e-10  0.957
b2.(Intercept) 3.484522e-02 -0.999 -0.958

 Formula: list(b0 ~ 1, b1 ~ 1, b2 ~ 1)
 Level: df2_subject %in% df2_condition
 Structure: General positive-definite, Log-Cholesky parametrization
               StdDev     Corr
b0.(Intercept) 0.05889506 b0.(I) b1.(I)
b1.(Intercept) 0.44963371  0.202
b2.(Intercept) 0.29031011 -0.046  0.689
Residual       0.97534731

Fixed effects: list(b0 ~ df2_cp, b1 ~ df2_cp, b2 ~ df2_cp)
                   Value Std.Error  DF  t-value p-value
b0.(Intercept)  3.934891 0.0586213  23 67.12396  0.0000
b0.df2_cp      -0.256744 0.1063055  23 -2.41515  0.0241
b1.(Intercept)  3.825480 0.1776343 469 21.53571  0.0000
b1.df2_cp      -0.927346 0.3221663 469 -2.87847  0.0042
b2.(Intercept)  0.951997 0.1262819 469  7.53867  0.0000
b2.df2_cp      -0.541911 0.2283152 469 -2.37352  0.0180
 Correlation:
               b0.(I) b0.d2_ b1.(I) b1.d2_ b2.(I)
b0.df2_cp      -0.903
b1.(Intercept)  0.076 -0.067
b1.df2_cp      -0.067  0.077 -0.873
b2.(Intercept) -0.338  0.306  0.650 -0.566
b2.df2_cp       0.307 -0.339 -0.568  0.648 -0.878

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
```

```
-2.17983514 -0.65517533 -0.06134234  0.65063955  3.03256620

Number of Observations: 500
Number of Groups:
                 df2_condition df2_subject %in% df2_condition
                             4                             28
```

IV. COMBINE THE 2 MODELS AND MAKE PLOTS

I tried to combined these 2 models into one and fit *nlme* function in R. However, the laptop just completely froze everytime it tried to optimise the nonlinear function.

So, I just combine these 2 models by merging the fitted values and the data together.

Level I of the population model is plotted below:

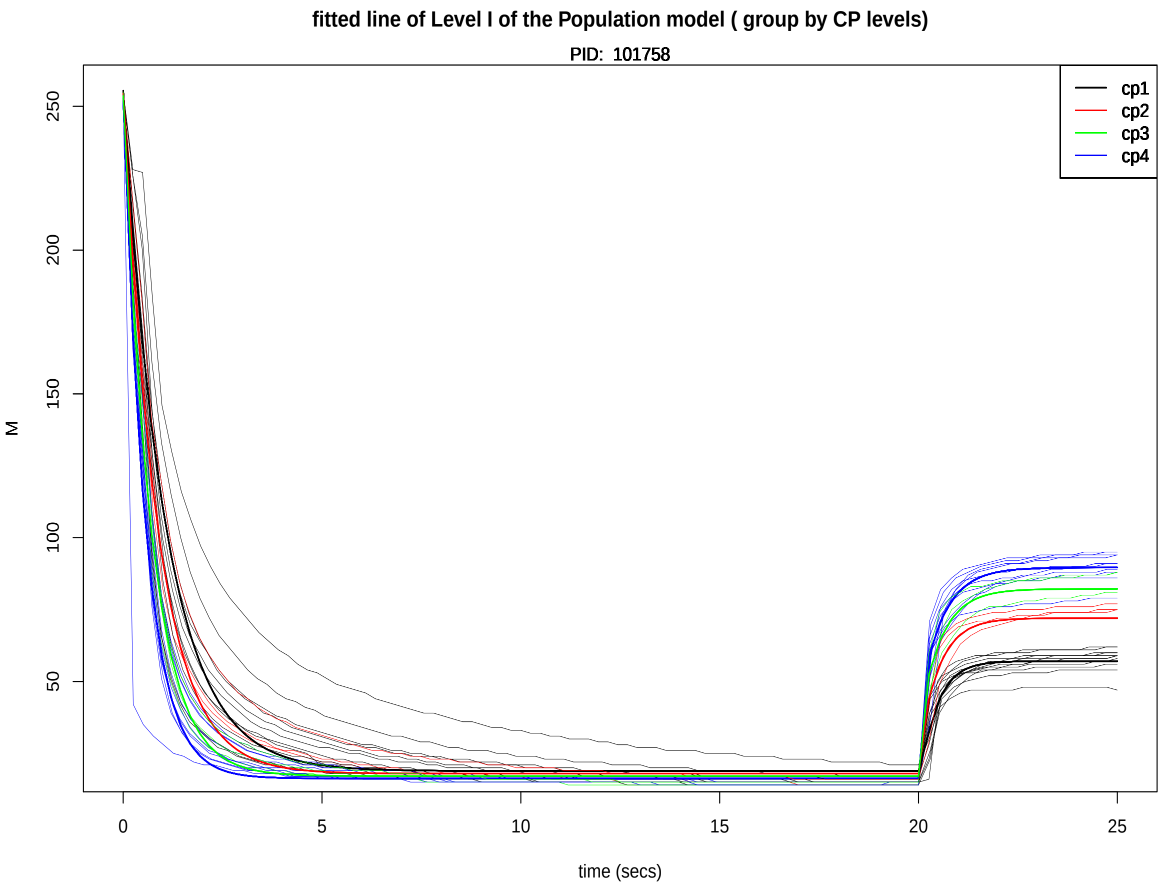

Figure IV.1: Superimpose fitted values of Level I of the nlme model on the data vectors

The thin lines represent the data vector, while the thick lines represent the level I fitted values from the nonlinear mixed-effects model. the color changes across different CP levels.

To examine level 2 of the population model and compare it to level I , we made multiple individual plots.

Note that in plot IV.2, the thick **cyan** colored line represents the fitted values of level I model, which **evaluates all the subjects under a particular** CP **level**.

The individual (subject) data is represented by unfilled triangles.

The fitted values of level II of the population model are represented by the black color.

This allows us to see how far off are the level II model fitted values from the level I model fitted values. See PlotIndivi.pdf. The first plot of PlotIndivi.pdf is shown belowIV.2. Note the plots are shown in chronological order (*ordered by timestamps*).

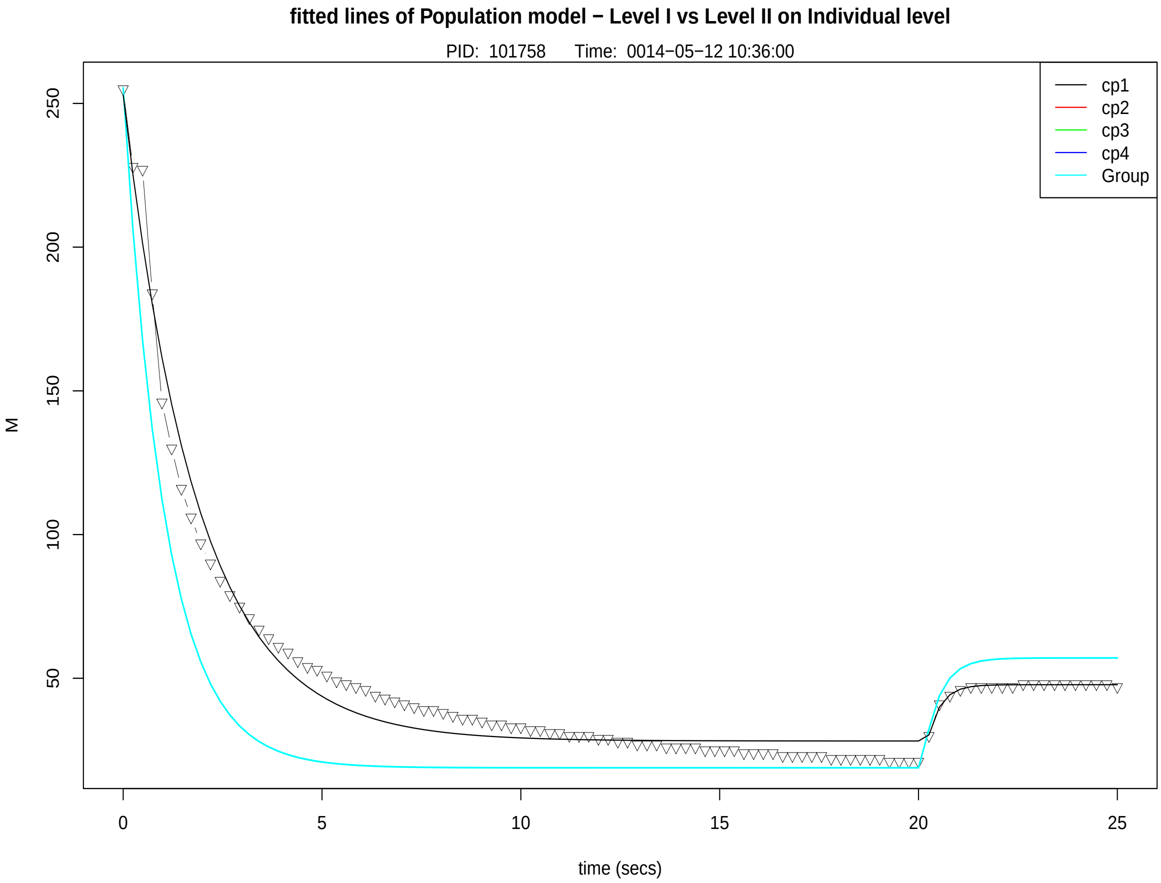

Figure IV.2: Level I fitted line (cyan) + Level II fitted line (black line) on the data vectors (unfilled triangles)

V. EVALUATION

We observe that if we leave the sensor idle for couple weeks and then use it, the sensor tends to produce inaccurate estimates in the first few runs. This can be seen in the PlotIndivi.pdf. For the first 2 to 3 CP observations, the Level II fitted line is quite far away from the Level I fitted line. As time goes on and the sensor is being used constantly, the sensor starts to give estimates that are more accurate than before.

This is shown, as the deviation between the level II fitted line and level I fitted line is closing.

Another issue is that over time, the sensor tends to give better estimates for low CP levels $(CP1, CP2)$ than for high CP levels $(CP3, CP4)$. We can see that the deviations between leve I and leve II fitted line tend to be smaller for $CP1$ and $CP2$ than for $CP3$ and $CP4$.

A. Looking ahead, if time permits $\cdots$

There is definitely room for improvements to fine tune these two nonlinear mixed-effects models. At the same time, we could start to think about:

- what kind of tests we can do to actually compare different *nlme* models with different random effect parameters ? Is ANOVA still valid in this case ?
- how to actually interpret the fixed-effects parameters correlation matrix ? What can we do to improve it ?
- How to check the random-effects parameters are statistically significant, since there are no p-values provided for these parameters except for the fixed-effects parameters?