M5MS06 – Bayesian Statistics

Homework 2 – Winter 2012

Due: 4:00pm, Monday, 18 February 2013

1. Suppose $y_j \sim \mathrm{N}(\theta_j, \sigma_j^2)$ for $j = 1, \ldots, J$ with $\theta_j \sim \mathrm{N}(\mu, \tau^2)$. Let $\theta = (\theta_1, \ldots, \theta_J)$ and $y = (y_1, \ldots, y_J)$ and assume that $\sigma_1^2, \ldots, \sigma_J^2$ are know.

   (a) Suppose that $p(\mu, \tau) \propto 1$. Specify the posterior $p(\theta, \mu, \tau^2|y)$ by first deriving $p(\theta|\mu, \tau^2, y)$ and then $p(\mu|\tau^2, y)$, and finally $p(\tau^2|y)$.

   (b) Now suppose $p(\mu, \tau) \propto 1/\tau$. Show that the posterior distribution, $p(\theta, \mu, \tau|y)$ is improper. *[Hint: Derive $p(\tau^2|y)$ as in (a) and examine its integrability near zero. Does $p(\tau^2|y)$ behave like τ^{-1}, τ^{-J}, or something else?]*

   (c) Reproduce the computations in Section 5.5 of Gelman et al. for the educational testing example. (The Section number is the same in both edition.)

   (d) For each school j, compute the probability that its coaching program is the best. For each pair of schools, j and k, compute the probability that the coaching program for school j is better than that for school k.

   (e) Suppose $\tau^2 = \infty$. What does this mean for the individual coaching programs? Under this assumption, repeat the computation of the probabilities in part (d).

   (f) Now suppose $\tau^2 = 0$. What does this mean for the individual coaching programs? Under this assumption, again repeat the computation of the probabilities in part (d).

   (g) Discuss how your answers to parts (d)–(f) differ.

2. **Problem 6.2 in Gelman et al.** (The problem number is the same in both editions.)