

Annual December Real Interest Rates

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Summary

In this project, we evaluate different models for forecasting one-year-ahead December inflation in order to build a time series of December real interest rates. In particular, we compare the predictive accuracy between two models: AR(1) and Simple Exponential Smoothing (SES). We evaluate the predictive accuracy of one-year-ahead forecasts using roll-forward partitioning, where the training window is expanded by one observation for every one-year-ahead forecast. Throughout this process, we attempt to find the optimal learning rate, alpha, for SES.

Our results reveal that there is no significant difference between an AR(1) and SES in terms of forecasting accuracy. No rate of alpha could alter these results.

Finally, we plot a times series of real interest rates, in which the values from the one-year-ahead AR(1) and SES forecasts are used as proxies for expected inflation.

Data Used

- December 10-, 5-, 3-, and 1-Year Treasury yields (Constant Maturity) from 1953 to 2017
- December CPI from 1947 to 2019
- December inflation from 1948 to 2019, which is computed by taking the difference of logs of consecutive January CPI's, like so:
$$\text{inflation}[y] = \log(\text{CPI}[y]) - \log(\text{CPI}[y-1])$$

Variables

We use the data frame *rates_yearly* to store 18 variables:

- *date*, which stores annual dates from December 1953 to December 2017

- *tcmnom_y10*, the 10-Year Treasury Yield (CM) used as a proxy for the nominal rate
- *tcmnom_y5*, the 5-Year Treasury Yield (CM)
- *tcmnom_y3*, the 3-Year Treasury Yield (CM)
- *tcmnom_y1*, the 1-Year Treasury Yield (CM)
- *pi.realized* stores the one-year-ahead inflation rate relative to the nominal rate (i.e. inflation at $t + 1$), from December 1954 to December 2018
- *r_y10*, the 10-Year realized real interest rate from December 1953 to December 2017, which is computed the following way: $tcmnom_y10 - pi.realized$
- *r_y5*, the 5-Year realized real interest rate
- *r_y3*, the 3-Year realized real interest rate
- *r_y1*, the 1-Year realized real interest rate
- *r_y10.ar1.roll* stores the 10-Y real rate forecasted using one-year-ahead AR(1) forecasts of inflation with roll-forward partitioning
- *r_y10.ses.roll* stores the 10-Year real rate forecasted using one-year-ahead Simple Exponential Smoothing forecasts of inflation with roll-forward partitioning
- *r_y5.ar1.roll* stores the forecasted 5-Y real rate using AR(1)
- *r_y5.ses.roll* stores the forecasted 5-Y real rate using SES
- *r_y3.ar1.roll* stores the forecasted 3-Y real rate using AR(1)
- *r_y3.ses.roll* stores the forecasted 3-Y real rate using SES
- *r_y1.ar1.roll* stores the forecasted 1-Y real rate using AR(1)
- *r_y1.ses.roll* stores the forecasted 1-Y real rate using SES

Methodology

In order to build a times series of real interest rates, we attempt to find the best model for predicting inflation based on literature and statistical testing. the Dickey-Fuller test suggests that December inflation follows a random walk. We wanted to test the AR(1) against a Simple Exponential Smoothing (SES) model knowing that stationary models of inflation tend to perform worse than models which account for a time varying mean ("Forecasting Inflation" by Faust and Wright). We try to to find the

optimal learning rate, alpha, for SES and set it to a default value of 0.2. It is important to note, however, that a high learning rate (alpha) leads to overfitting (“Practical Time Series Forecasting with R” by Schmueli and Lichtendahl Jr).

We first estimate an AR(1) and a SES model on inflation. The Diebold-Mariano test reveals no significant difference between the two models. This is true for any rate of alpha.

Then, we measure the long-term forecasting accuracy of AR(1) and SES using fixed partitioning (i.e. the training set is fixed throughout the forecast horizon). The 30-year-ahead forecast spans from December 1989 up to December 2018. No significant difference was found between the two forecasts at any level of alpha.

Roll-forward partitioning was used next to forecast one-year-ahead between 1954 and 2018. With roll-forward partitioning, the size of the training set increases by one for each one-year-ahead forecast as we add the next observed (realized) datapoint to the set. Roll-forward partitioning enables us to generate multiple one-year-ahead forecasts and thus gain a more accurate understanding of how the forecast performs on average. No significant difference was found between AR(1) and SES.

Finally, we plot four graphs for every type of interest rates (10Y, 5Y, 3Y, & 1Y), each of which contains the following curves from December 1953 to December 2017:

- the realized real rate (nominal rate minus one-year-ahead realized inflation)
- the forecasted real rate using AR(1) roll-forward forecasts of inflation
- the forecasted real rate using SES roll-forward forecasts of inflation

```
##  
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':  
##  
##   as.Date, as.Date.numeric
```

```
## Registered S3 method overwritten by 'quantmod':  
##   method           from  
##   as.zoo.data.frame zoo
```

```
##  
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:plyr':  
##  
##      arrange, count, desc, failwith, id, mutate, rename, summar  
ise,  
##      summarize
```

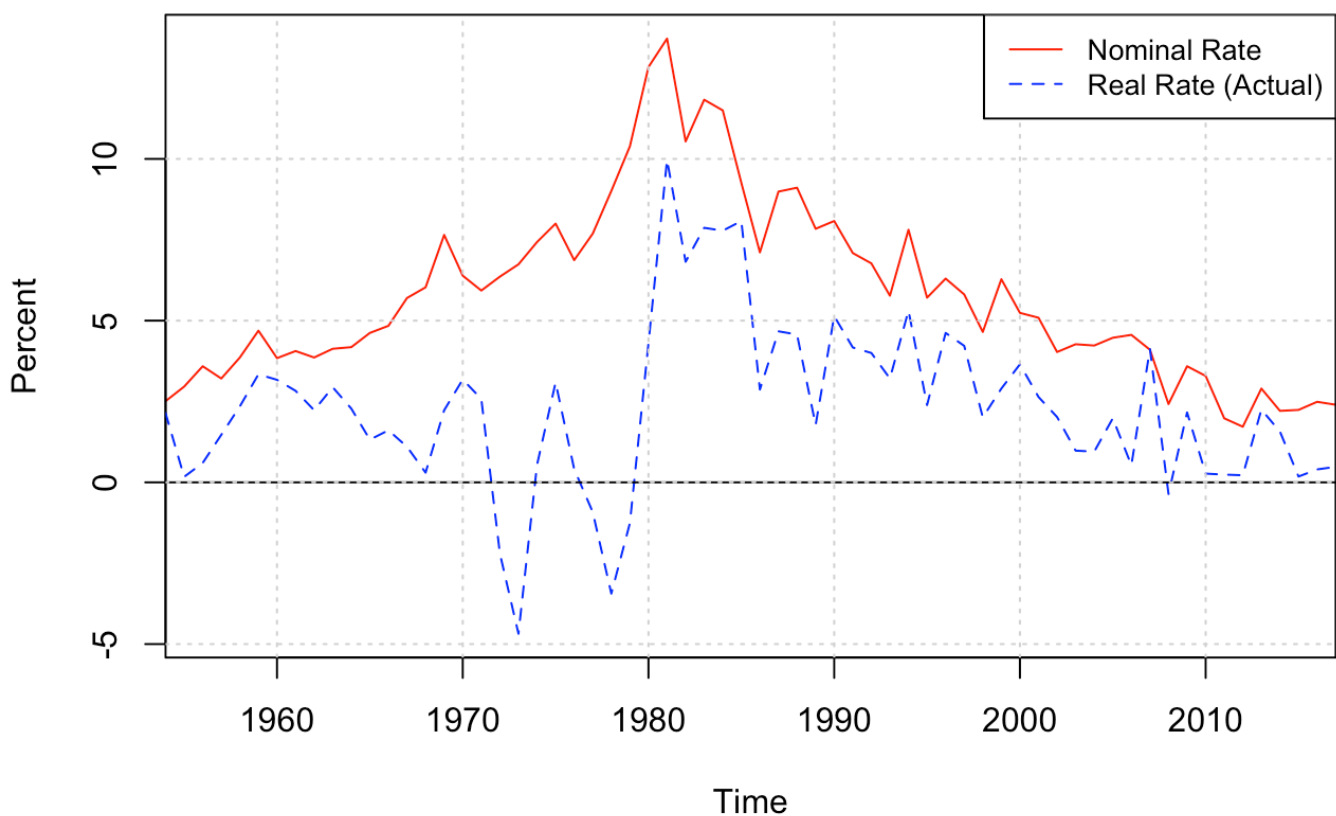
```
## The following objects are masked from 'package:stats':  
##  
##      filter, lag
```

```
## The following objects are masked from 'package:base':  
##  
##      intersect, setdiff, setequal, union
```

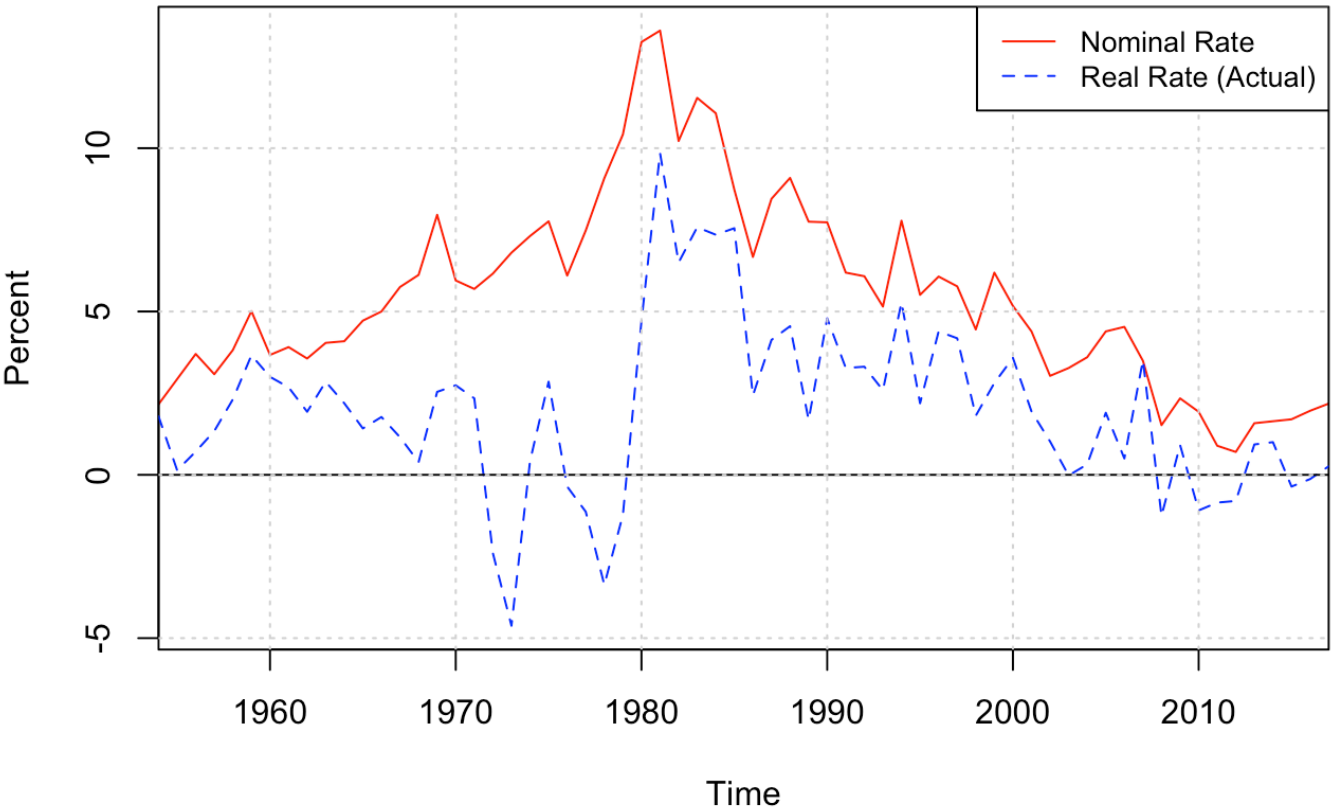
```
## [1] "Number of NA's: 0"
```

Plotting Real vs Nominal Interest Rates

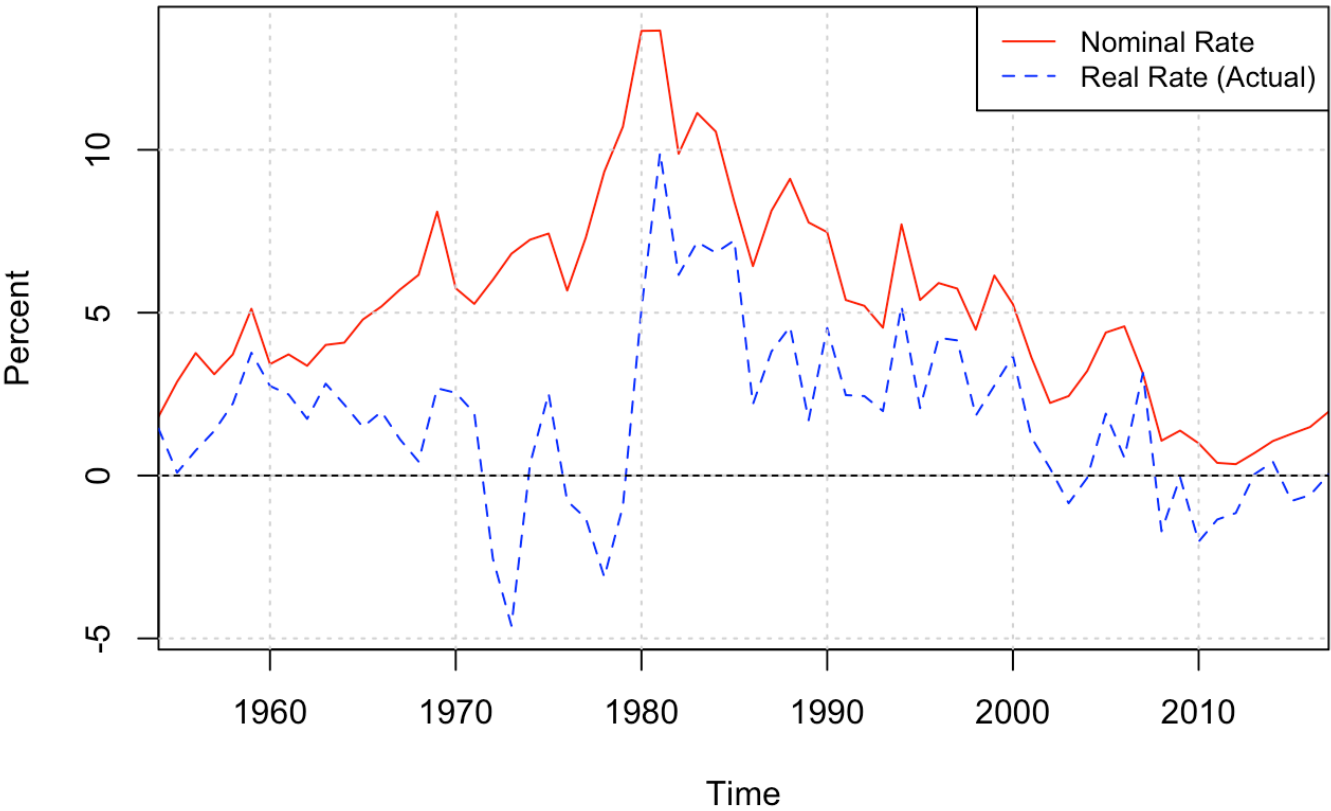
Dec 10-Year Nominal and Real U.S. Treasury Rates (1954 to 2017)



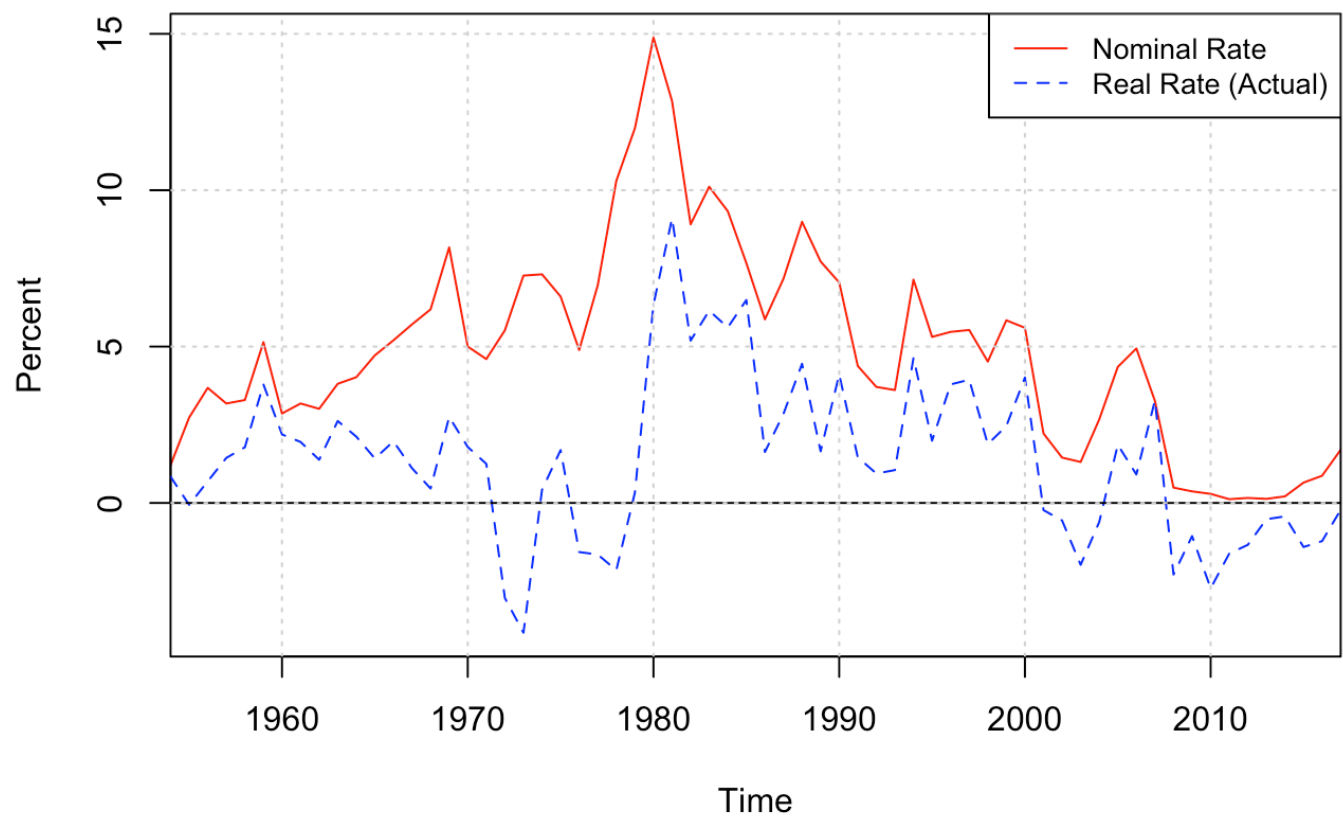
Dec 5-Year Nominal and Real U.S. Treasury Rates (1954 to 2017)



Dec 3-Year Nominal and Real U.S. Treasury Rates (1954 to 2017)



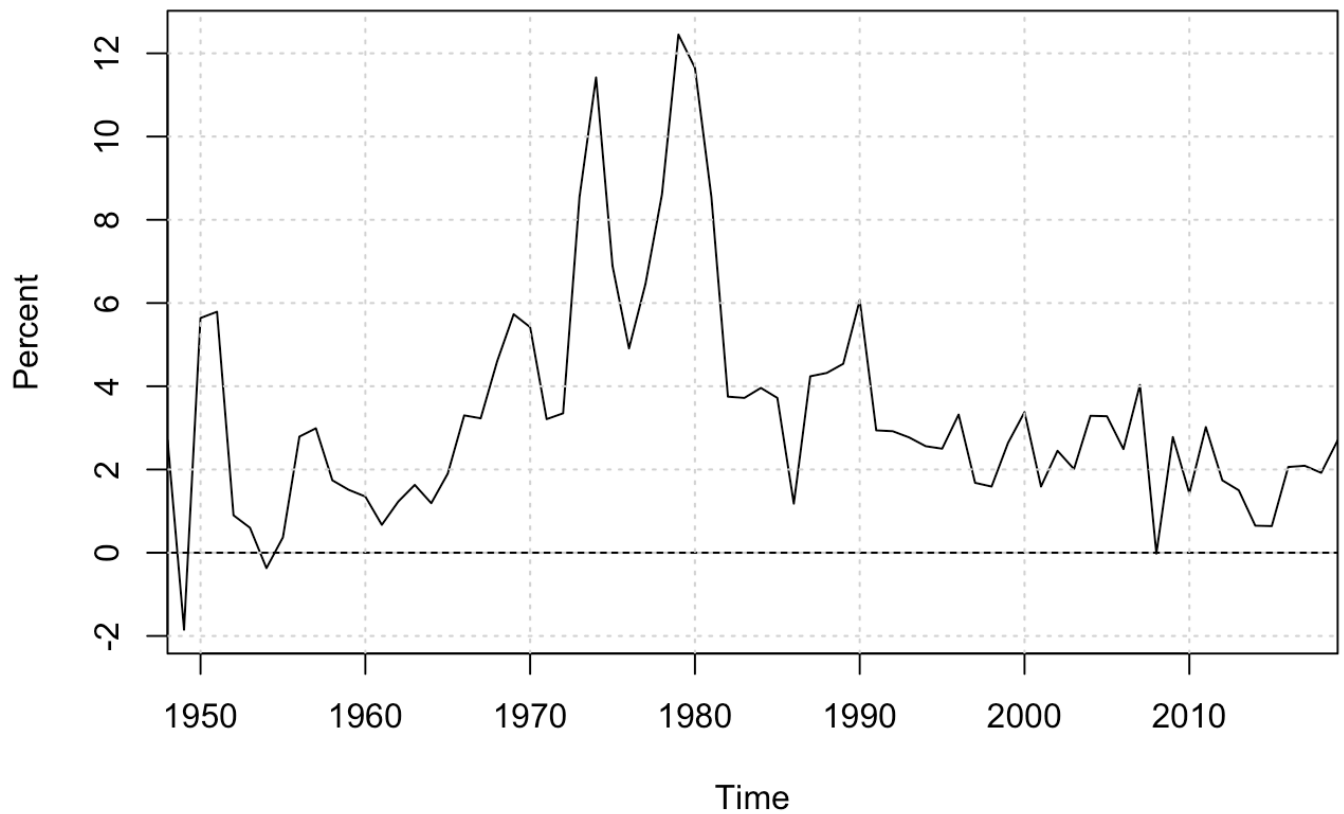
Dec 1-Year Nominal and Real U.S. Treasury Rates (1954 to 2017)



Model Estimation for Inflation

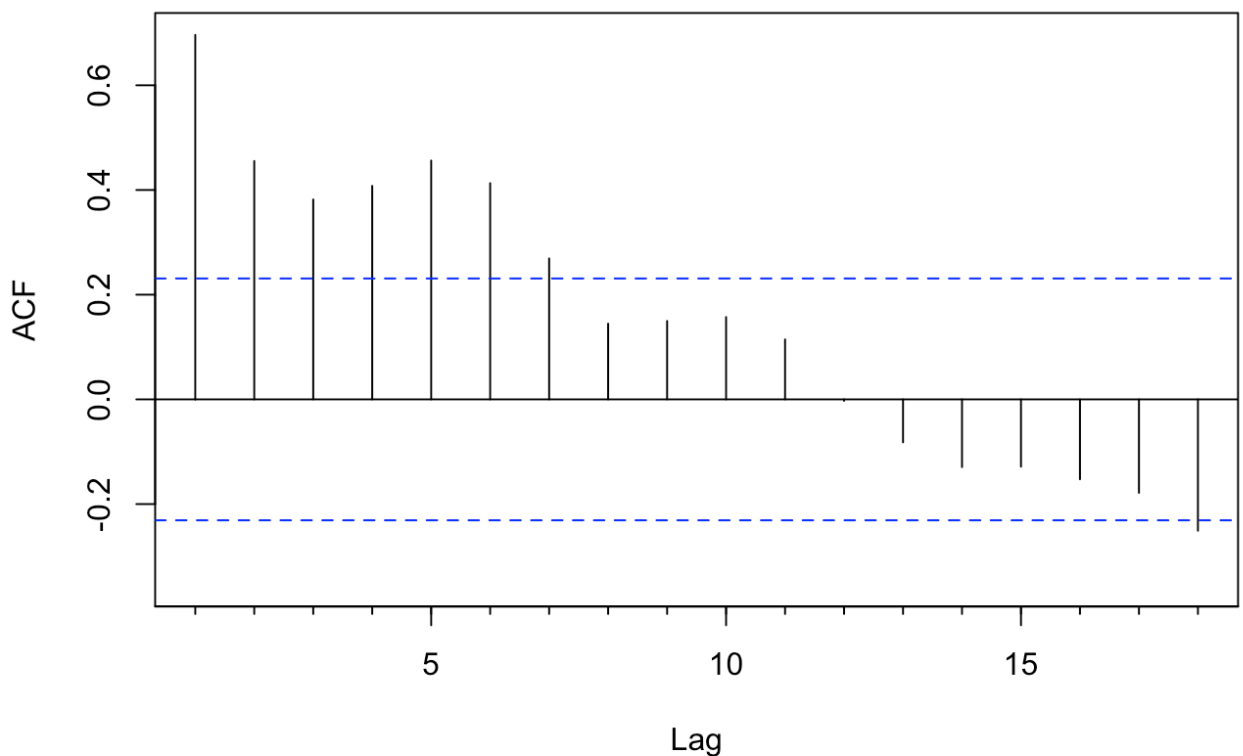
1. Plotting Realized Inflation

March Inflation at year = Y (1948 to 2019)



- Autocorrelations of inflation series:

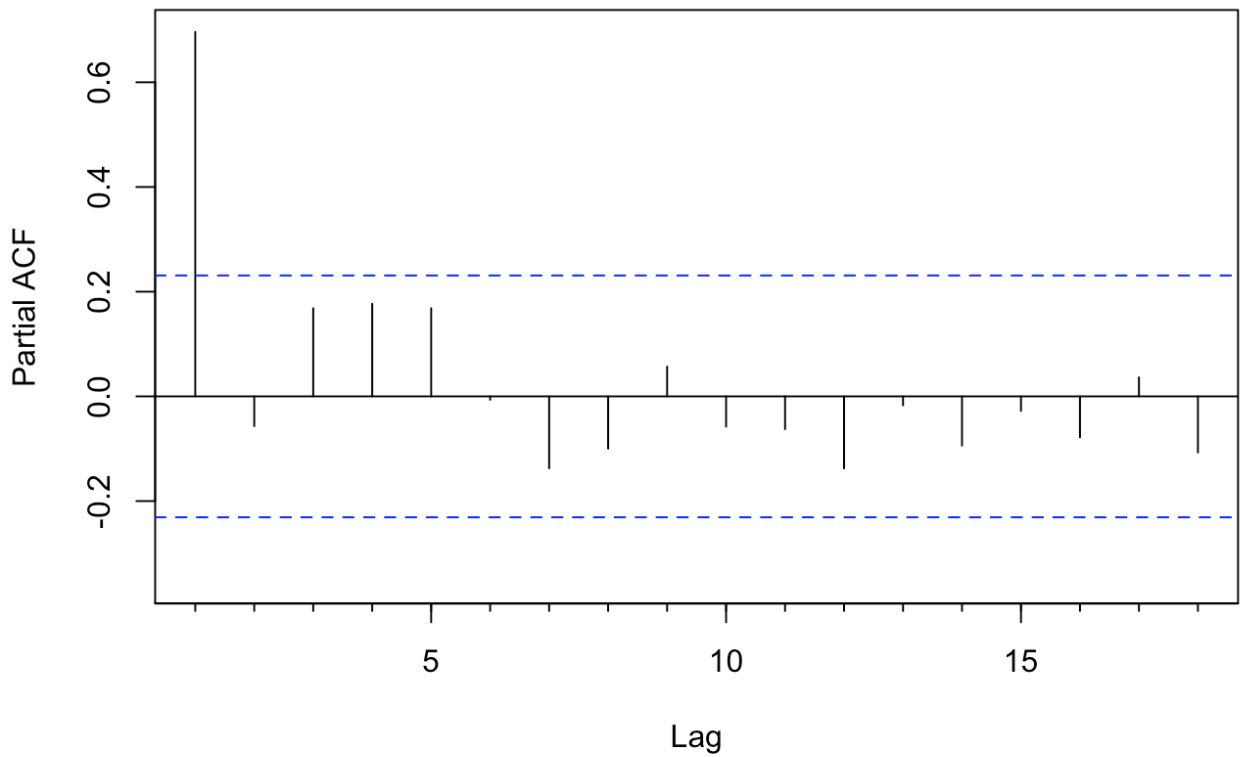
Autocorrelations of Inflation



Geometric decay of Acf suggests AR or ARMA process

- Partial Autocorrelations of inflation series:

Autocorrelations of Inflation



Large significant autocorrelation at lag 1 suggests $AR(1)$

- Dickey-Fuller Test#1 on inflation, where Null = unit root, Alternative = Stationary $AR(1)$:


```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.2861 -0.5056  0.3419  1.2105  7.0069
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1      -0.10263     0.05732  -1.791  0.0778 .
## z.diff.lag  -0.06446     0.11728  -0.550  0.5844
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.008 on 68 degrees of freedom
## Multiple R-squared:  0.05857,    Adjusted R-squared:  0.03088
## F-statistic: 2.115 on 2 and 68 DF,  p-value: 0.1285
##
##
## Value of test-statistic is: -1.7906
##
## Critical values for test statistics:
##      1pct   5pct 10pct
## tau1 -2.6 -1.95 -1.61
```

Dickey-Fuller Test#1 reveals a significant unit root at a 5% level, therefore, inflation series follows a random walk

- Model selection using Bayesian information Criterion:

```
## Series: pi.ts
## ARIMA(1,0,0) with non-zero mean
##
## Coefficients:
##          ar1      mean
##      0.6879  3.3041
## s.e.  0.0830  0.7033
##
## sigma^2 estimated as 3.785:  log likelihood=-149.39
## AIC=304.78   AICc=305.13   BIC=311.61
##
## Training set error measures:
##
##              ME      RMSE      MAE      MPE      MAPE
MASE
## Training set 0.008072358 1.918359 1.345409 250.7478 325.677 0.
9484115
##
##              ACF1
## Training set 0.04349333
```

Model selection using BIC suggests AR(1).

2. Comparing Ar(1) with Simple Exponential Smoothing (SES)

- Model accuracy:

```
## [1] "RMSE for AR(1):"
```

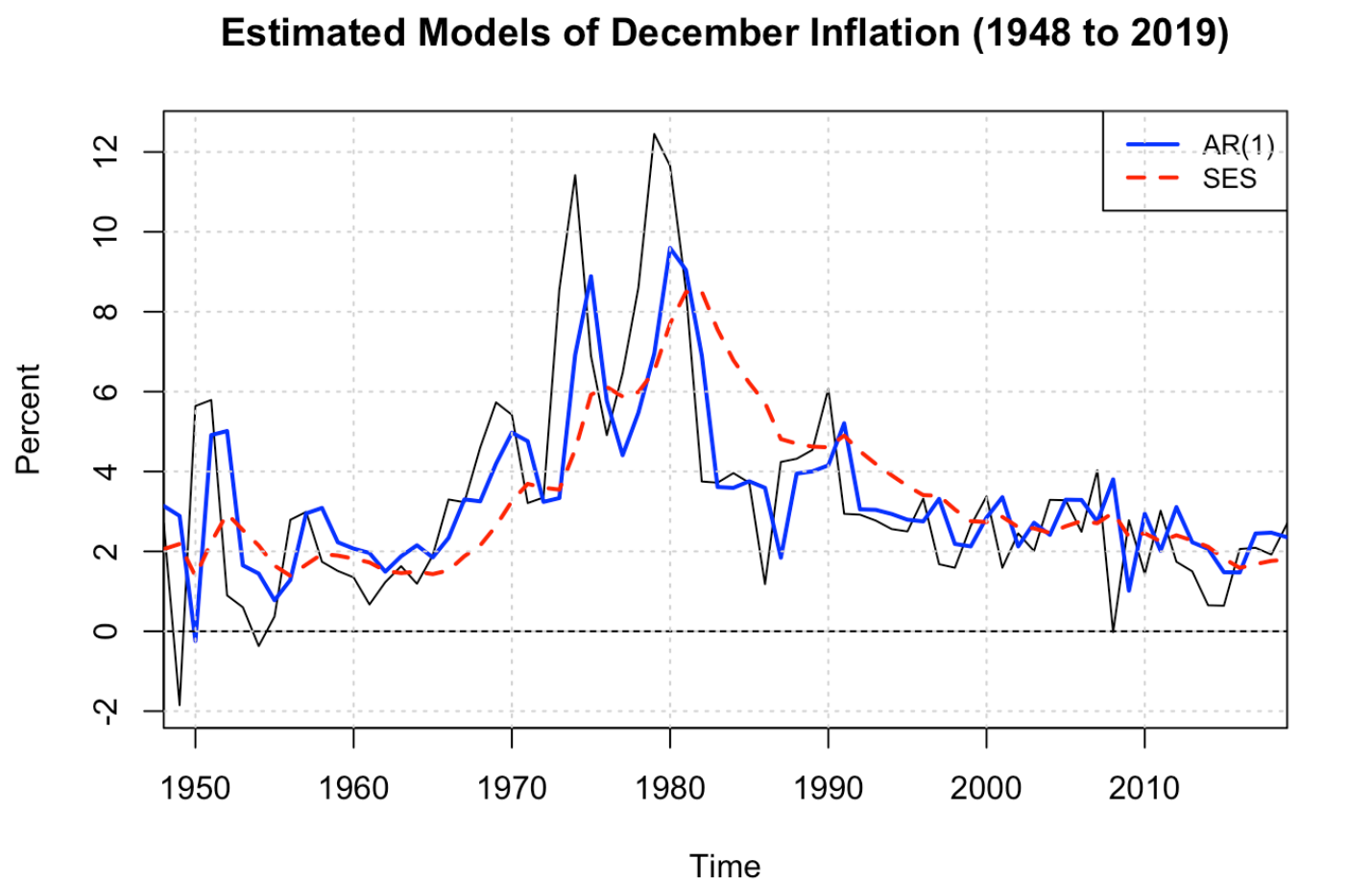
```
## [1] 1.918359
```

```
## [1] "RMSE for SES:"
```

```
## [1] 2.174499
```

```
##
## Diebold-Mariano Test
##
## data:  ar1.mod$residuals ses.mod$residuals
## DM = -1.5382, Forecast horizon = 1, Loss function power = 2, p
-value =
## 0.1285
## alternative hypothesis: two.sided
```

RMSE of AR(1) is 1.919 which is lower than for SES (RMSE= 2.174). The Diebold-Mariano test shows no significant difference in accuracy between both models at a 5% lvl (p -value for two-sided test = 0.129). No rate of alpha could change this.



Forecasting Inflation

1. 30-year-ahead Forecast with fixed partitioning (forecast horizon: 1990 to 2019)

- Accuracy of 30-year-ahead forecast of inflation:

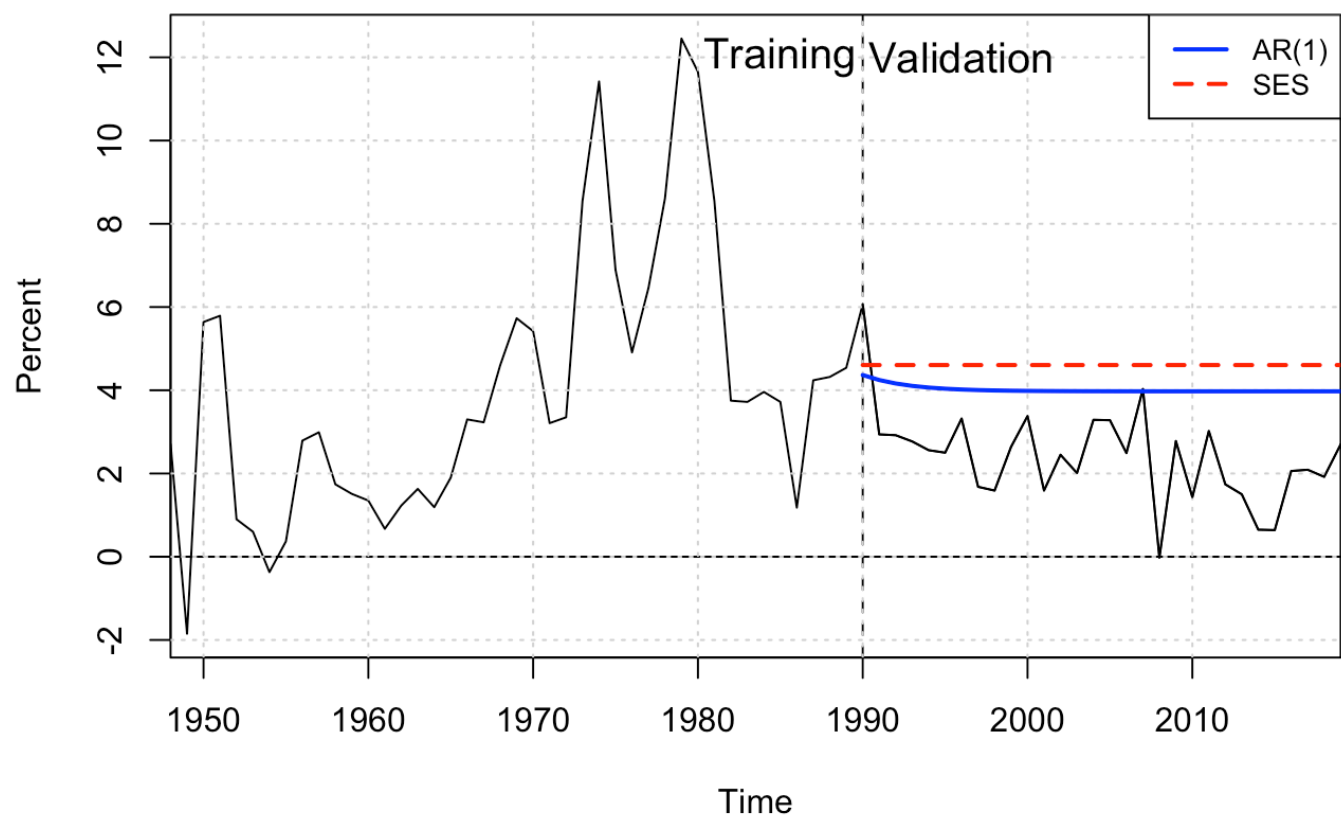
```
## [1] "RMSE of AR(1) forecast:"  
  
## [1] 1.93794  
  
## [1] "RMSE of SES forecast:"  
  
## [1] 2.472574
```

```
##
## Diebold-Mariano Test
##
## data: ar1.fixed$residuals ses.fixed$residuals
## DM = -1.7121, Forecast horizon = 1, Loss function power = 2, p
## -value =
## 0.09443
## alternative hypothesis: two.sided
```

RMSE for AR(1) is lower than for SES. D-M test reveals no significant difference in accuracy at a 5% lvl. Again, no significant value for alpha.

- Graphical representation:

30-year-ahead Fcast of Inflation using Fixed Part. (1990-2019)



2. One-year-ahead Forecasts with Roll-Forward partitioning (horizon: 1954 to 2018)

- Accuracy of one-year-ahead forecasts using roll-forward partitioning:

```
## [1] "RMSE for recursive AR(1)"
```

```
## [1] 1.945301
```

```
## [1] "RMSE for recursive SES"
```

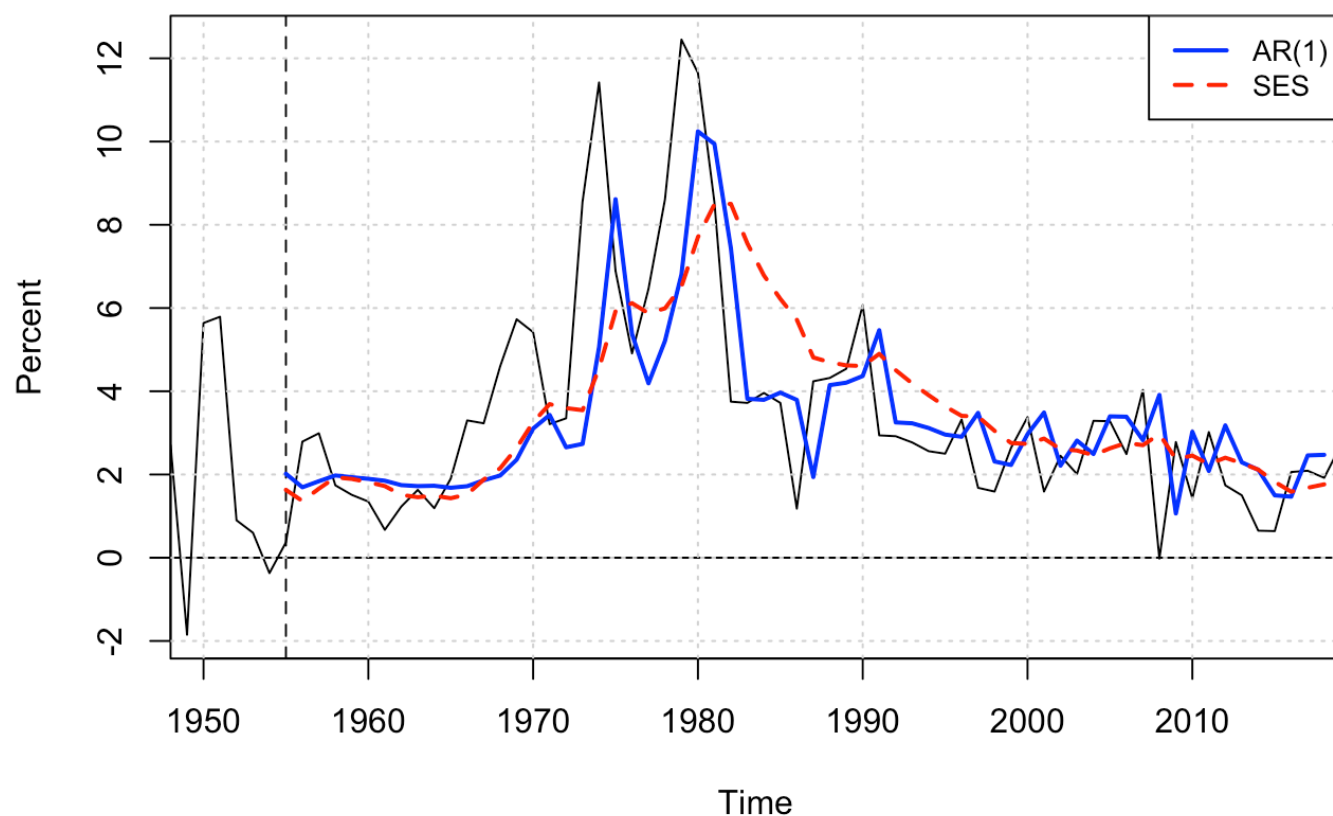
```
## [1] 2.083878
```

```
##  
## Diebold-Mariano Test  
##  
## data: valid.roll - ar1.rollvalid.roll - ses.roll  
## DM = -1.0906, Forecast horizon = 1, Loss function power = 2, p  
-value =  
## 0.2796  
## alternative hypothesis: two.sided
```

RMSE for AR(1) is lower than for SES, however, the difference is not significant at a 5% lvl. No value of alpha can change this.

- Graphical Representation:

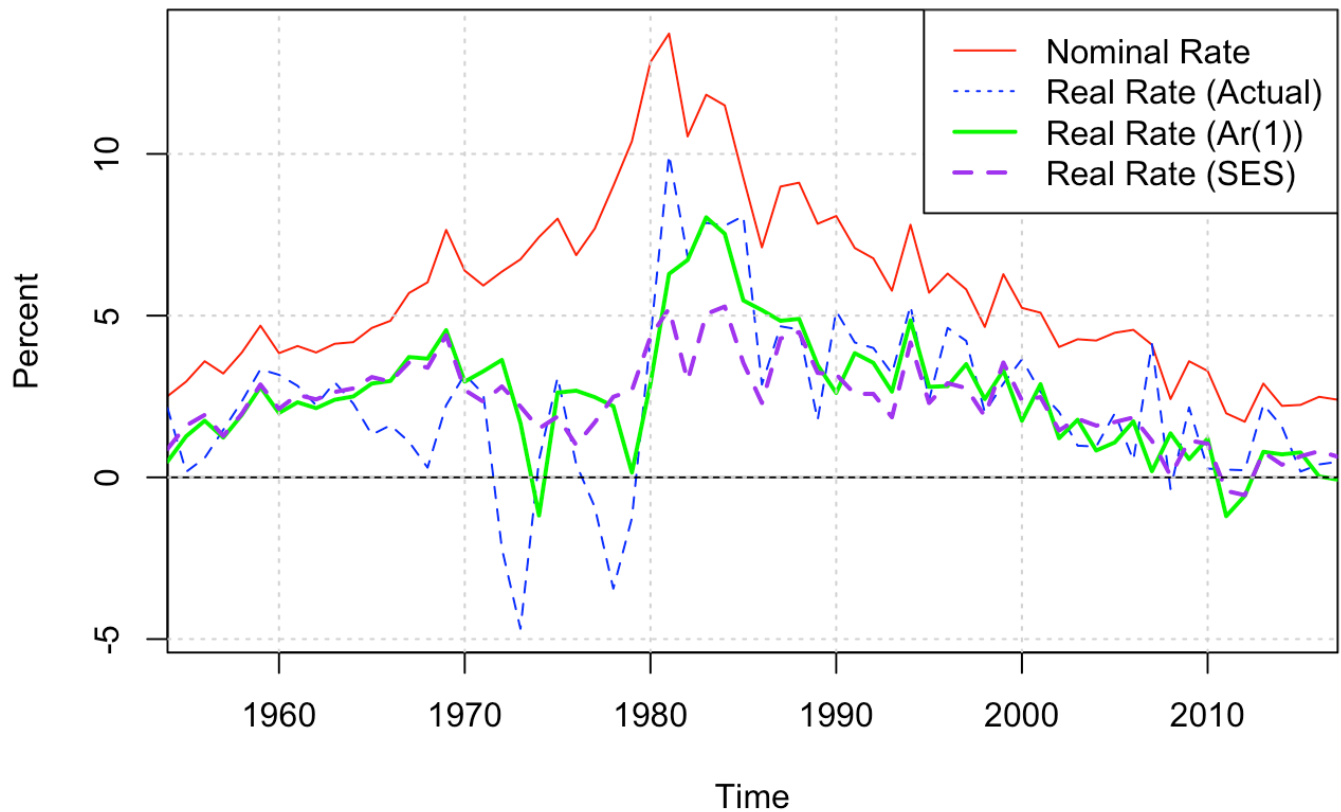
Fcasts of Inflation using Roll-Forward Part. (1955-2018)



Conclusion: There is no significant difference between AR(1) and SES long-term forecasts. This is also true for one-year-ahead forecasts.

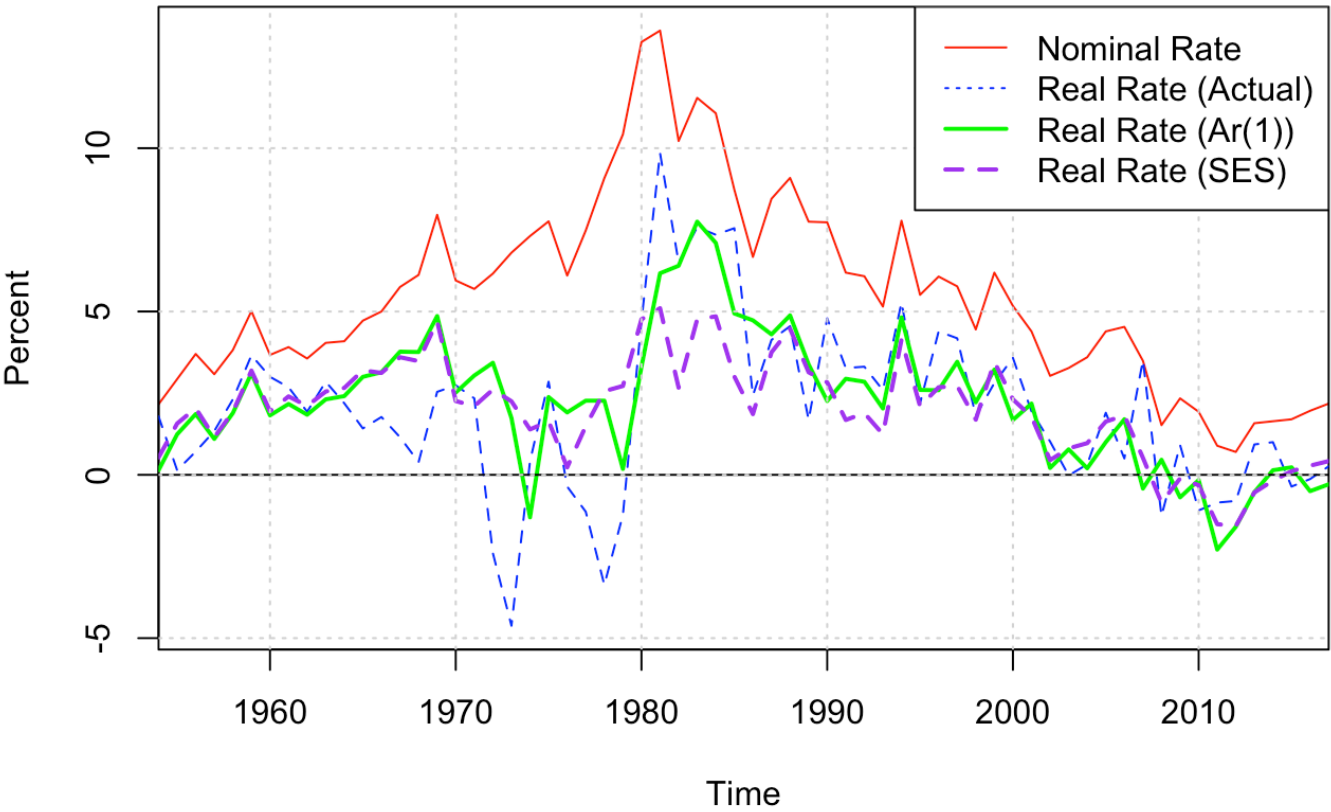
Plotting Real Interest Rates using Roll-Forward Forecasts of Inflation

December 10-Y Nominal and Real U.S. Treasury Rates (1954 to 2017)

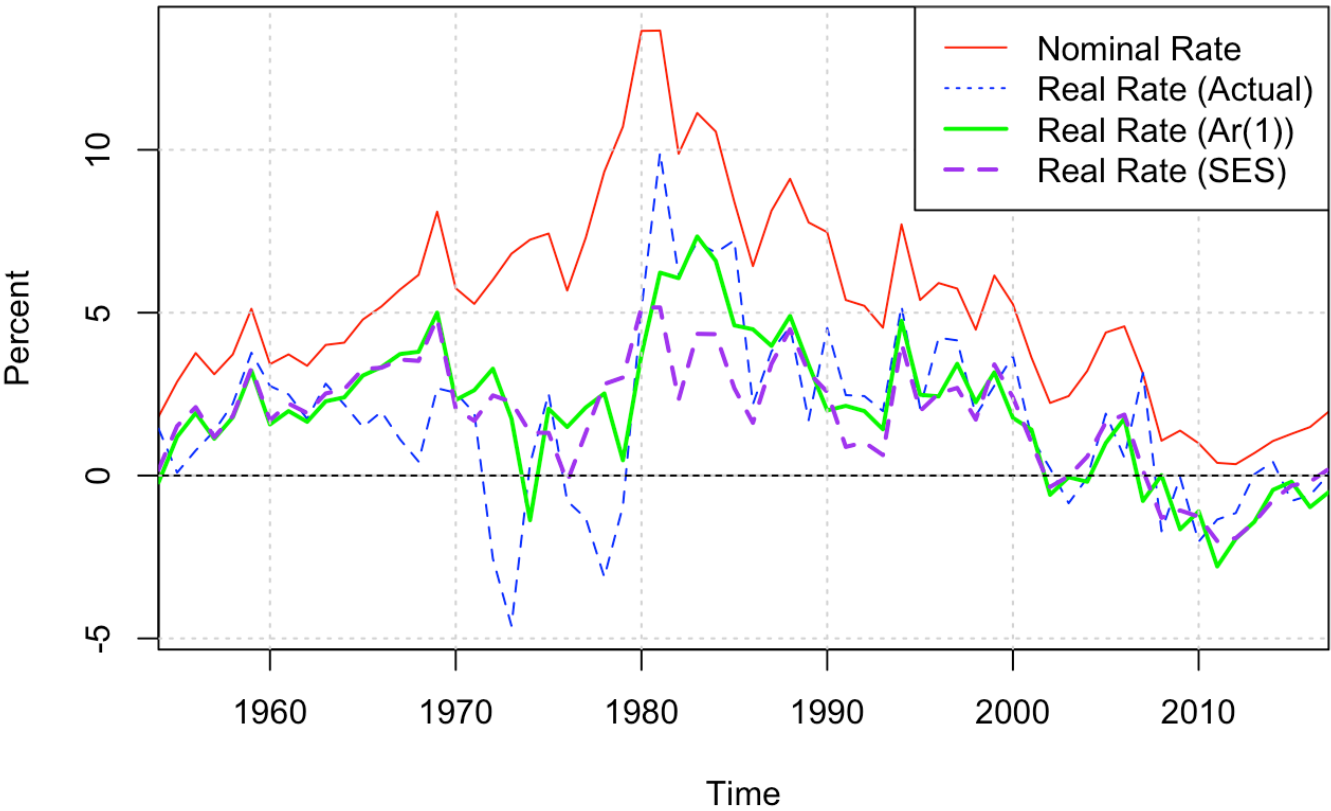


In the graph above, we can see the nominal rate plotted against the realized real rate as well as the forecasted real rates. For each forecasted real rate (AR(1) & SES), the value of r at year y is computed by subtracting forecasted inflation at year $y + 1$ from the nominal rate at year y .

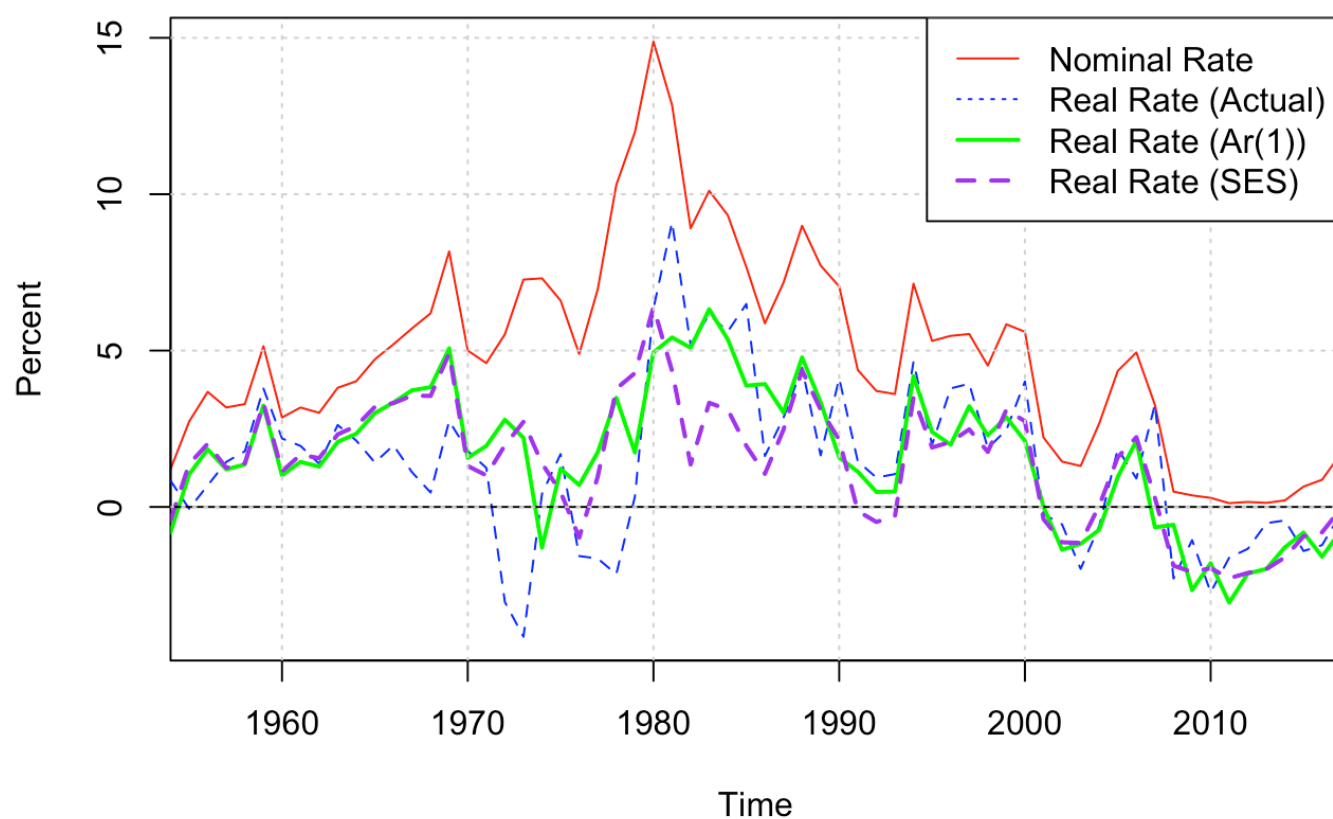
December 5-Y Nominal and Real U.S. Treasury Rates (1954 to 2017)



December 3-Y Nominal and Real U.S. Treasury Rates (1954 to 2017)



December 1-Y Nominal and Real U.S. Treasury Rates (1954 to 2017)



Sources

Forecasting Inflation: Faust & Wright, Chapter Chapter 1 in Handbook of Economic Forecasting, 2013, vol. 2, pp 2-56 from Elsevier

TS forecasting in R: Schmueli, G. C., & Lichtendahl, K. C. (2016). Practical Time Series Forecasting in R (2nd ed.). Axelrod Schnall.