Wrong-Way Risk in Funding Valuation Adjustments

Utrecht University, Rabobank

T. van der Zwaard

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Acknowledgements & Disclaimer

Joint work with Lech Grzelak and Kees Oosterlee [1].

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Outline

Goal: efficient Wrong-Way Risk (WWR) calculation for FVA.

Steps:

1. Introduce FVA and WWR;
2. Our contribution;
3. FVA equation;
4. Approximating FVA WWR;
5. Numerical results;
6. Conclusions.

FVA

- Suppose corporate C has a floating rate loan from bank I.
- To hedge IR risk, C often purchases an uncollateralized IR swap from I.
- I hedges in the interbank market, with perfect collateralization.
- I needs to fund itself in the money market at the cost of a funding spread $s_b(t)$ over $r(t)$.

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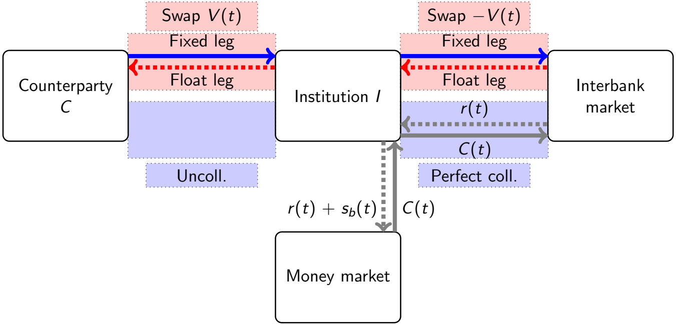

FVA

- Valuation Adjustments (xVAs): CVA, DVA, FVA, MVA, KVA. Economic value $=$ risk-neutral value $-$ xVA.
- FVA is the funding cost of eliminating market risk on non-perfectly collateralized deals.
- FVA can be split into FBA and FCA.

FVA WWR

- WWR occurs when *"exposure to a counterparty is adversely correlated with the credit quality of that counterparty"* [1].
- FVA WWR: increase in funding risk as a consequence of increased market risk.
- Adverse relationship between IR and funding spreads.

[1] Definition according to International Swaps and Derivatives Association.

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- FVA WWR: increase in funding risk as a consequence of increased market risk.
- Adverse relationship between IR and funding spreads.
- In our previous example of receiver swaps:
  - IR goes down.
  - Exposure goes up.
  - FVA goes up.
  - More negative funding spread sensitivity.
  - In addition, funding spreads will go up due to adverse relationship between IR and funding spreads.
- This happened during the March 2020 financial distress.

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- This happened during the March 2020 financial distress.
- FVA WWR difficult to hedge.

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Our contribution

- Demonstrate relevance of WWR in FVA modelling.
- Understand how various modelling choices affect FVA WWR.
- Propose efficient approximation of FVA WWR.
- Avoid simulating extra (correlated) dynamics for credit and funding spreads.

FVA equation

- Asymmetric funding spreads: $s_b(t) > 0$ and $s_l(t) = 0$, so $\mathrm{FBA}(t) = 0$, hence $\mathrm{FVA}(t) = \mathrm{FCA}(t)$.
- Choose correlated SDEs for processes $r(t)$, $\lambda_I(t)$ and $\lambda_C(t)$ $\Rightarrow \rho_{r,I}$ & $\rho_{r,C}$ & $\rho_{I,C}$.

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- Choose correlated SDEs for processes $r(t)$, $\lambda_I(t)$ and $\lambda_C(t)$ $\Rightarrow \rho_{r,I}$ & $\rho_{r,C}$ & $\rho_{I,C}$.

$$\begin{aligned}\mathrm{FVA}(t) &= \mathbb{E}\left[\int_{u=t}^{T\wedge\tau_I\wedge\tau_C} \mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\,(V(u))^+\,\mathrm{d}u \,\middle|\, \mathcal{G}(t)\right] \\ &= \int_{u=t}^{T} \mathbb{E}\left[\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v}\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\,(V(u))^+ \,\middle|\, \mathcal{F}(t)\right]\mathrm{d}u \\ &=: \int_{u=t}^{T} \mathrm{EPE}_{\mathrm{FVA}}(t;u)\mathrm{d}u,\end{aligned}$$

where we assumed conditional independence of defaults ($\rho_{I,C} = 0$) and no defaults before t.

FVA equation - exposure

Split $\text{EPE}_{\text{FVA}}(t;u)$ as follows:

$$\text{EPE}_{\text{FVA}}(t;u) = \text{EPE}^{\perp}_{\text{FVA}}(t;u) + \text{EPE}^{\text{WWR}}_{\text{FVA}}(t;u).$$

- Independent exposure $\text{EPE}^{\perp}_{\text{FVA}}(t;u)$ is taken from existing xVA engine where WWR is absent.
- WWR exposure $\text{EPE}^{\text{WWR}}_{\text{FVA}}(t;u)$ is the quantity we approximate.

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Similar split for FVA:

$$\begin{aligned}\mathsf{FVA}(t) &= \int_t^T \mathsf{EPE}^{\perp}_{\mathsf{FVA}}(t;u)\mathrm{d}u + \int_t^T \mathsf{EPE}^{\mathsf{WWR}}_{\mathsf{FVA}}(t;u)\mathrm{d}u \\ &=: \mathsf{FVA}^{\perp}(t) + \mathsf{FVA}^{\mathsf{WWR}}(t).\end{aligned}$$

FVA equation - credit adjustment effect

Including τ_I and/or τ_C in the FVA definition results in a credit adjustment effect:

$$\begin{aligned}\mathrm{FVA}(t) &= \mathbb{E}\left[\int_{u=t}^{T\wedge\tau_I\wedge\tau_C} \mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\left(V(u)\right)^+ \mathrm{d}u \,\middle|\, \mathcal{G}(t)\right] \\ &= \int_{u=t}^{T} \mathbb{E}\left[\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v} \mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\left(V(u)\right)^+ \,\middle|\, \mathcal{F}(t)\right]\mathrm{d}u.\end{aligned}$$

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In case all quantities are independent:

$$\begin{aligned}&\mathsf{EPE}_{\mathsf{FVA}}(t;u) \\ &= \mathsf{EPE}^{\perp}_{\mathsf{FVA}}(t;u) \\ &= \mathbb{E}_t\left[\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v} \mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\,(V(u))^+\right] \\ &= \mathbb{E}_t\left[\mathrm{e}^{-\int_t^u \lambda_I(v)\mathrm{d}v}\right]\cdot \mathbb{E}_t\left[\mathrm{e}^{-\int_t^u \lambda_C(v)\mathrm{d}v}\right]\cdot \mathbb{E}_t\left[s_b(u)\right]\cdot \mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\,(V(u))^+\right] \\ &= P_I(t,u)\cdot P_C(t,u)\cdot \mathbb{E}_t\left[s_b(u)\right]\cdot \mathbb{E}\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\,(V(u))^+ \,\middle|\, \mathcal{F}(t)\right].\end{aligned}$$

FVA equation - credit adjustment effect

The $\text{FVA}^{\perp}(t)$ reduction can be substantial, illustrated by a 74 basis point reduction in this example, which is approximately a 70% decrease:

	τ_I excl.	τ_I incl.
τ_C excl.	107.64	95.31
τ_C incl.	36.10	33.63

Table: $\text{FVA}^{\perp}(t)$ for the various choices of including/excluding τ_I and/or τ_C.

FVA equation - relevance of WWR

The WWR/RWR effects are non-negligible, as ratio $\frac{\mathrm{FVA}(t)}{\mathrm{FVA}^{\perp}(t)}$ is significantly different from 1 for non-zero correlations.

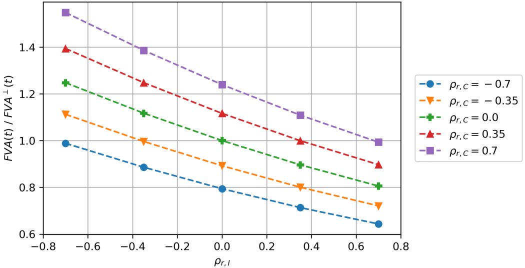

Figure: Correlation parameters effect for a receiver swap.

Model details - SDEs

Dynamics fit in the following generic setup:

$$\bar{z}(u) = x_z(u) + b_z(u),$$
$$x_z(u) = \mu_z(t,u) + y_z(t,u),$$
$$\int_t^u x_z(v)\mathrm{d}v = M_z(t,u) + Y_z(t,u),$$

where

- $\bar{z} \in \{r,\ \lambda_I,\ \lambda_C\}$ and $z \in \{r,\ I,\ C\}$.
- $b_z(u)$, $\mu_z(t,u)$ and $M_z(t,u)$ are deterministic quantities.
- $y_z(t,u)$ and $Y_z(t,u)$ are stochastic processes, with $\mathbb{E}_t\left[y_z(t,u)\right] = \mathbb{E}_t\left[Y_z(t,u)\right] = 0$.

Model details - funding spread

Credit-based funding spread, taking into account $\lambda_I(u)$ and liquidity adjustment $\ell(u)$:

$$\begin{aligned} s_b(u) &= \mathrm{LGD}_I\, \lambda_I(u) + \ell(u) \\ &= \mathrm{LGD}_I \left[x_I(u) + b_I(u)\right] + \ell(u) \\ &= \mathrm{LGD}_I \left[\mu_I(t,u) + b_I(u)\right] + \ell(u) + \mathrm{LGD}_I\, y_I(t,u) \\ &=: \mu_S(t,u) + \mathrm{LGD}_I\, y_I(t,u). \end{aligned}$$

WWR is introduced through the stochastic borrowing spread $s_b(u)$.

Model details - exposures

Now

$$
\begin{aligned}
&\mathrm{EPE}^{\perp}_{\mathrm{FVA}}(t;u)\\
&\quad = P_I(t,u)P_C(t,u)\mu_S(t,u)\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+\right]\\
&\qquad + \mathrm{LGD}_I\,\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v}y_I(t,u)\right]\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+\right],
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathrm{EPE}^{\mathrm{WWR}}_{\mathrm{FVA}}(t;u)\\
&= \mathbb{E}_t\left[\left(\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+ - \mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+\right]\right)\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v}s_b(u)\right].
\end{aligned}
$$

Model details - additional notation

- Some notation, with $\overline{z} \in \{r,\ \lambda_I,\ \lambda_C\}$ and $z \in \{r,\ I,\ C\}$:

$$\mathrm{e}^{-\int_t^u \overline{z}(v)\mathrm{d}v} = H_z(t,u)\mathrm{e}^{-Y_z(t,u)}$$
$$H_{z_1,\dots,z_n}(t,u) := H_{z_1}(t,u)\cdots H_{z_n}(t,u).$$

- Denote Taylor series expansions of e^{-x} as

$$T(x) := \sum_{j=0}^{\infty} \frac{(-x)^j}{j!},\ \ T_n^m(x) := \sum_{j=n}^{m} \frac{(-x)^j}{j!},$$

such that we can write $T(x) = T_0^n(x) + T_{n+1}^{\infty}(x)$.

Model details - FVA WWR exposure

Now apply the Taylor expansions:

$$\begin{aligned} \mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} &= H_r(t,u)T(Y_r(t,u)), \\ \mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v} &= H_{I,C}(t,u)\left[T_0^1(Y_I(t,u)+Y_C(t,u)) + T_2^\infty(Y_I(t,u)+Y_C(t,u))\right]. \end{aligned}$$

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Using the Taylor expansions and our model assumptions:

$$\begin{aligned}
&\mathrm{EPE}_{\mathrm{FVA}}^{\mathrm{WWR}}(t;u)\\
&\quad = H_{r,I,C}(t,u)\mu_S(t,u)\mathbb{E}_t\left[T_0^{n_j}(Y_r(t,u))(-Y_I(t,u)-Y_C(t,u))(V(u))^+\right]\\
&\qquad + \mathrm{LGD}_I\, H_{r,I,C}(t,u)\mathbb{E}_t\left[T_0^{n_j}(Y_r(t,u))y_I(t,u)(1-Y_I(t,u)-Y_C(t,u))(V(u))^+\right]\\
&\qquad + \mathrm{LGD}_I\, H_{I,C}(t,u)\mathbb{E}_t\left[Y_I(t,u)y_I(t,u)\right]\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}(V(u))^+\right]\\
&\qquad + \varepsilon^{\mathrm{WWR},1},
\end{aligned}$$

where $\varepsilon^{\mathrm{WWR},1}$ contains scaled truncation errors.

Model details - idea of approximation

- W.l.o.g. take $y_r(t,u)$ normally distributed (HW1F).
- IR swap payoff $V(u)$ can be written in terms of $y_r(t,u)$.
- Through a Jamshidian-like argument, $(V(u))^+$ is also expressed in terms of $y_r(t,u)$.
- Approximate $y_z(t,u)$ and $Y_z(t,u)$, $z \in \{r,\ I,\ C\}$, in terms of $y_r(t,u)$:

$$
\begin{aligned}
Y_z(t,u) &\approx \rho_{rz}\sqrt{\frac{\mathbb{V}\mathrm{ar}_t\left(Y_z(t,u)\right)}{\mathbb{V}\mathrm{ar}_t\left(y_r(t,u)\right)}}\,y_r(t,u) \\
&=: \rho_{r,z}\Sigma_t(Y_z(t,u))y_r(t,u)
\end{aligned}
$$

Model details - WWR exposure approximation

$$\gamma(t,u) := \rho_{r,I}\Sigma(y_I(t,u)),\ \ \alpha(t,u) := -\left[\rho_{r,I}\Sigma(Y_I(t,u)) + \rho_{r,C}\Sigma(Y_C(t,u))\right],$$
$$\nu(t,u) := \gamma(t,u)\alpha(t,u),\ \ \beta_j(t,u) := \frac{(-\Sigma(Y_r(t,u)))^j}{j!},$$

Approximate $\mathrm{EPE}^{\mathrm{WWR}}_{\mathrm{FVA}}(t;u)$ as follows.

$$\begin{aligned}
&\mathrm{EPE}^{\mathrm{WWR}}_{\mathrm{FVA}}(t;u)\\
&= H_{r,I,C}(t,u)\left(\mu_S(t,u)\alpha(t,u) + \mathrm{LGD}_I\,\gamma(t,u)\right)\sum_{j=0}^{n_j}\beta_j(t,u)\mathbb{E}_t\left[y_r^{j+1}(t,u)\left(V(u)\right)^+\right]\\
&\quad + \mathrm{LGD}_I\,H_{r,I,C}(t,u)\nu(t,u)\sum_{j=0}^{n_j}\beta_j(t,u)\mathbb{E}_t\left[y_r^{j+2}(t,u)\left(V(u)\right)^+\right]\\
&\quad + \mathrm{LGD}_I\,H_{I,C}(t,u)\mathbb{E}_t\left[Y_I(t,u)y_I(t,u)\right]\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+\right] + \varepsilon^{\mathrm{WWR},2}.
\end{aligned}$$

where $\varepsilon^{\mathrm{WWR},2} := \varepsilon^{\mathrm{WWR},1} + \varepsilon_{\mathrm{IV}}$ s.t. equality holds.

Recognize WWR and RWR.

Model details - WWR exposure approximation

- Until now, no assumptions have been made about product V. Write $\mathbb{E}_t\left[y_r^l(t,u)\,(V(u))^+\right]$ as a function of $y_r(t,u)$:

$$\mathbb{E}_t\left[y_r^l(t,u)\,(V(u))^+\right] = f\left(y_r(t,u)\right) + \varepsilon_{\mathrm{V}}.$$

  Product-level truncation error ε_{V} manifests itself after the application of the Gaussian approximation.
- For an IR swap under the HW1F model, $\mathbb{E}_t\left[y_r^l(t,u)\,(V(u))^+\right]$ can be written in terms of moments of a normal and truncated normal random variable.

Model details - approximation error

The approximation is a direct result of omitting overall error $\varepsilon^{\mathsf{WWR},3} := \varepsilon^{\mathsf{WWR},1} + \varepsilon_{\mathsf{IV}} + \varepsilon_{\mathsf{V}}$, where:

- $\varepsilon^{\mathsf{WWR},1}$ is a truncation error;
- $\varepsilon_{\mathsf{IV}}$ is the Gaussian approximation error;
- ε_{V} is the product-level truncation error.

Numerical results - exposure profile

Example for IR swap under HW1F for IR and CIR++ for credit processes.

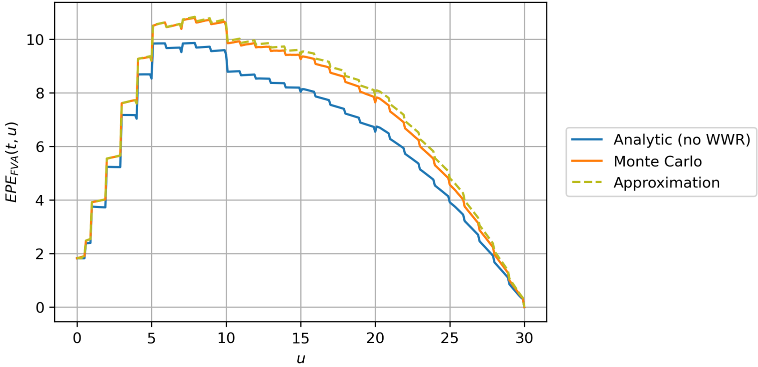

Figure: ITM receiver swap, $N = 10000$, EUR overnight yield curve, high credit rating for I, low credit rating for C, τ_I excluded, τ_C excluded.

Numerical results - WWR exposure

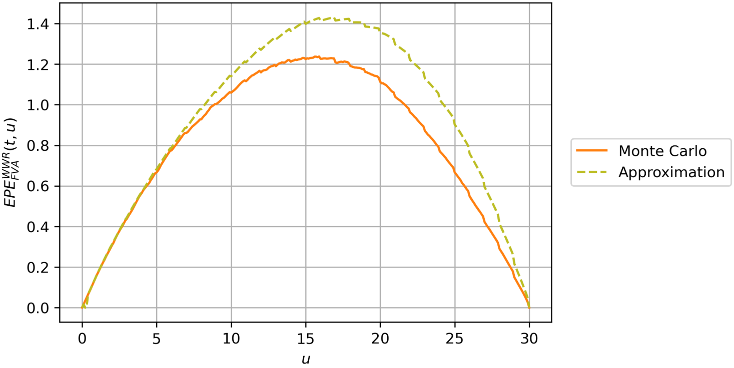

Figure: ITM receiver swap, $N = 10000$, EUR overnight yield curve, high credit rating for I, low credit rating for C, τ_I excluded, τ_C excluded.

Numerical results - FVA numbers

	FVA(t)	FVA$^{\text{WWR}}$(t)	WWR %	WWR runtime (sec)
Analytic (no WWR)	193.3481	0.0000	0.0000	0.00
Monte Carlo	217.8058	24.4577	12.6496	5.97
Approximation	221.6997	28.3516	14.6635	0.25

Table: ITM receiver swap, $N = 10000$, EUR overnight yield curve, high credit rating for I, low credit rating for C, τ_I excluded, τ_C excluded.

Conclusions

Conclusions

1. We demonstrated the relevance of WWR in FVA calculations.
2. We understand impact of various modelling choices.
3. We propose an efficient approximation:
    a. The approximation does not affect the no-WWR valuation.
    b. Build on top of existing xVA infrastructure, no extra simulation.
    c. Efficient method.
4. Example for IR swap under HW1F for IR and CIR++ for credit processes.
5. Extendable to other products and asset classes.

Wrong-Way Risk in Funding Valuation Adjustments

Utrecht University, Rabobank

T. van der Zwaard

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References I

[1] T. van der Zwaard, L. Grzelak, and C. Oosterlee.
Relevance of Wrong-Way Risk in Funding Valuation Adjustments.
arXiv Electronic Journal, May 2022.

FVA equation - relevance of WWR

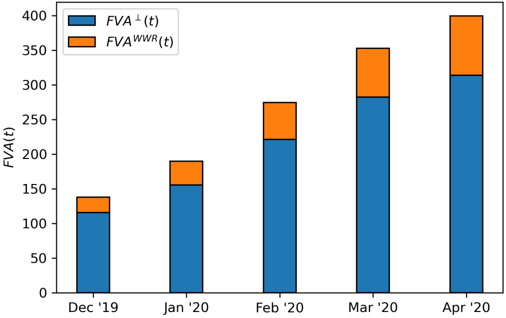

Figure: FVA through time for a receiver swap which is ATM in December 2019, with partially synthetic market data. There is a split between the independent part, $\mathrm{FVA}^{\perp}$, and the WWR part, $\mathrm{FVA}^{\mathrm{WWR}}$. Interest rates are negative, and decrease through time, such that the swap becomes ITM. The credit spreads are increasing through time.

Independence of defaults

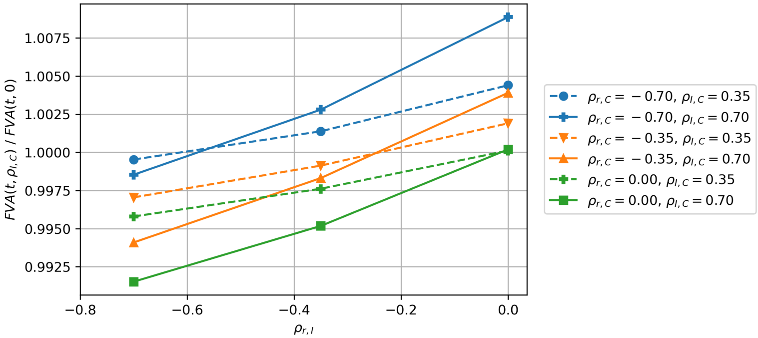

Figure: $\frac{\mathrm{FVA}(t,\rho_{I,C})}{\mathrm{FVA}(t,0)}$ for a stochastic funding spread, τ_I incl., τ_C incl.

Model details - stochastic integral approximation

$$y_r(t,u) \sim \mathcal{N}\left(0, \mathbb{V}\mathrm{ar}_t\left(y_r(t,u)\right)\right) \text{ and } Y_r(t,u) \sim \mathcal{N}\left(0, \mathbb{V}\mathrm{ar}_t\left(Y_r(t,u)\right)\right).$$

$$\begin{aligned} Y_r(t,u) &\stackrel{\mathrm{d}}{=} \sqrt{\frac{\mathbb{V}\mathrm{ar}_t\left(Y_r(t,u)\right)}{\mathbb{V}\mathrm{ar}_t\left(y_r(t,u)\right)}}\, y_r(t,u) \\ &=: \Sigma(Y_r(t,u))y_r(t,u). \end{aligned}$$

Approximate $Y_z(t,u)$, $z \in \{I,\ C\}$, by a normally distributed variable and write in terms of $y_r(t,u)$.

$$\begin{aligned} Y_z(t,u) &\approx \rho_{rz}\sqrt{\frac{\mathbb{V}\mathrm{ar}_t\left(Y_z(t,u)\right)}{\mathbb{V}\mathrm{ar}_t\left(y_r(t,u)\right)}}\, y_r(t,u) \\ &=: \rho_{r,z}\Sigma_t(Y_z(t,u))y_r(t,u) \end{aligned}$$

because the normal increments are correlated, i.e., $\mathrm{d}W_r \mathrm{d}W_z = \rho_{rz}\mathrm{d}t$.

Numerical results - parameters

- IR: $x_r(0) = 0.0$, $a_r = 1e-05$, $\sigma_r = 0.00284$, $P_r^{\text{mkt}}(0,T) =$ EUR1D, ATM $\sigma_{\text{imp},r} = 0.10$;
- Credit for I: $x_I(0) = 0.0016939$, $a_I = 0.05$, $\theta_I = 0.015390$, $\sigma_I = 0.02$, $\text{LGD}_I = 0.6$, $P_I^{\text{mkt}}(0,T) =$ AAA credit curve;
- Credit for C: $x_C(0) = 0.0063774$, $a_C = 0.2$, $\theta_C = 0.035447$, $\sigma_C = 0.08$, $\text{LGD}_C = 0.6049$, $P_C^{\text{mkt}}(0,T) =$ BBB credit curve;
- Correlation: $\rho_{r,I} = -0.35$, $\rho_{r,C} = -0.5$, $\rho_{I,C} = 0.0$;
- Grid: 10^5 paths, 10 dates per year, $t_0 = 0.0$, $T = 30.0$;
- Swap: forward starting payer swap, $T_{\text{exp}} = 1.0$, $T_{\text{mat}} = 30.0$, $\alpha = 1.0$, $m = 0.0$, $N = 10000$, swap rate is the par rate at $t = 0$;

Numerical results - market data

t	$\mathrm{DF}(t)$	$\mathrm{ZC}(t)$
0.00	1.000000	0.000000
0.25	1.001187	-0.004744
0.50	1.002448	-0.004891
0.75	1.003773	-0.005021
1.00	1.005158	-0.005145
1.50	1.008088	-0.005370
2.00	1.011132	-0.005535
2.50	1.014134	-0.005614
3.00	1.016990	-0.005616
4.00	1.022401	-0.005538

t	$\mathrm{DF}(t)$	$\mathrm{ZC}(t)$
5.00	1.026945	-0.005318
6.00	1.030583	-0.005021
7.00	1.033099	-0.004652
8.00	1.034654	-0.004258
9.00	1.035117	-0.003835
10.00	1.034622	-0.003404
12.00	1.031876	-0.002615
15.00	1.025681	-0.001690
20.00	1.021923	-0.001084
25.00	1.032268	-0.001270
30.00	1.053926	-0.001751

Table: Yield curve.

Numerical results - market data

t	DF(t)	ZC(t)
0.00	1.000000	0.000000
0.50	0.998984	0.002034
1.00	0.997659	0.002343
2.00	0.993528	0.003247
3.00	0.987626	0.004151
4.00	0.979424	0.005198
5.00	0.969391	0.006217
7.00	0.946630	0.007835
10.00	0.912382	0.009170
15.00	0.861670	0.009926
20.00	0.813199	0.010339
30.00	0.721512	0.010880

(a) Credit curve I.

t	DF(t)	ZC(t)
0.00	1.000000	0.000000
0.50	0.994676	0.010677
1.00	0.988348	0.011720
2.00	0.970999	0.014715
3.00	0.948562	0.017603
4.00	0.920897	0.020602
5.00	0.888371	0.023673
7.00	0.828067	0.026952
10.00	0.745380	0.029386
15.00	0.632957	0.030490
20.00	0.537460	0.031045
30.00	0.386538	0.031684

(b) Credit curve C.

Table: Credit curves.