Task 4 of the Magic Carpet Ride Task Sequence (Back to Top)

Handout 4: Linear Independence and Dependence: Creating Examples

Examples of Student Justification for Various Generalizations

A whole class discussion about "any set containing the zero vector is linearly dependent."

Instructor: I heard the table in the back say they were confident [that $\left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}1\\3\end{bmatrix},\begin{bmatrix}0\\0\end{bmatrix}\right\}$ is linearly independent]. Can I have a volunteer from your table to come up and tell us why you're confident?
Gabe: Basically, this breaks out a problem of number theory. If you believe zero is a number, you go with us. That's my campaign.
Justin: [Raises hand] Can I rebuttal?
Instructor: I don't understand the original one yet. Can somebody tell me what you said there?
Gabe: Zero vector, if <0, 0> can be a vector, ours is correct.
Nate: Yeah, but at the same time, if you say <0, 0> is a vector, by the linearly dependent definition, we say that you can use it if one of them, if one constant is not equal to zero, so we set c_1 and c_2 as zeroes and the c_3 is set to anything, and it's still a linearly dependent set.
Gabe: So the definition says that they all have to be zero.
Justin: And the last one can be any number you want.
Robert: Look, if you have a zero vector, then every, every set of vectors will be dependent.
Justin: Exactly!
Robert: If you put any coefficient in front of that <0, 0>, and it would be not all equal to zero.
Abraham: Including 2,564 or whatever that is up there.
...
Robert: If you put, if you want to say that all of them can be zero to get linear independence, then if you put any coefficient in front of that <0, 0> and say it's okay to have any coefficient in front of that, then it's not ever going to be linearly independent. Because you won't have a zero c in front of that zero vector.
Justin: Can I say that again?
Instructor: You can go ahead, you can say it again.
Justin: So what he's saying is, for it to be linearly independent, all the c's must be zero, they have to be zero. So if we chose zero for the first two, that's great, fine. But for the zero vector, we can chose any number, 8,462 if we want, and it's still going to be a solution that's zero.

Justin's justification for "Any set of vectors from $\mathbf{R}^n$ containing more than n vectors is linearly dependent":

> "So if you start in any $\mathbf{R}^n$, and you just start with 1 vector and keep adding more. So let's do $\mathbf{R}^3$, just for an example. So we start with 1 vector. So either, we have 2 choices: The next vector we add can either be on the same line, which means it's already linearly dependent, so we don't want that, so we're going to put it off somewhere else. Now the span of that is a plane in 3 dimensions. So now we're going to add another vector in. Our 3rd vector, now it can either be in that span or out of that span. And we want it to be linearly independent, so we're going to put it out of that span. But now that we have that going off of that plane, we just extended our span to all of $\mathbf{R}^3$. So our 4th vector, when we put it in, no matter where we put it, it's going to get us back home. Because just like in this case, we have to have the last one to get back home, we can get anywhere with those 1st 3 that we put in, but we have to have to have that 4th one to come back. And so it works like that in any dimension, because the more you, if you keep adding, eventually you're going to get the span of your dimensions, and then you're going to have that extra one bringing you back. Unless you have 2 vectors that are lying on the same line, then you won't have the span of all of your dimension, but it's negligible because those 2 will give you a linearly dependent set. Does that make sense?"