BU Pricing Challenge

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Overview/Abstract

Our valuation model is a Monte Carlo pricing method that can be simplified into three steps: generating Monte Carlo simulations to map the possible price paths of each underlying, calculating total payouts depending on price paths taken, and averaging those total payouts to derive a fair value. From our model, we calculated a fair value of $19,132.99.

Method

Monte Carlo Simulation

We built a stochastic model, in this case a Monte Carlo method, that traces possible paths an underlying may take up until its Final Redemption date. The process includes taking an underlying's price and adding a random shock calculated from historical daily volatility to simulate the next day's price. We used our function, `monteCarlo`, to build 1000 simulations for each of the three underlyings from 3/16/2020 to 1/17/2023. Additionally, we added market calendars to each underlying simulation for their respective markets to account for trading holidays.

To calculate the volatility of an underlying, we calculated the standard deviation of the daily percentage. Once the volatility was calculated we take a random walk for each trading day in the simulation. In other words, we simulate the daily price of each underlying through the following equation:
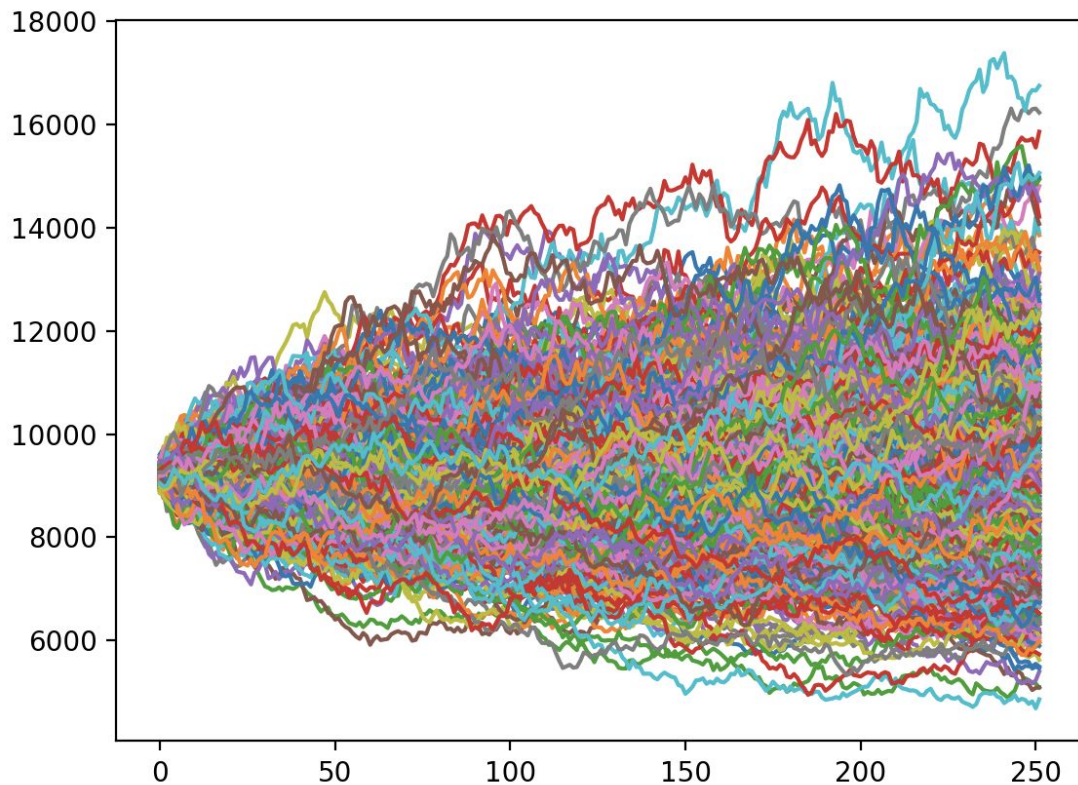
$$P_d = P_{d-1} * (1 + N(0, v^2))$$

Where:

P_d represents the publicly traded price of the underlying ticker from d days of the inception of the ELN.

$N(\mu, \sigma^2)$ represents a uniformly randomly sampled normal distribution.

v represents the volitility.

Once we model generated our series of Monte Carlo simulations, we proceeded to identify the payouts generated for each simulation.

Example of NDX Monte Carlo Simulations
(X axis is price from inception)
(Y axis is Price in $)

Calculating Total Payouts

        We calculated total payouts for each of the three conditions:

1. Early Redemption
   - Equal to 100% par + sum of payout amounts paid on payout dates before the Trigger Redemption date
2. Final Redemption where each underlying is at or above its Strike
   - Equal to 100% par + sum of payout amounts paid on payout dates before Final Redemption Date
3. Final Redemption where the Final Level of at least one underlying is below its Strike
   - Equal to Par × Final Level/Worst Strike Worst + sum of payout amounts paid on payout dates before Final Redemption Date

Payout Amounts were calculated by Par × (6.8%) × n/N. 'n' is equal to the number of days between a payout period where all underlyings were above their Payout Thresholds. 'N' is equal to the total number of payout observation dates during the relevant observation period.

Because market calendars differed, dates that had holidays for one underlying but not for another did not contribute to the count of 'n' or 'N'. For example, 4/5/2021, which is the Qingming Festival market holiday for the HKEX, did not contribute to the count of 'n' or 'N' n/N from 1/7/2020 to 3/16/2020, dates between the issue date and the pricing date, was also found to be 46/47 (not including market holidays) which was factored into the payout amount for the first payout period.

Our function, `payoutPath`, calculates the sum of payout amounts for every payout period up until its inputted Trigger Redemption date or Final Redemption date. If there is an Early Redemption, `payoutPath` adds the sum of payout amounts to 100% par. If the simulation path ends in a Final Redemption, the sum of payout amounts is added to the Final Redemption amount calculated in `getFinalRedemption`.
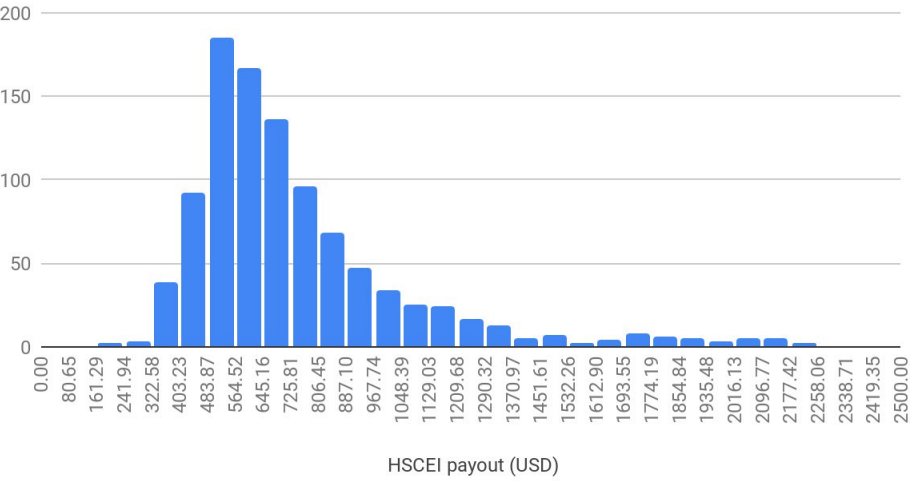
Averaging Total Payouts
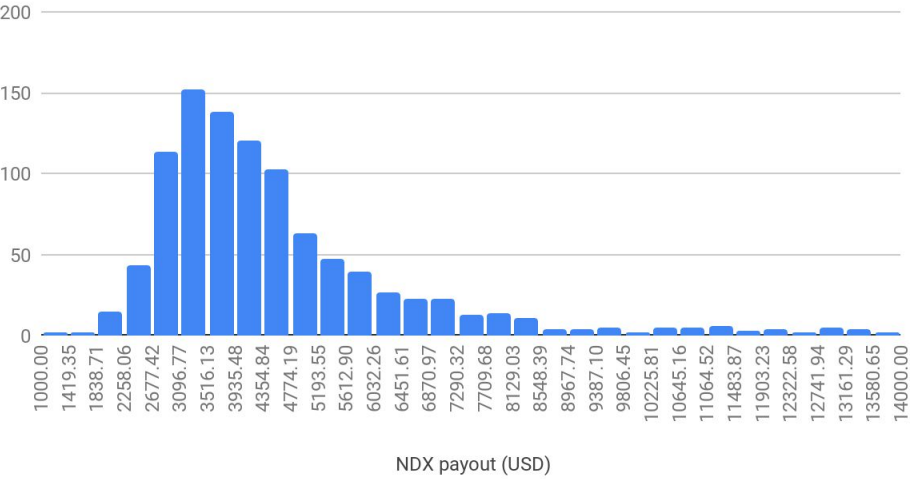We average the total payouts and convert to USD to return a fair value.
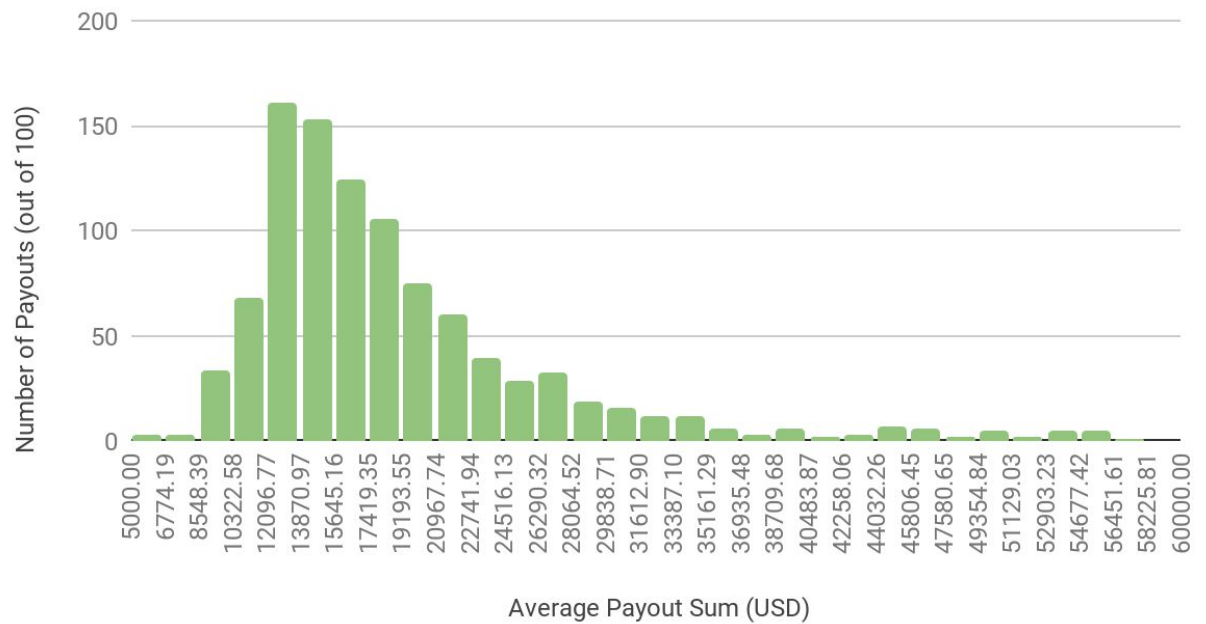
Results

Histogram of FTSEMIB payout (USD)
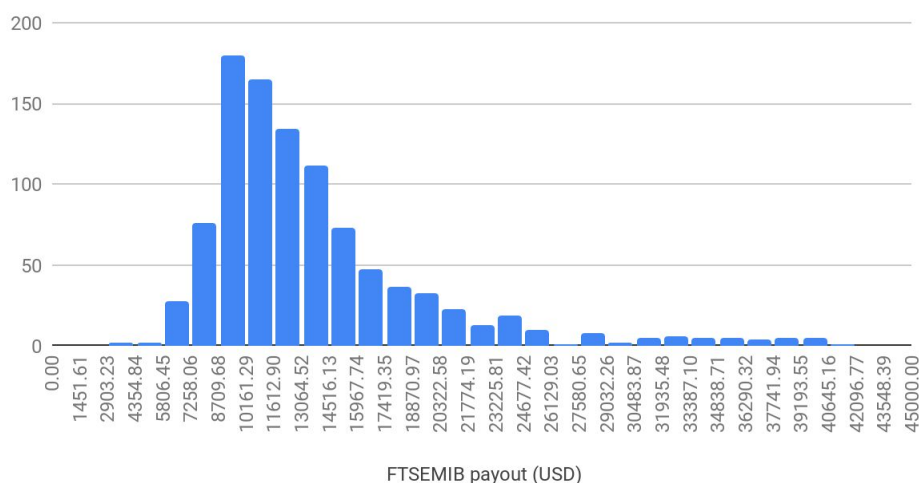
Histogram of HSCEI payout (USD)

HSCEI payout (USD)

Histogram of NDX payout (USD)


NDX payout (USD)

Histogram of Average Payout Sum (USD)

Final Comments and Assumptions

In a simulation of 10,000 paths, the number of triggers was 127. 1.27% trigger rate suggests that simulations almost entirely ran their full course. The drop in the stock market around 3/16/2020 brought these indexes to prices near or, in the case of FTSEMIB, below their Strike levels on the valuation date. With the FTSE MIB Index beginning at below its Payout Threshold on 3/16/2020, few runs saw consistent growth needed to return to 97% of their initial value needed to trigger.

Simulation 368
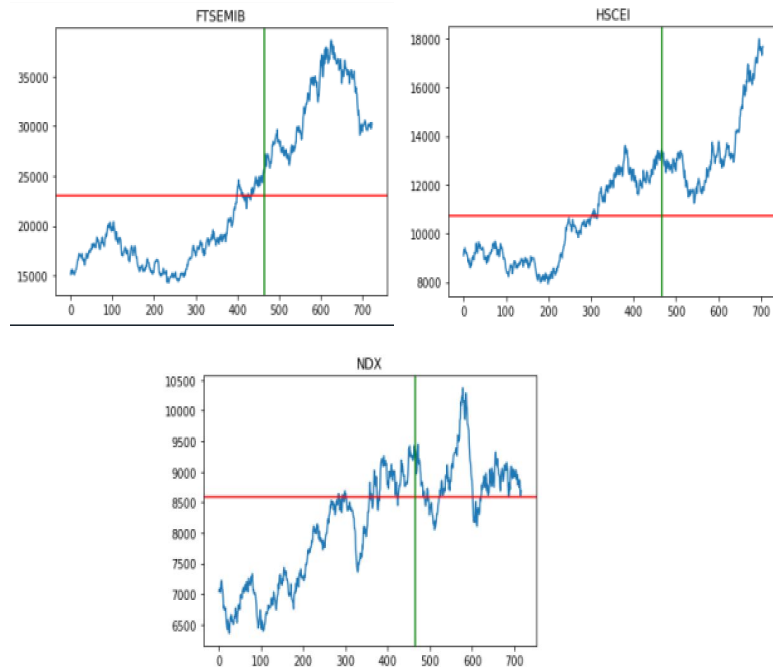
Green Line: Date Triggered   Red Line: Trigger Level
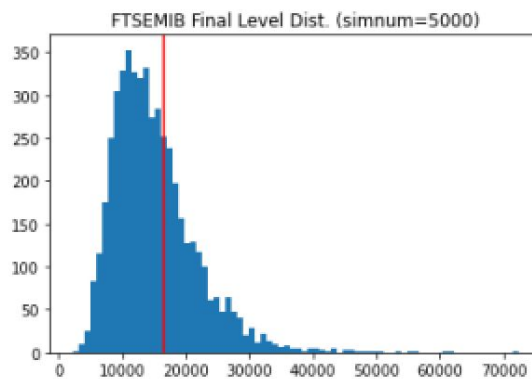
Shown in the above histogram, the final level of FTSEMIB was frequently below its strike level. With a median of 13774.95, well below its Strike level of 16,606.366, the Final Redemption value of this note was often only a fraction of its par.

The following assumptions were made:
- Assumption of a non-cyclical market
- Assumption that each underlying had no correlation and no covariance to the others
- Assumption that there is no currency risk and that opening exchange rates on 1/7/2020 are consistent throughout the period.
- Assumption that all future payout values are not discounted back to their present value (payouts in the future are worth as much as payouts now)