Preferential Attachment

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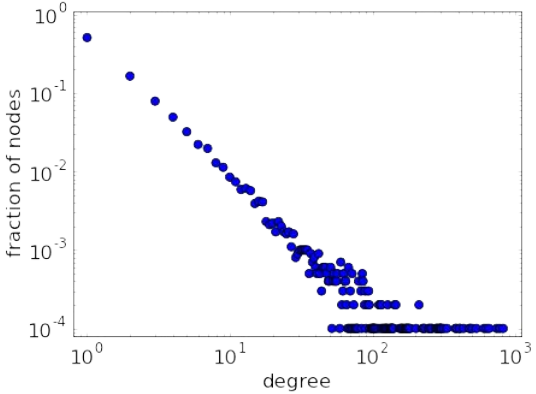
Background - Scale-Free Networks

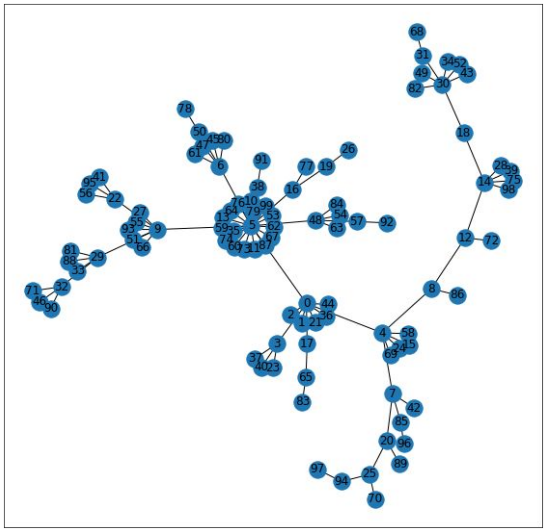
Power law degree distribution: $Pr(k) \sim k^{-\alpha}$

Typically α falls between 2 and 3

Some examples:

- Protein-Protein Interactions
- Citations
- World Wide Web
- Wealth Distribution



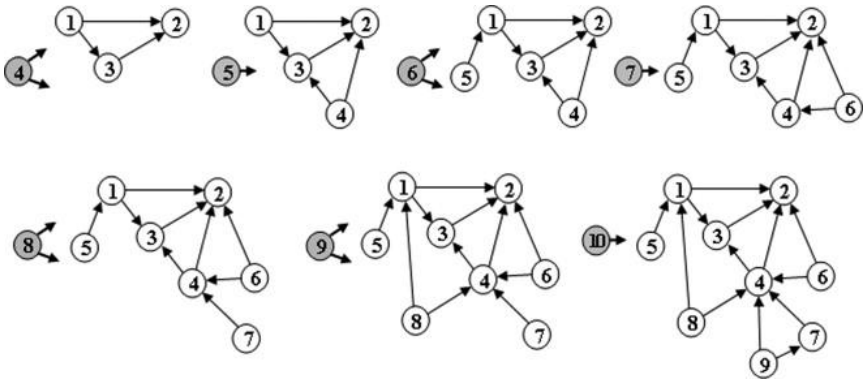


Background - Preferential Attachment

New nodes are more likely to attach to nodes with a high degree.

Examples:

- Wealth distribution
- Citation networks



It is well believed within the network science community that preferential attachment is the cause of scale-free networks.

“Emergence of Scaling in Random Networks” (Barabási, Albert 1999)

Question

Are there other attachment policies that create networks similar to preferential attachment networks, thus demonstrating other possible ways for scale-free networks to emerge?

Attachment policies

Preferential:

- Price Model - Attachment probability proportional to in-degree with some uniform random selection (Directed)
- Barabási-Albert - Attachment probability proportional to degree (Undirected)

Uniform: Equal probability to attach to all nodes

Temporal: More likely to attach to older nodes

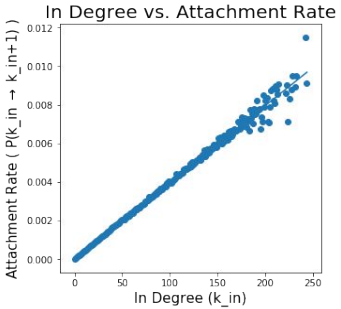
Mean Geodesic Distance:
More likely to attach to nodes with a large mean geodesic distance to other nodes

Inverse Mean Geodesic Distance:
More likely to attach to nodes with a small mean geodesic distance to other nodes

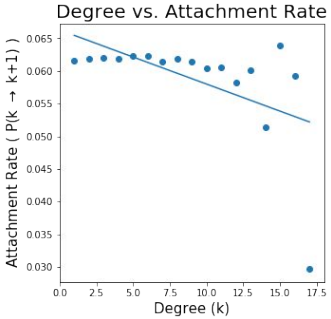
Methods

1. Create graphs using the various attachment policies
2. Collect data on each graph
    a. Node degree (k) vs Attachment rate ($Pr(k \rightarrow k+1)$)
    b. Degree Distribution (k vs. Pr(k))
    c. CCDF (k vs. $Pr(K \geq k)$)
3. Repeat steps 1,2 many (10,000) times for each graph
4. Determine if any attachment policies create graphs and data that mimic preferential attachment or scale-free networks

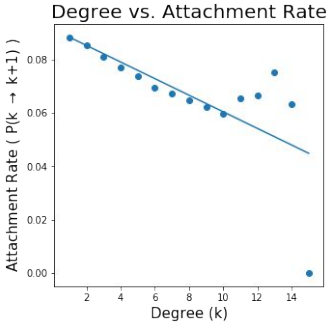
Results: Attachment Rate vs. Degree



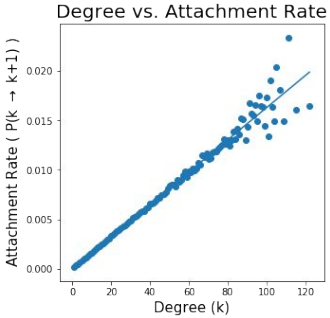
Price Model



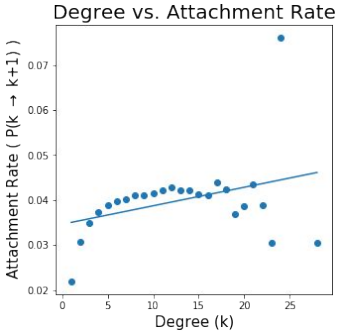
Uniform Attachment



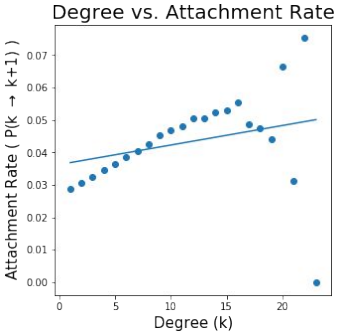
Mean Geodesic Distance



Barabasi-Albert Model

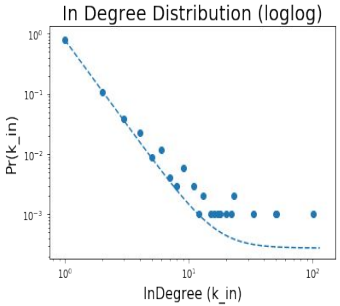


Temporal



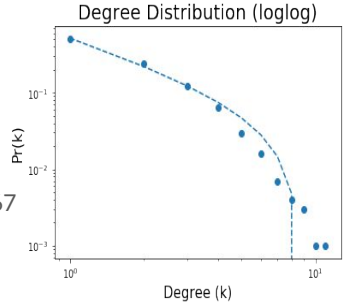
Inverse Mean Geodesic Distance

Results: Degree Distribution



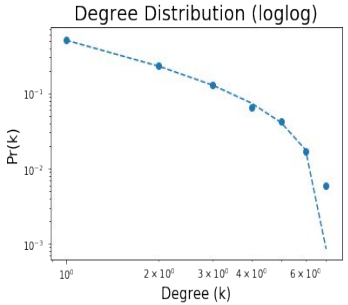
Price Model

$Pr(k\_in) \sim k\_in^{-2.67}$



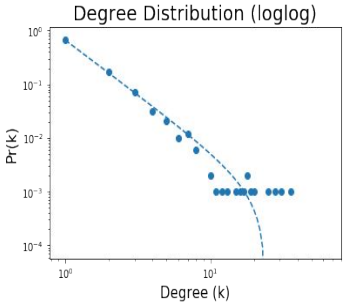
Uniform Attachment

$Pr(k) \sim k^{-0.73}$



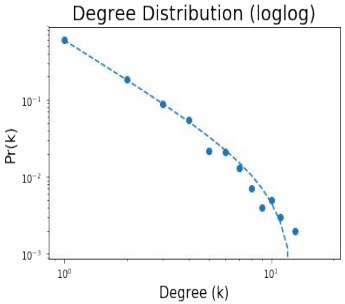
Mean Geodesic Distance

$Pr(k) \sim k^{-0.84}$



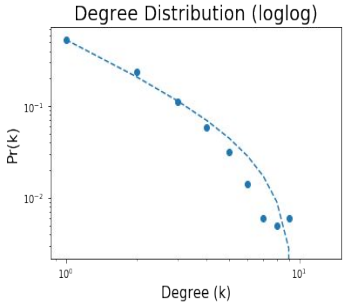
Barabasi-Albert Model

$Pr(k) \sim k^{-2.08}$



Temporal

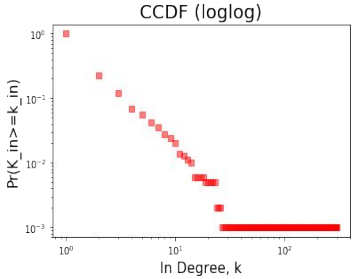
$Pr(k) \sim k^{-1.66}$



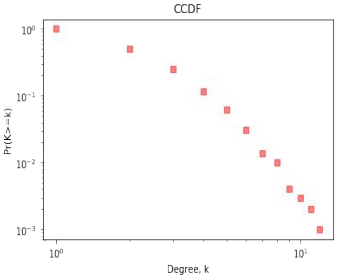
Inverse Mean Geodesic Distance

$Pr(k) \sim k^{-1.20}$

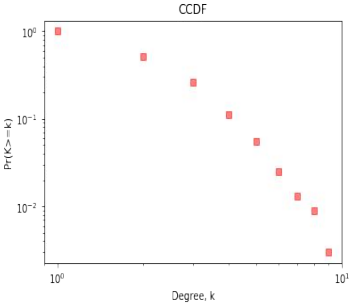
Results: CCDF



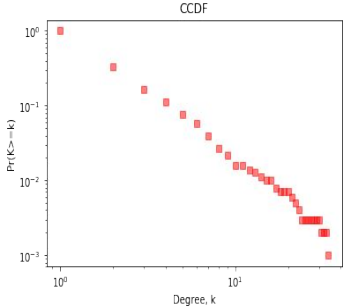
Price Model



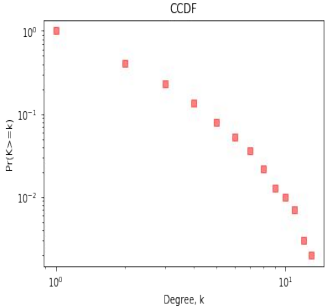
Uniform Attachment



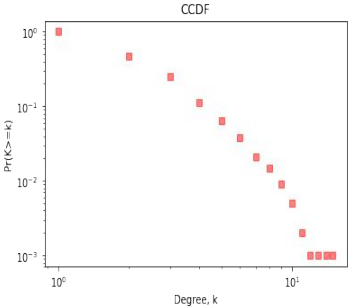
Mean Geodesic Distance



Barabasi-Albert Model



Time



Inverse Mean Geodesic Distance

Discussion

Power law degree distribution: $Pr(k) \sim k^{-\alpha}$

Current attachment policies do not imitate preferential attachment

- Inverse mean geodesic distance policy seemed similar
- **None** of the attachment policies created graphs with an α between 2 and 3

Applications of findings

- Ability to create accurate null models describing scale-free network formation
- Prediction of scale-free network growth

Future Plans

Test more models:

- Connect new node to multiple nodes when added
  - Local clustering coefficient
  - Triadic closure
- Find more network attributes which can be used in attachment policies
- Create more accurate best fit curve for degree distribution