I just copy all my code and result of problem3 into PDF version, if there is any doubt, please see all my ipynb files here: https://github.com/thomasyangrenqin/Home-work2-problem-3--Renqin-Yang.git

a)

```
import numpy as np
import matplotlib.pyplot as plt

def hh(I, dt):

##############################################################################
    # Simulate the membrane potential of a Hodgkin Huxley neuron with
    #   a given input current
    # Input:
    #    I = current in uA/mm^2a
    #    dt = time step between I measurments [ms]
    #
    # Output:
    #    Vm = membrane voltage in mV
    #    n = sodium activation
    #    m = potassium activation
    #    h = 1 - potassium inactivation

    # This function simulates a dynamical system with state variables
    #    DV = 1/C (I - Ik - Ina - Il)
    #    Il = gl(V-El)
    #    Ik = gk*n^4(V-Ek)
```

```
    #    Ina = gna*m^3*h(V-Ena)
    #    Dn = (ninf(V) - n)/taun(V) (same for m, h )

    # Summary of units: I = uA / mm^2; V = mV; g = mS; 
g*V = uA; C = uF; uA/uC*ms = mV


########################################################
###################
    # Constants:
    # Reversal potentials for various ions
    Ek = -77 #[mV]
    Ena = 50 #[mV]
    El = -54.402 #[mV]

    # Membrane capacitance:
    C = 0.01 #[uF/mm^2]

    # Maximum conductances [mS/mm^2]
    gna = 1.2
    gk = 0.36
    gl = 0.003


########################################################
###################
    # Gating variables:
    # activation K [n]
    alpha_n = lambda V: 0.01*(V + 55) / (1 - 
np.exp(-0.1*(V + 55)))
    beta_n = lambda V: 0.125 * np.exp(-0.0125*(V + 65))
    tau_n = lambda V: 1 /(alpha_n(V) + beta_n(V))
    n_inf = lambda V: alpha_n(V) * tau_n(V)
    # activation Na [m]
    alpha_m = lambda V: 0.1*(V + 40) / (1 - 
np.exp(-0.1*(V + 40)))
    beta_m = lambda V: 4 * np.exp(-0.0556*(V + 65))
    tau_m = lambda V: 1/(alpha_m(V) + beta_m(V))
```

```
    m_inf = lambda V: alpha_m(V) * tau_m(V)
    # inactivation Na [h]
    alpha_h = lambda V: 0.07*np.exp(-0.05*(V + 65))
    beta_h = lambda V:  1 /(1 + np.exp(-0.1*(V + 35)))
    tau_h = lambda V: 1/(alpha_h(V) + beta_h(V))
    h_inf = lambda V: alpha_h(V) * tau_h(V);

    # Initializations
    n = np.zeros(len(I)); m = np.zeros(len(I)); h =
np.zeros(len(I)); V = np.zeros(len(I))

    # Set initial conditions:
    Vstart = -65 #[mV] (starting membrane potential)
    V[0] = Vstart #[mV]
    n[0] = n_inf(Vstart); m[0] = m_inf(Vstart); h[0] =
h_inf(Vstart);


#######################################################
###################
    # Simulation: iteratatively update the variables
using the forward Euler method
    for ii in range(len(I)-1):
        # Update activation state variables
        n[ii+1] = n[ii] + dt*(n_inf(V[ii]) - n[ii])/
tau_n(V[ii])
        m[ii+1] = m[ii] + dt*(m_inf(V[ii]) - m[ii])/
tau_m(V[ii])
        h[ii+1] = h[ii] + dt*(h_inf(V[ii]) - h[ii])/
tau_h(V[ii])
        V[ii+1] = V[ii] + dt/C*(I[ii] - gl*(V[ii]-El) -
gk*n[ii]**4 *(V[ii]-Ek) - gna*m[ii]**3*h[ii]*(V[ii]-
Ena));

    return V, m, n, h

def get_aps(V):
    V_ap=[]
```

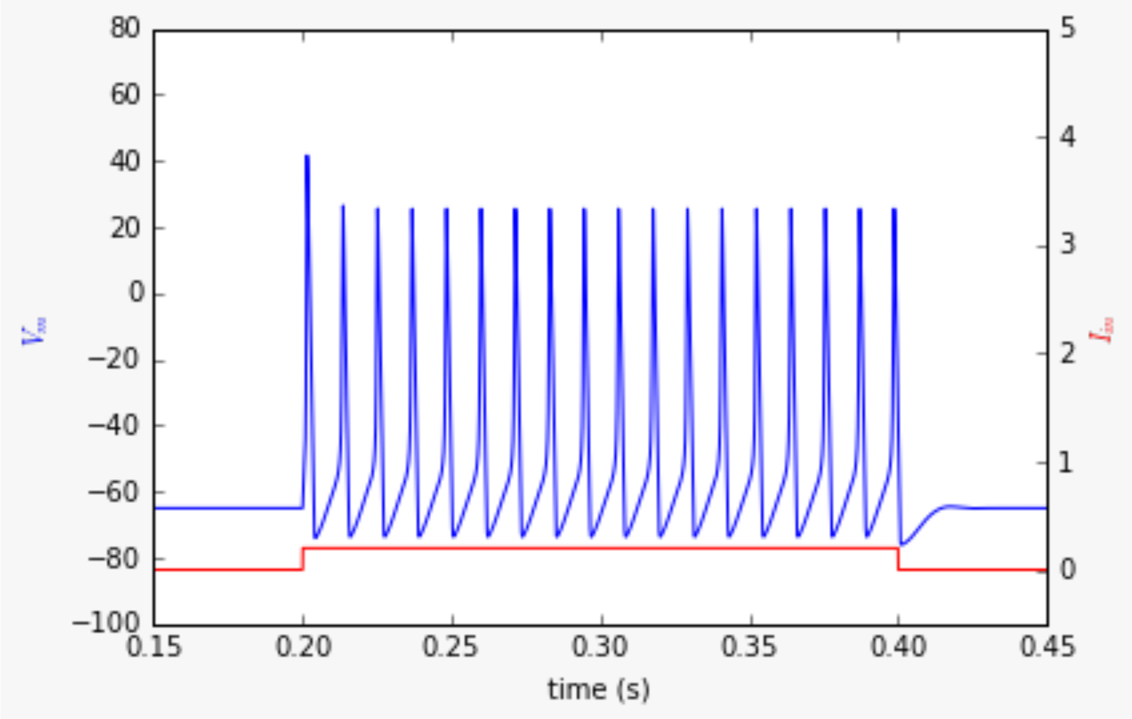
```
    for i in range(len(I)-1):
        if V[i]>0:
            V_ap.append(V[i])
    ap=0
    for i in range(len(V_ap)-1):
        if V_ap[i+1]-V_ap[i]>=0 and V_ap[i+2]-V_ap[i
+1]<=0:
            ap +=1
    return ap
```

```
dt=0.01
I=np.zeros(int(1000/dt))
I[int(200/dt):int(400/dt)]=0.2
V,m,m,h=hh(I,dt)
```

```
%matplotlib inline
fig, ax1 = plt.subplots()
t = np.arange(0, 1, dt / 1000)
ax1.plot(t, V, 'b-')
ax1.set_xlabel('time (s)')
ax1.set_ylabel(r'$V_{m}$', color='b')
ax1.set_ylim([-100, 80])
ax2 = ax1.twinx()
ax2.plot(t, I, 'r-')
ax2.set_ylim([-0.5, 5])
ax2.set_ylabel(r'$I_{in}$', color='r')
ax1.set_xlim([0.15, 0.45])
```

```
number_ap=get_aps(V)
print('generated spikes:%d'%(number_ap))
```

```
generated spikes:18
```



b)

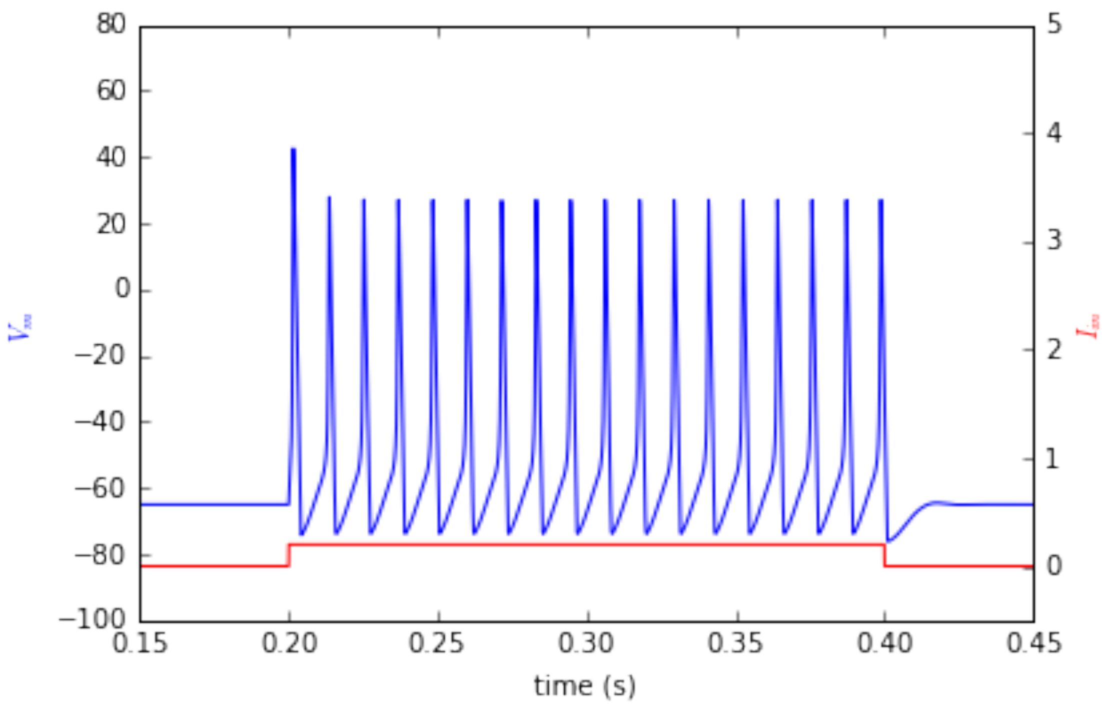
```
dt=0.05
I=np.zeros(int(1000/dt))
I[int(200/dt):int(400/dt)]=0.2
V,m,m,h=hh(I,dt)
```

```
%matplotlib inline
fig, ax1 = plt.subplots()
t = np.arange(0, 1, dt / 1000)
ax1.plot(t, V, 'b-')
ax1.set_xlabel('time (s)')
ax1.set_ylabel(r'$V_{m}$', color='b')
ax1.set_ylim([-100, 80])
```

```
ax2 = ax1.twinx()
ax2.plot(t, I, 'r-')
ax2.set_ylim([-0.5, 5])
ax2.set_ylabel(r'$I_{in}$', color='r')
ax1.set_xlim([0.15, 0.45])
```

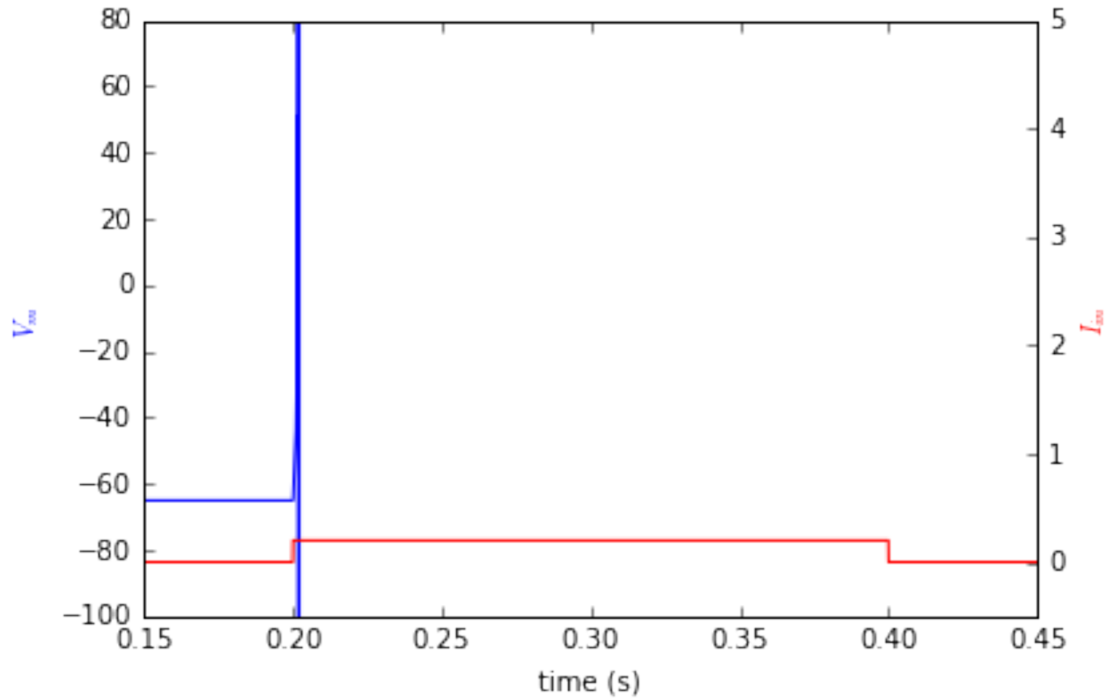
```
number_ap=get_aps(V)
print('generated spikes:%d'%(number_ap))
```

```
generated spikes:18
```

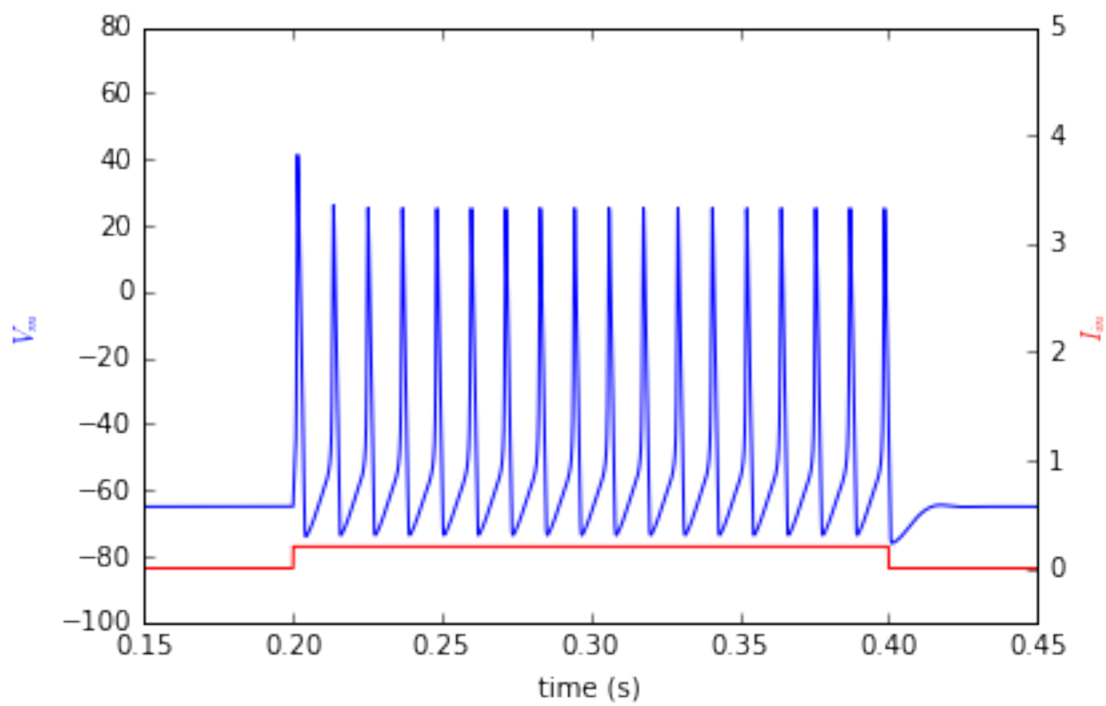



```
dt=0.1
```

Homework2,problem3




```
t=0.001
generated spikes:18
```



c)

I looked for a "knee" in the voltage curve, and estimated the threshold voltage is around -55mV.

To decrease and increase the threshold voltage by ~5 mV, I increase and decrease the gna(increase)/ gk(decrease) by around 7%,relatively and separately.

d)

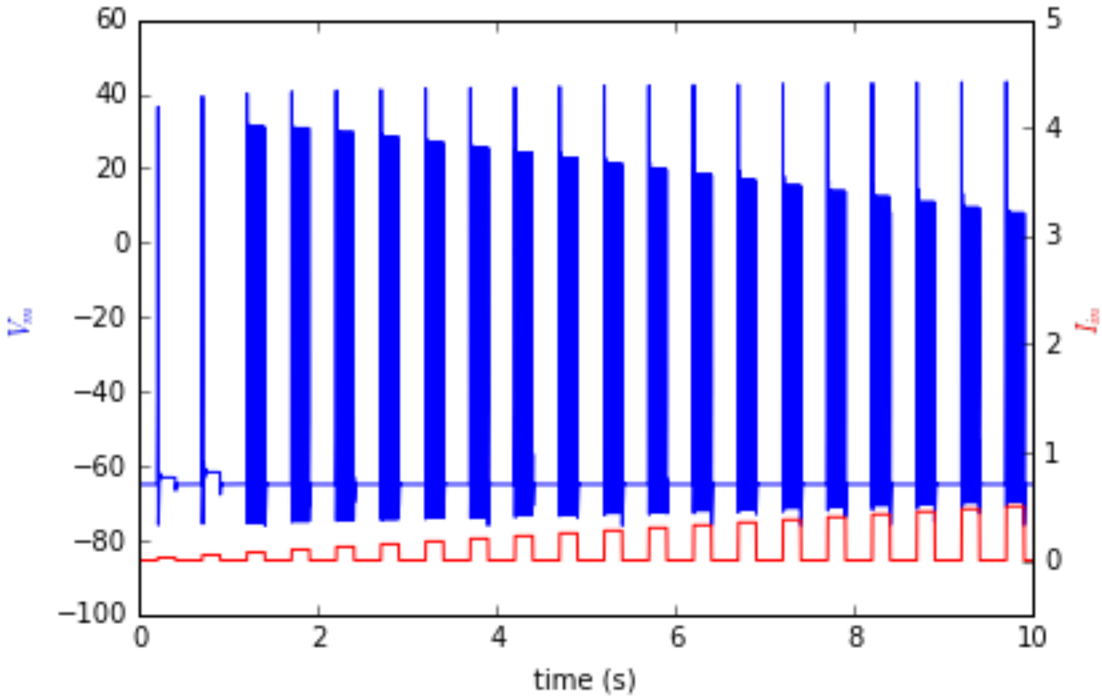
```
import numpy as np
import matplotlib.pyplot as plt

def hh(I, dt):

##########################################################################
    # Simulate the membrane potential of a Hodgkin Huxley neuron with
    #   a given input current
    # Input:
    #    I = current in uA/mm^2a
    #    dt = time step between I measurments [ms]
    #
    # Output:
    #    Vm = membrane voltage in mV
    #    n = sodium activation
    #    m = potassium activation
    #    h = 1 - potassium inactivation

    # This function simulates a dynamical system with state variables
    #    DV = 1/C (I - Ik - Ina - Il)
    #    Il = gl(V-El)
    #    Ik = gk*n^4(V-Ek)
```

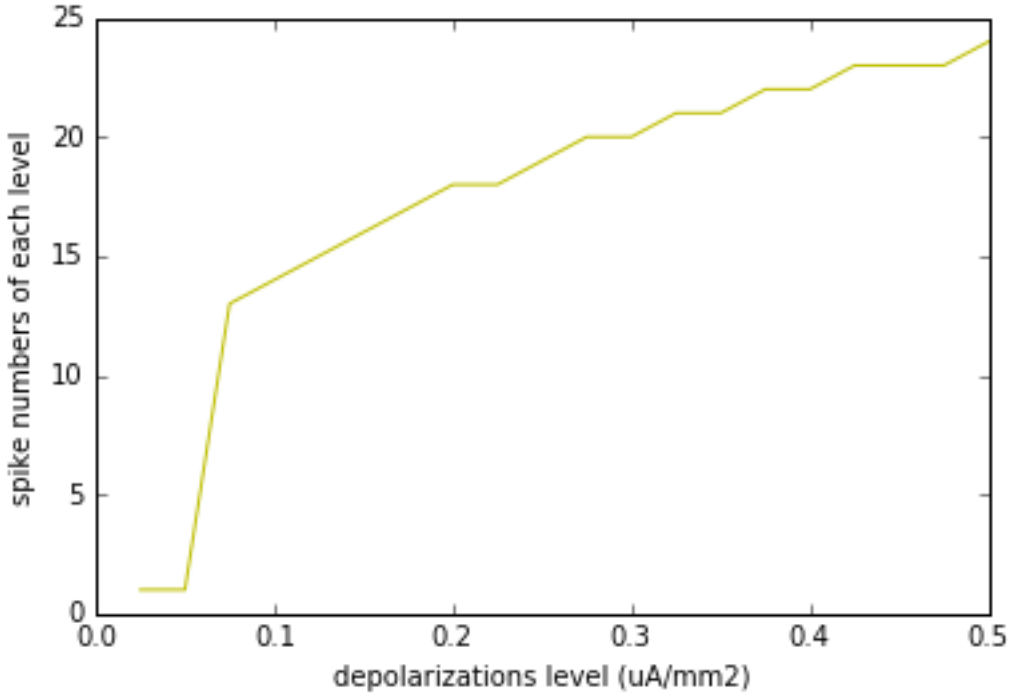
```
    #    Ina = gna*m^3*h(V-Ena)
    #    Dn = (ninf(V) - n)/taun(V) (same for m, h )

    # Summary of units: I = uA / mm^2; V = mV; g = mS;
g*V = uA; C = uF; uA/uC*ms = mV


##############################################################
###################
    # Constants:
    # Reversal potentials for various ions
    Ek = -77 #[mV]
    Ena = 50 #[mV]
    El = -54.402 #[mV]

    # Membrane capacitance:
    C = 0.01 #[uF/mm^2]

    # Maximum conductances [mS/mm^2]
    gna = 1.2
    gk = 0.36
    gl = 0.003


##############################################################
###################
    # Gating variables:
    # activation K [n]
    alpha_n = lambda V: 0.01*(V + 55) / (1 -
np.exp(-0.1*(V + 55)))
    beta_n = lambda V: 0.125 * np.exp(-0.0125*(V + 65))
    tau_n = lambda V: 1 /(alpha_n(V) + beta_n(V))
    n_inf = lambda V: alpha_n(V) * tau_n(V)
    # activation Na [m]
    alpha_m = lambda V: 0.1*(V + 40) / (1 -
np.exp(-0.1*(V + 40)))
    beta_m = lambda V: 4 * np.exp(-0.0556*(V + 65))
    tau_m = lambda V: 1/(alpha_m(V) + beta_m(V))
```

```
    m_inf = lambda V: alpha_m(V) * tau_m(V)
    # inactivation Na [h]
    alpha_h = lambda V: 0.07*np.exp(-0.05*(V + 65))
    beta_h = lambda V:  1 /(1 + np.exp(-0.1*(V + 35)))
    tau_h = lambda V: 1/(alpha_h(V) + beta_h(V))
    h_inf = lambda V: alpha_h(V) * tau_h(V);

    # Initializations
    n = np.zeros(len(I)); m = np.zeros(len(I)); h =
np.zeros(len(I)); V = np.zeros(len(I))

    # Set initial conditions:
    Vstart = -65 #[mV] (starting membrane potential)
    V[0] = Vstart #[mV]
    n[0] = n_inf(Vstart); m[0] = m_inf(Vstart); h[0] =
h_inf(Vstart);


#######################################################
###################
    # Simulation: iteratatively update the variables
using the forward Euler method
    for ii in range(len(I)-1):
        # Update activation state variables
        n[ii+1] = n[ii] + dt*(n_inf(V[ii]) - n[ii])/
tau_n(V[ii])
        m[ii+1] = m[ii] + dt*(m_inf(V[ii]) - m[ii])/
tau_m(V[ii])
        h[ii+1] = h[ii] + dt*(h_inf(V[ii]) - h[ii])/
tau_h(V[ii])
        V[ii+1] = V[ii] + dt/C*(I[ii] - gl*(V[ii]-El) -
gk*n[ii]**4 *(V[ii]-Ek) - gna*m[ii]**3*h[ii]*(V[ii]-
Ena));

    return V, m, n, h

def get_aps(V):
    V_ap=[]
```

```
    for i in range(len(V)-1):
        if V[i]>0:
            V_ap.append(V[i])
    ap=0
    for i in range(len(V_ap)-1):
        if V_ap[i+1]-V_ap[i]>=0 and V_ap[i+2]-V_ap[i
+1]<=0:
            ap +=1
    return ap

input_I = np.arange(0.025,0.525,0.025)
dt=0.01
I = np.zeros(int(10000/dt))
for i in range (20):
    I[int((i*500+200)/dt):int((i*500+400)/
dt)]=input_I[i]


V,m,m,h = hh(I,dt)
ap_numbers=np.zeros(20)
for i in range(20):
    ap_numbers[i]=get_aps(V[int((i*500))/
dt:int((i*500+499)/dt)])
print(ap_numbers)
%matplotlib inline
fig, ax1 = plt.subplots()
t = np.arange(0, 10, dt / 1000)
ax1.plot(t, V, 'b-')
ax1.set_xlabel('time (s)')
ax1.set_ylabel(r'$V_{m}$', color='b')
ax1.set_ylim([-100, 60])
ax2 = ax1.twinx()
ax2.plot(t, I, 'r-')
ax2.set_ylim([-0.5, 5])
ax2.set_ylabel(r'$I_{in}$', color='r')

fig, ax3 = plt.subplots()
ax3.plot(input_I,ap_numbers,'y')
```

```
ax3.set_xlabel('depolarizations level (uA/mm2)')
ax3.set_ylabel('spike numbers of each level’)
[  1.   1.  13.  14.  15.  16.  17.  18.  18.  19.  20.
20.  21.  21.  22.
  22.  23.  23.  23.  24.]
Out[22]:
<matplotlib.text.Text at 0x11e625160>
```





E)

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline


def CST(I, dt):

##########################################################
####################
    # Simulate the membrane potential of a CST neuron
with
    #   a given input current
    # Input:
    #    I = current in uA/mm^2a
    #    dt = time step between I measurments [ms]
    #
    # Output:
    #    Vm = membrane voltage in mV
    #    n = sodium activation
    #    m = potassium activation
    #    h = 1 - potassium inactivation

    # This function simulates a dynamical system with
state variables
    #    DV = 1/C (I - Ik - Ina - Il)
    #    Il = gl(V-El)
    #    Ik = gk*n^4(V-Ek)
    #    Ina = gna*m^3*h(V-Ena)
    #    IA=gA*a^3*b(V-EA)
    #    Dn = (ninf(V) - n)/taun(V) (same for m,
h,a,b )

    # Summary of units: I = uA / mm^2; V = mV; g = mS;
g*V = uA; C = uF; uA/uC*ms = mV
```

Homework2,problem3

```
##################################################################################
    # Constants:
    # Reversal potentials for various ions
    Ek = -72 #[mV]
    Ena = 55 #[mV]
    El = -17 #[mV]
    EA = -75 #[mV]

    # Membrane capacitance:
    C = 0.01 #[uF/mm^2]

    # Maximum conductances [mS/mm^2]
    gna = 1.2
    gk = 0.2
    gl = 0.003
    gA = 0.477


##################################################################################
    # Gating variables:
    # activation K [n]
    alpha_m = lambda V: 0.38*(V + 29.7) / (1 - 
np.exp(-0.1*(V + 29.7)))
    beta_m = lambda V: 15.2 * np.exp(-0.0556*(V + 
54.7))
    tau_m = lambda V: 1/(alpha_m(V) + beta_m(V))
    m_inf = lambda V: alpha_m(V) * tau_m(V)

    alpha_h = lambda V: 0.266*np.exp(-0.05*(V + 48))
    beta_h = lambda V:  3.8 /(1 + np.exp(-0.1*(V + 
18)))
    tau_h = lambda V: 1/(alpha_h(V) + beta_h(V))
    h_inf = lambda V: alpha_h(V) * tau_h(V);
```

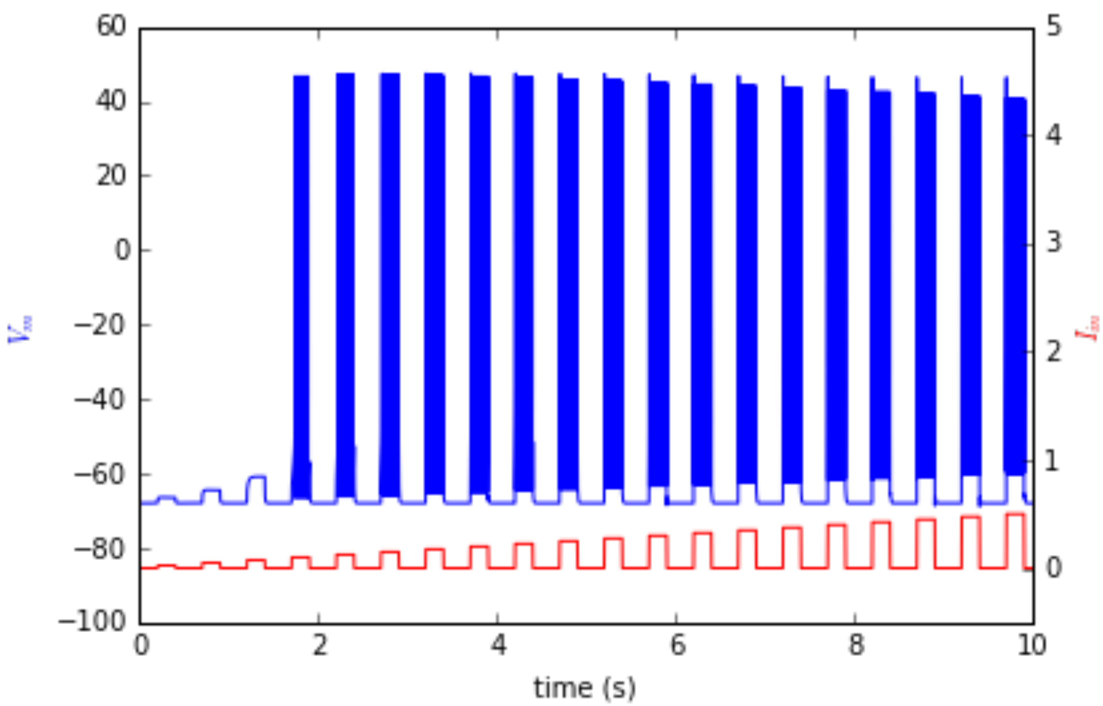
```
    alpha_n = lambda V: 0.02*(V + 45.7) / (1 -
np.exp(-0.1*(V + 45.7)))
    beta_n = lambda V: 0.25 * np.exp(-0.0125*(V +
55.7))
    tau_n = lambda V: 1 /(alpha_n(V) + beta_n(V))
    n_inf = lambda V: alpha_n(V) * tau_n(V)

    a_inf = lambda V: ((0.0761*np.exp(0.0314*(V
+94.22)))/(1 + np.exp(0.0346*(V+1.17))))**(1/3)
    b_inf = lambda V: 1/(1 + np.exp(0.0688*(V
+53.3)))**4
    tau_a = lambda V: 0.3632 + 1.158/(1 +
np.exp(0.0497*(V + 55.96)))
    tau_b = lambda V: 1.24 + 2.678/(1 +
np.exp(0.0624*(V + 50)))

    # Initializations
    n = np.zeros(len(I)); m = np.zeros(len(I)); h =
np.zeros(len(I));
    a = np.zeros(len(I)); b = np.zeros(len(I));
    V = np.zeros(len(I))

    # Set initial conditions:
    Vstart = -68 #[mV] (starting membrane potential)
    V[0] = Vstart #[mV]
    n[0] = n_inf(Vstart); m[0] = m_inf(Vstart); h[0] =
h_inf(Vstart);
    a[0] = a_inf(Vstart); b[0] = b_inf(Vstart);

#######################################################
###################
    # Simulation: iteratatively update the variables
using the forward Euler method
    for i in range(len(I)-1):
        n[i+1] = n[i] + dt*(n_inf(V[i]) - n[i])/
tau_n(V[i])
        m[i+1] = m[i] + dt*(m_inf(V[i]) - m[i])/
tau_m(V[i])
```

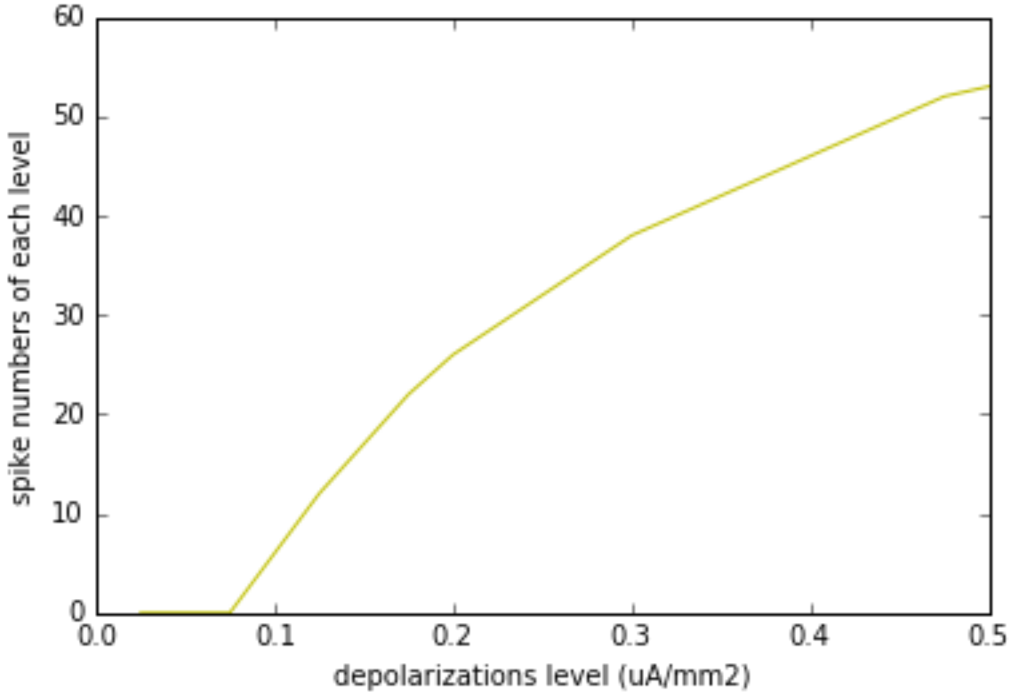
```
        h[i+1] = h[i] + dt*(h_inf(V[i]) - h[i])/
tau_h(V[i])
        a[i+1] = a[i] + dt*(a_inf(V[i]) - a[i])/
tau_a(V[i])
        b[i+1] = b[i] + dt*(b_inf(V[i]) - b[i])/
tau_b(V[i])
        V[i+1] = V[i] + dt/C*(I[i] - gl*(V[i] - El) -
gk*n[i]**4*(V[i] - Ek)-
                              gna*m[i]**3*h[i]*(V[i] -
Ena) - gA*a[i]**3*b[i]*(V[i] - EA))

    return V, m, n, h, a, b
def get_aps(V):
    V_ap=[]
    for i in range(len(V)-1):
        if V[i]>0:
            V_ap.append(V[i])
    ap=0
    for i in range(len(V_ap)-1):
        if V_ap[i+1]-V_ap[i]>=0 and V_ap[i+2]-V_ap[i
+1]<=0:
            ap +=1
    return ap

input_I = np.arange(0.025,0.525,0.025)
dt=0.01
I = np.zeros(int(10000/dt))
for i in range (20):
    I[int((i*500+200)/dt):int((i*500+400)/
dt)]=input_I[i]


V,m,n,h,a,b=CST(I,dt)
ap_numbers=np.zeros(20)
for i in range(20):
    ap_numbers[i]=get_aps(V[int((i*500))/
dt:int((i*500+499)/dt)])
print(ap_numbers)
```

Homework2,problem3

```
%matplotlib inline
fig, ax1 = plt.subplots()
t = np.arange(0, 10, dt / 1000)
ax1.plot(t, V, 'b-')
ax1.set_xlabel('time (s)')
ax1.set_ylabel(r'$V_{m}$', color='b')
ax1.set_ylim([-100, 60])
ax2 = ax1.twinx()
ax2.plot(t, I, 'r-')
ax2.set_ylim([-0.5, 5])
ax2.set_ylabel(r'$I_{in}$', color='r')

fig, ax3 = plt.subplots()
ax3.plot(input_I,ap_numbers,'y')
ax3.set_xlabel('depolarizations level (uA/mm2)')
ax3.set_ylabel('spikes numbers of each level')
```

```
[  0.   0.   0.   6.  12.  17.  22.  26.  29.  32.  35.  38.  40.  42.  44.
  46.  48.  50.  52.  53.]
Out[1]:
<matplotlib.text.Text at 0x1142c17b8>
```





F)

```
import numpy as np
import matplotlib.pyplot as plt

def hh(I, dt):

#############################################################
###################
    # Simulate the membrane potential of a Hodgkin
Huxley neuron with
    #   a given input current
    # Input:
    #    I = current in uA/mm^2a
    #    dt = time step between I measurments [ms]
    #
    # Output:
    #    Vm = membrane voltage in mV
    #    n = sodium activation
    #    m = potassium activation
    #    h = 1 - potassium inactivation

    # This function simulates a dynamical system with
state variables
    #    DV = 1/C (I - Ik - Ina - Il)
```

```
    #    Il = gl(V-El)
    #    Ik = gk*n^4(V-Ek)
    #    Ina = gna*m^3*h(V-Ena)
    #    Dn = (ninf(V) - n)/taun(V) (same for m, h )

    # Summary of units: I = uA / mm^2; V = mV; g = mS;
g*V = uA; C = uF; uA/uC*ms = mV


#######################################################
###################
    # Constants:
    # Reversal potentials for various ions
    Ek = -77 #[mV]
    Ena = 50 #[mV]
    El = -54.402 #[mV]

    # Membrane capacitance:
    C = 0.01 #[uF/mm^2]

    # Maximum conductances [mS/mm^2]
    gna = 1.2
    gk = 0.36
    gl = 0.003


#######################################################
###################
    # Gating variables:
    # activation K [n]
    alpha_n = lambda V: 0.01*(V + 55) / (1 -
np.exp(-0.1*(V + 55)))
    beta_n = lambda V: 0.125 * np.exp(-0.0125*(V + 65))
    tau_n = lambda V: 1 /(alpha_n(V) + beta_n(V))
    n_inf = lambda V: alpha_n(V) * tau_n(V)
    # activation Na [m]
    alpha_m = lambda V: 0.1*(V + 40) / (1 -
np.exp(-0.1*(V + 40)))
```

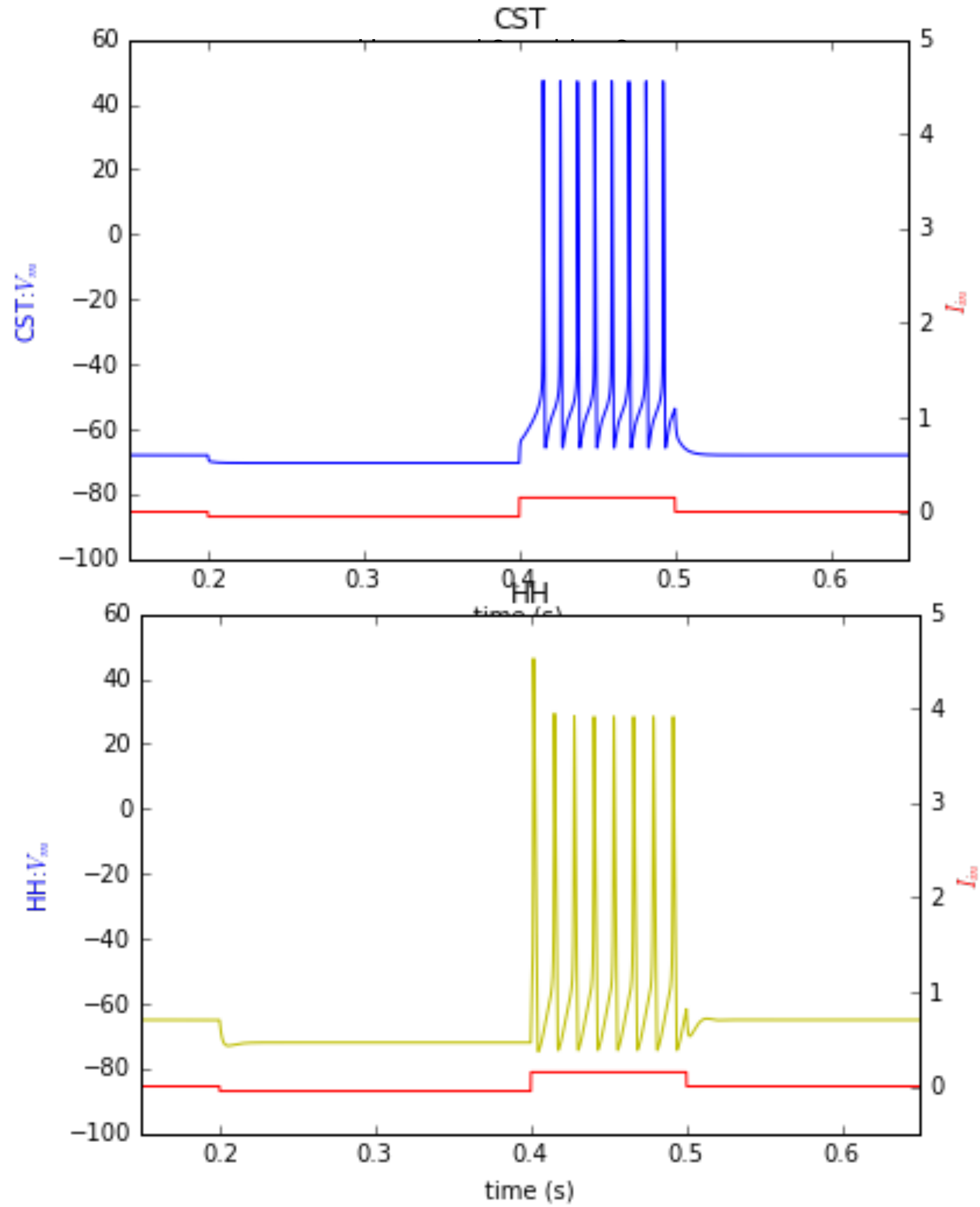
```
    beta_m = lambda V: 4 * np.exp(-0.0556*(V + 65))
    tau_m = lambda V: 1/(alpha_m(V) + beta_m(V))
    m_inf = lambda V: alpha_m(V) * tau_m(V)
    # inactivation Na [h]
    alpha_h = lambda V: 0.07*np.exp(-0.05*(V + 65))
    beta_h = lambda V:  1 /(1 + np.exp(-0.1*(V + 35)))
    tau_h = lambda V: 1/(alpha_h(V) + beta_h(V))
    h_inf = lambda V: alpha_h(V) * tau_h(V);

    # Initializations
    n = np.zeros(len(I)); m = np.zeros(len(I)); h =
np.zeros(len(I)); V = np.zeros(len(I))

    # Set initial conditions:
    Vstart = -65 #[mV] (starting membrane potential)
    V[0] = Vstart #[mV]
    n[0] = n_inf(Vstart); m[0] = m_inf(Vstart); h[0] =
h_inf(Vstart);


#######################################################
###################
    # Simulation: iteratatively update the variables
using the forward Euler method
    for ii in range(len(I)-1):
        # Update activation state variables
        n[ii+1] = n[ii] + dt*(n_inf(V[ii]) - n[ii])/
tau_n(V[ii])
        m[ii+1] = m[ii] + dt*(m_inf(V[ii]) - m[ii])/
tau_m(V[ii])
        h[ii+1] = h[ii] + dt*(h_inf(V[ii]) - h[ii])/
tau_h(V[ii])
        V[ii+1] = V[ii] + dt/C*(I[ii] - gl*(V[ii]-El) -
gk*n[ii]**4 *(V[ii]-Ek) - gna*m[ii]**3*h[ii]*(V[ii]-
Ena));

    return V, m, n, h
```

```
def CST(I, dt):

################################################################################
    # Simulate the membrane potential of a CST neuron with
    #   a given input current
    # Input:
    #    I = current in uA/mm^2a
    #    dt = time step between I measurments [ms]
    #
    # Output:
    #    Vm = membrane voltage in mV
    #    n = sodium activation
    #    m = potassium activation
    #    h = 1 - potassium inactivation

    # This function simulates a dynamical system with state variables
    #    DV = 1/C (I - Ik - Ina - Il)
    #    Il = gl(V-El)
    #    Ik = gk*n^4(V-Ek)
    #    Ina = gna*m^3*h(V-Ena)
    #    IA=gA*a^3*b(V-EA)
    #    Dn = (ninf(V) - n)/taun(V) (same for m, h,a,b )

    # Summary of units: I = uA / mm^2; V = mV; g = mS; g*V = uA; C = uF; uA/uC*ms = mV


################################################################################
    # Constants:
    # Reversal potentials for various ions
    Ek = -72 #[mV]
    Ena = 55 #[mV]
    El = -17 #[mV]
```

```
    EA = -75 #[mV]

    # Membrane capacitance:
    C = 0.01 #[uF/mm^2]

    # Maximum conductances [mS/mm^2]
    gna = 1.2
    gk = 0.2
    gl = 0.003
    gA = 0.477


#######################################################
###################
    # Gating variables:
    # activation K [n]
    alpha_n = lambda V: 0.02*(V + 45.7) / (1 -
np.exp(-0.1*(V + 45.7)))
    beta_n = lambda V: 0.25 * np.exp(-0.0125*(V +
55.7))
    tau_n = lambda V: 1 /(alpha_n(V) + beta_n(V))
    n_inf = lambda V: alpha_n(V) * tau_n(V)
    # activation Na [m]
    alpha_m = lambda V: 0.38*(V + 29.7) / (1 -
np.exp(-0.1*(V + 29.7)))
    beta_m = lambda V: 15.2* np.exp(-0.0556*(V + 54.7))
    tau_m = lambda V: 1/(alpha_m(V) + beta_m(V))
    m_inf = lambda V: alpha_m(V) * tau_m(V)
    # inactivation Na [h]
    alpha_h = lambda V: 0.266*np.exp(-0.05*(V + 48))
    beta_h = lambda V:  3.8 /(1 + np.exp(-0.1*(V +
18)))
    tau_h = lambda V: 1/(alpha_h(V) + beta_h(V))
    h_inf = lambda V: alpha_h(V) * tau_h(V);
    # activation A [a,b]
    a_inf= lambda V: ((0.0761*np.exp(0.0314*(V
+94.22)))/(1+np.exp(0.0346*(V+1.17))))**(1/3)
```

```
    tau_a=lambda V: 0.3632+(1.158)/(1+np.exp(0.0497*(V
+55.96)))
    b_inf=lambda V: (1/(1+np.exp(0.0688*(V+53.3))))**4
    tau_b=lambda V: 1.24+(2.678)/(1+np.exp(0.0624*(V
+50)))

    # Initializations
    n = np.zeros(len(I)); m = np.zeros(len(I)); h =
np.zeros(len(I));
    a = np.zeros(len(I)); b = np.zeros(len(I));
    V = np.zeros(len(I))

    # Set initial conditions:
    Vstart = -68 #[mV] (starting membrane potential)
    V[0] = Vstart #[mV]
    n[0] = n_inf(Vstart); m[0] = m_inf(Vstart); h[0] =
h_inf(Vstart);
    a[0] = a_inf(Vstart); b[0] = b_inf(Vstart);

#######################################################
###################
    # Simulation: iteratatively update the variables
using the forward Euler method
    for ii in range(len(I)-1):
        # Update activation state variables
        n[ii+1] = n[ii] + dt*(n_inf(V[ii]) - n[ii])/
tau_n(V[ii])
        m[ii+1] = m[ii] + dt*(m_inf(V[ii]) - m[ii])/
tau_m(V[ii])
        h[ii+1] = h[ii] + dt*(h_inf(V[ii]) - h[ii])/
tau_h(V[ii])
        a[ii+1] = a[ii] + dt*(a_inf(V[ii]) - a[ii])/
tau_a(V[ii])
        b[ii+1] = b[ii] + dt*(b_inf(V[ii]) - b[ii])/
tau_b(V[ii])
        V[ii+1] = V[ii] + dt/C*(I[ii] - gl*(V[ii]-El) -
gk*n[ii]**4 *(V[ii]-Ek) - gna*m[ii]**3*h[ii]*(V[ii]-
Ena) - gA*a[ii]**3*b[ii]*(V[ii]-EA));
```

```
    return V, m, n, h, a, b

dt=0.01
I=np.zeros(int(1000/dt))
I[int(200/dt):int(400/dt)]=-0.05
I[int(400/dt):int(500/dt)]=0.15
V,m,m,h,a,b=CST(I,dt)
V1=V
V,m,m,h,=hh(I,dt)
V2=V

%matplotlib inline
fig, ax1 = plt.subplots()
ax1.set_title('CST')
t = np.arange(0, 1, dt / 1000)
ax1.plot(t, V1, 'b-')
ax1.set_xlabel('time (s)')
ax1.set_ylabel(r'CST:$V_{m}$', color='b')
ax1.set_ylim([-100, 60])
ax2 = ax1.twinx()
ax2.plot(t, I, 'r-')
ax2.set_ylim([-0.5, 5])
ax2.set_ylabel(r'$I_{in}$', color='r')
ax1.set_xlim([0.15, 0.65])

fig, ax3=plt.subplots()
ax3.set_title('HH')
ax3.plot(t, V2, 'y-')
ax3.set_xlabel('time (s)')
ax3.set_ylabel(r'HH:$V_{m}$', color='b')
ax3.set_ylim([-100, 60])
ax4 = ax3.twinx()
ax4.plot(t, I, 'r-')
ax4.set_ylim([-0.5, 5])
ax4.set_ylabel(r'$I_{in}$', color='r')
ax3.set_xlim([0.15, 0.65])
```



G)

```
import numpy as np
import matplotlib.pyplot as plt

def CST(I, dt):
```

Homework2,problem3

```
######################################################################
##################
    # Simulate the membrane potential of a CST neuron
with
    #   a given input current
    # Input:
    #    I = current in uA/mm^2a
    #    dt = time step between I measurments [ms]
    #
    # Output:
    #    Vm = membrane voltage in mV
    #    n = sodium activation
    #    m = potassium activation
    #    h = 1 - potassium inactivation

    # This function simulates a dynamical system with
state variables
    #    DV = 1/C (I - Ik - Ina - Il)
    #    Il = gl(V-El)
    #    Ik = gk*n^4(V-Ek)
    #    Ina = gna*m^3*h(V-Ena)
    #    IA=gA*a^3*b(V-EA)
    #    Dn = (ninf(V) - n)/taun(V) (same for m,
h,a,b )

    # Summary of units: I = uA / mm^2; V = mV; g = mS;
g*V = uA; C = uF; uA/uC*ms = mV


######################################################################
##################
    # Constants:
    # Reversal potentials for various ions
    Ek = -72 #[mV]
    Ena = 55 #[mV]
    El = -17 #[mV]
    EA = -75 #[mV]
```

```
    # Membrane capacitance:
    C = 0.01 #[uF/mm^2]

    # Maximum conductances [mS/mm^2]
    gna = 1.2
    gk = 0.2
    gl = 0.003
    gA = 0.477


#######################################################
###################
    # Gating variables:
    # activation K [n]
    alpha_m = lambda V: 0.38*(V + 29.7) / (1 -
np.exp(-0.1*(V + 29.7)))
    beta_m = lambda V: 15.2 * np.exp(-0.0556*(V +
54.7))
    tau_m = lambda V: 1/(alpha_m(V) + beta_m(V))
    m_inf = lambda V: alpha_m(V) * tau_m(V)

    alpha_h = lambda V: 0.266*np.exp(-0.05*(V + 48))
    beta_h = lambda V:  3.8 /(1 + np.exp(-0.1*(V +
18)))
    tau_h = lambda V: 1/(alpha_h(V) + beta_h(V))
    h_inf = lambda V: alpha_h(V) * tau_h(V);

    alpha_n = lambda V: 0.02*(V + 45.7) / (1 -
np.exp(-0.1*(V + 45.7)))
    beta_n = lambda V: 0.25 * np.exp(-0.0125*(V +
55.7))
    tau_n = lambda V: 1 /(alpha_n(V) + beta_n(V))
    n_inf = lambda V: alpha_n(V) * tau_n(V)

    a_inf = lambda V: ((0.0761*np.exp(0.0314*(V
+94.22)))/(1 + np.exp(0.0346*(V+1.17))))**(1/3)
```

```
    b_inf = lambda V: 1/(1 + np.exp(0.0688*(V
+53.3)))**4
    tau_a = lambda V: 0.3632 + 1.158/(1 +
np.exp(0.0497*(V + 55.96)))
    tau_b = lambda V: 1.24 + 2.678/(1 +
np.exp(0.0624*(V + 50)))

    # Initializations
    n = np.zeros(len(I)); m = np.zeros(len(I)); h =
np.zeros(len(I));
    a = np.zeros(len(I)); b = np.zeros(len(I));
    V = np.zeros(len(I))

    # Set initial conditions:
    Vstart = -68 #[mV] (starting membrane potential)
    V[0] = Vstart #[mV]
    n[0] = n_inf(Vstart); m[0] = m_inf(Vstart); h[0] =
h_inf(Vstart);
    a[0] = a_inf(Vstart); b[0] = b_inf(Vstart);

#######################################################
###################
    # Simulation: iteratatively update the variables
using the forward Euler method
    for i in range(len(I)-1):
        n[i+1] = n[i] + dt*(n_inf(V[i]) - n[i])/
tau_n(V[i])
        m[i+1] = m[i] + dt*(m_inf(V[i]) - m[i])/
tau_m(V[i])
        h[i+1] = h[i] + dt*(h_inf(V[i]) - h[i])/
tau_h(V[i])
        a[i+1] = a[i] + dt*(a_inf(V[i]) - a[i])/
tau_a(V[i])
        b[i+1] = b[i] + dt*(b_inf(V[i]) - b[i])/
tau_b(V[i])
        V[i+1] = V[i] + dt/C*(I[i] - gl*(V[i] - El) -
gk*n[i]**4*(V[i] - Ek)-
```

```
                              gna*m[i]**3*h[i]*(V[i] - 
Ena) - gA*a[i]**3*b[i]*(V[i] - EA))

    return V, m, n, h, a, b
def get_aps(V):
    V_ap=[]
    for i in range(len(V)-1):
        if V[i]>0:
            V_ap.append(V[i])
    ap=0
    for i in range(len(V_ap)-1):
        if V_ap[i+1]-V_ap[i]>=0 and V_ap[i+2]-V_ap[i
+1]<=0:
            ap +=1
    return ap
u=[0,0.05,0.1,0.15,0.2]
sigma=[0,0.01,0.05,0.1]

spikes=[]
mean_spikes=np.zeros(4)
dt=0.01
I = np.zeros(int(600/dt))
for i in range(len(sigma)):
    for j in range(len(u)):
        if sigma[i]==0:
            s_0=0
            a = sigma[i]*np.random.randn(50000)+u[j]
            d = np.ones(100)
            c = np.convolve(a,d, "same")/sum(d)
            I[int((50)/dt):int((500+50)/dt)]=c
            V,m,n,h,a,b=CST(I,dt)
            s_o=get_aps(V)
            spikes.append(s_0)
        else:
            for l in range(25):
                s=0
                a = 
sigma[i]*np.random.randn(50000)+u[j]
```

```
                d = np.ones(100)
                c = np.convolve(a,d, "same")/sum(d)
                I[int((50)/dt):int((500+50)/dt)]=c
                V,m,n,h,a,b=CST(I,dt)
                s=get_aps(V)
                spikes.append(s)
    mean_spikes[i]=np.mean(spikes)

print(spikes)
print(len(spikes))
print(mean_spikes)

%matplotlib inline
fig,ax1=plt.subplots()
ax1.plot(sigma,mean_spikes,'r')
ax1.set_title(' mean spikes for each of the standard
deviations')
ax1.set_xlabel('standard deviation (μA/mm2)')
ax1.set_ylabel('spikes numbers’)
```

```
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16,
16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 45, 45,
45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45,
45, 45, 45, 45, 45, 45, 45, 45, 45, 65, 65, 65, 65, 65,
65, 65, 65, 65, 65, 65, 65, 65, 65, 65, 65, 65, 65, 65,
65, 65, 65, 65, 65, 65, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16,
16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 17, 16, 16, 16,
45, 45, 44, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45,
45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 66, 65, 65,
65, 65, 65, 65, 65, 65, 65, 65, 65, 65, 65, 65, 65, 65,
65, 65, 65, 65, 65, 66, 65, 65, 0, 0, 0, 0, 0, 0, 0, 0,
```

```
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 16, 16, 17, 16, 16, 16, 16, 16, 17,
17, 16, 16, 16, 16, 17, 16, 16, 17, 16, 16, 15, 16, 16,
16, 16, 45, 45, 44, 45, 45, 45, 44, 44, 45, 44, 44, 45,
44, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 66,
66, 65, 65, 65, 65, 65, 65, 65, 66, 66, 65, 66, 65, 65,
65, 65, 66, 66, 65, 66, 65, 65, 65, 66]
380
[  0.          24.23076923  24.71372549  24.89210526]
```

Out[1]:

```
<matplotlib.text.Text at 0x10ce93c88>
```