Problem1

a).
First, we have two independent poisson process, Poisson(λ1s) and Poisson(λ2s). And Poisson process is denote by :

$$Pr(N(s)=n)=\frac{(\lambda s)^n}{n!}\cdot e^{-\lambda s}$$

Given any time t0, we can assume that the next combined signal only appear after t0+t, which means at the interval(t0,t0+t), there isn't any spike within this time period. So we can plug n=0 into the above equation:

$$P\{next\ combined\ spike\ occurs\ after\ t\}=Pr(N_1(t)=0)\cdot Pr(N_2(t)=0)=e^{-(\lambda 1+\lambda 2)\cdot t}$$

So,
$$Pr(combined\ \mathrm{sign}al > t)=e^{-(\lambda 1+\lambda 2)\cdot t}$$

The probability that a spike has already occurred is 1 minus this result, i.e.

$$P\{next\ combined\ spike\ occurs\ before\ t\}=1-e^{-(\lambda 1+\lambda 2)\cdot t}$$

This is a cumulative distribution function for the probability of a spike occurring within the interval(t0,t0+t).

And The probability density function for the waiting time until the next spike is the derivative of the above cumulative distribution:

$$f_T(t)=\frac{d}{dt}\left(1-e^{-(\lambda 1+\lambda 2)\cdot t}\right)=(\lambda 1+\lambda 2)e^{-(\lambda 1+\lambda 2)\cdot t}$$

Thus, the interspike interval density for this Poisson spike train is an exponential function.

b).
Since the two neurons are spiking independently, following independent poisson process, Poisson(λ1s) and Poisson(λ2s). So, it a superposition situation, and as in a):

$$Pr(combined\ \text{signal} > t) = e^{-(\lambda 1+\lambda 2)\cdot t}$$

Therefore, the rate for the combined poisson process is: $\lambda 1+\lambda 2$

c).

$$P\{each\ neuron\ spikes\ once\ within\ the\ same\ 1\ ms\ interval\} = Pr(N_1(1ms)=1)\cdot\ Pr(N_2(1ms)=1)$$

so,

$$P = 0.001\lambda_1 e^{-0.001\lambda 1}\cdot 0.001\lambda_2 e^{-0.001\lambda 2} = 10^{-6}\cdot\lambda_1\cdot\lambda_2\cdot e^{-0.001\cdot(\lambda 1+\lambda 2)}$$