1.

a). Nernst equation:

$$\frac{RT}{Z_xF}\ln\frac{[x]_{out}}{[x]_{in}}=E_m$$

Ion	Extracellular Concentration	Intracellular Concentration	Permeability	Equilibrium Potential
K+	20mA	400mA	1	-75
Na+	440mA	50mA	0.04	+55
Cl-	560mA	52mA	0.045	-60
Mn++	2mA	0.01mA	0.01	

calculation example:
when room temperature(25), RT/F=25mV; and the z for K+ is 1, then we can have:

$$E_x=25mV\times\ln\frac{[x]_{out}}{[x]_{in}}=-75mV$$

b). When we taking all these ions into consideration, we have GHK equation:

$$V_m=\frac{RT}{F}ln\left(\frac{\Sigma P_c[C^+]_{out}-\Sigma P_A[A^-]_{in}}{\Sigma P_c[C^+]_{in}-\Sigma P_A[A^+]_{out}}\right)$$

in this case:

$$V_m=\frac{RT}{F}ln\left(\frac{P_K[K^+]_{out}+P_{Na}[Na^+]_{out}+P_{Cl}[Cl^-]_{in}}{P_K[K^+]_{in}+P_{Na}[Na^+]_{in}+P_{Cl}[Cl^-]_{out}}\right)$$

Therefore,

$$V_m=25mV\times ln\left(\frac{1\times20+0.04\times440+0.045\times52}{1\times400+0.04\times50+0.045\times560}\right)$$

Vm= -59.25mv

c). Increasing the conductivity of potassium channel, which leads to increase the Extracellular Concentration of potassium, and decrease its Intracellular Concentration.

From the GHK equation, we can simply see that, by bigger Extracellular Concentration of potassium and smaller its Intracellular Concentration, we can get bigger outcome of ln(), which increases the Vm.

d). (1) let Vm= -65mV, and all other parameter stay remained expect the Extracellular Concentration of potassium, which simply call x.

we have:

$$-65=25mV\times ln\left(\frac{1\times X+0.04\times440+0.045\times52}{1\times400+0.04\times50+0.045\times560}\right)$$

So, x=11.79mA

(2) In order to increase the Extracellular Concentration of potassium, the permeability of potassium will grow. Let Extracellular Concentration of potassium= 11.79mA, and the Vm return the original state, Vm= -59.25mv, and other parameter stay remained expect the permeability of potassium, which call Pk.

we have:

$$-59.25=25mV\times ln\left(\frac{P_K\times11.79+0.04\times440+0.045\times52}{P_K\times400+0.04\times50+0.045\times560}\right)$$

So, Pk=0.68

e). If this happened, extracellular chloride ions will flue into the cell, leading the Extracellular Concentration of chloride decrease and the its Intracellular Concentration increase.

Since,

$$V_m=\frac{RT}{F}ln\left(\frac{P_K[K^+]_{out}+P_{Na}[Na^+]_{out}+P_{Cl}[Cl^-]_{in}}{P_K[K^+]_{in}+P_{Na}[Na^+]_{in}+P_{Cl}[Cl^-]_{out}}\right)$$

we will get Vm increased.

2.

a).

$$E_{Mn}=\frac{25}{2}mV\times ln\frac{Mn_{out}}{Mn_{in}}=66.23mV$$

b).

$$j_A=\mu n_A P_A \frac{[A]_{out}-[A]_{in}e^{n\mu}}{1-e^{n\mu}}$$

let ja=0
we could have

$$\frac{1-e^{-\frac{FVm}{RT}}}{1-e^{-\frac{2FVm}{RT}}}\left(\left[P_KK_{in}-P_KK_{out}e^{-\frac{FVm}{RT}}+P_{NA}Na_{in}-P_{NA}Na_{out}e^{-\frac{FVm}{RT}}+P_{Cl}Cl_{out}-P_{Cl}Cl_{in}e^{-\frac{FVm}{RT}}+4P_{Mn}Mn_{in}-P_{Mn}Mn_{out}e^{-\frac{2FVm}{RT}}\right]\right)=0$$

let $e^{-\frac{FVm}{RT}}$ be x, rearrange the equation:

we can have:
ax^2+bx+c=0
where:

$$a=4P_{Mn}Mn_{out}+P_kK_{out}+P_{Na}Na_{out}+P_{Cl}Cl_{in}$$

$$b=-(P_KK_{in}-P_KK_{out}+P_{Na}Na_{in}-P_{Na}Na_{out}+P_{Cl}Cl_{out}-P_{Cl}Cl_{in})$$

$$c=-(P_KK_{in}+P_{Na}Na_{in}+P_{Cl}Cl_{out}+4P_{Mn}Mn_{in})$$

then, with the

(ignore the negative root)
$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

so we can get:

$$x=\frac{P_KK_{in}-P_KK_{out}+P_{Na}Na_{in}-P_{Na}Na_{out}+P_{Cl}Cl_{out}-P_{Cl}Cl_{in}+\sqrt{\left(P_KK_{in}+P_KK_{out}+P_{Na}Na_{in}+P_{Na}Na_{out}+P_{Cl}Cl_{out}+P_{Cl}Cl_{in}\right)^2+64P_{Mn}^2Mn_{out}Mn_{in}+16P_{Mn}P_KK_{in}Mn_{out}+16P_{Mn}P_KK_{out}Mn_{in}+16P_{Mn}P_{Na}Na_{out}Mn_{in}+16P_{Mn}P_{Na}Na_{in}Mn_{out}+16P_{Mn}P_{Cl}Mn_{out}Cl_{out}+16P_{Mn}P_{Cl}Mn_{in}Cl_{in}}}{2\left(4P_{Mn}Mn_{out}+P_KK_{out}+P_{Na}Na_{out}+P_{Cl}Cl_{in}\right)}$$

since x= $e^{-\frac{FVm}{RT}}$

so,

$$V_m=\frac{RT}{F}\ln\left(\frac{2\left(4P_{Mn}Mn_{out}+P_KK_{out}+P_{Na}Na_{out}+P_{Cl}Cl_{in}\right)}{P_KK_{in}-P_KK_{out}+P_{Na}Na_{in}-P_{Na}Na_{out}+P_{Cl}Cl_{out}-P_{Cl}Cl_{in}+\sqrt{\left(P_KK_{in}+P_KK_{out}+P_{Na}Na_{in}+P_{Na}Na_{out}+P_{Cl}Cl_{out}+P_{Cl}Cl_{in}\right)^2+64P_{Mn}^2Mn_{out}Mn_{in}+16P_{Mn}P_KK_{in}Mn_{out}+16P_{Mn}P_KK_{out}Mn_{in}+16P_{Mn}P_{Na}Na_{out}Mn_{in}+16P_{Mn}P_{Na}Na_{in}Mn_{out}+16P_{Mn}P_{Cl}Mn_{out}Cl_{out}+16P_{Mn}P_{Cl}Mn_{in}Cl_i}}\right)$$

c).

Form b) we have

$$V_m = \frac{RT}{F} \ln\left(\frac{2\left(4 P_{Mn} Mn_{out} + P_K K_{out} + P_{Na} Na_{out} + P_{Cl} Cl_{in}\right)}{P_K K_{in} - P_K K_{out} + P_{Na} Na_{in} - P_{Na} Na_{out} + P_{Cl} Cl_{out} - P_{Cl} Cl_{in} + \sqrt{\left(P_K K_{in} + P_K K_{out} + P_{Na} Na_{in} + P_{Na} Na_{out} + P_{Cl} Cl_{out} + P_{Cl} Cl_{in}\right)^2 + 64 P_{Mn}^2 Mn_{out} Mn_{in} + 16 P_{Mn} P_K K_{in} Mn_{out} + 16 P_{Mn} P_K K_{out} Mn_{in} + 16 P_{Mn} P_{Na} Na_{out} Mn_{in} + 16 P_{Mn} P_{Na} Na_{in} Mn_{out} + 16 P_{Mn} P_{Cl} Mn_{out} Cl_{out} + 16 P_{Mn} } } \right)$$

therefore, we put the membrane permeabilities and internal and external ionic concentrations in the table into this equation.

then:

finally, we have Vm= -59.20mV