SET10112 - Exercise sheet 5
Weakest Pre-Conditions 2

Paper-based exercises

Exercise 1 *Calculate the following weakest pre-conditions for the given sequential operations.*

1. $\{s := s + 2;\ n := n - 1\}(s \leq n)$.
2. $\{s := n;\ n := n + 1\}(s \leq n)$.
3. $\{s := n;\ n := s\}(s \leq n)$.
4. $\{s := n;\ n := m\}(s \leq n)$.
5. $\{s := s + n;\ n := s - n\}(s \leq n)$.
6. $\{x := x - 1;\ y := y + 1;\ H := H \cup \{x, y\}\}(H \subseteq \{x..y\})$

Exercise 2 *Calculate the following weakest pre-conditions for the given conditional operations.*

1. $\{\texttt{if}\ x > 7\ \texttt{then}\ x := x - 4\ \texttt{else}\ x := x - 3\}(x > 12)$.
2. $\{\texttt{if}\ x > 7\ \texttt{then}\ y := x - 4\ \texttt{else}\ y := x - 3\}(y > x)$.
3. $\{\texttt{if}\ s > n\ \texttt{then}\ s := s + 1\ \texttt{else}\ n := n + 1\}(s \leq n)$.
4. $\{\texttt{if}\ x > y\ \texttt{then}\ y := y - x\ \texttt{else}\ x := x - y\}(x + y = 2)$.
5. $\{\texttt{if}\ m \in A\ \texttt{then}\ A := A - \{a\}\ \texttt{else}\ A := A \cup \{m\}\}(n \in A)$.
6. $\{\texttt{if}\ m \in A\ \texttt{then}\ A := A - \{a\}\ \texttt{else}\ A := A \cup \{m\}\}(A = \{m, n, a\})$.
7. $\{\texttt{if}\ m \in A\ \texttt{then}\ A := A - \{a\}\ \texttt{else}\ A := A \cup \{m\}\}(A = \{m\})$.
8. $\{\texttt{if}\ x > 7\ \texttt{then}\ (x := x - 4;\ y := x + 2)\ \texttt{else}\ y := y + 3\}(y > x)$.

Exercise 3 *Consider the following operation P:*

```
case x is
   when x mod 2 = 0 ⟹ a := ⊤
   when x mod 2 ≠ 0 ⟹ a := ⊥
end case;
```

Describe in words what P does. Calculate $\{P\}(a = \top)$.

Coding exercises

We saw the files for the `TicketMachine` in the lectures. In these exercises you are going to create specifications and bodies for procedures that extends the functionality of the `TicketMachine`.

Exercise 4 *Write a procedure* `serve_next`*. The interpretation of this procedure is that you have served person* `n`*, and so will move on to serving person* `n+1`*. The post-condition for the function is* `serving`$\leq$`next`*.*

Exercise 5 *Write a procedure* `return_ticket`*. The interpretation is that someone has taken a ticket, but before being served they decide to leave. Being a thoughtful person, they return the ticket to the front of the reel. The post-condition for the function is* `serving`$\leq$`next`*.*

Exercise 6 *Write a function called* `waiting`*. The interpretation of this function is that it returns the number of people who have taken a ticket, but have not yet been served. Determine a sensible post-condition for this function, and use it to find the weakest pre-condition.*