Task 1

Step 1. Prisoner's Dilemma (plain-language)

Players:

Two people (call them A and B) who must each choose at the same time, without seeing the other's choice.

Strategy sets:

Each person has two options: Cooperate or Defect.

	B: (C)	B: (D)
A: (C)	(1, 1)	(3, 0)
A: (D)	(0, 3)	(2, 2)

Payoff structure:

If you defect while the other cooperates, you personally get the best outcome right now, and the other person gets the worst outcome.

If you both cooperate, you each get a good, stable outcome—better for both of you than mutual betrayal, though not as individually tempting as exploiting the other.

If you both defect, you each get a mediocre or poor outcome—worse than mutual cooperation.

If you cooperate while the other defects, you get the worst outcome and the other person gets the best.

Why it's a "dilemma":

Individually, defecting looks safer no matter what the other might do, so both people are tempted to defect—yet that leads to a result that's worse for both than if they had both cooperated.

Classic story version:

Two suspects are interrogated separately. Confessing (defecting) can free you or reduce your sentence if the other stays silent, but if both confess, both are punished more than if both had stayed silent. Because they can't coordinate, both often confess, even though both staying silent would have been better.

Step 2. Compute the Equilibrium — Prisoner's Dilemma (your payoffs)

Payoff setup (from best to worst: 3 > 2 > 1 > 0)

	B: Cooperate (C)	B: Defect (D)
A: C	(2, 2)	(0, 3)
A: D	(3, 0)	(1, 1) ★

★ means the equilibrium point

Result: The unique pure-strategy Nash equilibrium is (D, D) with payoffs (1, 1).

Reason: For each player, Defect beats Cooperate no matter what the other does:

If the other cooperates, defecting gives you 3 comparing to cooperating 2. If the other defects, defecting gives you 1 comparing to cooperating 0. Because “Defect” is each player’s best response in both cases, both choose D in equilibrium—even though (C, C) = (2, 2) is better for both. So choosing D is the dominant strategy.

Prisoner's Dilemma — Payoff Matrix & Nash Equilibrium

	B: Cooperate (C)	B: Defect (D)
A: Cooperate (C)	(2, 2)	(0, 3)
A: Defect (D)	(3, 0)	(1, 1) ★

Step 3
https://colab.research.google.com/drive/14Uy1RMTSbKAP66606Vtdz4omKfYeYSyh?usp=sharing

Task 2

Definition (Nash equilibrium)

▶ DEFINITION 14.1 A **Nash equilibrium of a strategic game** $\langle N, (A_i), (\succsim_i)\rangle$ is a profile $a^* \in A$ of actions with the property that for every player $i \in N$ we have

$$(a^*_{-i}, a^*_i) \succsim_i (a^*_{-i}, a_i) \text{ for all } a_i \in A_i.$$

(p. 14)

Paraphrase (what/why):

A Nash equilibrium is a steady state: each player's move is best given the others' moves.
No one can gain by changing alone—so there's no unilateral incentive to deviate.
It's the baseline solution concept used across game theory.

Citation: Osborne, M. J., & Rubinstein, A. (1994). A Course in Game Theory, Def. 14.1, p. 14. sites.math.rutgers.edu

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Existence theorem

Theorem 1.22 *(Debreu, Glicksberg, Fan theorem for existence of pure strategy Nash equilibrium) Let* $(I, (S_i : i \in I), (u_i : i \in I))$ *be a game in normal form such that* I *is a finite set and*

(i) S_i *is a nonempty, compact, convex subset of* $\mathbb{R}^{n_i}$.

(ii) $u_i(s)$ *is continuous on* $S = S_1 \times \cdots \times S_n$.

(iii) $u_i(s_i, s_{-i})$ *is quasiconcave in* s_i *for any* s_{-i} *fixed.*

Paraphrase (what/why):

If each player's action set is nonempty, compact, convex, payoffs are continuous, and each payoff is quasi-concave in own action, then a pure-strategy Nash equilibrium exists.
Guarantees we're not searching an empty set; a solution does exist under mild regularity.
Built on fixed-point logic (Kakutani), linking economics to topology.
Lets us study comparative statics and welfare without constructing the equilibrium explicitly.

Citation: Hajek, B. (2018). An Introduction to Game Theory, Thm. 1.22 (Debreu–Glicksberg–Fan), pp. 14. courses.grainger.illinois.edu

Computational complexity

people.csail.mit.edu (p. 62)

Theorem 13 ([7]) 2-NASH *is* PPAD-*complete.*

Proof. Let us define d-ADDITIVE GRAPHICAL NASH to be the problem d-GRAPHICAL NASH restricted to *bipartite graphical games with additive utility functions* defined next.

Paraphrase (what/why):

Finding a Nash equilibrium in a two-player (bimatrix) game is PPAD-complete。 So in general, no efficient algorithm is known (and one would have deep implications for complexity theory).

Existence ≠ tractability: equilibria exist but may be hard to compute.

Spurs focus on approximation, special structure (e.g., zero-sum), or alternative concepts (e.g., correlated equilibrium).

Citation: Daskalakis, C., Goldberg, P. W., & Papadimitriou, C. H. (2009). The Complexity of Computing a Nash Equilibrium, JACM, Thm. 13, p. 62 (citing Chen & Deng). people.csail.mit.edu

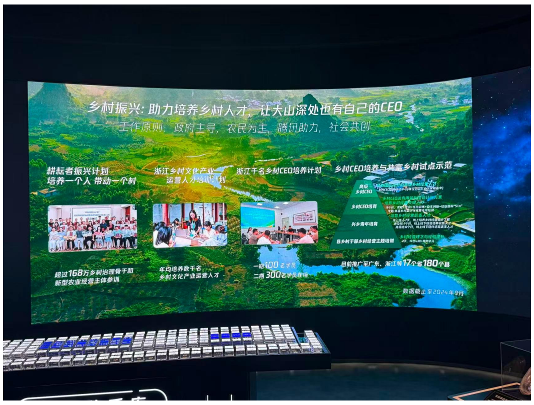
Flowchart (big picture)
Definition → Existence → Complexity

Nash equilibrium (what it is) ➜Conditions guaranteeing an equilibrium exists (DG-F theorem) ➜How hard it is to compute one in practice (PPAD-complete)

Existence theorems reassure us that strategic models are well-posed under broad regularity。 There is at least one equilibrium. But computational complexity results draw a sharp boundary around practicality. PPAD-complete for two-player games means that even though an equilibrium exists, we generally can't compute one efficiently.

For computational economics, this duality reshapes methodology. We either (i) exploit structure (zero-sum, potential games, convexity) where equilibria are computable; (ii) seek approximations with provable guarantees; or (iii) adopt alternative concepts (e.g., correlated equilibrium) that remain computationally tractable. In short, existence tells us equilibria are there; complexity tells us where to lookand when to change the question

Task 3



Computational game theory can design carbon-permit allocation that anticipates strategic emission, leakage, and collusion. **Promise:** efficient abatement, stable prices, fairness constraints inside the mechanism. **Peril:** opaque models amplify data gaps; firms game reports; cross-border effects misfire.

Convergence: both U.S. and China use AI simulation, cloud analytics, and near-real-time compliance. **Divergence:** the U.S. favors market auctions and open disclosure with civil-society scrutiny; China's ETS relies more on benchmarking, phased sectors, administrative coordination, and tighter data access.

Ethics & SDGs: embed distributional caps and transfers (SDG 10); require transparent algorithms, audit trails, and due process in enforcement (SDG 16). Publish stress tests, document assumptions, protect privacy, and include frontline communities. Done well, CGT delivers credible markets and cleaner air; done poorly, it scales inequity.

How can we together do better? Co-design carbon-market mechanisms with regulators, firms, and communities; publish reproducible code and data; run cross-border stress tests; adopt governance checklists (impact, privacy, fairness) and red-team the incentives.

Liberal arts for responsible innovation. Critical reflection asks "what are we optimizing, for whom, and at what cost?" Interdisciplinarity joins economics, CS, law, and ethics to align mechanism design with institutions. Ethics training normalizes transparency reports, bias audits, and stakeholder consent.

DKU's joint model. DKU's U.S.–China partnership builds bilingual, cross-cultural teams; studio courses link theory to field data; policy labs pair students with agencies and industry. Shared standards for replication and open communication cultivate leaders who can translate computational findings into equitable, lawful, and globally workable carbon-market policies.