GAMES 101 大作业

次表面散射

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求表面所有点对出射点的贡献

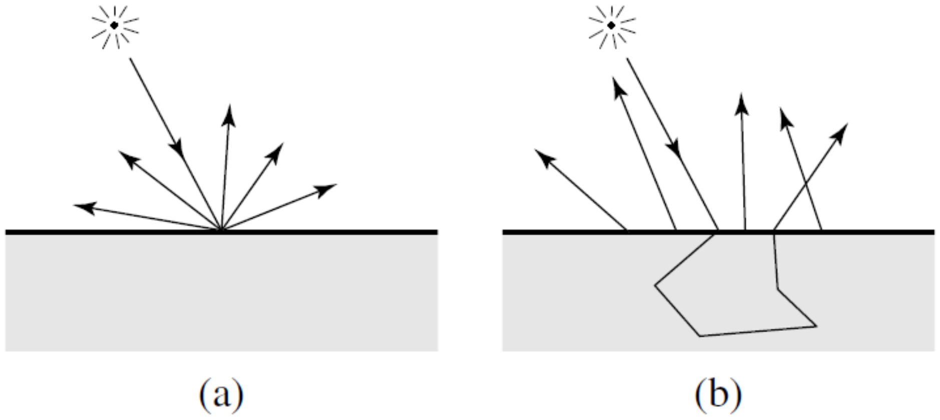


Figure 1: Scattering of light in (a) a BRDF, and (b) a BSSRDF.

蒙特卡洛方法求积分
S？
采样？

$$L_o(x_o,\vec{\omega}_o)=\int_A\int_{2\pi} S(x_i,\vec{\omega}_i;x_o,\vec{\omega}_o)\,L_i(x_i,\vec{\omega}_i)\,(\vec{n}\cdot\vec{\omega}_i)\,d\omega_i dA(x_i).$$

来自： Jensen, Henrik Wann, et al. [“A practical model for subsurface light transport.”] SIGGRAPH 2001.

Normalized Diffusion

$$S(x_i, w_i; x_o, w_o) = C\, F_t(x_i, w_i)\, R(|x_o - x_i|)\, F_t(x_o, w_o) \quad (1)$$

$$R(r) = \frac{e^{-r/d} + e^{-r/(3d)}}{8\,\pi\, d\, r} \quad .$$

Fresnel transmit

如何求d?

来自：Christensen, Per H., and Brent Burley. *“Approximate reflectance profiles for efficient subsurface scattering.”* Technical Report 15-04, Pixar, 2015.

dmfp as parameter

$\alpha' = \sigma_s' / \sigma_t'$ $\sigma_s' = \sigma_s \times (1-g)$ $\sigma_t' = \sigma_s' + \sigma_a$

$d = \frac{l_d}{3.5 + 100(A-0.33)^4}$。

平均散射距离 $l_d \approx 1/\sigma_{tr}$，而 $\sigma_t' = \frac{\sigma_{tr}}{\sqrt{3(1-\alpha')}}$。

A取diffuse color

Material	σ_s' [mm^{-1}]			σ_a [mm^{-1}]			Diffuse Reflectance			η
	R	G	B	R	G	B	R	G	B	
Apple	2.29	2.39	1.97	0.0030	0.0034	0.046	0.85	0.84	0.53	1.3
Chicken1	0.15	0.21	0.38	0.015	0.077	0.19	0.31	0.15	0.10	1.3
Chicken2	0.19	0.25	0.32	0.018	0.088	0.20	0.32	0.16	0.10	1.3
Cream	7.38	5.47	3.15	0.0002	0.0028	0.0163	0.98	0.90	0.73	1.3
Ketchup	0.18	0.07	0.03	0.061	0.97	1.45	0.16	0.01	0.00	1.3
Marble	2.19	2.62	3.00	0.0021	0.0041	0.0071	0.83	0.79	0.75	1.5

来自：https://zhuanlan.zhihu.com/p/21247702

投射采样法

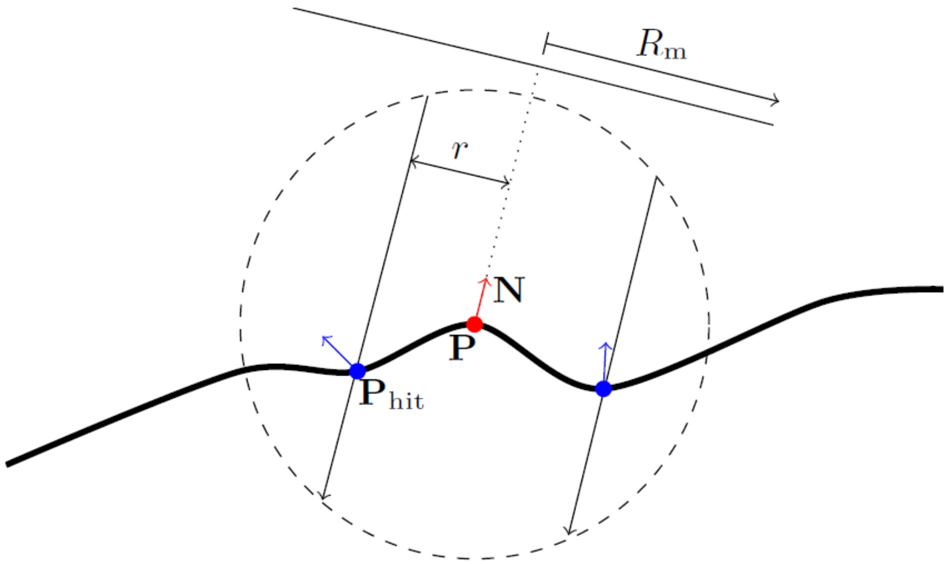


Figure 2: *Geometric setup for Equation 1.*

- 确定球面半径Rm，入射点必须在球体内。
- 在[0,Rm]中，以某种分布方式采样r。
- 以P圆心，r为半径，法线N为对称轴，作一个圆柱面，与物体表面相交。入射点就在这条交线上。
- 随机一个角度，确定入射点。

r？
Pdf？
项目最难处

$$\frac{R_d\left(\|\mathbf{P}_{\mathrm{hit}}-\mathbf{P}\|\right)}{\mathrm{pdf}_{\mathrm{disk}}(r)}\frac{1}{|\mathbf{V}\cdot\mathbf{N}_{\mathrm{hit}}|} \qquad (1)$$

来自：King, Alan, et al. *“BSSRDF importance sampling.”* SIGGRAPH Talks, 2013.

Inverse Sampling

Recall S_r:

$$S_r = \rho_{eff} \frac{e^{-r/d} + e^{-r/(3d)}}{8\pi d r} = \rho_{eff} S'_r \tag{16}$$

S'_r satisfies: (integration in polar coordinates is always 1)

$$\int\int_D S'_r(r)\,\mathrm{d}A = \int_0^\infty \int_0^{2\pi} S'_r(r) r\,\mathrm{d}r\mathrm{d}\phi = \int_0^\infty S(r)' 2\pi r\,\mathrm{d}r = 1 \tag{17}$$

So the desired PDF is proportional to S_r.

Assume $PDF = cS_r$,

$$\int\int_D PDF\,\mathrm{d}A = \int_0^\infty \int_0^{2\pi} c\,S_r\,r\,\mathrm{d}r\mathrm{d}\phi = \int_0^\infty cS_r 2\pi r\,\mathrm{d}r = c\rho_{eff} \int_0^\infty S'_r 2\pi r\,\mathrm{d}r = c\rho_{eff} = 1 \tag{18}$$

$$c = 1/\rho_{eff}$$

So the PDF is $cS_r = S'_r$

And the CDF is:

$$\begin{aligned} CDF &= \int\int_D S'_r \mathrm{d}A \\ &= \int_0^r \int_0^{2\pi} r S'_r \mathrm{d}r\mathrm{d}\phi \\ &= \int_0^r 2\pi r S'_r \mathrm{d}r \\ &= \frac{1}{4}(4 - e^{-r/d} - 3e^{-r/(3d)}) \\ &= 1 - \frac{1}{4}e^{-r/d} - \frac{3}{4}e^{-r/(3d)} \end{aligned}$$

However, the CDF is not analytically invertible.

2. Precompute the $CDF^{-1}(r)$ when $d = 1$, and multiply d in rendering.

$$\begin{aligned} CDF &= 1 - \frac{1}{4}e^{-r/d} - \frac{3}{4}e^{-r/(3d)} \\ &\overset{d=1}{=\!=} 1 - \frac{1}{4}e^{-r} - \frac{3}{4}e^{-r/3} \end{aligned}$$

$CDF^{-1} : \xi => r$

[0,1]中随机一个数u, r = $cdf^{-1}(u)$ * d
计算pdf_{disk} = Sr'(r)

随机一个channel
计算$pdf_{channel}$ = 1/3

最终Pdf = $pdf_{disk}\ pdf_{channel}\ |V \cdot N_{hit}|$

来自：https://gao-duan.github.io/blogs/bssrdf/index.html

Multiple Importance Sampling

$$\int_0^{R_{max}} p_1(r)\, 2\pi\, r\, dr = 1$$

$$\int_0^{R_{max}} c\, \frac{e^{-r/d}}{8\pi\, d\, r} 2\pi\, r\, dr = \int_0^{R_{max}} \frac{c}{4d} e^{-r/d}\, dr = \frac{-c}{4}(e^{\frac{-R_{max}}{d}} - 1) = 1$$

$$c = \frac{4}{1 - e^{\frac{-R_{max}}{d}}} \implies p_1(r) = \frac{4}{1 - e^{\frac{-R_{max}}{d}}} \frac{e^{-r/d}}{8\pi\, d\, r} = \frac{1}{1 - e^{\frac{-R_{max}}{d}}} \frac{e^{-r/d}}{2\pi\, d\, r}$$

$$\begin{aligned} P_1(r) &= \int_0^r p_1(r')\, 2\pi\, r'\, dr' \\ &= \int_0^r \frac{1}{1 - e^{\frac{-R_{max}}{d}}} \frac{e^{-r'/d}}{d} dr' \\ &= \frac{-1}{1 - e^{\frac{-R_{max}}{d}}} (e^{\frac{-r}{d}} - 1) \end{aligned}$$

策略1

$$r = \log(1 - \xi(1 - e^{\frac{-R_{max}}{d}})) \times -d$$

$$p_2(r) = \frac{e^{\frac{-r}{3d}}}{2\pi\, d\, r} \frac{1}{3(1 - e^{\frac{-R_{max}}{3d}})},$$

$$r = \log(1 - \xi(1 - e^{\frac{-R_{max}}{3d}})) \times -3d$$

策略2

确定Rm
以w1，w2为权值随机产生策略
根据策略采样出对应的r

$$w_1 = 1 - e^{\frac{-R_{max}}{d}}$$

$$w_2 = 3(1 - e^{\frac{-R_{max}}{3d}})$$

$pdf_{disk} = P_1(r) + P_2(r)$
策略1：$pdf_{strategy} = w1 / (w1 + w2)$
策略2：$pdf_{strategy} = w2 / (w1 + w2)$

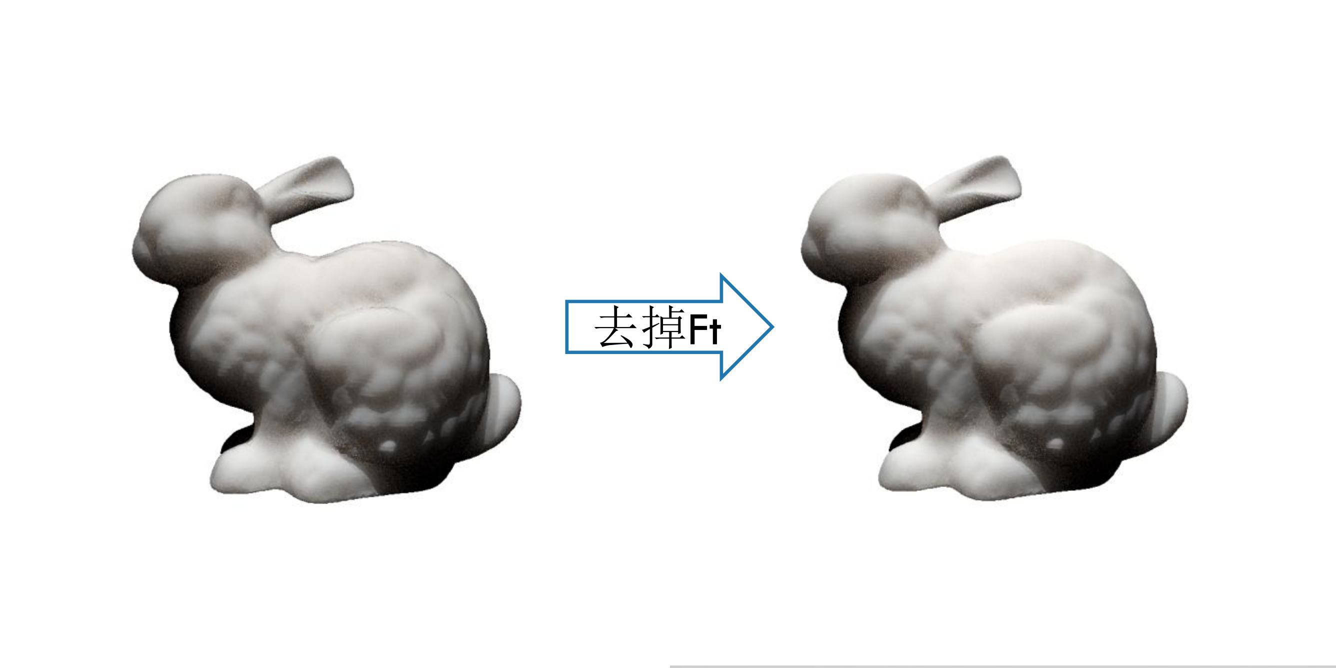
随机一个channel
计算$pdf_{channel} = 1/3$

最终Pdf = $pdf_{disk}\, pdf_{strategy}\, pdf_{channel}\, |V \cdot N_{hit}|$

来自： http://shihchinw.github.io/2015/10/bssrdf-importance-sampling-of-normalized-diffusion.html

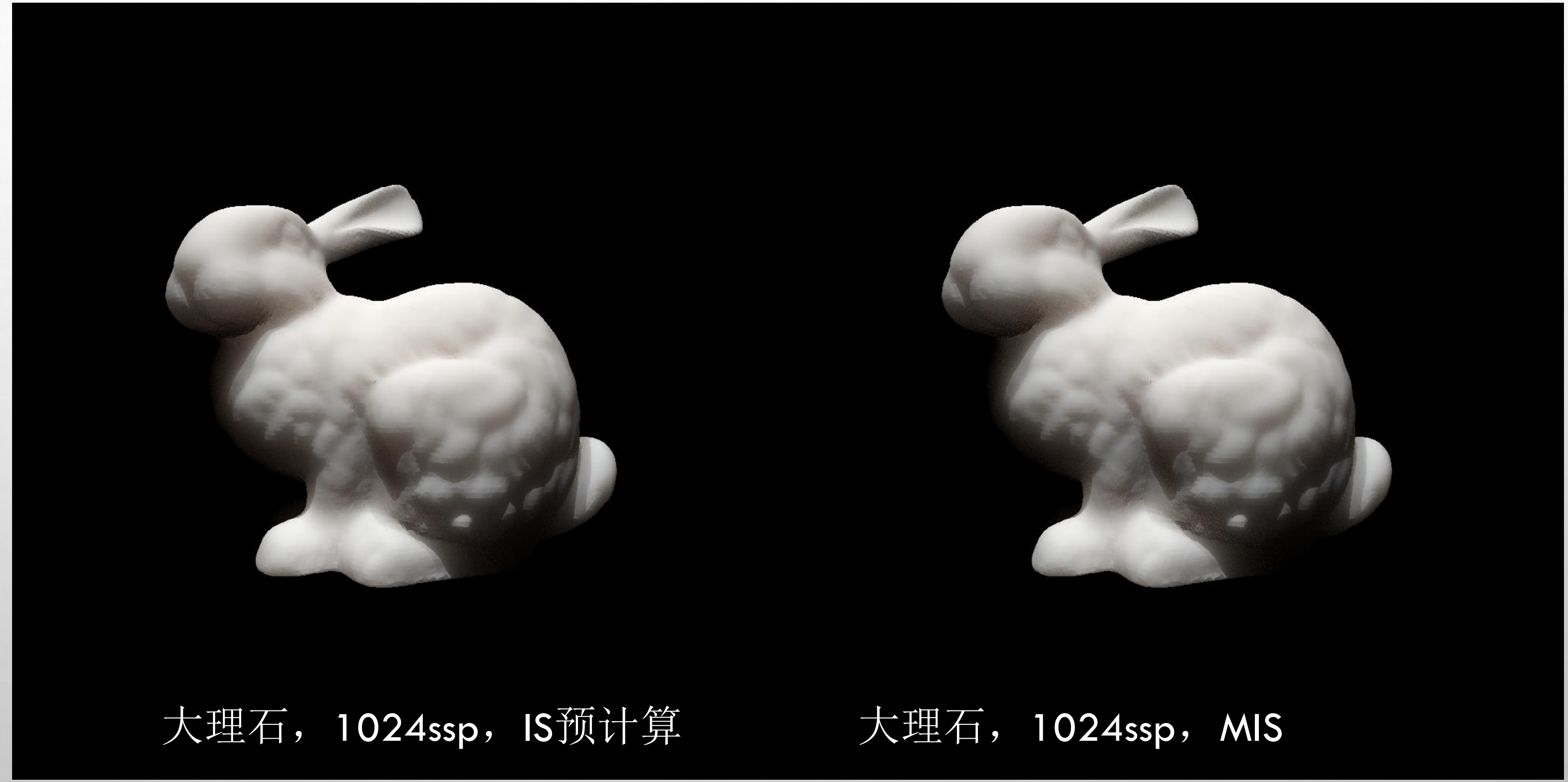
出射Fresnel transmit

$$S(x_i, w_i; x_o, w_o) = C\,\cancel{F_t(x_i, w_i)}\,R(|x_o - x_i|)\,F_t(x_o, w_o) \quad (1)$$



图像周围有黑边，符合原理，因为掠射角的Reflect很高，Transmit很小。要想得到更好的效果，需要配合反射模型。此次项目简单的忽略出射Ft。

最终效果



参考图片来自：Christensen, Per H., and Brent Burley. ["Approximate reflectance profiles for efficient subsurface scattering."]() Technical Report 15-04, Pixar, 2015.

谢谢！