Boat lab

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Contents

Introduction

In this lab we were assigned multiple problems associated with control of a simulated ship.
The ship is simulated in MATLAB and Simulink. Control of the ship is done through MATLAB scripts and Simulink.
We were assigned different problems raging from identifying the boat parameters, creating a wave spectrum model of wave disturbance. We made a PD controller for the ship and did an analysis of observability for the ship. In the end we created, and tuned, a Discrete Kalman Filter for estimating the states of the ship.
Labs like this are important in education of control engineers. Practical experience is essential to obtain a good understanding of a subject, and getting problems like this is therefore good. This lab grants practical experience on how to apply theory, tuning, and control systems in general.
This report is organized into multiple parts. The report follows the assignment manual closely, and answers the questions prompted.

1 Part 5.1 Identification of the boat parameters

1.a Calculating the transfer function

In this part of the assignment we want to look at the system without wave disturbances or measurement noise. We want to find the transfer function, $H(s)$, from δ to ψ, where ψ heading angle of the ship, relative to north and δ is the rudder angle relative to the body. From [1], and assuming no disturbances, we get the equations

$$\dot{\psi} = r \tag{1a}$$

$$\dot{r} = -\frac{1}{T}r + \frac{K}{T}\delta \tag{1b}$$

where r is the rate of change for the average heading, T is the time constant and K is a constant gain. Taking the Laplace transform of eq. (1b) we get

$$\begin{aligned}
\mathcal{L}\{\dot{r}\}(s) &= \mathcal{L}\{-\frac{1}{T}r + \frac{K}{T}\delta\}(s) \\
sr &= -\frac{1}{T}r + \frac{K}{T}\delta \\
r &= \frac{K}{T}\frac{\delta}{s + \frac{1}{T}}
\end{aligned} \tag{2}$$

and taking the Laplace tranform of eq. (1a) we get

$$\begin{aligned}
\mathcal{L}\{\dot{\psi}\}(s) &= \mathcal{L}\{r\}(s) \\
\psi s &= r
\end{aligned} \tag{3}$$

Substituting eq. (3) into eq. (2) we get

$$H(s) = \frac{\psi}{\delta}(s) = \frac{K}{s(Ts+1)} \tag{4}$$

where $H(s)$ is the transfer function from δ to ψ. The transfer function consist of one pure integrator and a first order dynamic. Having a pure integrator means that for a constant rudder angle the heading will increase linearly and the ship would sail in circles. The first order dynamic represents the fact that a ship has inertia and a change in rudder angle doesn't change the heading immediately.

1.b Estimating the model parameters

Now we want to identify the parameters K and T. We can do this by applying two different sine inputs with different frequencies. Thus, using the transfer function and the measured amplitudes of the response, we can calculate the parameters. This is done in smooth water conditions, meaning

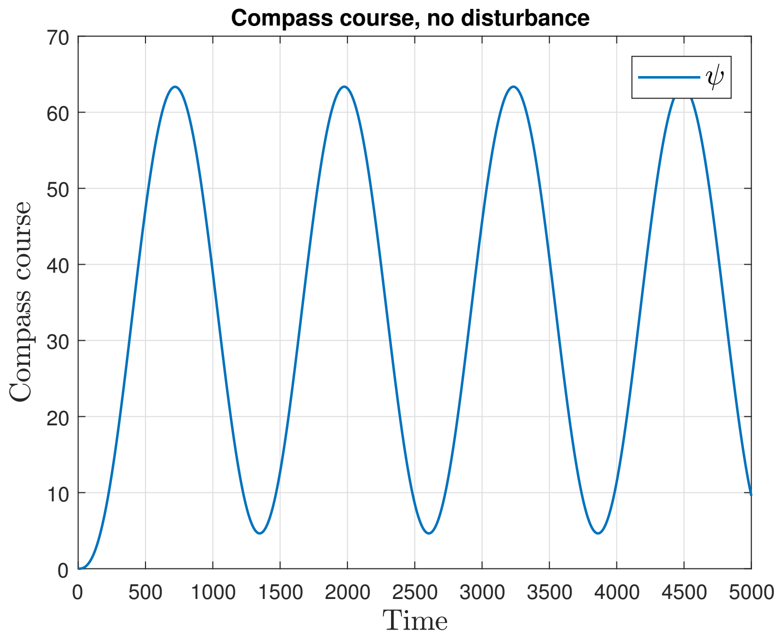

Figure 1: Compass course response. Rudder input is sine with the frequency of ω_1

that there is no wave or current disturbance on the system. We apply the two sines with frequencies of

$$\omega_1 = 0.005$$
$$\omega_2 = 0.05$$

and measure the amplitudes as seen in fig. 1 and fig. 2.

$$\left|H(j\omega_1)\right| = A_1 = 29.35$$
$$\left|H(j\omega_2)\right| = A_2 = 0.838$$

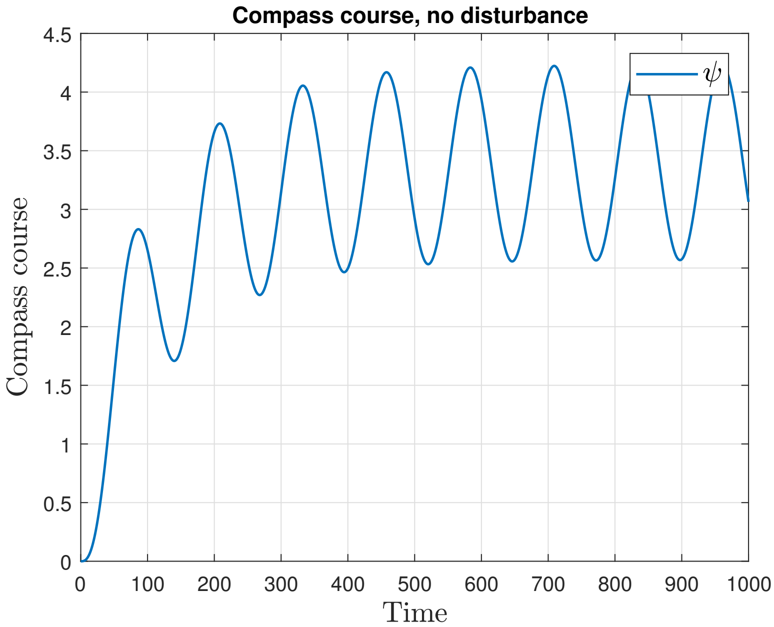

Figure 2: Compass course response. Rudder input is sine with the frequency of ω_2

where A_1 and A_2 are the amplitudes of the transfer function at ω_1 and ω_2. By solving the transfer function for K we get

$$
\begin{aligned}
\left|H(j\omega_1)\right| &= A_1 \\
\left|\frac{K}{j\omega_1(Tj\omega_1 + 1)}\right| &= A_1 \\
\frac{K}{\left|j\omega_1(Tj\omega_1 + 1)\right|} &= A_1 \\
\frac{K}{\left|(-T\omega_1^2 + j\omega_1)\right|} &= A_1 \\
\frac{K}{\left|(-T\omega_1^2 + j\omega_1)\right|} &= A_1 \\
K &= A_1\sqrt{T^2\omega_1^4 + \omega_1^2} \\
K &= A_1\omega_1\sqrt{T^2\omega_1^2 + 1}
\end{aligned}
$$

Inserting for K and solving for T we get

$$
\begin{aligned}
\left|H(j\omega_2)\right| &= A_2 \\
\left|\frac{K}{j\omega_2(Tj\omega_2+1)}\right| &= A_2 \\
\frac{K}{\left|(-T\omega_2^2+j\omega_2)\right|} &= A_2 \\
A_1\omega_1\sqrt{T^2\omega_1^2+1} &= A_2\omega_2\sqrt{T^2\omega_2^2+1} \\
A_1^2\omega_1^2(T^2\omega_1^2+1) &= A_2^2\omega_2^2(T^2\omega_2^2+1) \\
(A_1^2\omega_1^4 - A_2^2\omega_2^4)T^2 &= A_2^2\omega_2^2 - A_1^2\omega_1^2 \\
T &= \sqrt{\frac{A_2^2\omega_2^2 - A_1^2\omega_1^2}{A_1^2\omega_1^4 - A_2^2\omega_2^4}}
\end{aligned}
$$

Thus, inserting numerical values, we get that

$$K = 0.1559 \tag{5a}$$
$$T = 71.6865 \tag{5b}$$

1.c Estimating with waves measurement noise

Now we are to repeat the procedure of calculating T and K, but now in rough water conditions, with wave and current disturbance turned on. Setting the measurement cursors at the peak without noise. Repeating the procedure we get

$$K = 0.3572 \tag{6a}$$
$$T = 444.6904 \tag{6b}$$

When trying to estimate the model parameters in rough weather conditions, mainly when we apply an input with the frequency $\omega_2 = 0.05$ it is very hard to distinguish the signal from the wave disturbance as we can see in fig. 4. When the system has an input with the frequency of $\omega_1 = 0.005$ it is much easier to distinguish the signal from the wave disturbance. Thus the resulting parameters don't match up with the estimated estimated parameters in calm waters and must be considered useless.

1.d Comparing the estimated model to the ship

We apply a step of rudder angle on the ship model eq. (4) as well a a step on the full system model in Simulink. The step is of one degree. The responses are plotted and compared in Figure 5. To begin with the model is pretty

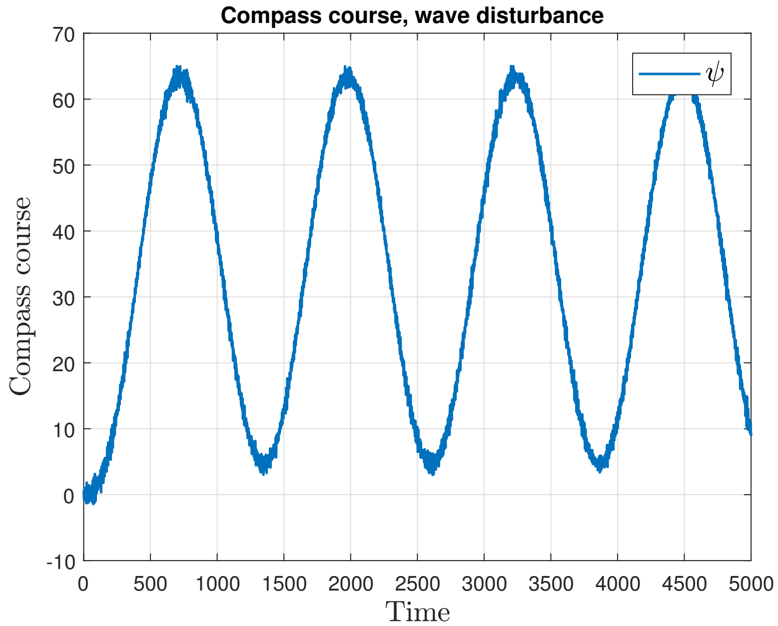

Figure 3: Compass course response, with wave and current disturbance. Rudder input is sine with frequency of ω_1

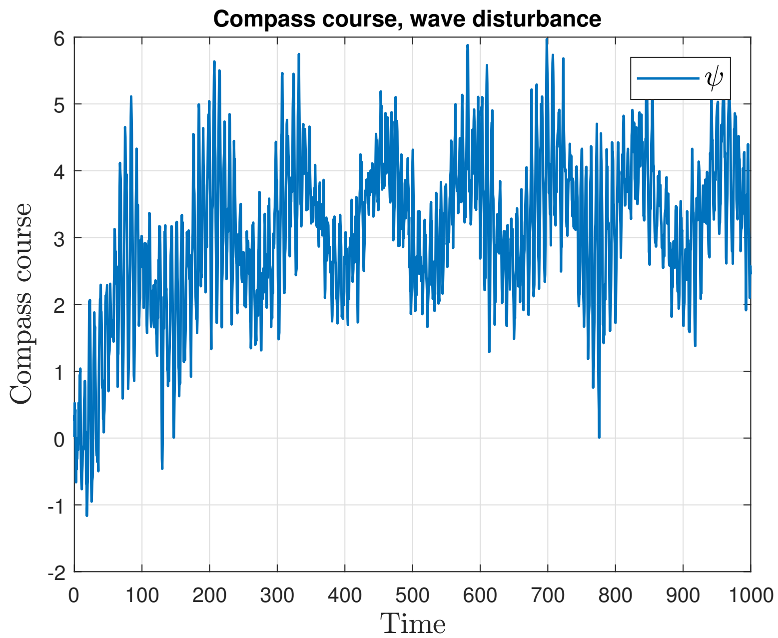

Figure 4: Compass course response, with waves and current disturbance. Rudder input is sine with frequency of ω_2

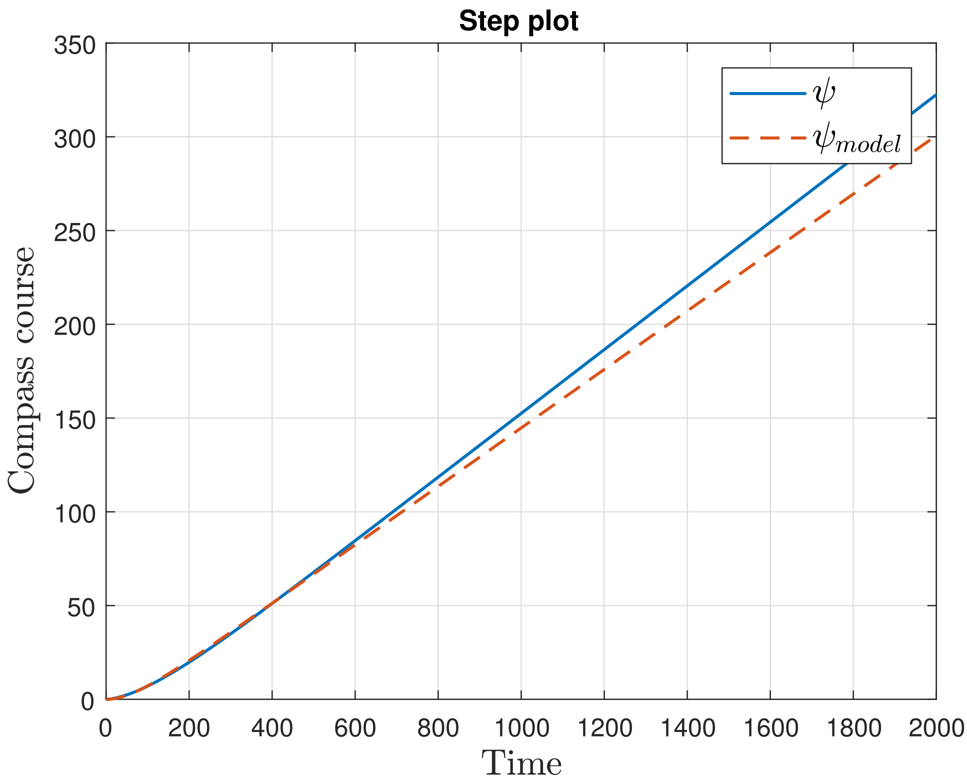

Figure 5: Step response of the ship and the ship model.

accurate, but as time goes the model starts to deviate from the response of the ship. For our use the model is a good enough approximation since its most important that the model holds for small values of time. If we did need a more accurate model we could model the physical system in more detail.

2 Part 5.2 Identification of the wave spectrum model

2.a Estimating the Power Spectral Density function of ψ_w

In this part we wish to estimate the Power Spectral Density- (PSD) function of how the waves impact the heading. Given data of how the waves influence the compass measurement, ψ_w , we can find the PSD-function, named $S_{\psi_w}(\omega)$, using MATLAB. In listing 1 we show how we calculated the PSD-function.
psi_w is a data series of ψ_w, given in degrees. We scale it to radians.

Listing 1: Calculating Power Spectral Density function

```
x = psi_w(2,:)*pi/180;
fs = 10;
[pxx,f] = pwelch(x,window, [], [], fs);
pxx=pxx./(2*pi);
f=f.*2*pi;
```

Plotting (pxx, f), which is S_{ψ_w}, we get Figure 6.

2.b Analytical expression for the Power Spectral Density

Analytical expression for the transfer function of the wave response model From [1] we have the Namoto equations

$$\dot{\xi}_w = \psi_w \tag{7a}$$
$$\dot{\psi}_w = -\omega_0^2 \xi_w - w\omega_0 \psi_w + K_w w_w \tag{7b}$$

Taking the Laplace transform of these and inserting 7a into 7b we get

$$\begin{aligned}
s\xi_w &= \psi_w \\
s\psi_w &= -\omega_0^2 \frac{\psi}{s} - w\omega_0\psi_w + K_w w_w \\
(s^2 + 2\lambda\omega_0 + \omega_0^2)\psi_w &= K_w w_w \\
\frac{\psi_w}{w_w}(s) &= \frac{K_w s}{s^2 + 2\omega_0 s + \omega_0^2} \\
\hat{H}(j\omega) &= \frac{K_w s}{s^2 + 2\omega_0 s + \omega_0^2}
\end{aligned}$$

where $\hat{H}(j\omega)$ is the transfer function from w_w to ψ_w. Using the Wiener-Khinchin theorem [4] we can find an analytical expression for the Power Spectral Density function for ψ_w. Using that w_w is a zero mean white noise, we know that the Power Spectral Density function of w_w, called P_{w_w}, is 1.

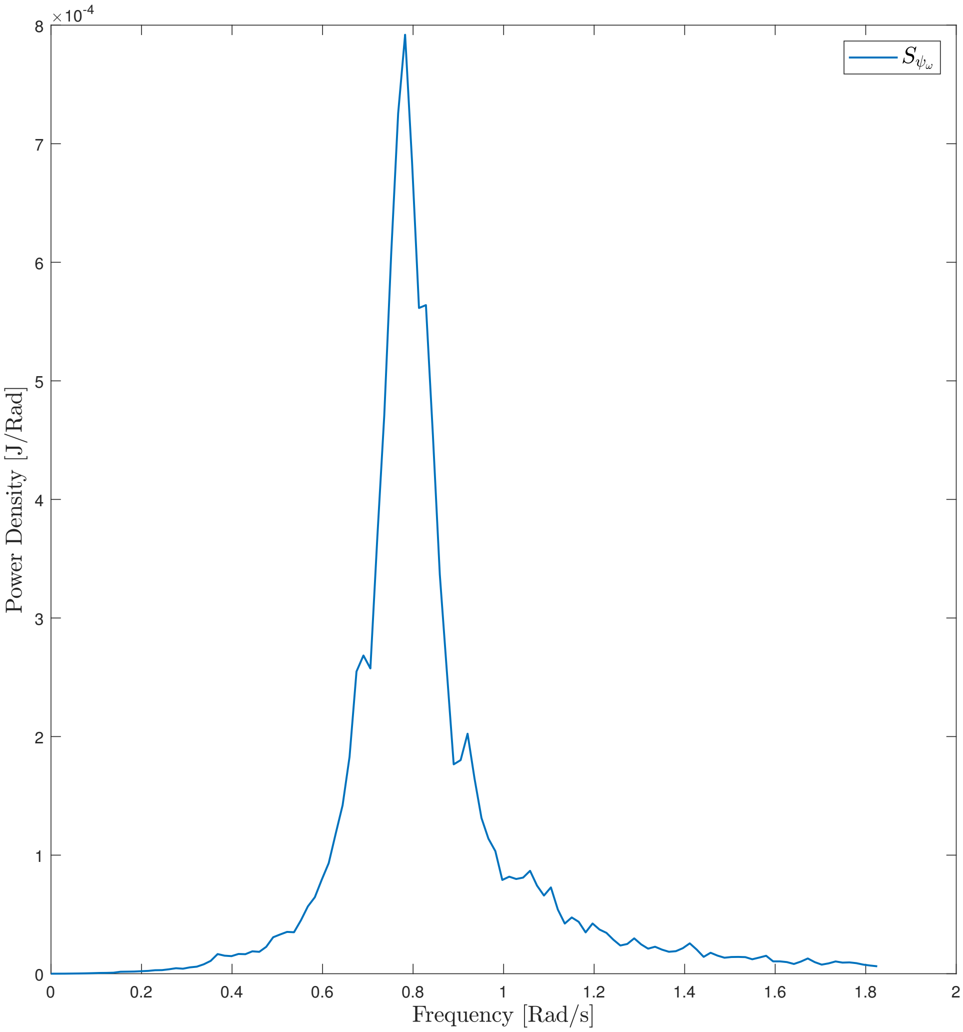

Figure 6: Power Spectral Density function for compass measurement

Thus we get that

$$\begin{aligned}
P_{\psi_w} &= P_{w_w} \left| \hat{H}(j\omega) \right|^2 \\
&= \hat{H}(-j\omega)\hat{H}(j\omega) \\
&= \frac{(K_w j\omega)(-K_w j\omega)}{((j\omega)^2 + 2\omega_0 j\omega + \omega_0^2)(-j\omega)^2 - 2\omega_0 j\omega + \omega_0^2)} \\
&= \frac{K_w^2 \omega^2}{\omega^4 + \omega^2\omega_0^2(4\lambda^2 - 2) + \omega_0^4}
\end{aligned}$$

2.c Finding ω_0 and σ^2

Reading the values from fig. 6 we get

$$\omega_0 = 0.07823$$
$$\sigma = 0.0281$$

where ω_0 is the frequency of peak intensity, and σ^2 is the peak frequency.

2.d Identifying the dampening factor λ

We want to identify the damping factor λ. Using

$$K_w = 2\lambda\omega_0\sigma$$

we get

$$P_{\psi_w} = \frac{4\lambda^2\omega_0^2\sigma^2\omega^2}{\omega^4 + (4\lambda^2 - 2)\omega_0^2\omega^2 + \omega_0^4}$$

By plotting P_{ψ_w} versus S_{ψ_w} and adjusting λ until the plots overlay we find an estimate for λ. Thus we found that a decent value for lambda is $\lambda = 0.09$. This value gave P_{ψ_w} shown in Figure 7 plotted versus S_{ψ_w}.

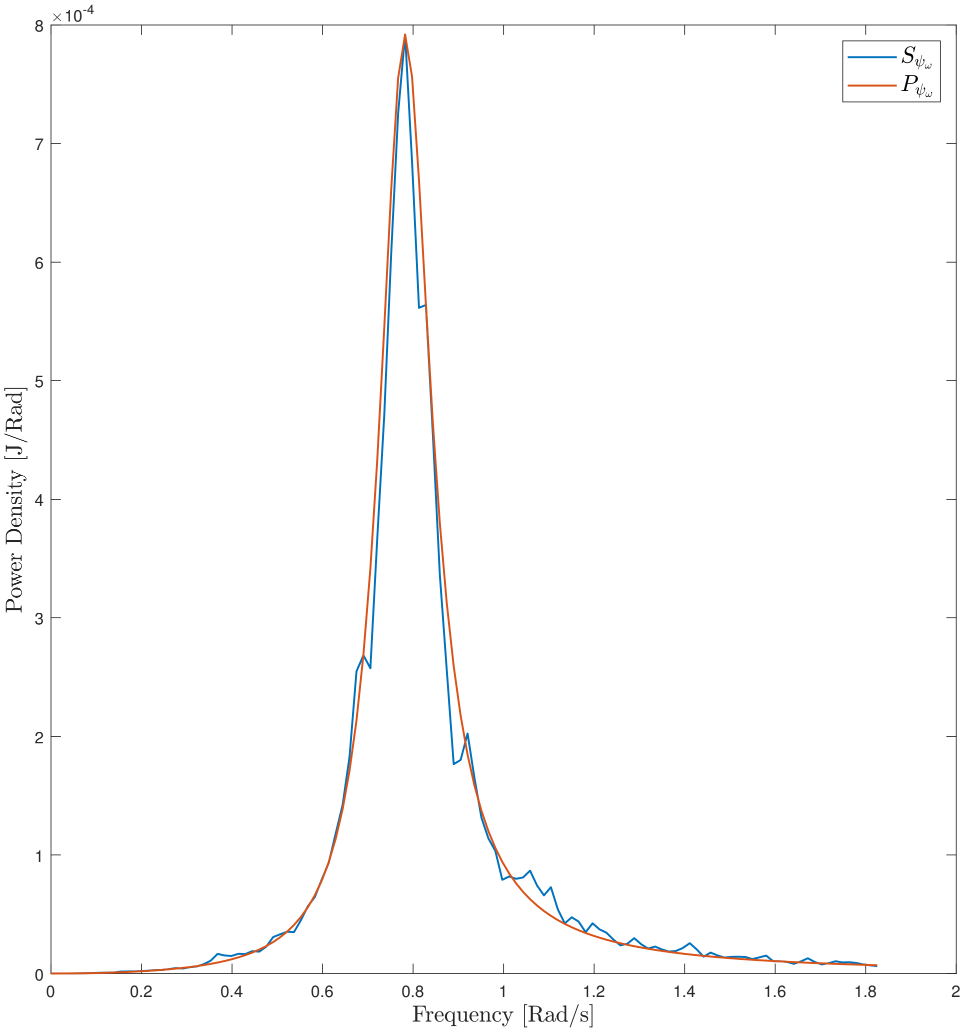

Figure 7: P_{ψ_w} plotted versus S_{ψ_w}, $\lambda = 0.09$

3 5.3 Control system design

3.a Design of a PD controller

A PD controller is given by the transfer function $H_{pd}(s) = K_{pd}\frac{1+T_d s}{1+T_f s}$. Using the transfer function eq. (4) from δ to ψ without disturbances we define the open loop system as $H(s) \cdot H_{pd}(s)$. We chose $T_d = T$ to cancel the transfer function time constant. This reduces the order of the transfer function because the two terms cancel out. The open loop transfer function then becomes

$$H(s) = \frac{K_{pd}K}{s(1+T_f s)} \tag{8}$$

We wish to chose K_{pd} such that the open loop system has $\omega_c = 0.10\,\mathrm{rad/s}$ and a phase margin of 50°.

$$\begin{aligned} \left|H(j\omega_c)\right| &= 0dB = 1 \\ \left|\frac{K_{pd}K}{j\omega_c(1+T_f j\omega_c)}\right| &= 1 \\ \frac{K_{pd}K}{\omega_c\sqrt{1+T_f^2\omega_c^2}} &= 1 \end{aligned} \tag{9}$$

The phase margin ϕ is defined as

$$\phi = \angle H(j\omega_c) - (-180°) \tag{10}$$

and by setting ϕ equal to 50° we get

$$\begin{aligned} 50° &= -90° - \arctan(T_f\omega_c) \\ T_f = \frac{\tan(50° - 180° + 90°}{\omega_c} &= 8.4\,\mathrm{s} \end{aligned} \tag{11}$$

Now by inserting the obtained T_f into eq. (9) we can solve for K_{pd}.

$$K_{pd} = \frac{\omega_c\sqrt{1+T_f^2\omega_c^2}}{K} = 0.839\,\mathrm{s}^{-1} \tag{12}$$

3.b Implementation of PD controller and simulation without disturbances

We implement the PD controller in Simulink as seen in fig. B.2 and fig. B.1. We set the reference $\psi_r = 30°$ and simulate the system with only measurement noise, no disturbances. As we can see in fig. 8 the ship reaches the reference pretty quickly and is able to keep the course stable. Thus the

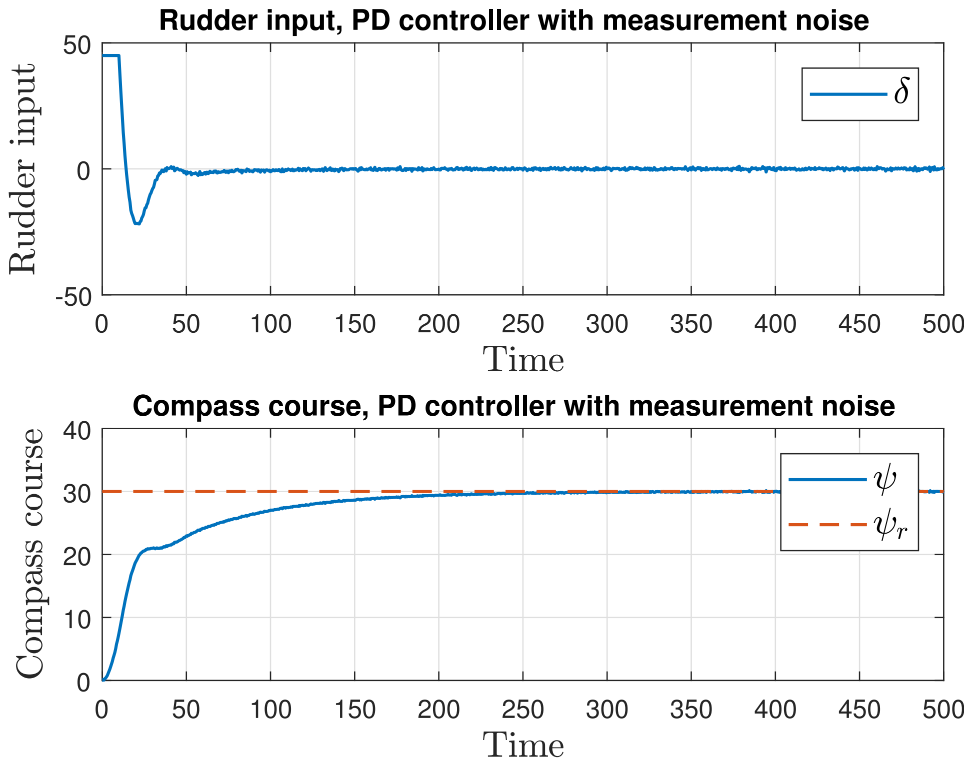

Figure 8: Rudder input and compass course response with a PD controller.

autopilot does its job quite satisfyingly. The rudder input is high in the beginning but as the system stabilizes the size of the rudder input input moves to zero. It does however have some small vibrations due to the controller being sensitive to the measurement noise. The ship dynamics has a quite negative phase which results in either a slow controller or an oscillating response due to a small phase margin. A PD-controller lifts the phase of the full system and allows for higher ω_c resulting in quicker and more robust response than a P-controller.

3.c Simulation with a current disturbance

Now we simulate the system with current disturbance and measurement noise but no wave disturbance. As we can see in the fig. 9, the autopilot is no longer able to reach the reference of 30°. A stationary deviation of about 3.5° is present. This is expected as we have no integral effect and the PD controller will only change the rudder input if the error changes. The rudder input never reaches zero but constantly tries to counteract the current disturbance.

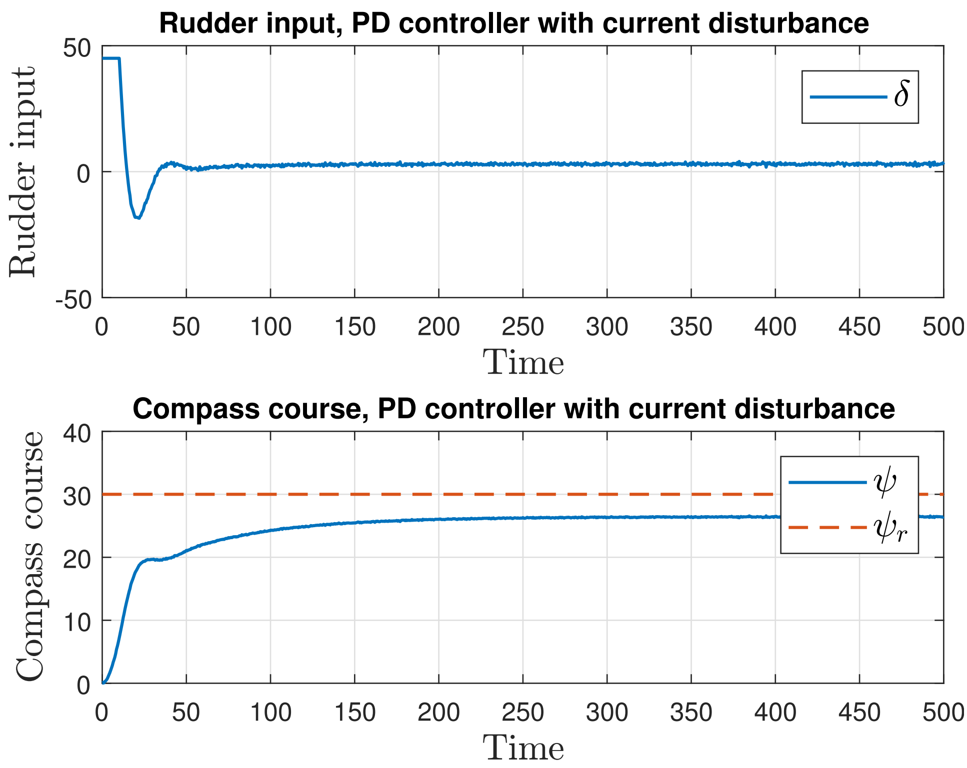

Figure 9: Rudder input and compass course response with a PD controller. Simulated with current disturbance.

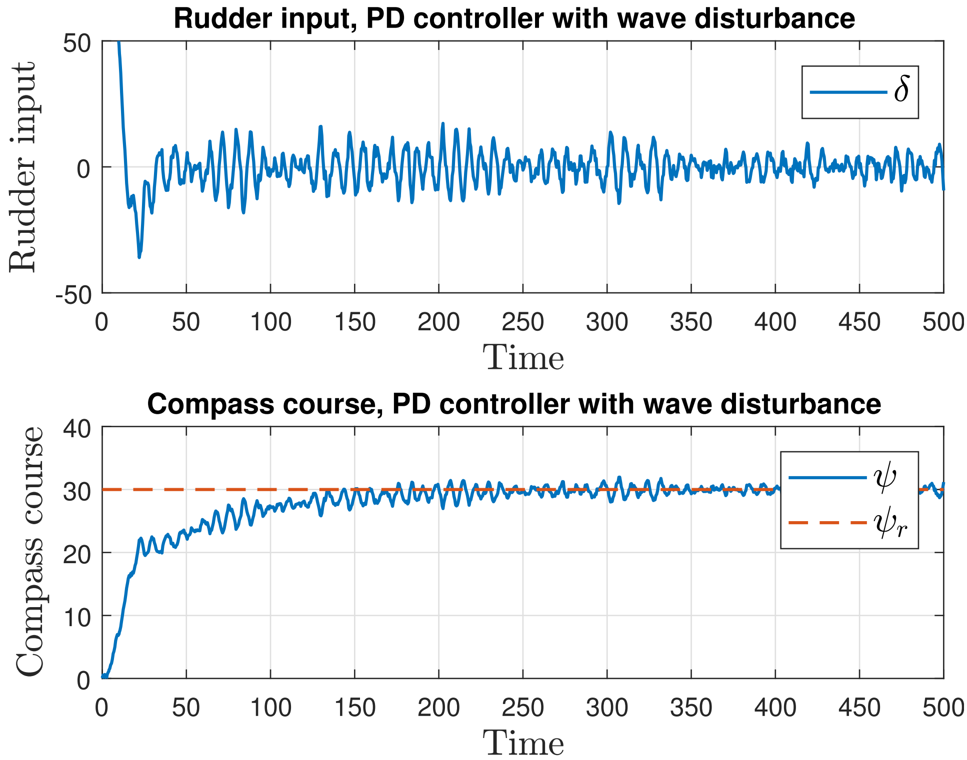

Figure 10: Rudder input and compass course response with a PD controller. Simlutated with wave disturbance.

3.d Simulation with a wave disturbance

When simulating without current disturbance but with wave disturbance the system reaches the reference of 30°. With the wave disturbance the signal has a lot of high frequency oscillations as we can see in fig. 10. This is due to the high frequency nature of the waves that affect the system.
The oscillations are quite small compared to the state and the reference. Therefore it will not affect the system too much. Moreover the noise has a mean of zero, so it will not affect the path of the ship in the long run. The actuation of the rudder is quite oscillatory. This is not ideal and would cause wear on the physical system. Later we will look at how a Kalman filter reduces the use of actuation.

4 5.4 Observability

4.a Creation of a state space model

We want to find the system on the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{E}\mathbf{w}, y = \mathbf{C}\mathbf{x} + v$$

The equations for the complete system are given in [1] and are given by

$$\dot{\xi} = \psi_\omega \tag{13a}$$
$$\dot{\psi}_\omega = -\omega_0^2 \xi_\omega - 2\lambda\omega_0\psi_\omega + K_\omega\omega_\omega \tag{13b}$$
$$\dot{\psi} = r \tag{13c}$$
$$\dot{r} = -\frac{1}{T}r + \frac{K}{T}(\delta - b) \tag{13d}$$
$$\dot{b} = \omega_b \tag{13e}$$
$$y = \psi + \psi_\omega + v. \tag{13f}$$

Using the full model for the system we get the following

$$\begin{bmatrix} \dot{\xi} \\ \dot{\psi}_w \\ \dot{\psi} \\ \dot{r} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_0^2 & -2\lambda\omega_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{T} & -\frac{K}{T} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_w \\ \psi_w \\ \psi \\ r \\ b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K}{T} \\ 0 \end{bmatrix} \delta + \begin{bmatrix} 0 & 0 \\ K_w & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_w \\ w_b \end{bmatrix} \tag{14}$$

and

$$y = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_w \\ \psi_w \\ \psi \\ r \\ b \end{bmatrix} + v \tag{15}$$

Thus we can identify the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{E}$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_0^2 & -2\lambda\omega_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{T} & -\frac{K}{T} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K}{T} \\ 0 \end{bmatrix} \tag{17}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix} \tag{18}$$

$$\mathbf{E} = \begin{bmatrix} 0 & 0 \\ K_w & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{19}$$

4.b Observability without disturbances

A system is observable if and only if the observability matrix

$$\mathcal{O} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA^{n-1}} \end{bmatrix} \tag{20}$$

has full rank [2]. To check the observability we created a small MATLAB script. We check if the following statement is true `rank(obsv(A, C)) == length(A)`. When the system has no disturbances, $\mathbf{A}$ and $\mathbf{C}$ is reduced to

$$\mathbf{A_{none}} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T} \end{bmatrix}, \mathbf{C_{none}} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

This gives us a obersvability matrix

$$\mathcal{O}_{\mathbf{none}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{21}$$

which has rank equal to the dimension of $\mathbf{A_{none}}$ and the system is thus observable.

4.c Observability with current disturbance

With current disturbance, $\mathbf{A}$ and $\mathbf{C}$ is reduced to

$$\mathbf{A_c} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{T} & -\frac{K}{T} \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{C_c} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

This gives us a obersvability matrix

$$\mathcal{O}_{\mathbf{c}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{T} & -\frac{K}{T} \end{bmatrix} \tag{22}$$

which has rank equal to the dimension of $\mathbf{A_c}$ and the system is thus observable.

4.d Observability with wave disturbance

With wave disturbance, **A** and **C** is reduced to

$$\mathbf{A_w} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_0^2 & -2\lambda\omega_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{T} \end{bmatrix}, \mathbf{C_w} = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix}$$

This gives us a obersvability matrix

$$\mathcal{O}_{\mathbf{w}} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -\omega_0^2 & -2\lambda\omega_0 & 0 & 1 \\ 2\lambda\omega_0^3 & (4\lambda^2-1)\omega_0^2 & 0 & -\frac{1}{T} \\ (4\lambda^2-1)\omega_0^4 & (4\lambda^2-1)2\lambda\omega_0^3 & 0 & \frac{1}{T^2} \end{bmatrix} \tag{23}$$

which has rank equal to the dimension of $\mathbf{A_w}$ and the system is thus observable.

4.e Observability with both disturbances

With current and wave disturbance we use **A** and **C** given in eq. (16) and eq. (18). This gives us a observability matrix

$$\mathcal{O} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ -\omega_0^2 & -2\lambda\omega_0 & 0 & 1 & 0 \\ 2\lambda\omega_0^3 & (4\lambda^2-1)\omega_0^2 & 0 & -\frac{1}{T} & -\frac{K}{T} \\ (4\lambda^2-1)\omega_0^4 & (4\lambda^2-1)2\lambda\omega_0^3 & 0 & \frac{1}{T^2} & \frac{K}{T^2} \\ (2\lambda^2-1)4\lambda\omega_0^5 & (4\lambda^4-12\lambda^2+1)\omega_0^4 & 0 & -\frac{1}{T^3} & -\frac{K}{T^3} \end{bmatrix} \tag{24}$$

which has rank equal to the dimension of **A** and the system with both disturbances is thus observable. We can conclude that the system is observable no matter the weather conditions. Especially important is the observabillity when both disturbances are present because this means that from knowledge of the output of the system we are able to determine the states [2].This enable us to use an estimator, more specifically the Kalman filter, to estimate states which we don't measure.

5 5.5 Discrete Kalman filter

5.a Discretization of the state space model

Discretizing a linear system, given the continuous system, is given by the following equations

$$\mathbf{A_d} = e^{\mathbf{A}T_s} \tag{25a}$$

$$\mathbf{B_d} = \int_0^{T_s} e^{\mathbf{A}\tau}\mathbf{B}\mathrm{d}\tau \tag{25b}$$

$$\mathbf{E_d} = \int_0^{T_s} e^{\mathbf{A}\tau}\mathbf{E}\mathrm{d}\tau \tag{25c}$$

$$\mathbf{C_d} = \mathbf{C} \tag{25d}$$

and is easily computed with MATLAB. We have the continuous state space model from (14) in subsection 4.a. A sampling frequency of 10 Hz gives an sampling time of 0.1 s. The output matrices C and D remain unchanged under discretization as the relationship between the state and output does not change. We have to use the `c2d`-function twice, because it only accepts one input to the system. This is fine because the system is linear and the superposition principle holds. The discrete state space model can thus be found as in Listing 2.

Listing 2: MATLAB script to discretize a continuous state space model.

```
T_s = 0.1;
[~, Bd] = c2d(A,B,T_s);
[Ad, Ed] = c2d(A,E,T_s);
Cd = C;
Dd = D;
```

This results in the following discretized system

$$\mathbf{A_d} = \begin{bmatrix} 0.9969 & 0.1006 & 0 & 0 & 0 \\ -0.0616 & 1.0111 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0.0999 & 0 \\ 0 & 0 & 0 & 0.9986 & -0.0002 \\ 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{26a}$$

$$\mathbf{B_d} = \begin{bmatrix} 0 \\ 0 \\ 1.09 \cdot 10^{-5} \\ 2.173 \cdot 10^{-4} \\ 0 \end{bmatrix} \tag{26b}$$

$$\mathbf{E_d} = \begin{bmatrix} 0 & 0 \\ 0.0004 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.1000 \end{bmatrix} \tag{26c}$$

5.b Estimating the variance of the measurement noise

To estimate the variance of the measurement noise, we simulate the ship model and export the measurement noise to the MATLAB workspace as a time series. The MATLAB function `var` can then be used to find the empirical variance. The ship model gives a measurement noise in degrees, but we want to use radians in our Kalman filter implementation. The full expression is thus `variance = var(measurement_noise*pi/180)`. This results in a variance $\sigma^2 = 6.0825 \times 10^{-7}\,\mathrm{rad}^2$

5.c Implementation of the Kalman filter

The Kalman filter was implemented as a S-function in MATLAB and the full implementation is given in section A. Since the model operates in continuous time, we have to discretize the input to the discrete Kalman filter. We use zero-order hold block with the sample time T_s. The measurement and rudder input also has to be converted to radians. Output from the Kalman filter is the optimal estimated state. In this application we are only interested in two of the states, ψ and b. The first time the S-function is called the a priori state and error covariance estimate is initialized, and the consecutive times the S-function calculates the Kalman gain and updates the state and error covariance according to the update equations for a discrete Kalman filter. The algorithm as given in [3] and implemented as following. First we update

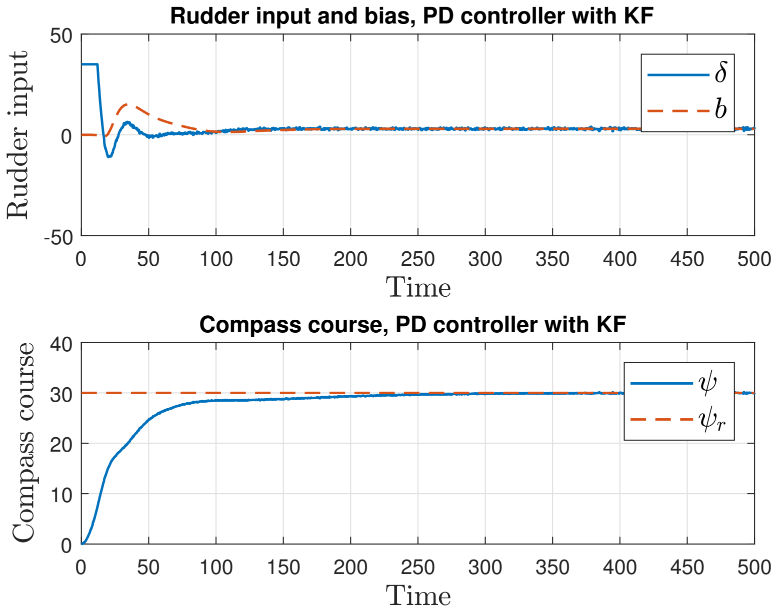

Figure 11: Rudder input together with bias estimate from KF and compass course of ship

the Kalman gain, state and error covariance

$$\mathbf{L}[k] = \mathbf{P}^{-}[k]\mathbf{C}[k]^T(\mathbf{C}[k]\mathbf{P}^{-}[k]\mathbf{C}[k]^T + \mathbf{R}[k])^{-}1 \tag{27a}$$

$$\hat{\mathbf{x}}[k] = \hat{\mathbf{x}}^{-}[k] + \mathbf{L}[k](\mathbf{y}[k] - \mathbf{C}[k]\hat{\mathbf{x}}^{-}[k]) \tag{27b}$$

$$\mathbf{P}[k] = (\mathbf{I} - \mathbf{L}[k]\mathbf{C}[k])\mathbf{P}^{-}[k](\mathbf{I} - \mathbf{L}[k]\mathbf{C}[k])^T + \mathbf{L}[k]\mathbf{R}[k]\mathbf{L}[k]^T \tag{27c}$$

and then we project ahead by computing the a priori estimates for the next iteration

$$\hat{\mathbf{x}}^{-}[k+1] = \mathbf{A_d}[k]\hat{\mathbf{x}}[k] + \mathbf{B_d}[k]\mathbf{u}[k] \tag{28a}$$

$$\mathbf{P}^{-}[k+1] = \mathbf{A_d}[k]\mathbf{P}[k]\mathbf{A_d}[k]^T + \mathbf{Q}[k] \tag{28b}$$

5.d Simulation with bias from the Kalman filter

Using the estimated bias in a feed forward to to the rudder input, as seen in Figure B.1, we manage to remove the stationary deviation the current disturbance causes. Without the feed forward from the estimator the stationary deviation was significant, as seen in Figure 9. Figure 11 shows that the estimate for the bias current converges fairly quickly and we are thus able to correct the input rudder angle to remove any stationary deviation.

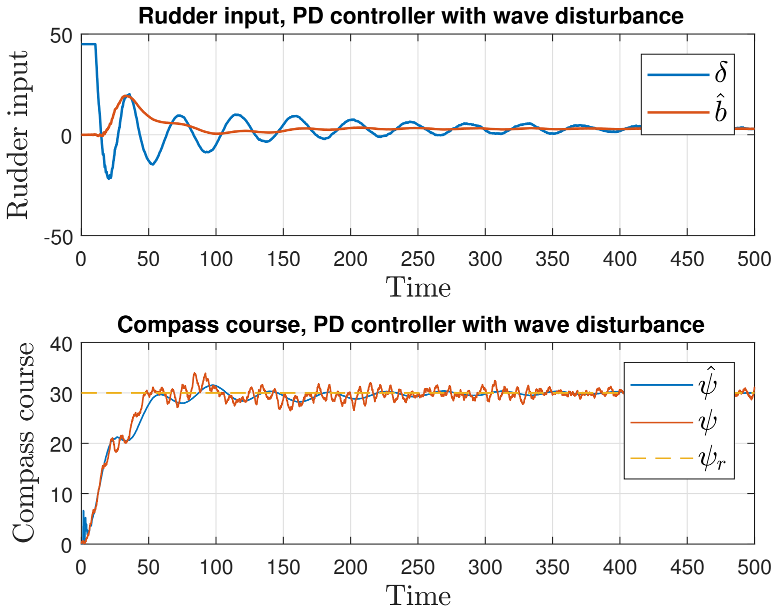

Figure 12: Rudder input and compass course when using the estimated state in the autopilot.

5.e Simulation with heading and bias from the Kalman filter

Simulating the ship using the filtered heading in the autopilot instead of the measurement results in a response that is way faster but also quite oscillatory. The system uses some time to reach its desired state with large swings in amplitude of the rudder input. Comparing fig. 12 to fig. 10 the swings are however with a much slower frequency compared to using the measurement in the autopilot saving the rudder from extensive wear. However the state estimate of ψ_w takes some time before it able to correctly estimate the wave disturbance. The reason for discrepancies could be caused by wrongly estimating the PSD. This would result in reduction of accuracy in the model of the disturbance. Also tuning of the Q matrix could lead to improved results.

5.f Discussion of the Q matrix

The **Q** matrix is the disturbance covariance. A large **Q** results in the Kalman filter weighting the measurement more than the model prediction. If the Q is small the Kalman filter trust the model prediction more than the measurement. The **Q** matrix given in [1] is

$$\mathbf{Q} = \begin{bmatrix} 30 & 0 \\ 0 & 10^{-6} \end{bmatrix}$$

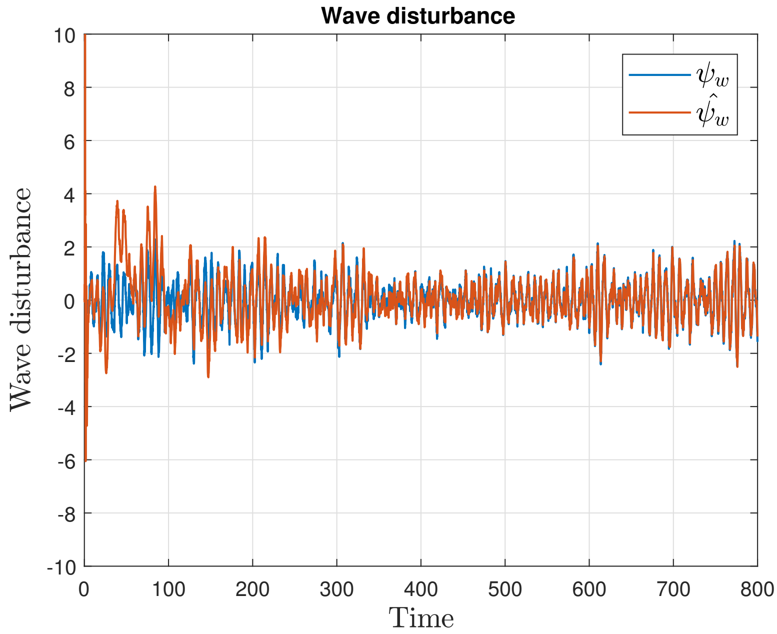

Figure 13: Estimated and actual wave influence on heading.

When simulating with current disturbance only the second element in Q alters the behavior of the system. As previously stated, increasing the covariance will increase the dependency on the measurement. By setting the second element multiple orders of magnitude higher the bias estimate has a much faster dynamic. This means it picks up on the high frequency measurement noise. If we set the second element of Q towards zero the controller is no longer able to eliminate the stationary deviation because it relies too heavily on the model prediction. It turns out 10^{-6} is quite reasonable. When simulating with both disturbances and the estimated heading in the feedback loop we vary the first element. Setting the first element of Q very low results in oscillations on the estimated states from the Kalman filter as the filter relies heavily on the compass measurements. Heavy oscillations with high frequency as seen in fig. 15 is not good for actuator health. As Q is increased the rudder input becomes less oscillatory but the system uses more time to reach its desired state as seen in fig. 16. All of this is done by leaving the controller untouched. Some tuning of the PD controller could have given a smoother response without oscillations, albeit a slower one. But the underlying problem that causes the oscillations originates in that the Kalman filter utilizes a model of lower order than the ship. The filter is not able to estimate the states quick enough and leaves us with a delay in our feedback loop. The delay is introduced because the model does not incorporate the disturbances and thus the Kalman filter favors the model rather than the noise measurements.

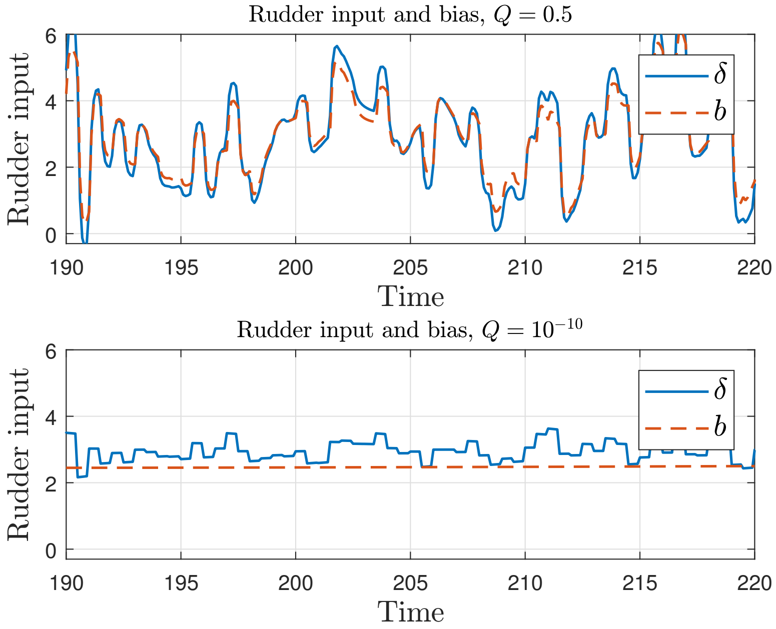

Figure 14: Comparison of part 5 D with different Q matrices.

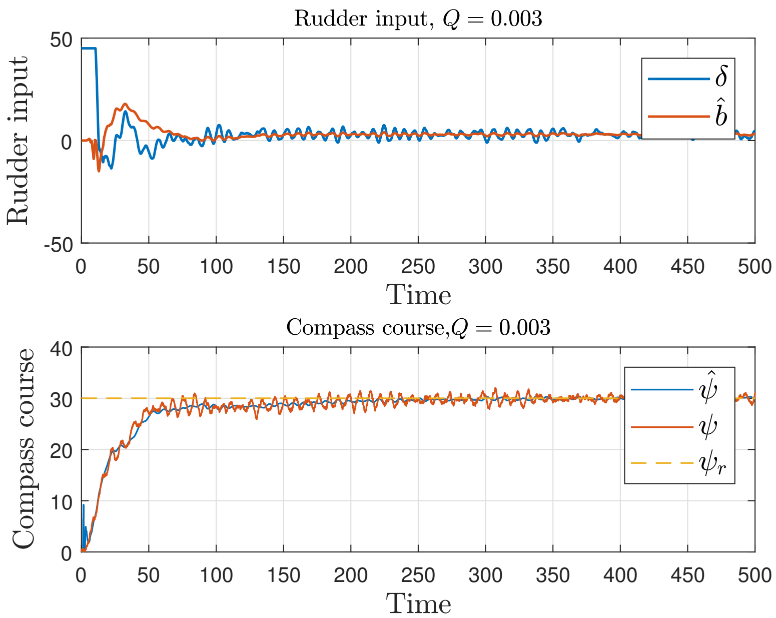

Figure 15: Rudder input and compass course setting Q low

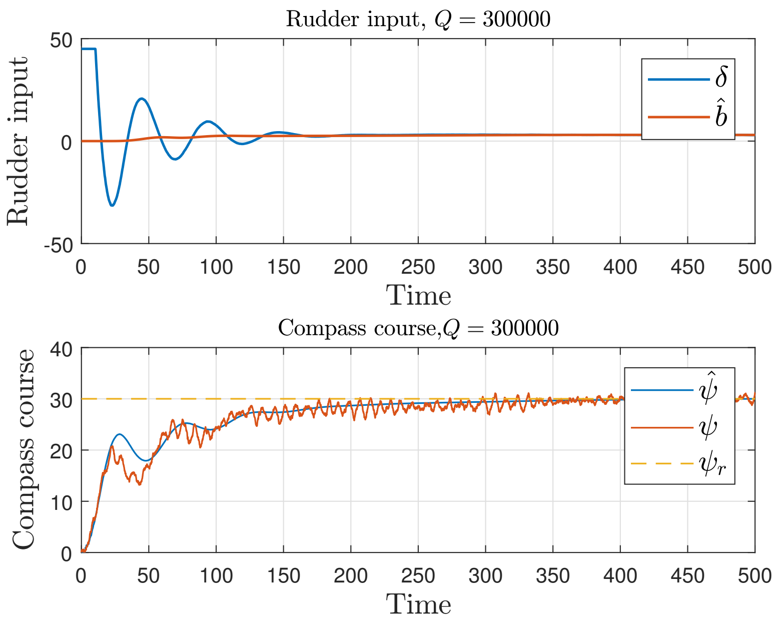

Figure 16: Rudder input and compass course setting Q high

6 Summary

This lab was about the control of a simulated ship. The lab consisted of 5 different problems. In Part 1 we identified essential parameters by using transfer function and step response. In Part 2 we estimated a wave spectrum density for the system given wave disturbance. This was used to identify certain parameters. Part 3 consisted of creating PD-control for the ship heading, given rudder input. Part 4 an analysis of the observability of the system. In Part 5 we designed a discrete Kalman filter and controlled the ship using the Kalman filter. Using the Kalman filter we got an oscillatory response, although much faster than without the Kalman filter.

A Implementation of a discrete Kalman filter

The Kalman filter was implemented as a MATLAB S-function for use with Simulink.

kalman.m

```
function [sys,x0,str,ts] = DiscKal(t,x,u,flag,data)
    % Shell for the discrete kalman filter
    % made by J rgen Spjtvold

  switch flag

  % Initialization %
  case 0
    [sys,x0,str,ts]=mdlInitializeSizes(data);

  % Outputs   %
  case 3
    sys=mdlOutputs(t,x,u,data);

  % Terminate %
  case 2
    sys=mdlUpdate(t,x,u,data);

  case {1,4,}
    sys=[];

  case 9
      sys=mdlTerminate(t,x,u);

  % Unexpected flags %
  otherwise
    error(['Unhandled flag = ',num2str(flag)]);
  end
end

function [sys,x0,str,ts]=mdlInitializeSizes(data)
    sizes = simsizes;
    sizes.NumContStates   = 0;
    sizes.NumDiscStates   = 35;
    sizes.NumOutputs      = 3;
    sizes.NumInputs       = 2;
    sizes.DirFeedthrough  = 0;
```

```
    sizes.NumSampleTimes = 1;
    sys = simsizes(sizes);
    x0  = data.x_0;
    str = [];
    ts  = [-1 0]; % Sample time.
end

function x=mdlUpdate(t,x,u,data)
    I = eye(5);
    P_pri = reshape(x(11:35),5,5);
    x_pri = x(1:5);

    % Kalman gain, state error, error covariance
    L = P_pri*(data.Cd)'*(data.Cd*P_pri*(data.Cd)'+...
        data.R)^-1;
    x_post = x_pri + L*(u(2)-data.Cd*x_pri);
    P_post = (I-L*data.Cd)*P_pri*(I-L*data.Cd)'+...
        L*data.R*L';

    % A priori estimates for next iteration
    x_next = data.Ad*x_post + data.Bd*u(1);
    P_next = data.Ad*P_post*(data.Ad)' +...
        data.Ed*data.Q*(data.Ed)';

    x(1:5) = x_next;
    x(6:10) = x_post;
    x(11:35) = P_next;
end

function sys=mdlOutputs(t,x,u, data)
    sys=[x(8); x(10); x(7)]; % psi, b, psi_w
end

function sys=mdlTerminate(t,x,u)
    sys = [];
end
```

B Simulink Diagrams

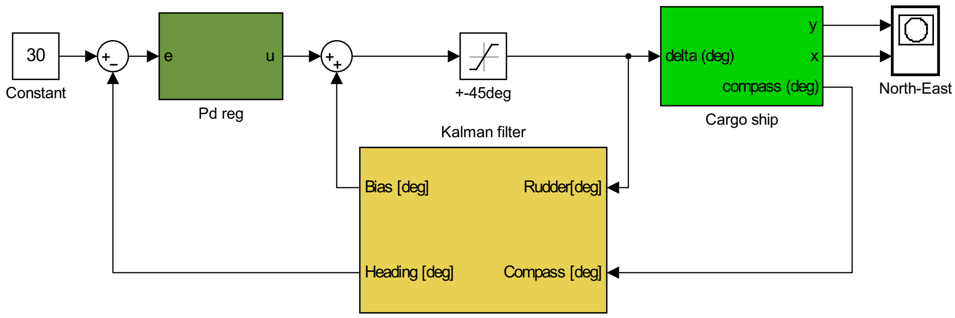

Figure B.1: The complete system using the estimates from the Kalman filter.

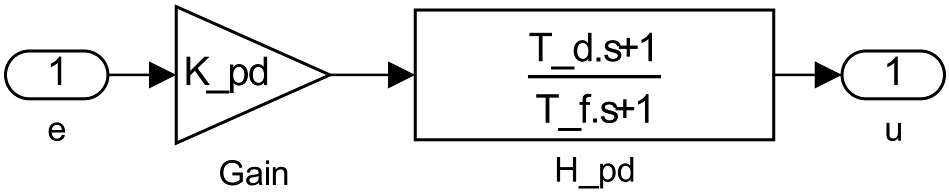

Figure B.2: PD regulator.

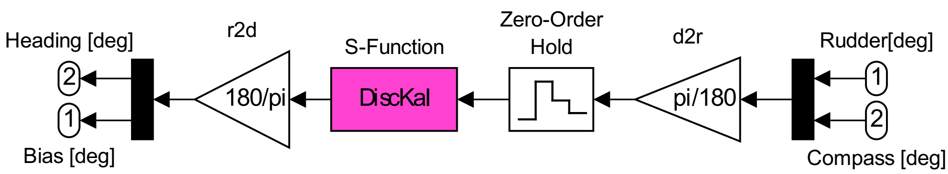

Figure B.3: Conversions and discrete Kalman filter s-function block.

References

[1] TAJ et al. *Boat lab assignment Version 1.9*. Department of Engineering Cybernetics, NTNU, 2018.

[2] Chi-Tsong Chen. *Linear System Theory and Design*. Oxford University Press, Incorporated, 2014.

[3] Robert Grover Brown Patrick Y. C. Hwang. *Introduction to Random Signals and Applied Kalman Filtering*. John Wiley Sons, Inc., 2012.

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. `https://authors.library.caltech.edu/11363/2/FISjns08supp.pdf`.
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