HW #3 (Feb. 9, 2016): Computational Physics

deadline: Feb. 16, 2016 5pm

1. Write a program to solve the following linear algebraic equations using the LU decomposition method with implicit partial pivoting. Compare your numerical result with the answer, $x = 224/1545$, $y = -268/1545$, $z = 83/515$, and $u = 589/1545$. Don't forget carrying out the same row permutations (for pivoting) to the right-hand side vector $\mathbf{b}$. This step is important for obtaining the solution. In the report, the following six issues should be addressed. (i) Write down the constructed lower $\mathbf{L}$ and upper triangular matrices $\mathbf{U}$. (ii) Check if $\mathbf{L} \cdot \mathbf{U} = \mathbf{A}$. If you have to carry out row permutations or operations, one needs to check if $\mathbf{L} \cdot \mathbf{U} = \mathbf{P} \cdot \mathbf{A}$, where $\mathbf{P}$ is a permutation matrix. You may use the 3×3 matrix example used in the class or other simple examples for debugging. (iii) Write down a permutation matrix $\mathbf{P}$ if it was used. (iv) Write down the solution $\mathbf{x} = (x, y, z, u)$. (v) Confirm if your numerical solution satisfies the given answer. (vi) Now turn off the pivoting procedure and see if you obtain the same answer with the pivoting. If so, compare the two answers up to seventh digits (10^{-7}). If not, explain the reason. **(30 pts)**

$$
\begin{aligned}
2x + 3y + 10z - u &= 1 & (1) \\
10x + 15y + 3z + 7u &= 2 & (2) \\
-4x + y + 2z + 9u &= 3 & (3) \\
15x - 3y + z + 3u &= 4 & (4)
\end{aligned}
$$