Thomas Keller
SDSU ID 828545348

Homework 2

COMP 670
October 13, 2022

Problem

Consider example elliptic1D.m
1. Please, change the order of accuracy to four.
2. Set the number of cells to ten.
3. Keep the Robin boundary condition.
4. Change the force function to f (x) = 1 + x.
5. Run the example with all the changes you have made.
6. Verify that the plot is appropriate (titles, axes values, etc.).
7. Please, include all m files you have modified and a document (named elliptic1DWork.docx) describing explicitly the PDE to a Canvas message.

Calculation of the PDE with poisson equation and force function f(x):

$$\begin{aligned} \Delta u &= f(x) \\ \nabla^2 u &= 1 + x \end{aligned} \tag{1}$$

In 1D: the ∇^2 means second order derivative of the variable.

→ our function depends only on x
→ ∇^2 u = $\frac{d^2}{dx^2}$u = 1+x

In order to solve this differential equation we integrate both sides in respect to x:

$$\begin{aligned} u &= \int\int (1+x)dx^2 \\ &= \int (\frac{1}{2}x^2 + x + A)dx \\ &= (\frac{1}{6}x^3 + \frac{1}{2}x^2 + Ax + B) \end{aligned} \tag{2}$$

The robin boundaries conditions gives us the following equations:

$$a \cdot u\Big|_0 - b \cdot u'\Big|_0 = \mu \tag{3}$$

$$a \cdot u\Big|_1 + b \cdot u'\Big|_1 = \lambda \tag{4}$$

From example, we have the two boundaries: $\mu = 0$ and $\lambda = 2{\cdot}\exp(1)$. The two variables a and b are equal to 1 because of the robin boundary conditions. The minus in equation 5 is from the normal vector which points in the different direction.

$$1 \cdot u\Big|_0 - 1 \cdot u'\Big|_0 = 0 \tag{5}$$

$$1 \cdot u\Big|_1 + 1 \cdot u'\Big|_1 = 2 \cdot exp(1) \tag{6}$$

Solving the first equation 5, we get the following:

$$\begin{aligned} B - A &= 0 \\ B &= A \end{aligned} \tag{7}$$

Solving the second equation 6, we get the following:

$$\begin{aligned} \frac{1}{2} + \frac{1}{6} + A + B + (1 + \frac{1}{2} + A) &= 2 \cdot exp(1) \\ \frac{4}{6} + A + B + \frac{3}{2} + A &= 2 \cdot exp(1) \\ \frac{13}{6} + 3A &= 2 \cdot exp(1) \\ A &= \frac{12 \cdot exp(1) - 13}{18} \end{aligned} \tag{8}$$

The result of A and B filled in equation 2 for u(x):

$$u(x) = \frac{12 \cdot exp(1) - 13}{18} \cdot (1 + x) + \frac{1}{2}x^2 + \frac{1}{6}x^3 \tag{9}$$

The equation 9 is the analytical solution and is filled in the Matlab code line 35.

The Matlab code includes the following changes:

- line 11: order of accuracy to 4
- line 12: minimum number of cells to 10
- line 28: U = 1+(grid)', set right hand side of the poisson eq. to the force function
- line 35: analytical solution: Eq. 9

```
1  % Solves the 1D Poisson's equation with Robin boundary conditions
2  clc
3  close all
4
5  addpath('../mole_MATLAB')
6
7  % Domain's limits
8  west = 0;
9  east = 1;
10
11 k = 4;  % Operator's order of accuracy %NEW 6->4
12 m = 2*k+2;  % Minimum number of cells to attain the desired accuracy %NEW 1->2
13 dx = (east-west)/m;  % Step length
14
15 L = lap(k, m, dx);  % 1D Mimetic laplacian operator
16
17 % Impose Robin BC on laplacian operator
```

```
    a = 1;
    b = 1;
    L = L + robinBC(k, m, dx, a, b);

    % 1D Staggered grid
    grid = [west west+dx/2 : dx : east-dx/2 east];
    grid

    % RHS
    % U = exp(grid)';
    U = 1+(grid)'; % NEW exp(grid)' -> 1+(grid)'
    U(1) = 0;  % West BC
    U(end) = 2*exp(1);  % East BC

    U = L\U;  % Solve a linear system of equations with robin boundaries
    U

    anal = ((12*exp(1)-13)/18).*(1+(grid))+(((grid).^2)./2)+(((grid).^3)./6);
   % analytical solution
    anal

    % Plot result
    plot(grid, U, 'o')
    hold on
    plot(grid, anal)
    legend('Approximated', 'Analytical', 'Location', 'NorthWest')
    title('Poisson''s equation with Robin BC')
    xlabel('x')
    ylabel('u(x)')
```

Listing 1: changed Matblab code for file elliptic1D.m

The following figure 1 shows the plot of the Matlab file elliptic1D.m. It is good to see that the approximation and analytical solution are congruent.

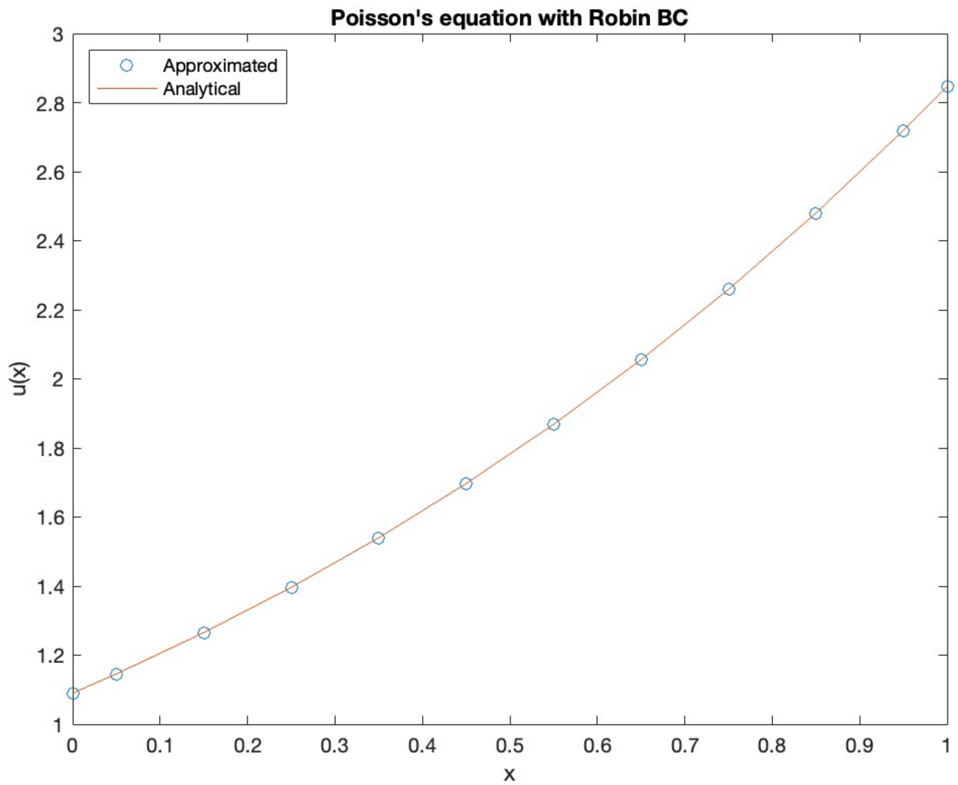

Figure 1: Plot of the Matlab file: elliptic1D.m, Poisson's equation with robin boundaries 5 and 6, analytical solution 9.