Perona Malik equation with mimetic method

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Introduction

This project is about solving the Perona-Malik diffusion equation with the mimetic method in 2D. The Perona-Malik diffusion is a non-linear anisotropic process that was presented in 1987 by Perona and Malik [1]. In principle, the equation smoothes or blurs the image, except around image areas that are likely to have edges. This is in contrast to a linear diffusion, which smoothes the entire image in the same way (such as the Gaussian diffusion). As a result, the equation is used to smoothly denoise or blur images. The equation is the following:

$$I_t = \boldsymbol{\nabla}\cdot\left(c(x,y,t)\nabla I\right) \tag{1}$$

Diffusion coefficient:

$$c\left(\|\nabla I\|\right) = \frac{1}{1+\left(\frac{\|\nabla I\|}{K}\right)^2} \tag{2}$$

where:

I = Image 512×512px [double]
I_t = Image derivative
c = diffusion coefficient [double]
K = constant, controls sensitivity to edges [int]
∇ = Gradient
$\boldsymbol{\nabla}\cdot$ = Divergence

Mimetic Method

The mimetic discretization method uses a staggered grid, which is defined at the beginning of the code with the MOLE library, which was developed by Corbino-Castillo [2]. With the grid, the differential operators for gradient and laplacian can be defined. Instead of using the finite difference method by manually creating matrices or loops, the mimetic method creates them by just defining the grid and mimetic operators. As a result, the calculation of the PDE is clear and straightforward. The grid is exactly related to the number of pixels in the image. From there, it follows that the pixels and the grid are matching, and the values from the pixel centers are used for the calculations[3].

Image quality

A measure of the denoise quality is the mean square error (MSE) and the peak signal-to-noise ratio (PSNR). The two graphs (Figures 5 and 6) show that about 90 iterations should result in the best image quality.

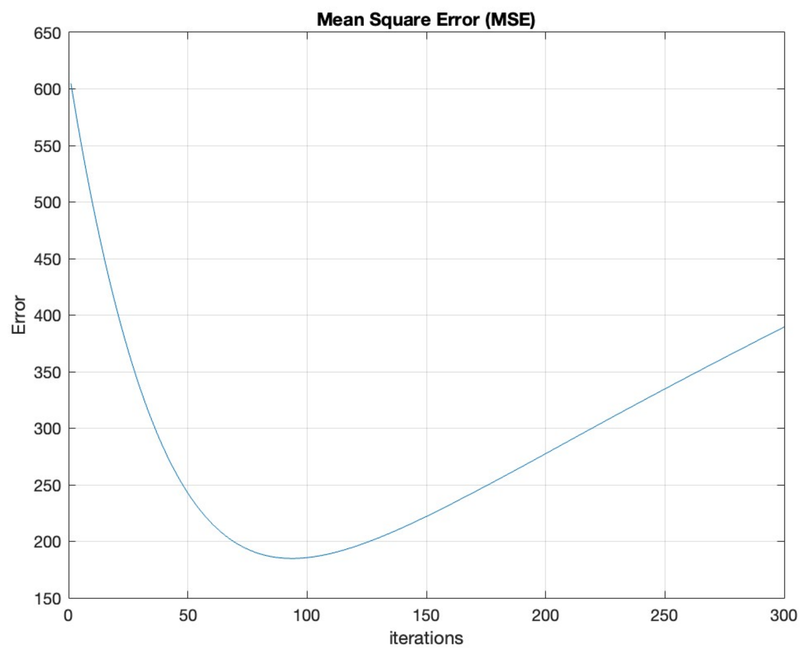

Figure 5: MSE for 300 iterations, tom with Gaussian noise (noise with variance of 0.01)

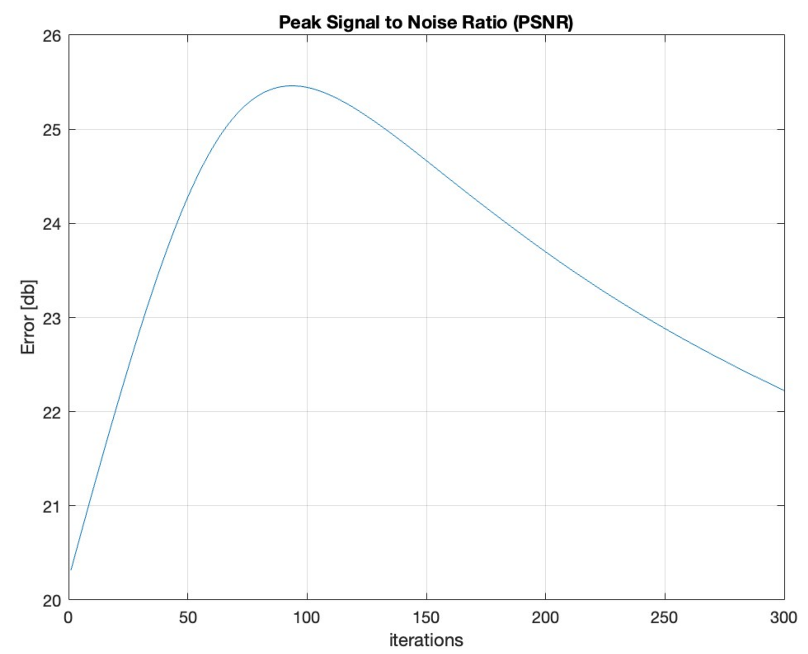

Figure 6: PSNR for 300 iterations, tom with Gaussian noise (noise with variance of 0.01)

Image Outputs

The result of the Perona-Malik diffusion for a noisy input image (Figure 1) is shown in Figures 2, 3, and 4. The number of iterations varies between 50, 90, and 140. The comparison with the noisy image shows that the edges are sharper and the entire image is denoised. It is good to see that the 90 iterations that were evaluated in Figures 5 and 6 result in the subjectively best output. As expected, the image in Figure 2 is still noisy, and the one in Figure 4 has lost many details.

Figure 1: Input image: Noisy Image

Figure 2: Output image: 50 iterations

Figure 3: Output image: 90 iterations

Figure 4: Output image: 140 iterations

Future research

Because the algorithm is working fine in 2D, the next step will be to convert the code into a three-dimensional space. This would include processing color, 3D medical, or seismic images with the goal of capturing changes in tissue or changes in rock type, respectively.

References

[1] P. Perona and J.Malik. Scale-space and edge detection using anisotropic diffusion.

[2] J.Corbino and J.E. Castillo. High-order mimetic finite-difference operators satisfying the extended gauss divergence theorem. 2020.

[3] M. Abouali J. E. Castillo, C. Bazan and P. Blomgren. Mimetic finite difference methods in image processing. *Computational & Applied Mathematics*, 2011.

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