COP4020.02 **Brandon Thompson**

Dr. Muhammed Abid Due: 3/31/21

Assignment 5: Chapter 10, 11, 12

PROBLEM #1 (15 POINTS):

Use the pumping lemma to show that this language is nonregular:

$$\{a^nb^na^n\} = \{aba, aabbaa, aaabbbaaa, \ldots\}$$
$$\{a^nba^n\} = \{aba, aabaa, aaabaaa, \ldots\}$$
$$\{a^nb^{2n} = \{abb, aabbbb, aaabbbbbb, \ldots\}$$

SOLUTION:

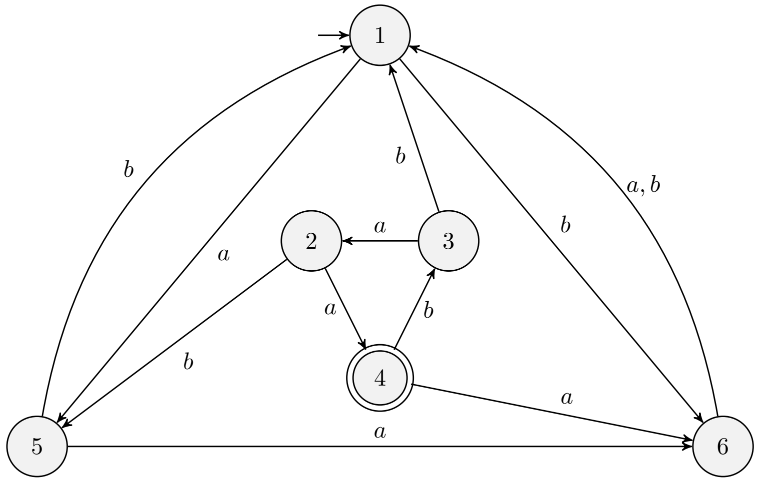
1. First we assume that L is regular and n is the number of states.
   Let $w = a^nb^na^n$. Thus $|w| = 3n \geq n$.
   By pumping lemma, let $w = xyz$, where $|xy| \leq n$.
   Let $x = a^p, y = a^q, z = a^rb^na^n$, where $p + q + r = n, p \neq 0, q \neq 0, r \neq 0$.
   Let $k = 2$. Then $xy^2z = a^pa^{2q}a^rb^na^n$.
   The number of a's is :$(p + 2q + r) + n = (p + q + r) + q + n = 2n + q$
   Hence, $xy^2z = a^{n+q}b^na^n$. Since $q \neq 0, xy^2z$ is not of the form $a^nb^na^n$.
   Thus, xy^2z is not in L making L not regular.

2. First we assume that L is regular and n is the number of states.
   Let $w = a^nba^n$. Thus $|w| = 2n + 1 \geq n$.
   By pumping lemma, let $w = xyz$, where $|xy| \leq n$.
   Let $x = a^p, y = a^q, z = a^rba^n$, where $p + q + r = n, p \neq 0, q \neq 0, r \neq 0$.
   Let $k = 2$. Then $xy^2z = a^pa^{2q}a^rba^n$.
   The number of a's is: $(q + 2q + r) + n = (p + q + r) + q + n = 2n + q$
   Hence, $xy^2z = a^{n+q}ba^n$. Since $q \neq 0, xy^2z$ is not of the form a^nba^n.
   Thus xy^2z is not in L making L not regular.

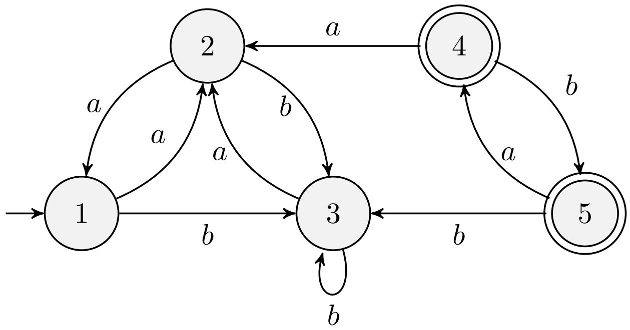
3. First we assume that L is regular and n is the number of states.
   Let $w = a^nb^{2n}$. Thus $|w| = 3n \geq n$.
   By pumping lemma, let $w = xyz$, where $|xy| \leq n$.
   Let $x = a^p, y = a^q, z = a^rb^{2n}$, where $p + q + r = n, p \neq 0, q \neq 0, r \neq 0$.
   Let $k = 2$. Then $xy^2z = a^pa^{2q}a^rb^{2n}$.
   The number of a's is: $(q + 2q + r) = (p + q + r) + q = n + q$
   Hence, $xy^2z = a^{n+q}b^{2n}$. Since $q \neq 0, xy^2z$ is not of the form a^nb^{2n}.
   Thus xy^2z is not in L making L not regular.

Problem #2 (10 points):

By using blue paint, determine which of the following FA's accept any words:

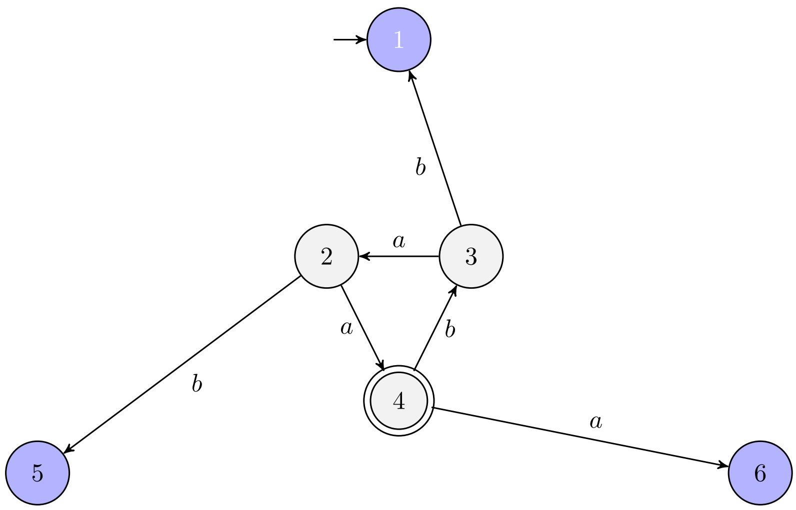


a)



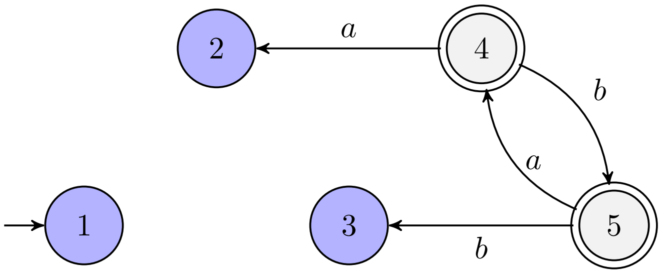
b)

Solution:



1.

2.



PROBLEM #3 (18 POINTS):

Describe the language generated by the following context free grammar (CFG) in English and regular expressions:

a) $S \to SS$
$S \to ZZZ$
$Z \to bZ$
$Z \to Zb$
$Z \to a$

b) $S \to aS$
$S \to bb$

c) $S \to XYX$
$S \to aX$
$S \to bX$
$S \to \Lambda$
$S \to bbb$

SOLUTION:

Problem #4 (15 points):

Find CFG for the following languages over the alphabet $\Sigma = \{a, b\}$:

a) All words that have different first and last letters.

b) All words in which the letter b is never tripled.

c) All words that do not have substring ab.

Solution:

PROBLEM #5 (10 POINTS):

Show that the CFG below is ambiguous by finding a word with two distinct syntax trees. Show both syntax trees.

a) $S \to Sbb$
$S \to Sbbb$
$S \to b$

b) $S \to AA$
$A \to AAA|a|bA|Ab$

SOLUTION: