COP4020.02 **Brandon Thompson**
Dr. Muhammed Abid

Notes COP4020: Programming Languages

1 Chapter 1: Introduction

Mathematical models called "machines." Collection of successful inputs are called the language of the machines.

1.1 History

- Cantor (1845-1918) - Theory of sets (union, intersections, cardinality, etc) Discovered paradoxes such as the idea that infinity comes in different sizes, some set is bigger than the universal set.
- Hilbert (1906-1978) Methodology for finding proofs. Each true proposition is provided with a rigorous proof in which every line is either an axiom or follows from the axioms and previously proved theorems by a specified small set of rules of inference.
- Godel (1906-1978) Incompleteness theorem. There was no algorithm to provide proofs for all true statements in mathematics. He showed that either there were some true statements in mathematics that had no proofs, or else there were some false statements that did have proofs.
- Church, Kleene, Post, Markov, von Neumann, Turing - Which statements have proofs? Building blocks of mathematical algorithms. Turing proved that there were mathematically definable fundamental questions about the machine itself that the machine could not answer.

1.2 Languages and Machines

The term computer is never used in this course. We study computers by building mathematical models called **machines**, and then studying their limitations by analyzing the types of inputs on which they operate successfully. The collection of successful inputs is called the **language** of the machine.

	Language Defined by	Corresponding Accepting Machine	Nondeterminism= Determinism	Language Closed Under	What Can Be Decided?	Examples of Applications
I.	Regular expression	Finite automaton, transition graph	Yes	Union, product, Kleene star, intersection, complement	Equivalence, emptiness, finiteness, membership	Text editors, sequential circuits, verification
II.	Context-free grammar	Pushdown automaton	No	Union, product, Kleene star	Emptiness, finiteness, membership	Parsing, compilers
III.	Type 0 grammar	Turing machine, Post machine, Pushdown automaton	Yes	Union, product, Kleene star	Not much	Computers

Figure 1: Page 434 in textbook.

2 Chapter 2: Languages

- Mathematical models of computers
  - Analysis of the input **language**
    - study of their limitations

2.1 Definitions

- alphabet - a finite set of symbols, denoted Σ
- letter - an element of an alphabet
- word - a finite sequence of letters from the alphabet
- Λ (empty string) - a word without letters
- language - a set of words formed from the alphabet

Two words are considered the same if all their letters are the same and in the same order. There is a difference between the word that has no letters (Λ), and the language that has no words (Φ). It is not true that Λ is a word in the language Φ since this language has no words at all.

	English-Words	English-Sentences
alphabet	Σ={a,b,c,d,...}	Γ=words in dictionary + space + punctuation marks
letter	letter	word
word	word	sentence
language	all the words in the dictionary	all English sentences

Figure 2: Examples of the definitions.

Languages can be defined as a list of all of the words, or as a set of the rules **(grammar)**. The alphabet is always finite, but the set of words can be infinite. It must be possible in a finite time to determine if a word is in a language or not (algorithm).

$$\Sigma = x \quad L_1 = \{x, xx, xxx, xxxx, \ldots\}$$

or

$$L_1 = \{X^n | n = 1, 2, 3, \ldots\}$$

Concatenation is when two words, written down side by side, create a new word:

$$a = xxx \qquad b = xxx \qquad ab = xxxxx$$

When a word is concatenated, we are able to factor out the individual words.

$$\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$L_2 = \{\text{words that do not start with the letter0}\}$$

Length is the number of letters in a word:

$$\text{length}(xxxxx) = 5$$
$$\text{length}(1025) = 4$$
$$\text{length}(\Lambda) = 0$$

Reverse will reverse the letters in a word:

$$\text{reverse}(xxx) = xxx$$
$$\text{reverse}(1570) = 751$$

A palindrome is a word that is the same reading forwards as it is reading backwards:

$$\begin{aligned}\Sigma &= \{a, b\}\\ \text{Palindrome} := &\{\Lambda \text{ and } x|\text{reverse}(x) = x\}\\ &= \{\Lambda, a, b, aa, bb, aaa, aba, bab, bbb, \ldots\}\end{aligned}$$

2.2 Kleene Closure (Star)

Given an alphabet Σ, the closure of Σ (or Kleene star), denoted as Σ^*, is the language containing all words made up of finite sequences of letters from Σ, including the empty string Λ.

$$\begin{aligned}&\Sigma = \{x\} && \Sigma^* = \{\Lambda, x, xx, xxx, \ldots\}\\ &\Sigma = \{0, 1\} && \Sigma^* = \{\Lambda, 0, 1, 00, 01, 10, 11, 000, 001, \ldots\}\\ &\Sigma = \{a, b, c\} && \Sigma^* = \{\Lambda, a, b, c, aaa, aab, aac, aba, abb, abc, \ldots\}\end{aligned}$$

Let S be a set of words. S^* is the language formed by concatenating words from S, including the empty string (null string) Λ.

$$S = \{a, ab\} \qquad S^* = \{\Lambda, a, aa, ab, aaa, aab, aba, aaaa, \ldots\}$$

$$abaaababa \in S^*$$

$$ab|a|a|ab|ab|a \leftarrow \text{factors}$$

$$\begin{aligned}S^* = \{&\Lambda \text{ plus all sequences of a's and b's except those}\\ &\text{that start with b and those that contain a double b}\}\end{aligned}$$

Factoring is not always unique, if $S = \{xx, xxx\}$ and $xxxxxxx \in S^*$, then there are multiple ways to break up the word between the words in S.

3 Chapter 7: Kleene's Theorem

Theorem 1. *Let* A, B, B *be sets such that* $A \subset B, B \subset C, C \subset A$.
Then $A = B = C$.

Proof. $\square$