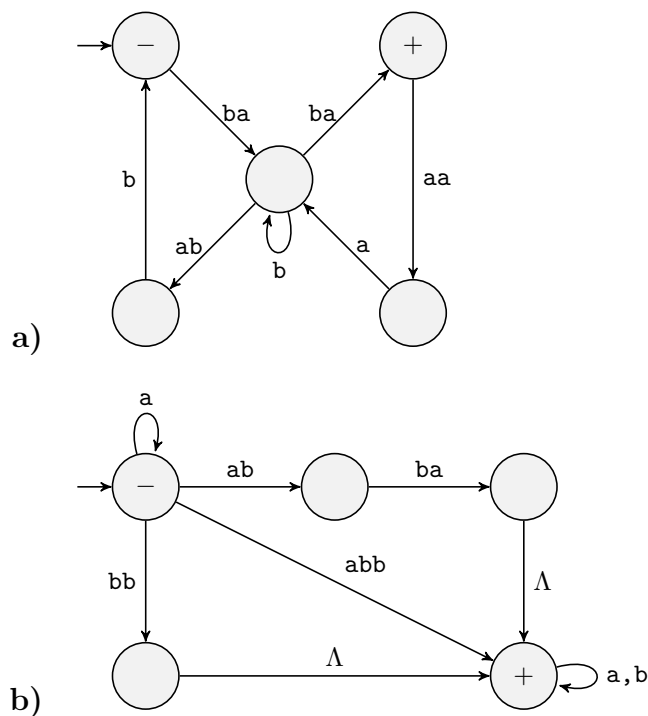


Assignment 4: Chapter 7, 8, 9

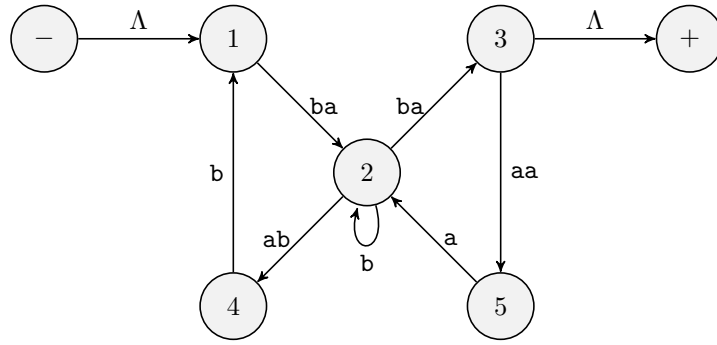
PROBLEM #1 (20 POINTS):

Using the bypass algorithm taught in class, convert each of the following TGs into regular expressions.



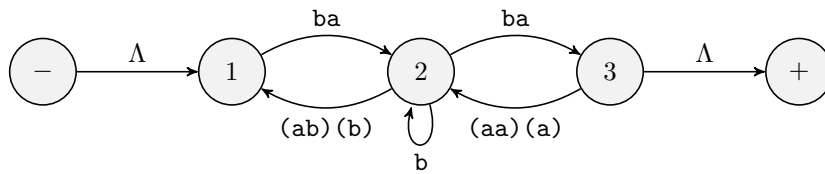
SOLUTION:

- a) 1: Start states should only have outgoing edges and end states should only have incoming edges. Move start and end states to fit this criteria. Label nodes that are not the start and end states.



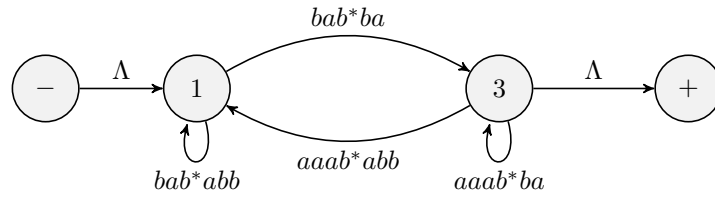
Step 1: moving the start and end states.

2: Remove / bypass states 4 and 5.



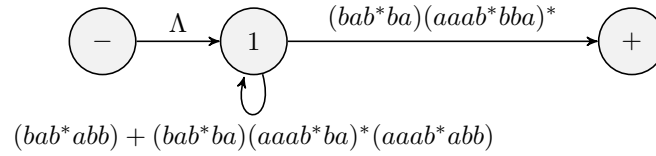
Step 2: Bypass states 4 and 5.

3: Remove / bypass state 2.



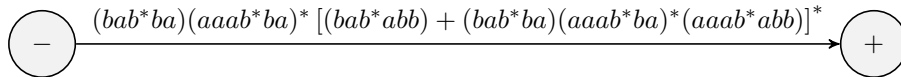
Step 3: Bypass state 2.

4: Remove / bypass state 3.

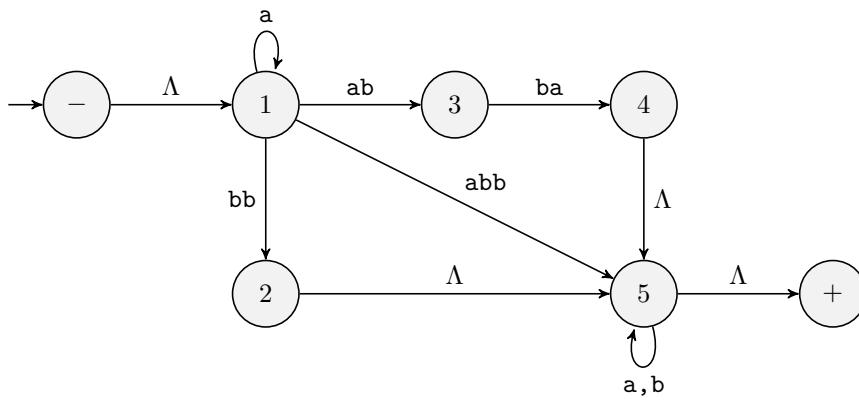


Step 4: Removing state 3.

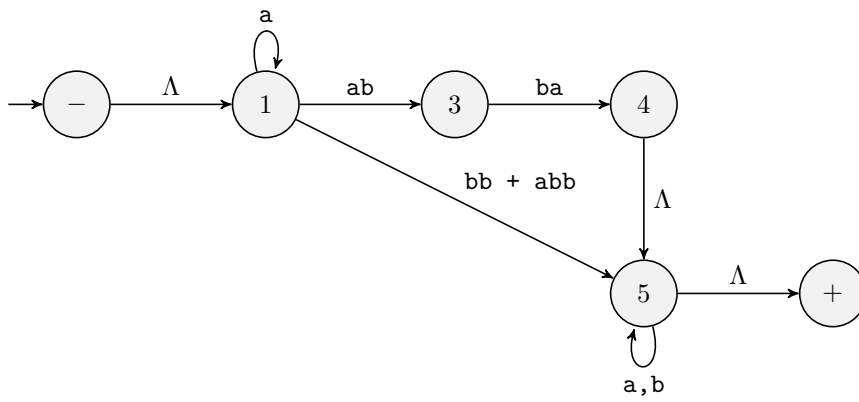
5: Remove / bypass state 1.



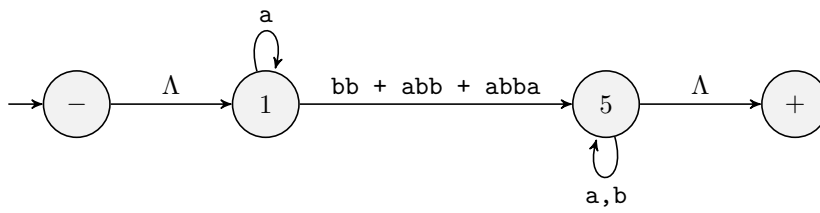
Step 5: Removing state 1, completed regular expression.



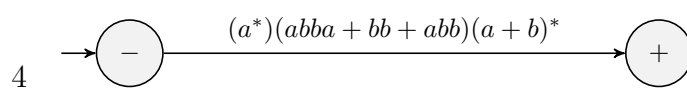
b) 1



2



3



4

PROBLEM #2 (110 POINTS):

- a) Given a TG, called $TG1$, that accepts the language $L1$ and a TG called $TG2$ that accepts the language $L2$, show how to build a new TG (called $TG3$) that accepts exactly the language $L1L2$.
- b) Let the language L be accepted by the transition graph T and let L not contain the word Λ . Show how to build a new TG that accepts all the words in L and the word Λ .

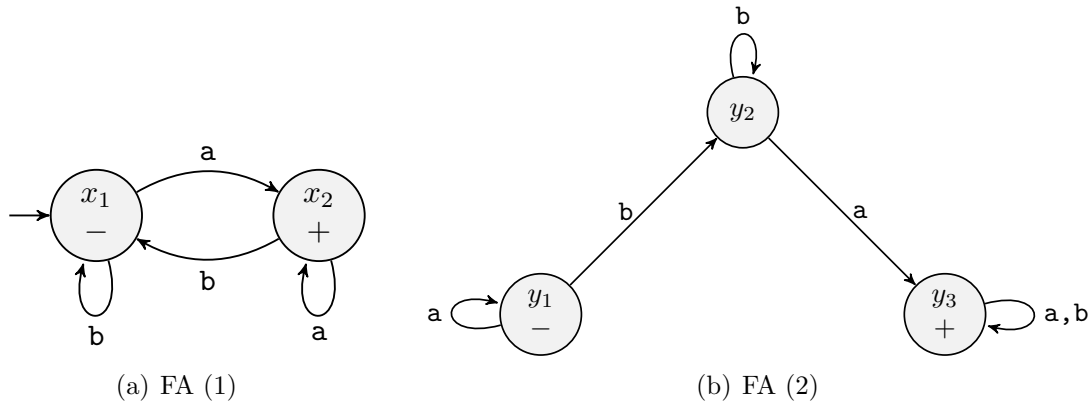
SOLUTION:

- a) In order to create a new TG that accepts $L1L2$ you would need to create a z state for every non final state in $TG1$ reached before hitting a final state in $TG1$. Once reaching a final state of $TG1$, determine if you are continuing in $TG1$ or moving to $TG2$.
- b) If a TG does not accept Λ the start state and the final state need to be connected with a path that has no words. So you could either create a new edge from the start state to the end state with λ , or make the start state also be the final state. This way the empty string will be accepted.

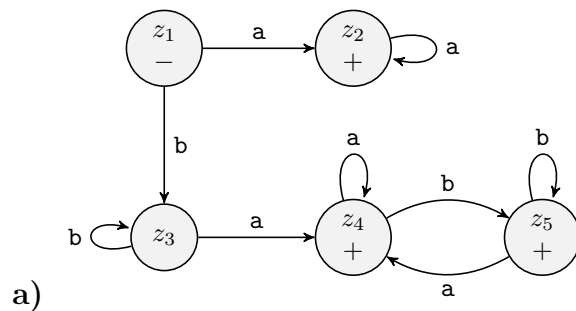
PROBLEM #3 (16 POINTS):

Consider FA (1) and FA (2) at the end of this question. Let $L1$ be the language accepted by FA (1) and let $L2$ be the language accepted by FA (2).

- Using the algorithm of Kleene's theorem, Lemma 3, Rule 2, construct an FA for the union language $L1 + L2$.
- Give an example of a word in the language $L1 + L2$ that is also in both languages $L1$ and $L2$.
- Give a word in the language $L1 + L2$ that is in $L1$ but not in $L2$.
- Give a word in the language $L1 + L2$ that is in $L2$ but not in $L1$.



SOLUTION:



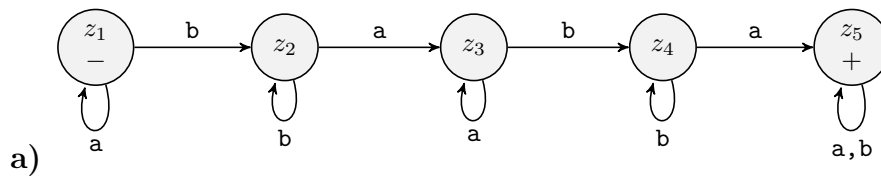
- The word ba is a member of both $L1$ and $L2$.
- The word a is a member of $L1 + L2$ and $L1$ but not $L2$.
- The word bab is a member of $L1 + L2$ and $L2$ but not $L1$.

PROBLEM #4 (14 POINTS):

Consider FA (2) of Q3 that accepts language $L2$.

- a) Using the algorithm of Kleene's theorem, Lemma 3, Rule 3, construct an FA for the product language $L2L2$.
- b) Describe (in English phrases) the language $L2L2$ (the language accepted by part (a)).
- c) Give an example of a word with at least 5 letters that is not in the language $L2L2$.

SOLUTION:



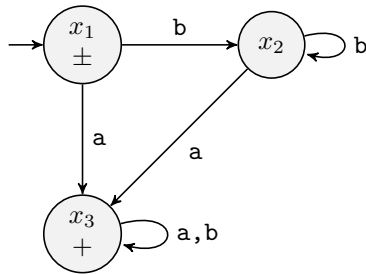
- b) The description of the language $L2L2$ is alternating sets of letters. could have $aabbaabb$ with any numbers of letters after.
- c) A word with 5 letters not in $L2L2$ is $baabb$. If there is an a after this word it will be accepted.

PROBLEM #5 (15 POINTS):

Consider FA (1) that accepts language $L1$ from Question 3.

- a) Using the algorithm of Kleene's theorem, Lemma 3, Rule 4, construct an FA for the language $L1^*$.
- b) Is the language $L1$ the same as the language $L1^*$? If so justify your answer with a brief explanation. If not, give an example of a word the is in one language but not the other.

SOLUTION:



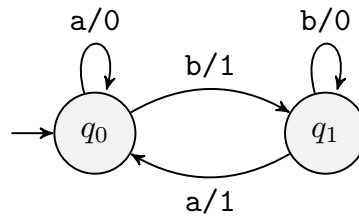
a)

- b) No the language $L1$ and $L1^*$ are not the same language, namely the empty string Λ is present in $L1^*$ but not in $L1$.

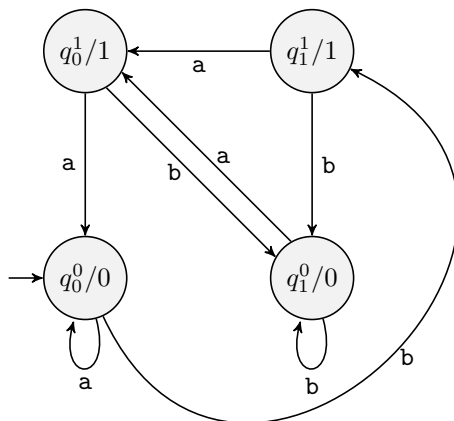
PROBLEM #6 (14 POINTS):

Consider the following Mealy machine $\Sigma = \{a, b\}$ and $\Gamma = \{0, 1\}$.

- a) Convert this Mealy machine into a Moore machine.
- b) For input $bbaa$, what is the output of the original Mealy machine?
- c) For input $bbaa$ what is the output of your Moore machine?



SOLUTION:



- a)
- b) The original mealy machine outputs 1010.
- c) The moore machine outputs 1010.