COP4020.02 **Brandon Thompson**
Dr. Muhammed Abid Due: 3/31/21

Assignment 6: Chapter 13, 14, 15

PROBLEM #1 (20 POINTS):

The following grammar has Λ-productions, but Λ is not in the language generated by this grammar. Using the algorithm in chapter 13, find another grammar for the same language that does not have any Λ-productions.

a) $S \to bZ$
$Z \to ABAB$
$A \to a|\Lambda$
$B \to b|\Lambda$

b) $S \to aX|bX$
$X \to a|b|\Lambda$

SOLUTION:

a) $S \to bZ$
$Z \to ABAB|BAB|ABA|BA|AB|A|B$
$A \to a$
$B \to b$

b) $S \to aX|bX|a|b$
$X \to a|b$

PROBLEM #2 (20 POINTS):

The following context-free grammar (CFG) has unit productions. Using the algorithm presented in chapter 13, find a CFG for the same language that does not have any unit productions.

a) $S \to aXZa|aXa|aZa|aa$
$X \to Y|a$
$Y \to Z|b$
$Z \to bZ|b$

b) $S \to aX|Yb$
$X \to S$
$Y \to bY|b$

SOLUTION:

a) $S \to aXZa|aXa|aZa|aa$
$X \to a|b|bZ$
$Z \to bZ|b$

b) $S \to aX|Yb$
$X \to aX|Yb$
$Y \to bY|b$

PROBLEM #3 (20 POINTS):

Convert the grammar you obtained as a result of Question 2 into Chomsky Normal Form (CNF).

SOLUTION:

a) $S \to R_1R_2|R_1A|AR_2|AA$
$X \to a|b|BZ$
$Z \to b|BZ$
$R_1 \to AX$
$R_2 \to ZA$
$A \to a$
$B \to b$

b) $S \to AX|YB$
$X \to AX|YB$
$Y \to YB|b$
$A \to a$
$B \to b$

PROBLEM #4 (20 POINTS):

Convert the following CFG into CNF.

a) $S \to aXX$
$X \to aS|bS|a$

b) $E \to E + E$
$E \to E * E$
$E \to (E)$
$E \to 7$

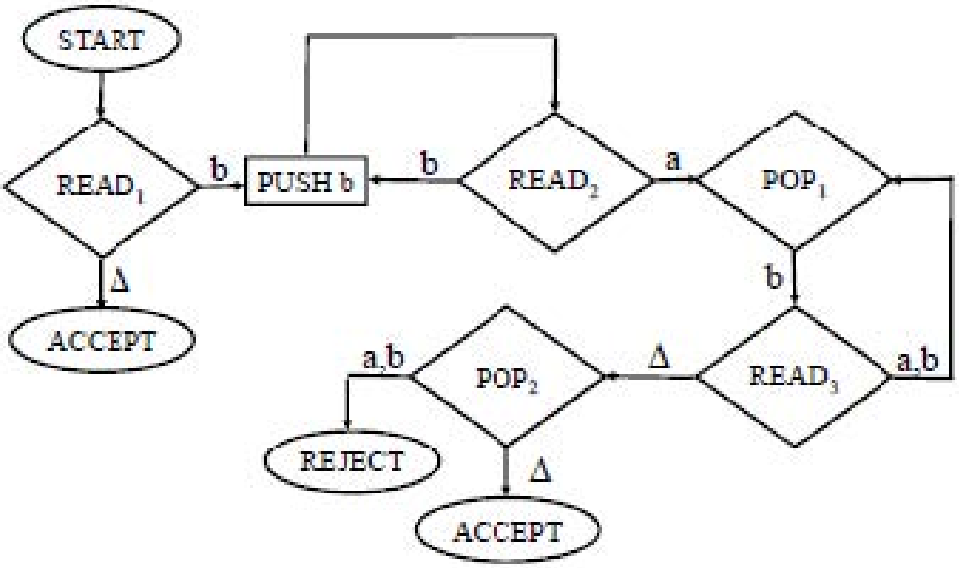
SOLUTION:

a) $S' \to AR_1$
$S \to AR_1$
$X \to AS|BS|a$
$R_1 \to XX$
$A \to a$
$B \to b$

b) $E' \to R_1E|R_2E|R_3E|7$
$E \to R_1E|R_2E|R_3)|7$
$R_1 \to E+$
$R_2 \to E*$
$R_2 \to (E$

PROBLEM #5 (15 POINTS):

Consider the following push down automation (PDA).



a) Using a trace table like those in Chapter 14, show what happens to the input tape and stack as each of the following words proceeds through the machine.

   1) *bbbaaa*

   2) *bbbbaa*

b) What is the language accepted by this PDA? Write English description of the language.

SOLUTION:

STATE	STACK	TAPE
START	Δ	bbbaaaΔ
$READ_1$	Δ	~~b~~bbaaaΔ
PUSH b	bΔ	~~b~~bbaaaΔ
$READ_2$	bΔ	~~bb~~baaaΔ
PUSH b	bbΔ	~~bb~~baaaΔ
$READ_2$	bbΔ	~~bbb~~aaaΔ
PUSH b	bbbΔ	~~bbb~~aaaΔ
$READ_2$	bbbΔ	~~bbb~~aaaΔ

STATE	STACK	TAPE
POP_1	bbΔ	~~bbb~~aaaΔ
$READ_3$	bbΔ	~~bbb~~aaaΔ
POP_1	bΔ	~~bbb~~aaaΔ
$READ_3$	bΔ	~~bbb~~aaaΔ
POP_1	Δ	~~bbb~~aaaΔ
$READ_3$	Δ	~~bbb~~aaaΔ
POP_2		~~bbb~~aaaΔ
ACCEPT		~~bbb~~aaaΔ

a) 1)

STATE	**STACK**	**TAPE**
START	Δ	bbbbaaΔ
READ_1	Δ	~~b~~bbbaaΔ
PUSH b	bΔ	~~b~~bbbaaΔ
READ_2	bΔ	~~bb~~bbaaΔ
PUSH b	bbΔ	~~bb~~bbaaΔ
READ_2	bbΔ	~~bbb~~baaΔ
PUSH b	bbbΔ	~~bbb~~baaΔ
READ_2	bbbΔ	~~bbbb~~aaΔ

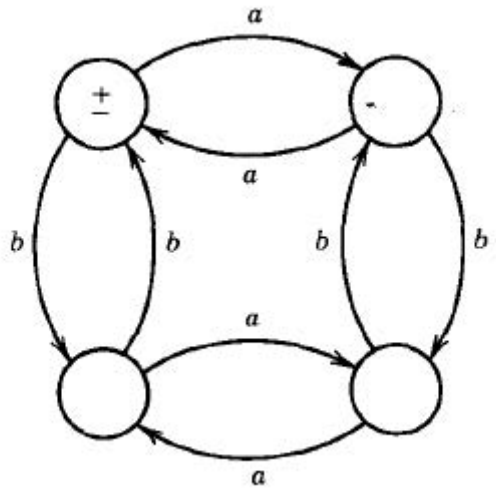
STATE	**STACK**	**TAPE**
PUSH b	bbbbΔ	~~bbbb~~aaΔ
READ_2	bbbbΔ	~~bbbba~~aΔ
POP_1	bbbΔ	~~bbbba~~aΔ
READ_3	bbbΔ	~~bbbbaa~~Δ
POP_1	bbΔ	~~bbbbaa~~Δ
READ_3	bbΔ	~~bbbbaaΔ~~
POP_3	bΔ	~~bbbbaaΔ~~
REJECT	bΔ	~~bbbbaaΔ~~

2)

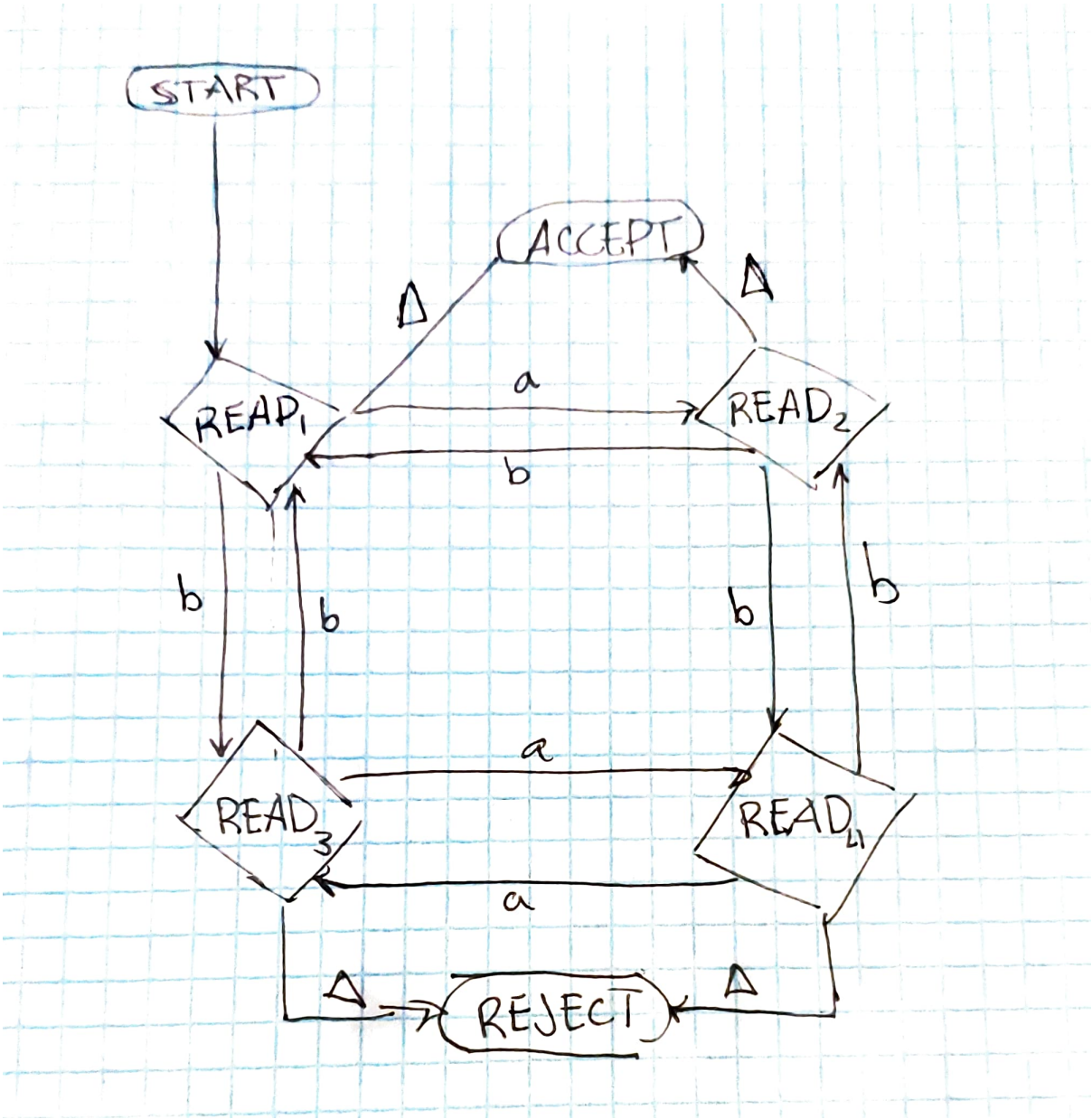
b) The language accepted by this PDA is any number of b's followed by the same number of a's.

Problem #6 (10 points):

Convert the following FA into equivalent PDA.



Solution:


START
ACCEPT
Δ
Δ
READ1
a
READ2
b
b
b
b
b
a
READ3
READ4
a
Δ
REJECT
Δ

Problem #7 (20 points):

For a given CFG, construct a PDA that accepts the same language they generate, using the algorithm in chapter 15?

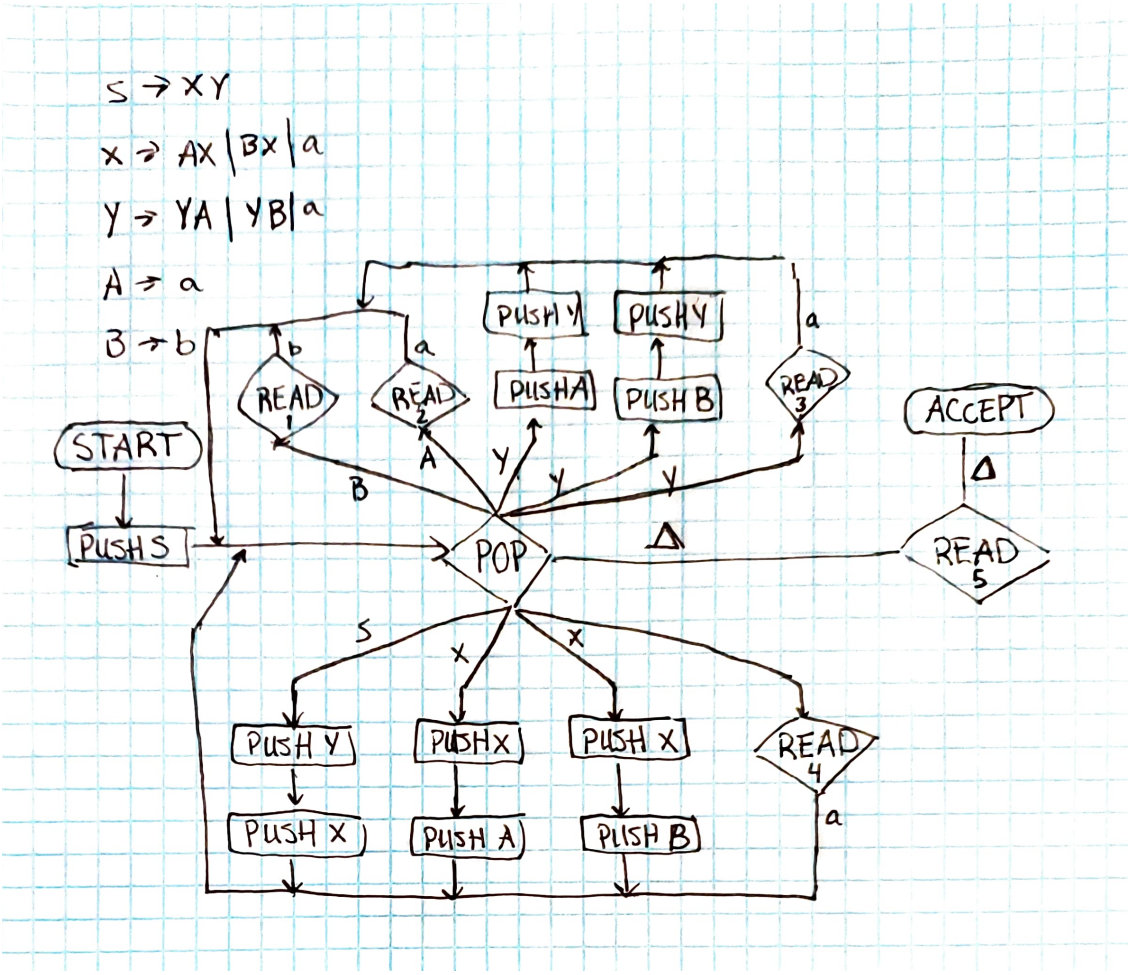
Note: Make sure that CFG in proper format to convert it into PDA.

a) $S \to XY$
$X \to aX|bX|a$
$Y \to Ya|Yb|a$

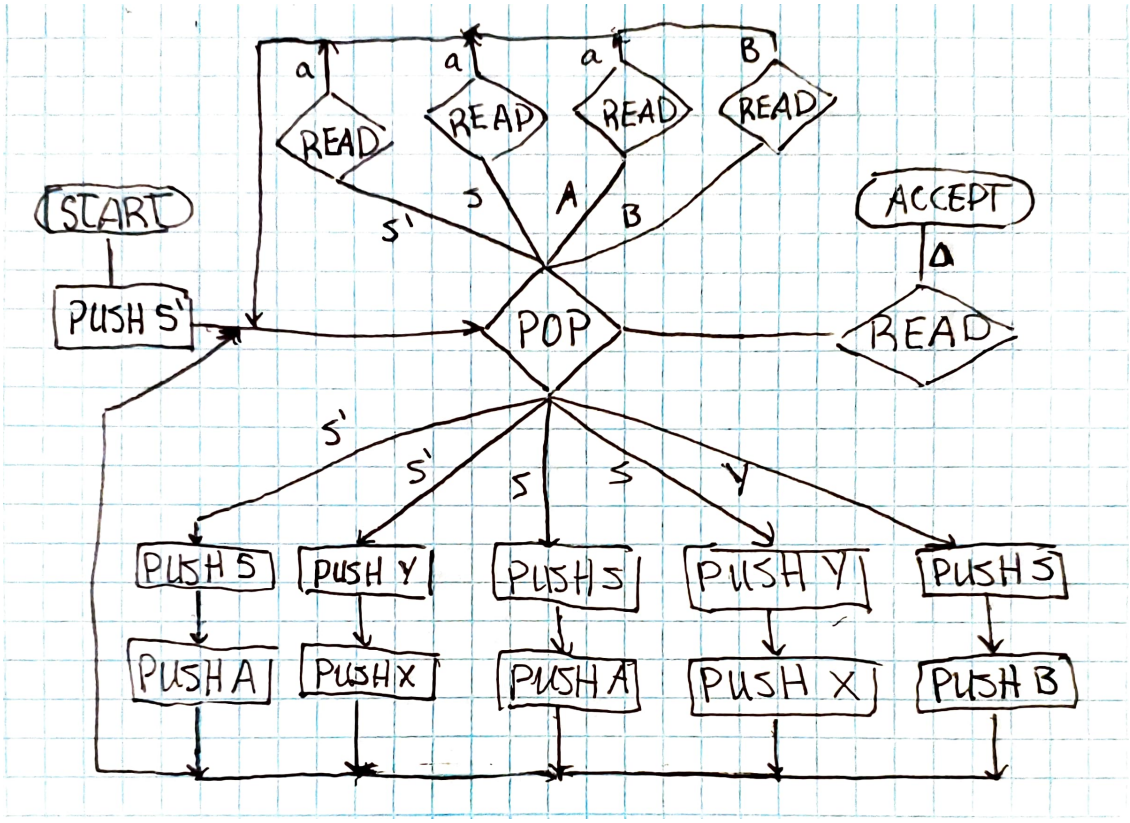
b) $S \to aS|aSbS|a$

Solution:

a)



b)


START
PUSH S'
READ
READ
READ
READ
a
a
a
B
S'
S
A
B
POP
ACCEPT
Δ
READ
S'
S'
S
S
Y
PUSH S
PUSH Y
PUSH S
PUSH Y
PUSH S
PUSH A
PUSH X
PUSH A
PUSH X
PUSH B