COP4020.02 **Brandon Thompson**

Dr. Muhammed Abid Due: 3/31/21

Assignment 5: Chapter 10, 11, 12

PROBLEM #1 (15 POINTS):

Use the pumping lemma to show that this language is nonregular:

$$\{a^n b^n a^n\} = \{aba, aabbaa, aaabbbaaa, \ldots\}$$
$$\{a^n b a^n\} = \{aba, aabaa, aaabaaa, \ldots\}$$
$$\{a^n b^{2n} = \{abb, aabbbb, aaabbbbbb, \ldots\}$$

SOLUTION:

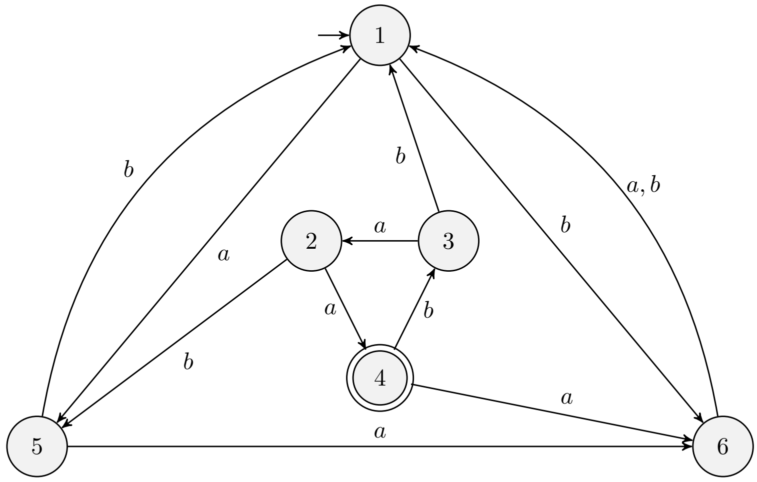
1. First we assume that L is regular and n is the number of states.
   Let $w = a^n b^n a^n$. Thus $|w| = 3n \geq n$.
   By pumping lemma, let $w = xyz$, where $|xy| \leq n$.
   Let $x = a^p, y = a^q, z = a^r b^n a^n$, where $p + q + r = n, p \neq 0, q \neq 0, r \neq 0$.
   Let $k = 2$. Then $xy^2z = a^p a^{2q} a^r b^n a^n$.
   The number of a's is :$(p + 2q + r) + n = (p + q + r) + q + n = 2n + q$
   Hence, $xy^2z = a^{n+q} b^n a^n$. Since $q \neq 0, xy^2z$ is not of the form $a^n b^n a^n$.
   Thus, xy^2z is not in L making L not regular.

2. First we assume that L is regular and n is the number of states.
   Let $w = a^n b a^n$. Thus $|w| = 2n + 1 \geq n$.
   By pumping lemma, let $w = xyz$, where $|xy| \leq n$.
   Let $x = a^p, y = a^q, z = a^r b a^n$, where $p + q + r = n, p \neq 0, q \neq 0, r \neq 0$.
   Let $k = 2$. Then $xy^2z = a^p a^{2q} a^r b a^n$.
   The number of a's is: $(q + 2q + r) + n = (p + q + r) + q + n = 2n + q$
   Hence, $xy^2z = a^{n+q} b a^n$. Since $q \neq 0, xy^2z$ is not of the form $a^n b a^n$.
   Thus xy^2z is not in L making L not regular.

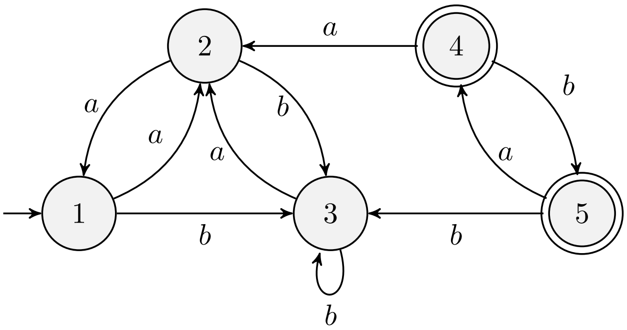
3. First we assume that L is regular and n is the number of states.
   Let $w = a^n b^{2n}$. Thus $|w| = 3n \geq n$.
   By pumping lemma, let $w = xyz$, where $|xy| \leq n$.
   Let $x = a^p, y = a^q, z = a^r b^{2n}$, where $p + q + r = n, p \neq 0, q \neq 0, r \neq 0$.
   Let $k = 2$. Then $xy^2z = a^p a^{2q} a^r b^{2n}$.
   The number of a's is: $(q + 2q + r) = (p + q + r) + q = n + q$
   Hence, $xy^2z = a^{n+q} b^{2n}$. Since $q \neq 0, xy^2z$ is not of the form $a^n b^{2n}$.
   Thus xy^2z is not in L making L not regular.

Problem #2 (10 points):

By using blue paint, determine which of the following FA's accept any words:

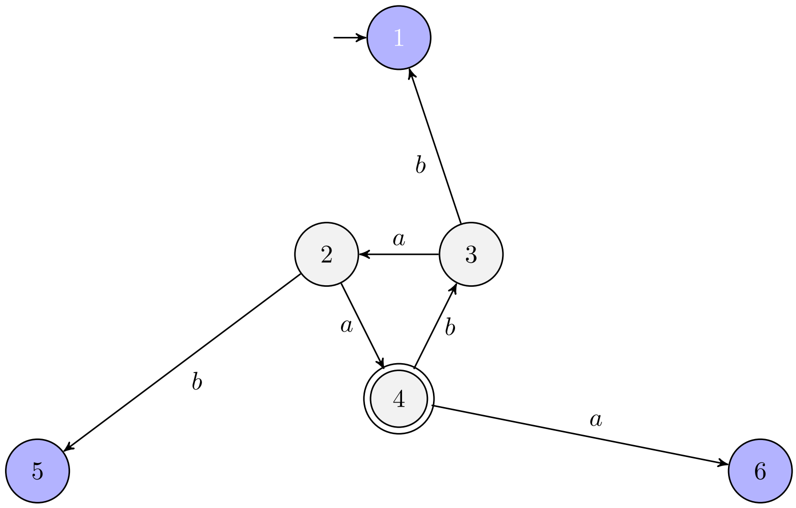


a)



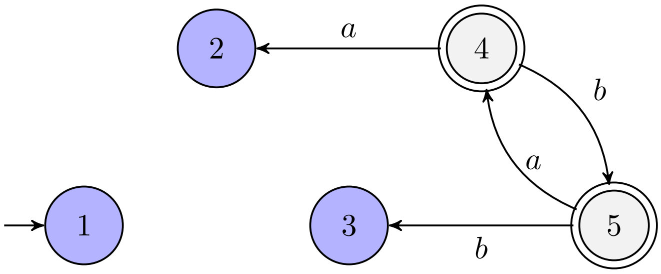
b)

Solution:



1.

2.



Problem #3 (18 points):

Describe the language generated by the following context free grammar (CFG) in English and regular expressions:

a) $S \to SS$
$S \to ZZZ$
$Z \to bZ$
$Z \to Zb$
$Z \to a$

b) $S \to aS$
$S \to bb$

c) $S \to XYX$
$X \to aX$
$X \to bX$
$X \to \Lambda$
$Y \to bbb$

Solution:

a) This CFG can be described as any number of b's with a multiple of 3 a's.
Regular Expression: $(b^*ab^*ab^*ab^*)^*$

b) This CFG can be described as any number of a's followed by two b's.
Regular Expression: a^*bb

c) This CFG can be described as any string of a's and b's containing the string bbb.
Regular Expression: $(a+b)^*bbb(a+b)^*$

PROBLEM #4 (15 POINTS):

Find CFG for the following languages over the alphabet $\Sigma = \{a, b\}$:

a) All words that have different first and last letters.

b) All words in which the letter b is never tripled.

c) All words that do not have substring ab.

SOLUTION:

a) $S \to aXb \mid bXa$
$X \to aX \mid bX \mid a \mid b \mid \Lambda$

b) $S \to \Lambda \mid b \mid bb \mid Sa \mid aS \mid Sab \mid Sabb$

c) $S \to \Lambda \mid bS \mid bX \mid X$
$X \to aX \mid \Lambda$

PROBLEM #5 (10 POINTS):

Show that the CFG below is ambiguous by finding a word with two distinct syntax trees. Show both syntax trees.

a) $S \to Sbb$
$S \to Sbbb$
$S \to b$

b) $S \to AA$
$A \to AAA|a|bA|Ab$

SOLUTION:

1. The CFG is ambiguous because the word *bbbbbb* can follow:

Rule	**Application**	**Result**
Start → S	Start	S
S → Sbbb	**S**	**Sbbb**
S → Sbb	**S**bbb	**Sbb**bbb
S → b	**S**bbbbb	**b**bbbbb
Start → S	Start	S
S → Sbb	**S**	**Sbb**
S → Sbbb	**S**bb	**Sbbb**bb
S → b	**S**bbbbb	**b**bbbbb

2. The CFG is ambiguous because the word *aba* can follow:

Rule	**Application**	**Result**
Start → S	Start	S
S → AA	**S**	**AA**
A → Ab	**A**A	**Ab**A
A → a	**A**bA	**a**bA
A → a	ab**A**	ab**a**
Start → S	Start	S
S → AA	**S**	**AA**
A → a	**A**A	**a**A
A → bA	a**A**	a**bA**
A → a	ab**a**	ab**a**