COP4020.02 **Brandon Thompson**
Dr. Muhammed Abid Due: 2/7/21

Assignment 2: Chapter 4

PROBLEM #1 (15 POINTS):

Construct a regular expression defining each of the following languages over the alphabet $\Sigma = \{a, b\}$:

a) The language of all words that do not begin with *ba*.

b) The language of all words in which the total number of b's is divisible by 3 no matter how they are distributed, such as *bbabbaabab*.

c) All words that contain exactly 2 b's or exactly 3 b's, not more.

d) All words in which a appears tripled, if at all. This means that every clump of a's contains 3 or 6 or 9 or 12 ... a's.

e) All words that contain at least one of the strings $S_1, S_2, S_3, \text{or} S_4$.

SOLUTION:

a) a^*b^*
Contains only a's or only b's or a's followed by b's, therefore the string *ba* should never appear.

b) $(a^*ba^*ba^*ba^*)^*$

c) $a^*(b+\Lambda)a^*ba^*ba^*$

d) $b^*(aaa)^*b^*$
Nothing was said about there being no a's so $(aaa)^*$ allows for only strings of b's or an empty string.

e) $(S_1+S_2+S_3+S_4)(S_1+S_2+S_3+S_4)^*$

Problem #2 (15 points):

Construct a regular expression defining each of the following languages over the alphabet $\Sigma = \{a, b\}$:

a) All strings that end in a double letter.

b) All strings that do not end in a double letter.

c) All strings that have exactly one double letter in them.

d) All words in which the letter b is never tripled. This means that no word contains the substring bbb.

e) All words in which a is tripled or b is tripled, but not both. This means each word contains the substring aaa or the substring bbb but not both.

Solution:

a) $(a+b)^*(aa+bb)$

b) $(a+b)^*(ab+ba)+a+b+\Lambda$

c) $(b+\Lambda)(ab)^*aa(ba)^*(b+\Lambda)+(a+\Lambda)(ba)^*bb(ab)^*(a+\Lambda)$

d) $(\Lambda+b+bb)(a+ab+abb)^*$

e) $(\Lambda+b+bb)(a+ab+abb)^*aaa(\Lambda+b+bb)(a+ab+abb)^* +$
$(\Lambda+a+aa)(b+ba+baa)^*bbb(\Lambda+a+aa)(b+ba+baa)^*$

PROBLEM #3 (10 POINTS):

Let us consider the regular expression

$$(a+b)^*a(a+b)^*b(a+b)$$

Show that this is equivalent to

$$(a+b)^*ab(a+b)^*$$

In the sense that they define the same language.

SOLUTION:

$(a+b)^* = \{\Lambda, a, b, aa, ab, bb, aaa, aab, aba, abb, bbb, \ldots\}$
$a(a+b)^* = \{a, aa, ab, aaa, aab, aba, \ldots\}$
$b(a+b) = \{ba, bb\}$

$(a+b)^* = \{\Lambda, a, b, aa, ab, bb, aaa, aab, aba, abb, bbb, \ldots\}$
$ab(a+b)^* = \{ab, aba, abb, abaa, abab, abbb, \ldots\}$

$a(a+b)^*b(a+b) = ab(a+b)^*$

Problem #4 (5 points):

If the only difference between L and L^* is the word Λ, is the only difference between L^2 and L^* the word Λ? Show by example.

Solution:

No, if $L = \{a, b, aa, ab, ba, bb, \ldots\}$ then $L^* = \{\Lambda, a, b, aa, ab, ba, bb, \ldots\}$.
But $L^2 = \{aa, bb, aaaa, abab, baba, bbbb\}$ which does not include $\{a, b\}$.

Problem #5 (12 points):

Describe in English phrases the languages associated with the following regular expressions.

a $(a+b)^*a(\Lambda + bbbb)$

b $(a(a+bb)^*)^*$

c $a(aa)^*b(bb)^*$

d $((a+b)a)^*$

Solution:

a The set of strings over the alphabet $\{a, b\}$ that ends with a or $bbbb$ and must have at least one a.

b The set of strings over the alphabet $\{a, b\}$ with any number of a's followed by none or even numbers of b's.

c The set of strings over the alphabet $\{a, b\}$ that contains only odd numbers of a's followed by odd numbers of b's

d The set of strings over the alphabet $\{a, b\}$ with any number of a's or b's ending with an a (Λ included)