

## Assignment 5: Chapter 10, 11, 12

PROBLEM #1 (20 POINTS):

Consider the following CFG:

$S \rightarrow AY|BZ|AA|BB$

$Y \rightarrow SA$

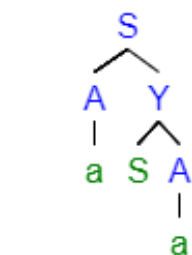
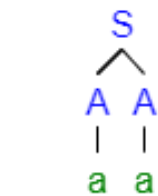
$Z \rightarrow SB$

$A \rightarrow a$

$B \rightarrow b$

- Find a derivation tree that does not have a self-embedded nonterminal.
- Find a derivation tree that contains a self-embedded nonterminal.

SOLUTION:



PROBLEM #2 (20 POINTS):

Decide whether the following grammar generates any words using the algorithm of Theorem 43 (page 403) Chapter 18.

1.  $S \rightarrow AB$   
 $A \rightarrow BC|b$   
 $C \rightarrow DA$   
 $B \rightarrow CD$   
 $D \rightarrow a$
2.  $S \rightarrow AB$   
 $A \rightarrow BSB$   
 $B \rightarrow AAS$   
 $A \rightarrow CC$   
 $B \rightarrow CC$   
 $C \rightarrow SS$   
 $A \rightarrow a|b$   
 $C \rightarrow b|bb$

SOLUTION:

1. First we replace the  $D$  production everywhere with  $a$ .

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow BC|b \\ C &\rightarrow aA \\ B &\rightarrow Ca \end{aligned}$$

Next we replace the  $A$  production everywhere with  $b$ .

$$\begin{aligned} S &\rightarrow bB \\ C &\rightarrow ab \\ B &\rightarrow Ca \end{aligned}$$

Now we replace the  $C$  production everywhere with  $ab$

$$\begin{aligned} S &\rightarrow bB \\ B &\rightarrow aba \end{aligned}$$

Lastly we replace the  $B$  production everywhere with  $aba$ .

$$S \rightarrow baba$$

There is a production of the form  $S \rightarrow t$  so the language is not empty.

2. First we replace the  $C$  production with  $b$

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow BSB \end{aligned}$$

$$B \rightarrow AAS$$

$$A \rightarrow bb$$

$$B \rightarrow bb$$

$$A \rightarrow a|b$$

Next we replace all of the  $B$  productions with  $bb$ .  $S \rightarrow Abb$

$$A \rightarrow bbSbb$$

$$A \rightarrow bb$$

$$A \rightarrow a|b$$

Lastly we replace all of the  $A$  productions with  $a$ .  $S \rightarrow abb$

There is a production of the form  $S \rightarrow t$  so the language is not empty.

PROBLEM #3 (20 POINTS):

Consider the following grammar for arithmetic expressions.

$$S \rightarrow E$$

$$E \rightarrow T \mid E + T \mid E - T \mid -T$$

$$T \rightarrow F \mid T * F \mid T / F$$

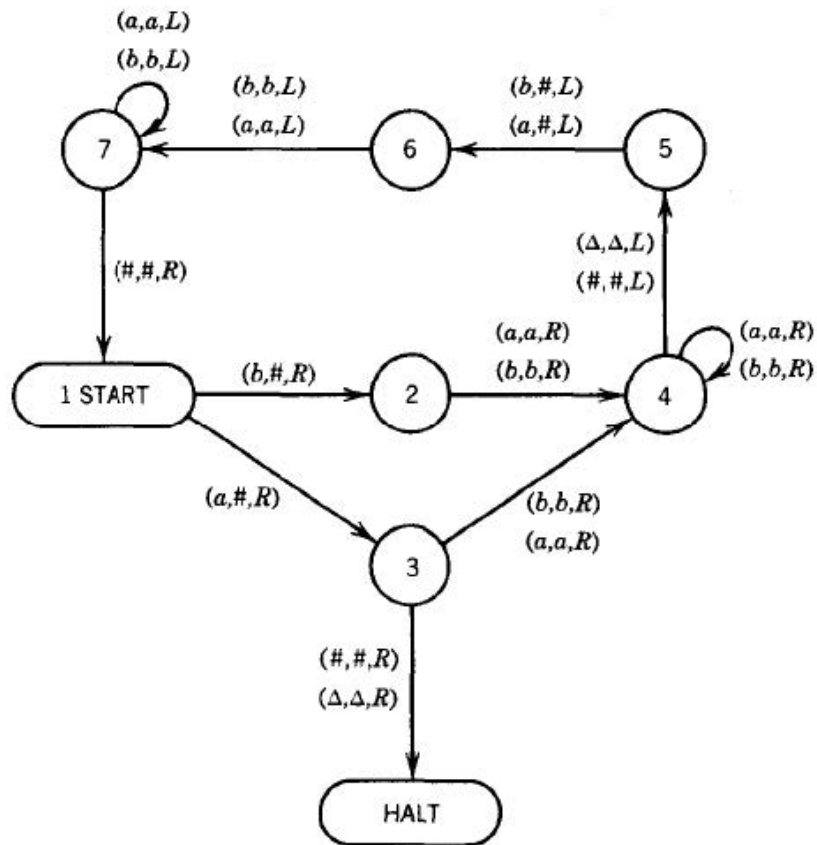
$$F \rightarrow (E) \mid i$$

Using top-down parsing, find a leftmost derivation in this grammar for the expression  $i/i + i$ . Show your work.

SOLUTION:

PROBLEM #4 (20 POINTS):

a) Consider the following Turing Machine (TM).



Trace the execution chains of the following input strings on this machine.

- 1) baaba
- 2) ababb

SOLUTION:

<b>START</b>	<b>2</b>	<b>4</b>	<b>4</b>	<b>4</b>
<u>b</u> aaba	# <u>a</u> aba	#a <u>a</u> ba	#a <u>a</u> b	#a <u>a</u> b

<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>7</b>
#aaba <u>Δ</u>	#a <u>a</u> b	#a <u>a</u> b#	#a <u>a</u> b#	# <u>a</u> ab#

<b>7</b>	<b>1</b>	<b>3</b>	<b>4</b>	<b>4</b>
# <u>a</u> ab#	# <u>a</u> ab#	## <u>a</u> b#	## <u>a</u> b#	## <u>a</u> b#

<b>5</b>	<b>6</b>	<b>7</b>	<b>1</b>	<b>3</b>
## <u>a</u> b#	## <u>a</u> ##	## <u>a</u> ##	## <u>a</u> ##	### <u>a</u> ##

**HALT**  
a) #####

<b>START</b>	<b>3</b>	<b>4</b>	<b>4</b>	<b>4</b>
<u>a</u> babb	# <u>b</u> abb	#b <u>a</u> bb	#b <u>a</u> bb	#b <u>a</u> bb

<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>7</b>
#babb <u>Δ</u>	#b <u>a</u> bb	#b <u>a</u> b#	#b <u>a</u> b#	# <u>b</u> ab#

<b>7</b>	<b>1</b>	<b>2</b>	<b>4</b>	<b>4</b>
# <u>b</u> ab#	# <u>b</u> ab#	## <u>a</u> b#	## <u>a</u> b#	## <u>a</u> b#

<b>5</b>	<b>6</b>	<b>7</b>	<b>1</b>	<b>3</b>
## <u>a</u> b#	## <u>a</u> ##	## <u>a</u> ##	## <u>a</u> ##	### <u>a</u> ##

**HALT**  
b) #####