COP4020.02 **Brandon Thompson**

Dr. Muhammed Abid Due: 1/31/21

Assignment 1: Chapters 1, 2, 3

PROBLEM #1 (7 POINTS):

a) Consider the language S^*, where $S = \{a, ab, ba\}$. Is the string $(abbba)$ a word in the language?

b) Write out all of the words in this language with 3 letters.

c) Write 5 words from this language with 4 letters.

d) What is English description of this language?

SOLUTION:

a) The String $(abbba)$ is not in the language S^*.
If we factor the word we get: $ab|b|ba$ and b is not a letter in the alphabet, meaning that this cannot be a word in S^*.

b) (aaa, aab, aba, baa)

c) $(aaaa, aaab, aaba, abaa, baaa, abba)$

d) All words of a's, and b's with no more than 2 b's in a row.

PROBLEM #2 (6 POINTS):

Let $\Sigma = \{a, b\}$

a) Consider the language S^*, where $S = \{aa, ab, ba, bb\}$. Give an English description of this language.

b) Give an example of a set S such that S^* only contains all possible strings of a's and b's that have length divisible by 3.

c) Let S be all strings of a's and b's with odd length. What is S^*?

SOLUTION:

a) All combinations of a's and b's excluding odd amounts of just a's or odd amounts of just b's.

b) $S = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$, with a language of only three letter words the Kleene closure is guaranteed to only have combinations divisible by 3.

c) S^* is all strings of a's and b's. Because there is a single 'a' and 'b', any string in S^* could be factored with these two.

Problem #3 (10 points):

a) Let $S = \{b, bb\}$ and $T = \{ab, bb, bbbb$. Show that $S^* = T^*$.

b) Let $S = \{ab, bb\}$ and $T = \{ab, bb, bbb\}$. Show that $S^* \neq T^*$ but that $S^* \subset T^*$

c) What principle does this illustrate?

Solution:

1. $bbbb \in T$ can be factored by $bb \in S$. Therefore, any combination of $bbbb$ is a combination of bb meaning that $S^* = T^*$.

2. Because $S \subset T$ then by definition, every word in S^* is in T^*, but not every word in T^* is in S^*. The word $ab|bbb \in T^*$ but this cannot be factored with only ab, bb so $ab|bbb \notin S^*$.

3. This shows the principle of equality of sets.

PROBLEM #4 (15 POINTS):

Let $\Sigma = \{a, b\}$. Give recursive definitions for the following languages over Σ.

a) The language BB of all words containing the substring bb.

b) The language NOTBB of all words not containing the substring bb.

c) Give recursive definition of the set EVEN. Show that all the numbers in it end in the digits 0, 2, 4, 6, or 8.

d) Give a recursive definition of the set ODD $= \{1, 3, 5, 7, \ldots\}$.

e) Give a recursive definition for the set of strings of digits 0,1,2,3,..., 9 that cannot start with the digit 0.

SOLUTION:

a) BB is defined by the rules:

1 The string bb is in BB.

2 If x is in BB, any string of a' or b's concatenated with x is in BB.

3 The only elements of BB are those that are constructed by following rules 1 and 2.

b) NOTBB is defined by the rules:

1 $a \in$ NOTBB and $b \in$ NOTBB

2 If $x \in$ NOTBB and x ends with a, add a or b. If x ends with b add a.

3 The only elements in NOTBB are the ones that are constructed by following rules 1 and 2

c) EVEN is defined by the rules:

1 $2 \in$ EVEN

2 If $x \in$ EVEN, $x + 2 \in$ EVEN.

3 The only elements in EVEN are the ones that are constructed by following rules 1 and 2.

EVEN $= \{2, 4, 6, 8, 10, 12, \ldots\}$, any element of EVEN will end in $(0, 2, 4, 6, 8)$.

d) ODD is defined by the rules:

1 $1 \in$ ODD

2 If $x \in$ ODD, $x + 2 \in$ ODD/

3 The only elements in ODD are the ones that are constructed by following rules 1 and 2.

e) String of digits not starting with 0 is defined by the rules:

1 $(1, 2, 3, 4, 5, 6, 7, 8, 9) \in$ DIGITS

2 If w is any word in DIGITS, then the concatenations:
$w0, w1, w2, w3, w4, w5, w6, w7, w8, w9$ are also in DIGITS.

3 The only elements in DIGITS are the ones that are constructed by following rules 1 and 2.

Problem #5 (5 points):

In the given recursive definition for the language PALINDROME over the alphabet $\Sigma = \{a, b\}$:

1. $a, b \in$ PALINDROME

2. if x is in PALINDROME, then axa and bxb are also in PALINDROME.

All the words in the language defined above have an odd length and so it is not all of PALINDROME. Fix this problem.

Solution: Adding aa and bb to the base case will fix the odd length issue. aa and bb will create only even members of PALINDROME, $(aa, bb, aaaa, baab, abba, bbbb, \ldots)$. The combination of even PALINDROME and odd PALINDROME creates a complete set of PALINDROME.