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Monte Carlo and Central Limit Theorem Short Report

Objective:

The goal of this homework is to estimate the value of pi using the Monte Carlo method. Once this is implemented, then we explore the Law of Large Numbers and the Central Limit Theorem.

Method:

π is estimated using the ratio of points that fall inside a unit circle to the total number of points uniformly generated points inside a unit square.

1. Generate n random points in the square [-1,1] x [-1,1]
2. Count how many points are inside the unit circle:

$$\sqrt{x^2 + y^2} \leq 1$$

3. Estimate pi using the equation:

$$\pi_{est} = 4 * \frac{Points\ in\ Circle}{Total\ Points}$$

Task 2:

The estimating pi methodology was repeated for values from 1 to 10000.

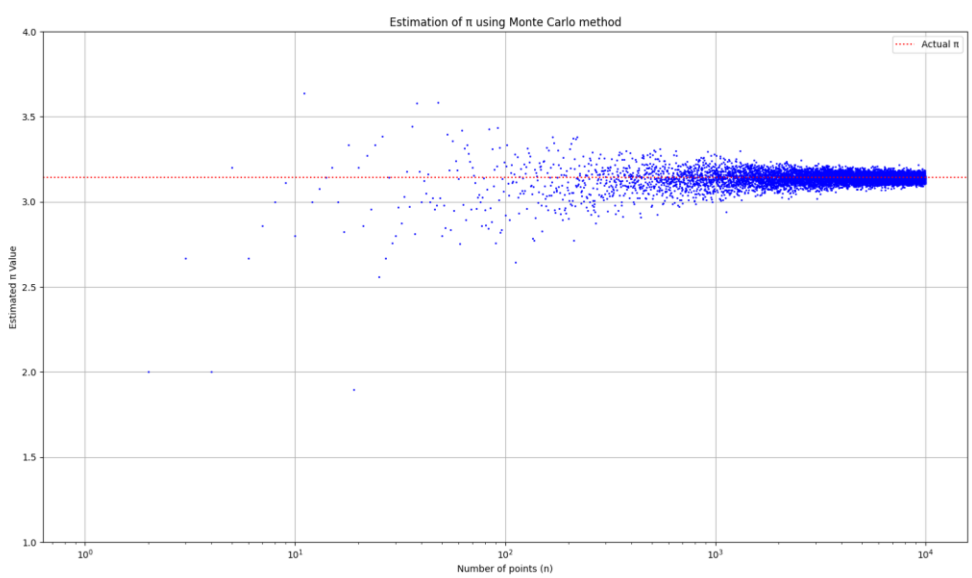

Figure 1. Estimated π vs. Number of points (n)

At small n, the estimate has much higher variance. The estimated value becomes more stable and accurate as n increases. As n increases, the estimated value converges towards the true value of pi (in accordance with the law of large numbers).

Task 3:

The estimating pi methodology was then repeated 500 times for three increasing n values.

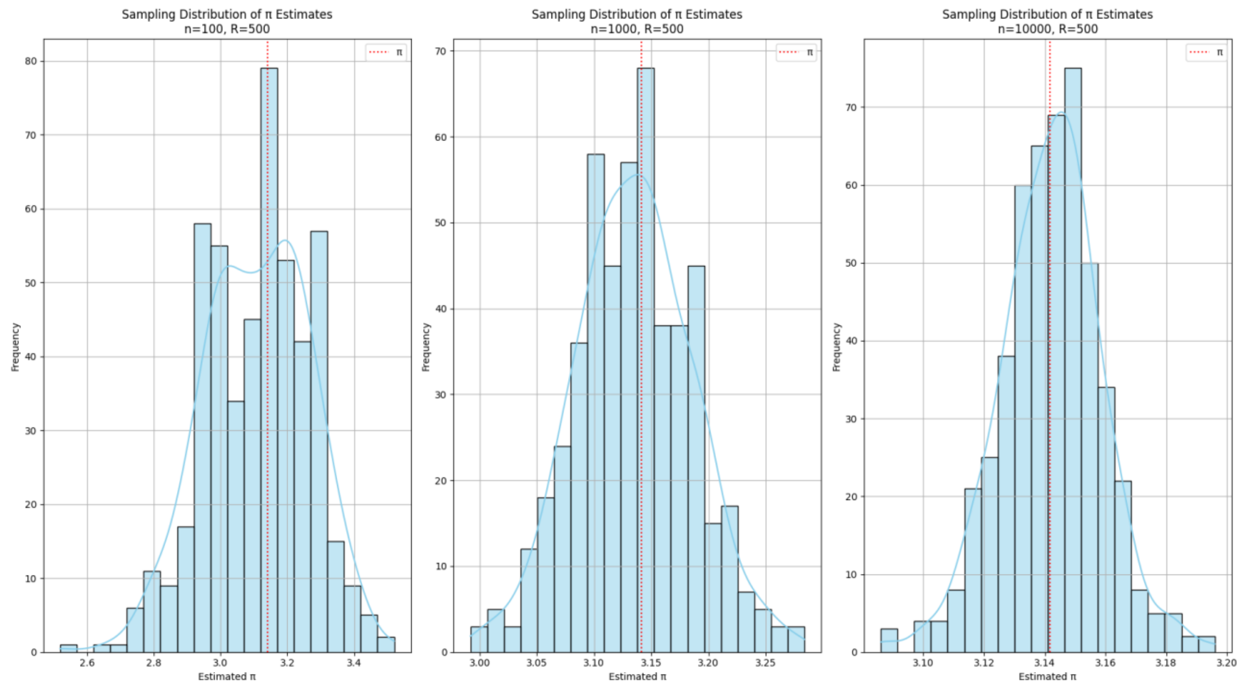

Figure 2. Sampling Distributions of Estimated π for Increasing Sample Sizes. Each histogram is based on 500 runs, with n = 100, 1000, and 10,000.

A kernel density estimation is overlaid on top of these histograms to demonstrate how the distribution of estimated values resembles a normal distribution. Unfortunately, due to computational limitations, higher values of n could not be tested, but the trend is still observable. The distribution can be seen getting smoother and more concentrated around the actual value as individual trial accuracy increases. The bell curve is getting narrower.

Conclusion:

This simple program demonstrated the Monte Carlo method for estimating π and how increasing number of samples improved accuracy. Furthermore, by repeating it multiple times, it was shown that the distribution of estimates approached a normal distribution – as predicted by the Central Limit Theorem.