Breaking Rotacrypt: Cryptanalysis Breakdown of Rotational Mechanics as a Cryptographic Primitive

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Abstract

...TBD...

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1 Introduction

Encryption, the process of encoding information in a way that only authorized parties can decipher it, lies at the heart of secure digital communication. It serves as a fundamental tool for safeguarding our messages, files, and personal data from unauthorized access and eavesdropping.

1.1 Outline

In the ever-evolving landscape of encryption, innovative approaches continue to emerge, each inspired by unique and unexpected sources. I propose the worst of them all, Rotacrypt, a new cryptosystem that employs the mechanics of the Rubik's Cube to create a fresh paradigm in secure communication.

Rotacrypt uses Rubik's Cube mechanics to construct a mathematically intricate and seemingly secure encryption scheme. Despite its promise, the system contains an inherent flaw related to the cube's cyclic rotational limitations. The latter part of this paper will perform a cryptanalysis of this flaw, showing its capacity to compromise Rotacrypt's security.

This paper first introduces Rotacrypt, a nearly flawless cryptosystem, and then proceeds to break it due to a singular flaw in the scheme's cryptographic primitive.

1.1.1 Rotacrypt Outline

In Section 2, I start by providing a brief overview of the cube's mechanics. I state the goals of my approach, the avenues that I've chosen, and limitations that arise. The following logical workflow is used in Section 2:

1 `Physical breakdown of the cube and its core mechanics`

2 `Review of the notation used to describe the cube's movements`

3 `Discussion of the cube's state space and the combinatorial explosion that occurs when multiple cubes are chained together`

4 `Review of the constraints imposed by the cube's rotational mechanics and the new resulting state space`

5 `Explanation of the scheme's overall goal and primary avenue of approach`

6 `Summary of the key points of the section`

Before jumping into the technicalities of the scheme, I provide a brief high-level diagram overview in Section 3 accompanied by explanatory text.

In Section 4, I detail the programmed implementation of the *3x3x3* cube and its associated data structure with an emphasis on its linearity. This includes subsections on the augmented *SPEFFZ* map transformations and the cyclic transformations.

In Section 5, I discuss the deserialization algorithm(s) and the overall key generation protocol for primary-key and sub-key generation.

In Section 6, I finally dive into the encryption process. To understand Section 6 fully, it is recommend to read through all previous sections, but it is not required. The sections build on each other, but are designed to be understandable in isolation.

In Section 7, I discuss the complementing decryption process. In Section 8, ...

1.2 Purpose

There is no purpose. I was advised not to create a cryptosystem, so I did the opposite. As it turns out, the other person was right—so I will be the one to break my own scheme.

2 The Rubik's Cube

The Rubik's Cube, created by Ernő Rubik in 1974, is more than a mind-bending puzzle; it serves as a direct application of group theory principles. Its intricate configurations provide a robust framework for in-depth studies in both cryptography and algorithm design.

2.1 Review of Rubik's Cubes

Max Park, the current world record holder (11 June 2023) for the 2-handed solve, obliterated the cube in a mind-blowing 3.13 seconds. Lucky scramble? Hardly. Yiheng Wang, who holds the world record average-of-five, clocks in at a dizzying 4.48 second mean solve time throughout the five solves.[1] And no, I won't depress you by mentioning his tender age of nine years old. But let's not divert. The Rubik's Cube–a mathematical marvel and a plaintext[2] waiting to be encrypted.

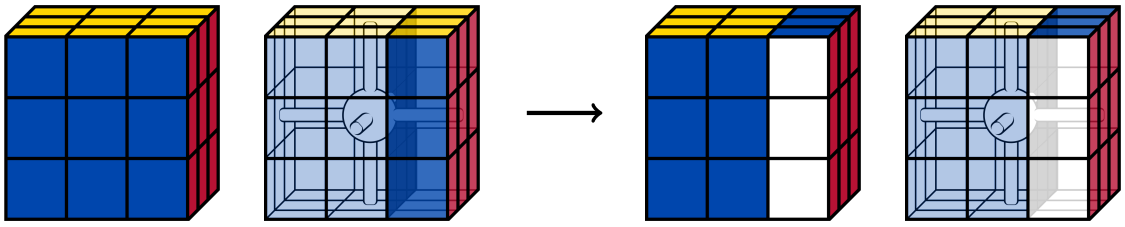
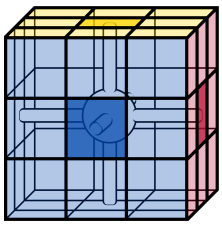


Figure 1: A visualization of the R operation (rotating the right layer clockwise).

Let's dive into the mechanics of the 3x3x3 puzzle. The cube boasts centers, edges, and corners. The single-colored center pieces serve as the invariant axis around which the peripheral smaller unit cubes rotate. The six unit colors are yellow, blue, red, green, orange, and white.



A visualization of a 3x3x3 cube. The cube has 6 faces, each with 9 stickers. Notably, this means that there exists 54 different unit tiles on the cube. The cube has 43,252,003,274,489,856,000 possible states.[3]

For instance, if the cube is held with a yellow top and blue front, the red and orange faces will invariably be to the right and left, respectively. In fact, the red and orange center pieces will *always* be opposites, as will the blue-green and white-yellow pairs of center pieces.

[1] An average-of-five is determined by taking the average of the three "middle" solves in a session of five scramble. In other words, the worst and best solve time are dropped from the calculation for the average.

[2] In cryptography, plaintext refers to the original readable message, while ciphertext is the scrambled message produced through encryption.

[3] This number is calculated by considering the 8 corners, each with 3 orientations, and the 12 edges, each with 2 orientations.

2.2 Notation for Cube Operations

The notation used to describe the movements and algorithms for solving the Rubik's Cube is standardized to facilitate easy understanding and sharing of solutions. Each face of the cube is designated by an uppercase letter:

- **U** - Up (Top Layer)
- **D** - Down (Bottom Layer)
- **L** - Left (Left Layer)
- **R** - Right (Right Layer)
- **F** - Front (Front Layer)
- **B** - Back (Back Layer)

The following symbols are appended to these letters to indicate the direction of rotation:

- No symbol - 90-degree clockwise rotation
- **'** (apostrophe) - 90-degree counterclockwise rotation
- **2** - 180-degree rotation (either direction)

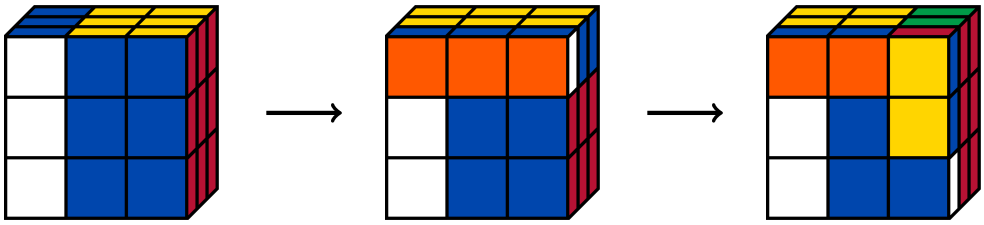


Figure 2: A visualization of the *L' U' R'* operation(s).

For example, the sequence *L' U' R'* would indicate a counterclockwise rotation of the left layer, followed by a counterclockwise rotation of the top layer, and finally, a counterclockwise rotation of the right layer.

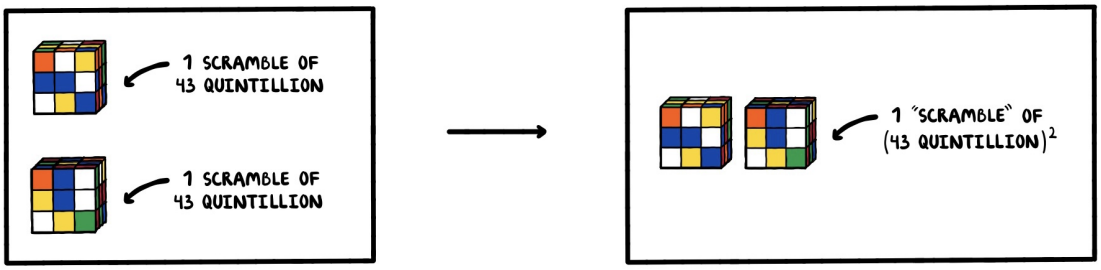
2.3 God's Number

In the realm of 3x3x3 Rubik's Cubes, *God's Number* is a term used to denote the maximum number of moves required to solve any scrambled cube. It has been proven that any cube can be solved in 20 moves or fewer (the citation can be found in the *References* section). This concept is an intriguing insight into the mathematical efficiency of the cube's design.

Furthermore, this means that any scramble can be reached with 20 moves or fewer. This is a key point to keep in mind when considering the security of the cube as a cryptographic primitive as well as for key size considerations.

2.4 Combinatorial Explosion with Multiple Cubes

Let us consider the number of possible states for a single 3x3x3 Rubik's Cube, which is $43,252,003,274,489,856,000$. This is not large enough for a secure cryptographic state space. However, when chaining[4] together the combinations of two different cubes, the number of combined states is $(43,252,003,274,489,856,000)^2$.



This squaring occurs because each state of the first cube can pair with every state of the second cube, yielding $43,252,003,274,489,856,000 \times 43,252,003,274,489,856,000$. Mathematically, the set of possible states becomes the Cartesian product of the two sets of states, leading to an exponential increase in complexity.

The intriguing aspect of chaining multiple Rubik's Cubes is the notable exponential growth in the state space[5]. Let N represent the number of unique states for a single Rubik's Cube. For k chained Rubik's Cubes, the total number of unique states becomes N^k. This exponential increase serves a critical function: it substantially minimizes the likelihood of collisions whilst simultaneously increasing the difficulty of brute-force attacks.[6]

2.5 Edge and Corner Constraints

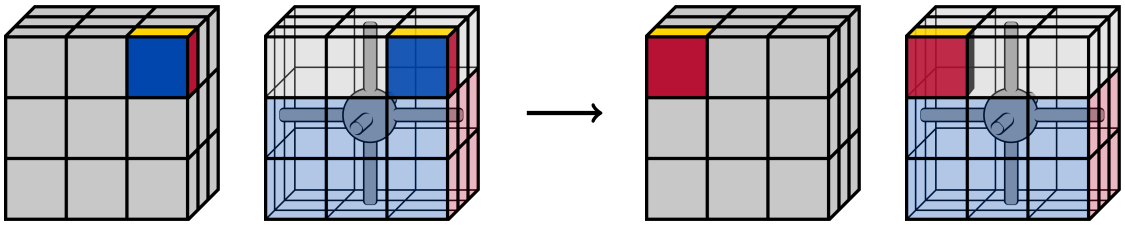


Figure 3: A visualization of categorical immutability. Corners will always map to corners.

One of the fundamental constraints of a Rubik's Cube is that edges and corners are immutable in their categories; edges cannot morph into corners and vice versa. Due to this

[4]Linking the encryption of one block with the next, making the entire data set more secure and diffused by causing a ripple effect throughout the blocks.

[5]The state space refers to the total number of possible configurations or states that a system can be in; for a cube, it's a very large number.

[6]In cryptography, a brute-force attack involves trying all possible combinations to decrypt a message, which is computationally expensive.

constraint, it becomes inappropriate to assign all 48 movable units (54 total units minus 6 centers) to represent bits, as doing so would leave patterns in trivial plaintext-to-ciphertext cryptanalysis attacks.[7] If a bit mapped to a corner unit can never be encrypted into an edge piece no matter how many layer operations occur, this serves as a trivial pattern in which cryptanalysists can exploit.

While the utilization of multiple Rubik's Cubes in the encryption scheme introduces a combinatorial explosion in the state space, it is essential to consider the limitations imposed by the unique and contained mapping of plaintext bits to chosen cube pieces. Each plaintext bit (either a 0 or 1) is mapped to a specific piece on the cube, which can either be an edge or a corner for the entirety of the scheme.

2.6 Per-Cube 24-bit State Space

As a result of the aforementioned limitations, the encryption scheme opts for a more constrained assignment by sticking to either edges or corners to represent the bits. This decision narrows down the number of units used for bit representation to 24, thus limiting the state space to 2^{24} for each cube.

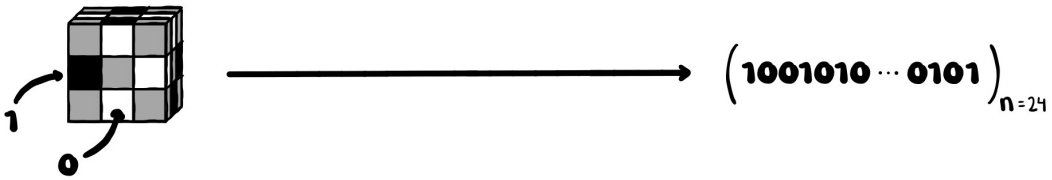


Figure 4: The serialization of a plaintext-mapped cube state. Black units represent 1s, while white units represent 0s.

A serialized 24-bit bitstring yields a state space of 2^{24} *because there are* 2^{24} *different ways to pick and choose 0s and 1s across 24 elements. For the first bit, there are 2 possible choices for the value. For each of those choices, the second bit can also have 2 possible values. This pattern continues for all 24 bits, leading to* $2 \times 2 \times 2 \times \ldots$ *(24 times)* $\rightarrow 2^{24}$.

Given the inherent limitations in the state space of a single Rubik's Cube, capped at 2^{24} due to geometric constraints, a novel approach to enlarging this space involves chaining multiple cubes together. Specifically, by employing a tuple of six Rubik's Cubes[8] for each block, the scheme effectively increases the state space to $2^{(24\times6)}$.

In this enhanced scheme, each block of plaintext is split into six portions, each of which is mapped onto a separate cube. The combined state of all six cubes serves as a unique

[7] Cryptanalysis attacks involve mathematical and computational techniques to analyze and possibly break an encryption scheme.

[8] In this scheme, a tuple refers to an ordered set of six individual Rubik's Cubes, each contributing to the encryption process.

representation of the original block. This approach leverages the geometric diversity across multiple cubes to create a more complicated and less predictable state space.

With a state space of $2^{(24\times6)}$, the algorithm becomes computationally prohibitive for brute-force attacks, even when accounting for potential parallelization.[9]

While the increase in state space significantly boosts the algorithm's resilience against attacks, it also introduces additional computational complexity. However, the impact on efficiency is deemed acceptable given the substantial increase in cryptographic security.[10]

2.7 Summary

The *Introduction* can be summarized with the following points:

- **3x3x3 Mechanics**: The cube boasts centers, edges, and corners. The six unique single-colored center pieces serve as the invariant axis around which the peripheral cubies rotate.
- **State Space**: The cube has 6 faces, each with 9 stickers. Notably, this means that there exists 54 different unit tiles on the cube. The cube has 43,252,003,274,489,856,000 possible states.
- **Increasing State Space**: Let N represent the number of unique states for a single Rubik's Cube. For k chained Rubik's Cubes, the total number of unique states is N^k.
- **No More Colors**: In this scheme, colors are not mapped to units on the cube. Instead, each plaintext bit (either a 0 or 1) is mapped to a specific piece on the cube. As centers are immutable, they are not used to represent bits (only 48 movable units).
- **Mapping Constraints**: One should note that edges cannot morph into corners and vice versa. Therefore, bits are only assigned exclusively to either edges or corners (24 potential bit mappings per cube), thus limiting the state space to 2^{24} for each cube.
- **State Space Again**: By employing a tuple of six Rubik's Cubes for each block, the scheme effectively increases the state space to $2^{(24\times6)}$ (yielding 2^{144} possible states).

[9] Parallelization refers to the process of dividing a task into sub-tasks that are solved concurrently, often used to speed up computational tasks.

[10] The term refers to the resilience of a cryptographic system against unauthorized access or data breaches.

3 Overview

The proposed encryption scheme is a block cipher[11] that leverages the mathematical complexities and state space of Rubik's Cubes. Each block in the scheme utilizes a tuple of six 3x3x3 Rubik's Cubes, effectively rendering a block size of 144 bits (24 bits per cube; 6 cubes).

3.1 Initial Round

At the start of the encryption process, each cube is initialized to the solved state and then a corresponding-plaintext-bit-to-3x3x3-layer-operation-mapping is applied. The details of this progress can be found in the *Encryption / Cube Mapping Procedure* section. This will yield six unique cube states.

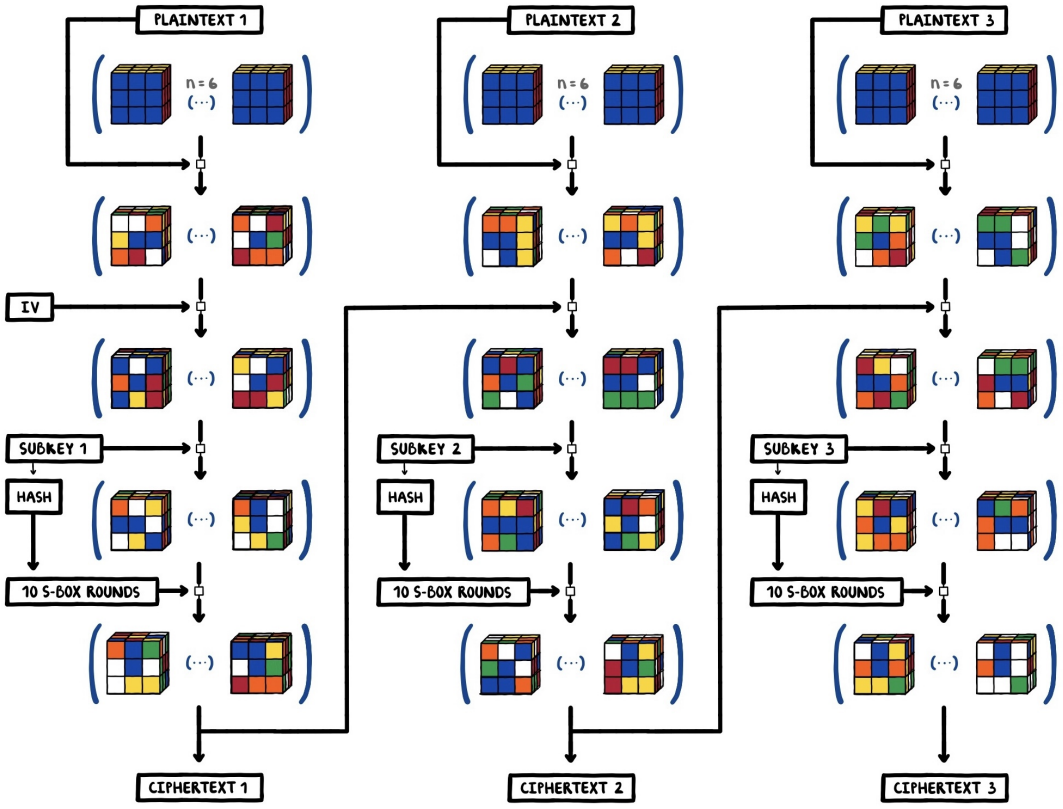


Figure 5: Rotacrypt Encryption Overview

The initial round begins by applying a cryptographically secure pseudorandom number generated Initialization Vector[12] (IV) deserialized into six unique scrambles. Each cube in the

[11] Encrypts chunks of plaintext data (blocks) using unique subkeys for each block.

[12] An Initialization Vector is a cryptographically secure random number that is applied during initial encryption, ensuring distinct encrypted data every time (even with equivalent plaintext inputs). This technique disrupts pattern recognition attempts, particularly in chosen-plaintext attacks.

tuple will have their own unique scramble at this stage, and a dynamic subkey[13] is then generated from the original key using a secure key derivation function. The subkey is likewise deserialized[14] into six unique scrambles. Each scramble is applied to their respective cube. The cubes finally undergo a series of deterministic Rubik's moves, mimicking the traditional S-box[15] and diffusion[16] operations in AES like SubBytes, ShiftRows, and MixColumns. These Rubik's moves are termed as SubCubes, ShiftFaces, and MixEdges respectively. It should be noted that the nature of layer operations intrinsically involves a form of diffusion, making the scheme's diffusion already naturally high without S-box operations. The resultant state of the cubes serves as the ciphertext for that block.

One may question the intent of an S-box operation if the layer operations already provide a high degree of diffusion. Layer operations are linear transformation[17] (this becomes evident in *3x3x3 Implementation*), and the S-box operation adds a layer of non-linearity to the encryption process. This non-linearity is critical to the security of the scheme, as it prevents the encryption process from being modeled as a linear system of equations.

3.2 Subsequent Rounds

For subsequent rounds, a part of the cube state from the previous round is extracted and used as the IV for the next block. New subkeys are generated dynamically by hashing the original key along with a salt (detailed in the *Key Generation / Sub-Key Generation* section), which is then converted into a Rubik's Cube scramble sequence. The same sequence of operations: SubCubes, ShiftFaces, and MixEdges, are then applied to these new blocks, followed by scrambling with the round-specific subkey.

The state of the cubes after the S-box operation is used to link subsequent blocks, ensuring that the entire encryption process influences each block. This design decision not only maximizes the use of the large Rubik's Cube state space but also provides strong cryptographic properties.

We are chaining blocks to increase diffusion. A blockchain if you must.

3.3 Final Round

(MAYBE... TBD) In the final round, the processed cubes go through an S-box transformation to further improve the security of the encrypted data. This S-box is carefully designed to maximize non-linearity and is dynamically generated based on the cube's final state.

[13] A subkey is a key derived from a master key for use in a particular cryptographic algorithm. In this scheme, subkeys are generated dynamically for each block

[14] Turning the cube's state into a simple format like a string or array for easier data handling.

[15] A substition box (S-box) is a cryptographic component that performs fixed, non-linear substitutions on input bits to produce output bits.

[16] Diffusion is a cryptographic concept that refers to the spreading of plaintext information throughout the ciphertext, making it difficult to decipher.

[17] Linear transformations are mathematical operations that preserve the structure of the underlying space, such as rotations and reflections.

4 3x3x3 Implementation

In selecting an implementation approach, the conventional choice often centers around the 3x3x3 matrix representation, which involves managing multidimensional arrays and employing matrix operations to facilitate cube rotations. However, I opted for an alternative method: a one-dimensional array consisting of 54 elements.

This choice was driven by the desire for a more streamlined and memory-efficient approach to represent the Rubik's Cube in the context of our encryption scheme.

The one-dimensional representation not only simplifies the management of the cube's state but also spotlights the intriguing abstract cycles that underlie the cube's mechanics.

4.1 Data Structure Breakdown

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4.2 Augmented SPEFFZ Mapping

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4.3 Cyclic Transformations

5 Key Generation

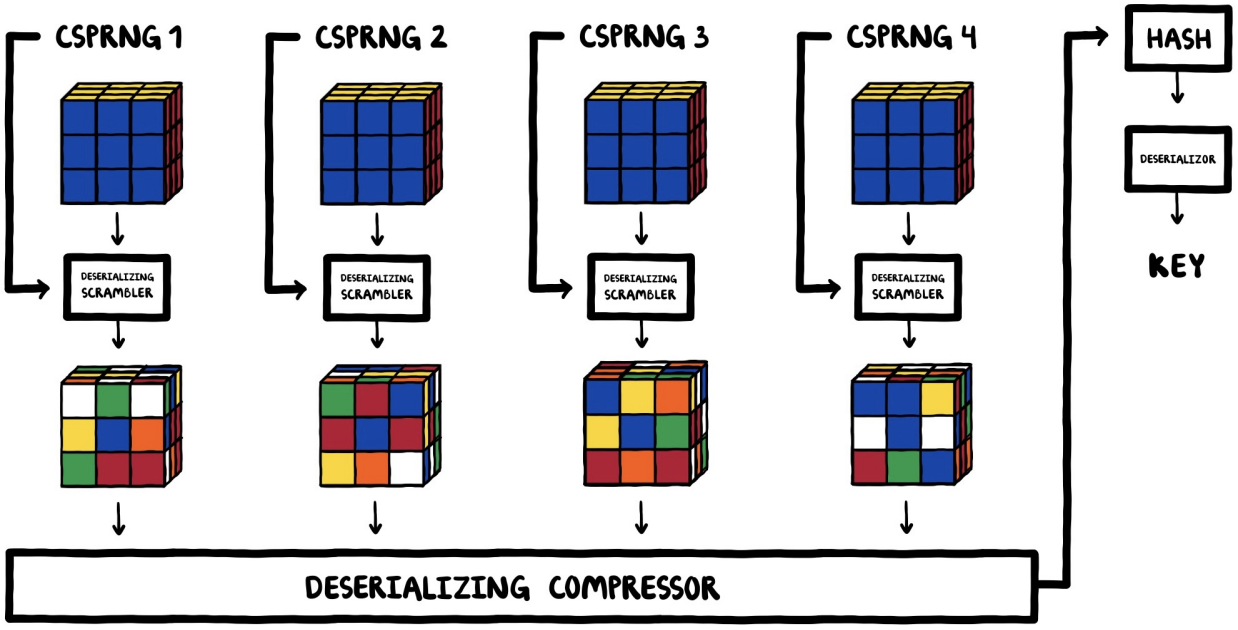

Figure 6: beep boop

5.1 4-Cube Initialization

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5.2 Master-Key Serialization

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5.3 Sub-Key Generation

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6 Encryption

¡...¿

6.1 Plaintext Setup With S-Box Transformations On Chunks

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6.2 Cube Mapping Procedure

¡...¿

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