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Lab 5 - Switch-Mode DC-DC Converters

Lab Section 021

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Introduction:

The objective of this lab is to understand the principles of operation of buck and buck-boost DC-DC converters. We will learn how discontinuous current mode (DCM) affects converters. We will know how to design DC-DC converters that operate in continuous current mode (CCM). We will learn how the non-linear switching operation can be replaced by the linear average CCM-DCM modeling, and its advantages and disadvantages. We will understand the need for system feedback control in the design/development of switch-mode regulators.

Theory:

PART 1: Buck Converter

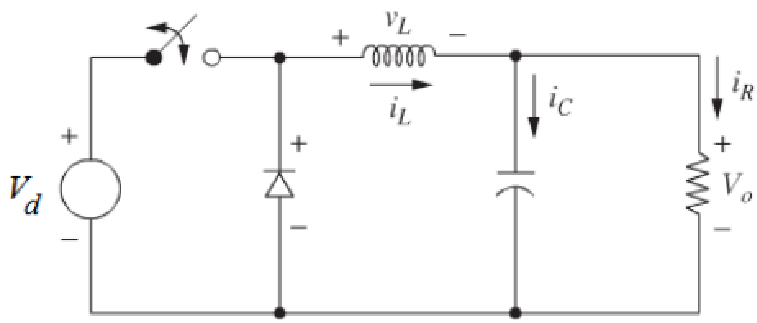


Figure 1.1 The Buck converter

A buck converter produces a DC voltage Vo less or equal to the input voltage Vd depending on the duty cycle of the switch D. This means that it steps-down voltage.The average capacitor current is zero so, the average output current is equal to the average inductor current, $I_o = I_R = I_L$. The average voltage across the inductor is zero, so the average output voltage is equal to the average capacitor voltage $V_o = V_C$. It can operate in continuous current mode (CCM) (I_L is never 0) or discontinuous current mode (DCM) (I_L is 0 for some part).

CCM Equations

Ouput Voltage

(1.1) $$V_o = DV_d$$

Maximum and Minimum inductor current

(1.2) $$I_{\max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

(1.3) $$I_{\min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

Inductor

(1.6) $$L = \frac{V_o(1-D)}{\Delta i_L\, f}$$

Ripple Voltage

(1.7) $$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2}$$

Capacitor

(1.8) $$C = \frac{1-D}{8L\left(\frac{\Delta V_o}{V_o}\right)f^2}$$

Boundary between CCM and DCM

(1.4) $$I_{\min} = 0 = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$
$$(Lf)_{\min} = \frac{(1-D)R}{2}$$

(1.5) $$L_{\min} = \frac{(1-D)R}{2f}$$

DCM Equations
Output Voltage

(1.9) $$V_o = V_d\frac{2D}{D+\sqrt{D^2+8\frac{Lf}{R}}}$$

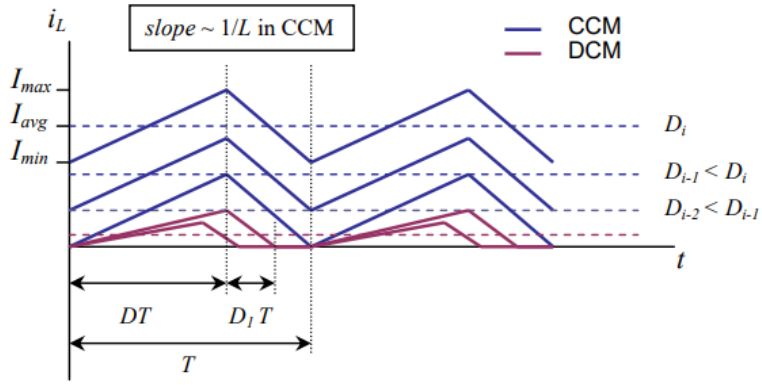


Figure 1.2 Inductor current in CCM and DCM as a function of duty cycle D

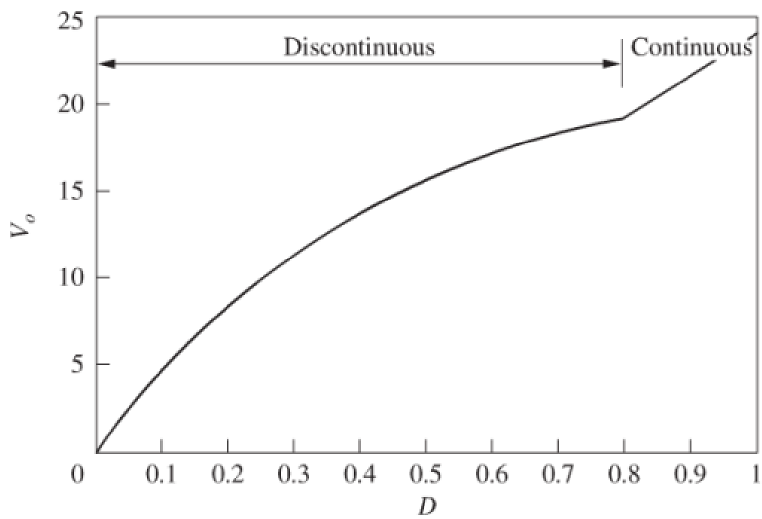


Figure 1.3 The output buck converter voltage vs duty cycle

As a note, ideal inductors. Capacitors, and switches do not consume power and power is only consumed by the load R. Ideally, the power efficiency of switching power supplies η is 100%. In reality we can expect 96-98%.

Power Conversion Efficiency

$$\eta = \frac{\text{power consumed by the load}}{\text{power generated}} \times 100\% = \left| \frac{P_{out}}{P_{in}} \right| \times 100\% \quad (1.10)$$

PART 2: Buck-Boost Converters

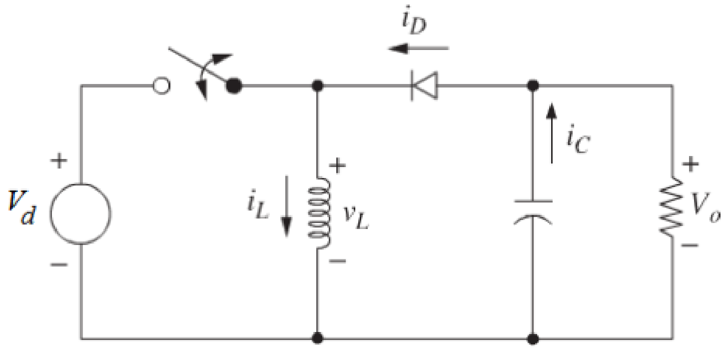


Figure 2.1 The Buck-Boost converter

Buck-Boost Converters allow either a step-up or a step-down of the output voltage depending on the value of the duty cycle. The output voltage is the opposite polarity of the input voltage. If the duty cycle is greater than 0.5 then the output voltage is stepped up. If the duty cycle is less than 0.5 then the output voltage is stepped down.

Output Voltage

$$(2.1) \quad V_o = -V_d \frac{D}{1-D}$$

Duty cycle in terms of input and output voltage

$$(2.2) \quad D = \frac{|V_o|}{V_d + |V_o|}$$

Average, Maximum and Minimum inductor currents

$$(2.3) \quad I_L = \frac{V_d D}{R(1-D)^2}$$

$$(2.4) \quad I_{max} = I_L + \frac{V_d D}{2Lf}$$

$$(2.5) \quad I_{min} = I_L - \frac{V_d D}{2Lf}$$

The boundary between CCM and DCM

$$(2.6) \quad (Lf)_{min} = \frac{(1-D)^2 R}{2}$$

Minimum Inductance

$$(2.7) \quad L_{min} = \frac{(1-D)^2 R}{2f}$$

Output Voltage Ripple

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf} \quad (2.8)$$

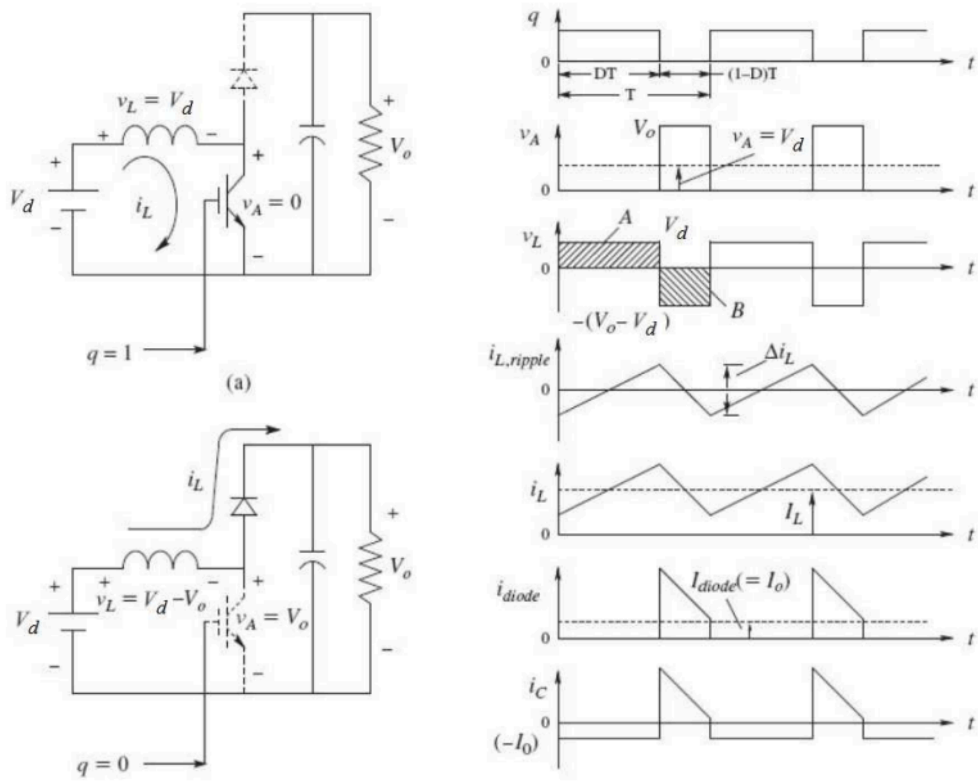


Figure 2.2 Waveforms of the buck-boost converter where the transistor acts as an ideal switch when **a)** closed, **b)** open. Note that the ground node here is where the positive output node in **Figure 2.1** is.

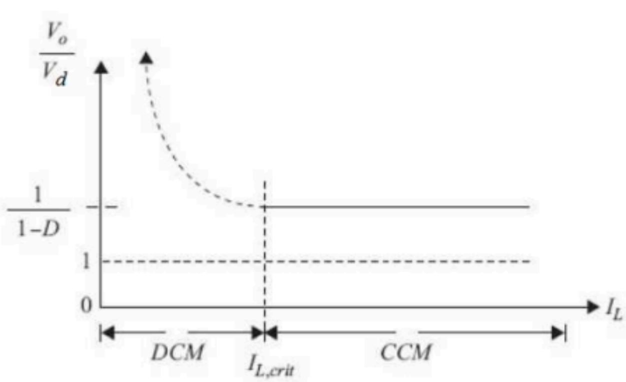


Figure 2.3 Voltage transfer ratio for the Buck-Boost converter with fixed D

Note that a combination of a switch and a diode is called a power-pole which can be represented as an ideal transformer with the turn ratio corresponding to Duty Cycle

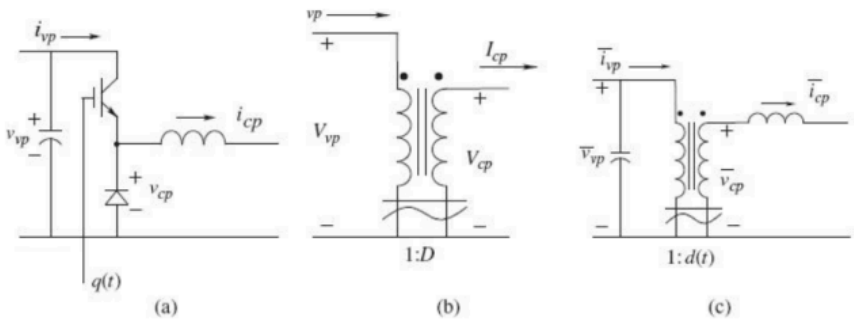


Figure 2.4 Dynamic averaging with an ideal transformer in CCM mode: **a)** switching model; **b)** an ideal transformer equivalent of the switch combination, aka "power-pole", when operating in CCM; **c)** dynamic average modeling for small variations in operating parameters, aka "small-signal" analysis.

Voltage and Current ports

$$V_{cp} = DV_{vp} \qquad (2.9)$$
$$I_{vp} = DI_{cp}$$

Assuming further that all variables (Vvp, Vcp, Ivp, Icp, D) vary slowly with respect to the switching frequency

$$V_{cp}(t) = \bar{v}_{cp}(t), \quad V_{vp}(t) = \bar{v}_{vp}(t) \qquad (2.10)$$
$$I_{cp}(t) = \bar{i}_{cp}(t), \quad I_{vp}(t) = \bar{i}_{vp}(t)$$
$$D(t) = d(t)$$

Therefore (2.9) can be written as (2.11):

$$\bar{v}_{cp}(t) = d(t)\,\bar{v}_{vp}(t) \qquad (2.11)$$
$$\bar{i}_{vp}(t) = d(t)\,\bar{i}_{cp}(t)$$

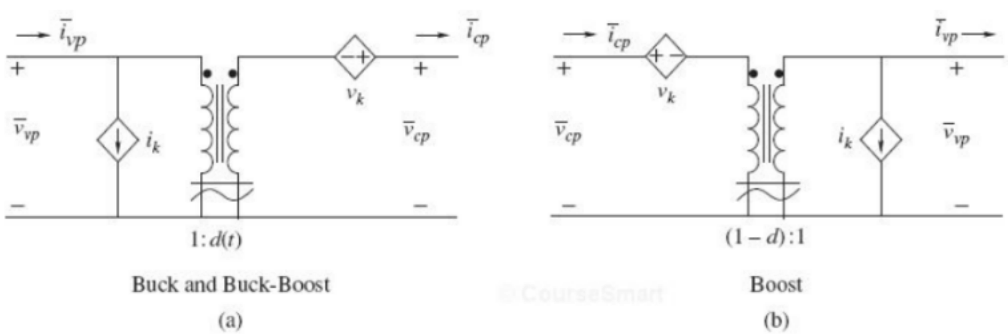


TABLE 2.1 v_k and i_k in DCM

Converter	v_k	i_k
Buck	$\left(1-\frac{2Lf\,\bar{i}_L}{(V_d-\bar{v}_o)d}\right)\bar{v}_o$	$\frac{d^2}{2Lf}(V_d-\bar{v}_0)-d\bar{i}_L$
Boost	$\left(1-\frac{2Lf\,\bar{i}_L}{V_d d}\right)(V_d-\bar{v}_0)$	$\frac{d^2}{2Lf}V_d-d\bar{i}_L$
Buck-Boost	$\left(1-\frac{2Lf\,\bar{i}_L}{V_d d}\right)\bar{v}_o$	$\frac{d^2}{2Lf}V_d-d\bar{i}_L$

Note that v_k and i_k are zero in CCM and The DCM condition can be determined during run-time by checking the value of the inductor current i_L.

PART 3:

Because switching operations are non-linear, analyzing these are highly complicated. By linearizing switching functions using ideal CCM/DCM transformers. This allows for simpler analysis since it ignores small waveform ripples that are not essential during design. It allows the development of robust regulators using well-established system control methods. It is critical to understand the importance of system control analysis.

Prelab:

2. For the buck converter in Part 1, determine theoretically the critical load resistance value RL, crit at which the mode of operation is at the boundary between CCM and DCM. Assume D, Lmin, f to be known. Hint: use (1.5).

$$L_{\min}=\frac{(1-D)R}{2f} \tag{1.5}$$

D = 0.75
f = 100 kHz
Lmin = 100 uH

(1-D)R= Lmin*2f
R = Lmin * 2f / (1-D) = 80 Ω

3. Derive inductor current ripple ∆iL for the buck converter in CCM mode from (1.2), (1.3).

$$I_{\max}=V_o\left(\frac{1}{R}+\frac{1-D}{2Lf}\right) \tag{1.2}$$

$$I_{\min}=V_o\left(\frac{1}{R}-\frac{1-D}{2Lf}\right) \tag{1.3}$$

$$(1.6) \qquad L = \frac{V_o(1-D)}{\Delta i_I f}$$

$I_{max} = V_o$ ((1/R) + ((1-D)/(2Lf))) => R = ((2fLV_o) / (2fI_{max}L + DV_o -V_o))
Plug in R into I_{min}
$I_{min} = V_o$ ((1/R) - ((1-D)/(2Lf))) = V_o ((1/((2fLV_o) / (2fI_{max}L + DV_o -V_o))) - ((1-D)/(2Lf)))
I_{min} = (fLI_{max} - DV_o + V_o) / fL = I_{min} - I_{max} = (V_o(1 -D)) / (fL)
=> Δi_L = (V_o/L) * ((1-D)/f)

4. Derive inductor current ripple ∆iL for the buck-boost converter in CCM mode from (2.4), (2.5).

$$(2.3) \qquad I_L = \frac{V_d D}{R(1-D)^2}$$

$$(2.4) \qquad I_{max} = I_L + \frac{V_d D}{2Lf}$$

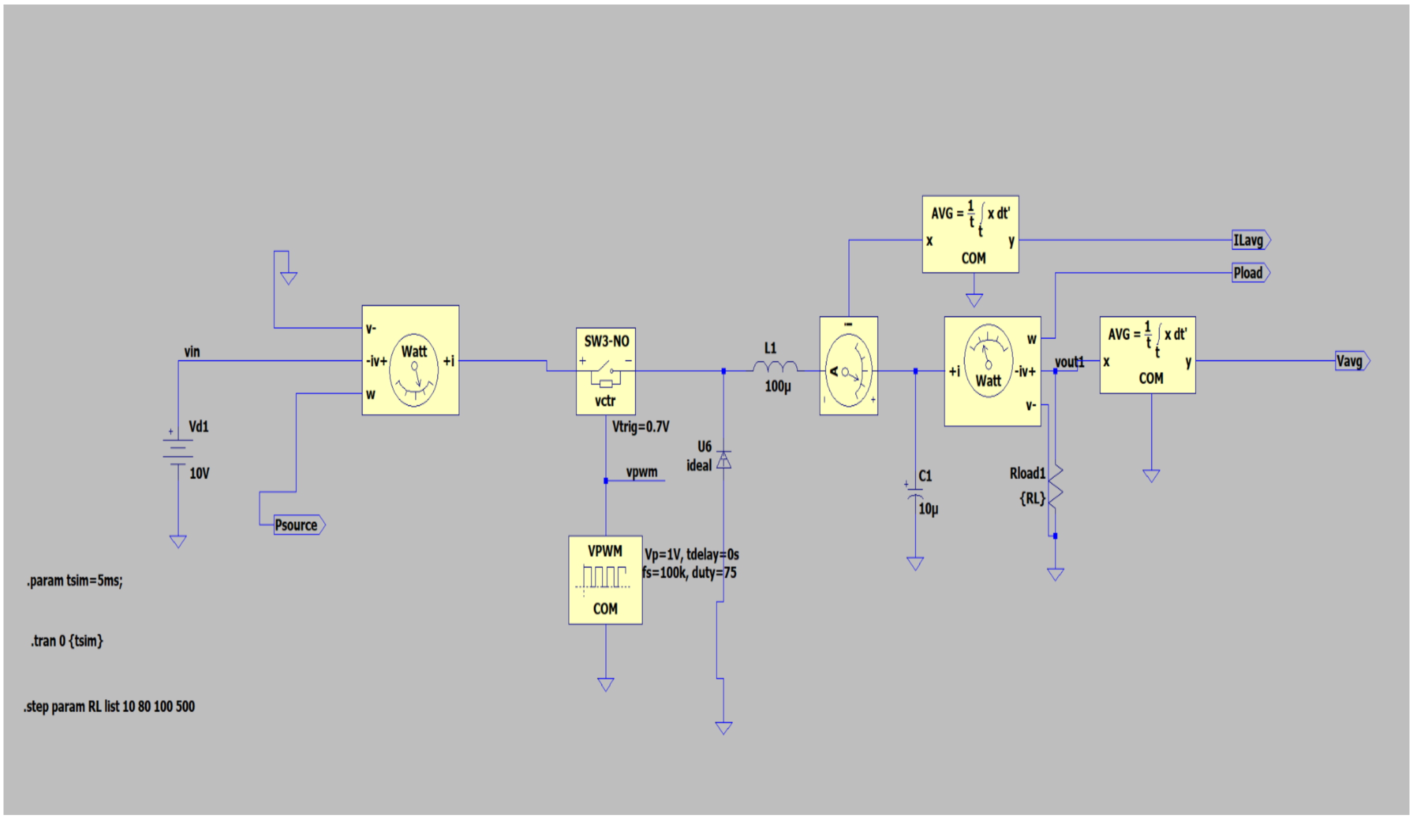
$$(2.5) \qquad I_{min} = I_L - \frac{V_d D}{2Lf}$$

$I_{max} = I_L$ + ((V_dD)/(2Lf)) => I_L = ((2fI_{max}L - DV_d) / (2fL))
Plug I_L into I_{min}
$I_{min} = I_{max} = I_L$ + ((V_dD)/(2Lf)) = ((2fI_{max}L - DV_d) / (2fL)) + ((V_dD)/(2Lf)) = (fI_{max}L - DV_d) / (fL)
I_{min} - I_{max} = (-DV_d) / (fL)
= Δi_L = (V_dD)/(fL)

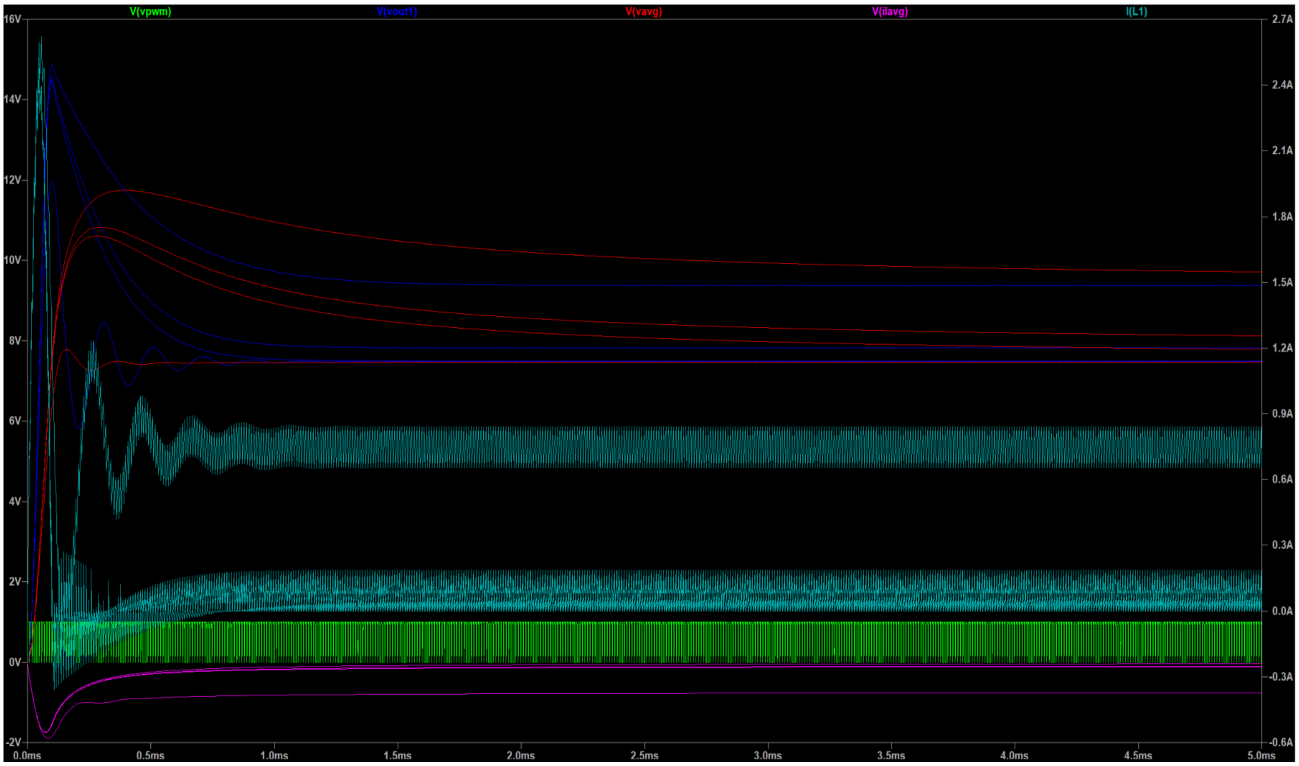
Design Calculations and Circuit Schematic, including Experimental Data and Data Analysis:

1.2:

1)

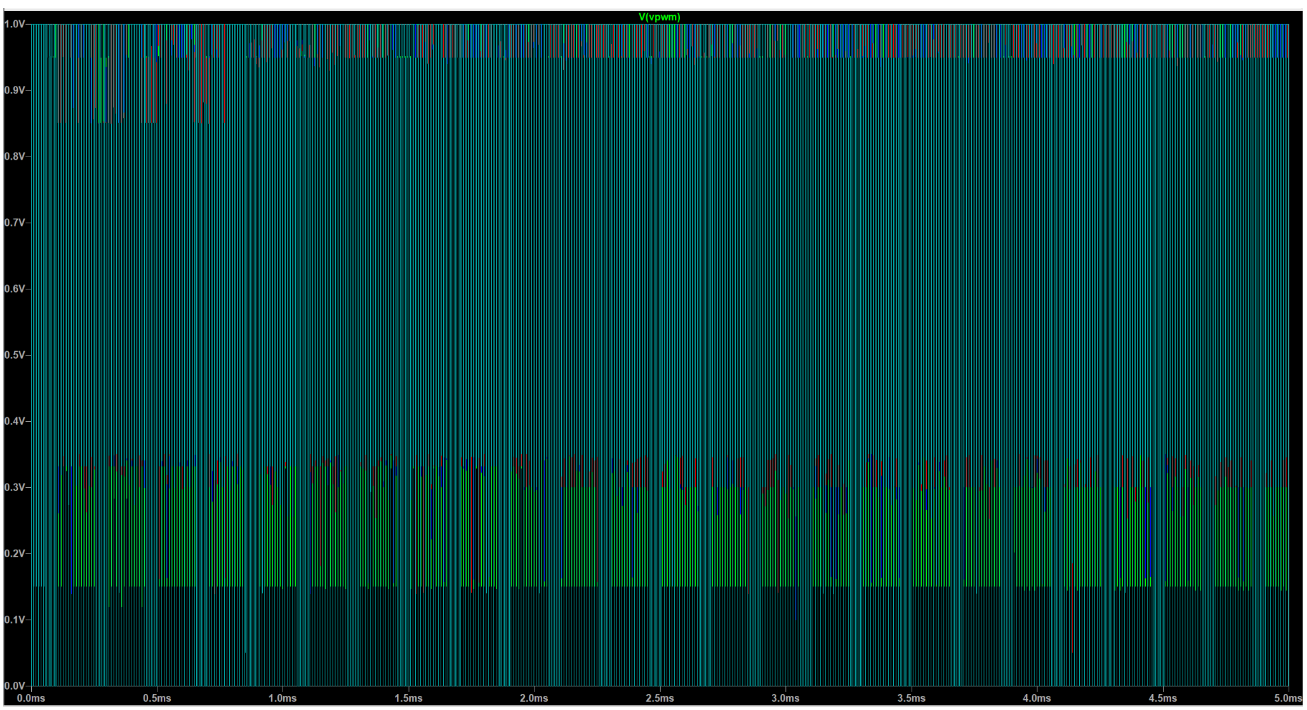


3)



V(vpwm)
V(vout1)
V(vavg)
V(ilavg)
I(L1)
16V
14V
12V
10V
8V
6V
4V
2V
0V
-2V
2.7A
2.4A
2.1A
1.8A
1.5A
1.2A
0.9A
0.6A
0.3A
0.0A
-0.3A
-0.6A
0.0ms
0.5ms
1.0ms
1.5ms
2.0ms
2.5ms
3.0ms
3.5ms
4.0ms
4.5ms
5.0ms

Vpwm

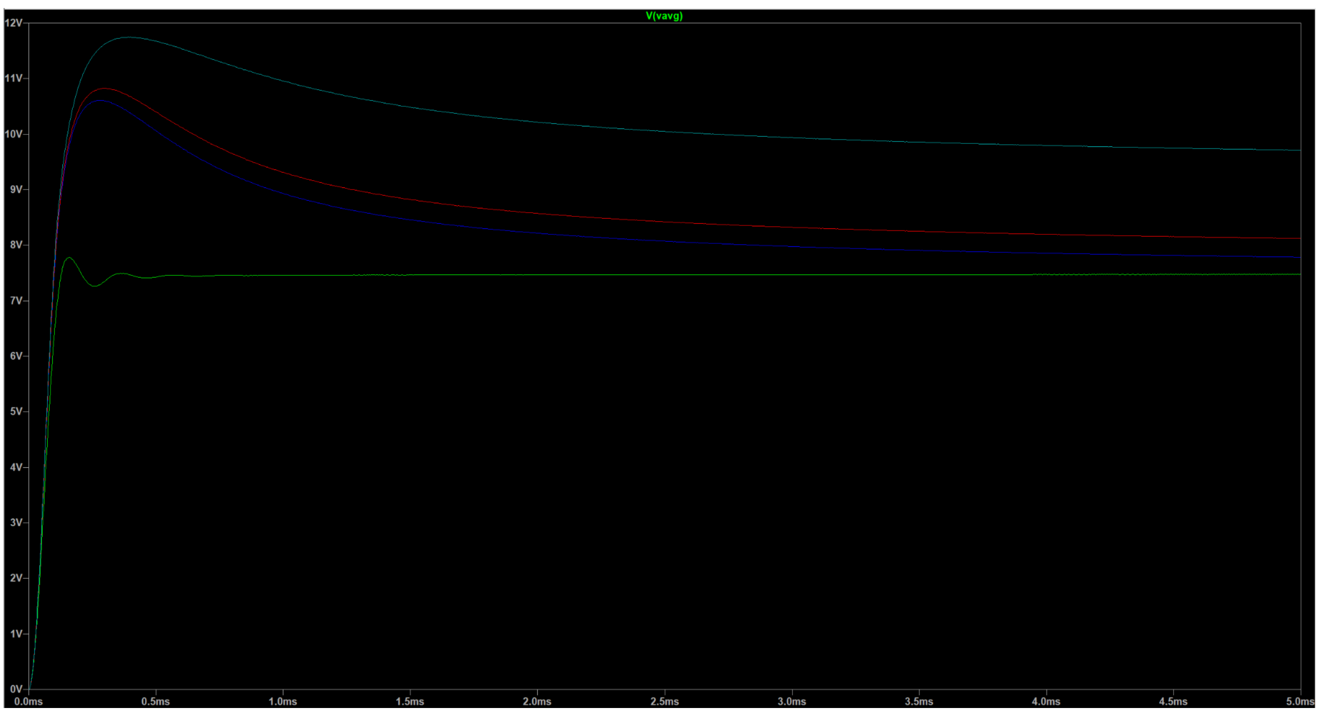


V(vpwm)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
0.0ms
0.5ms
1.0ms
1.5ms
2.0ms
2.5ms
3.0ms
3.5ms
4.0ms
4.5ms
5.0ms

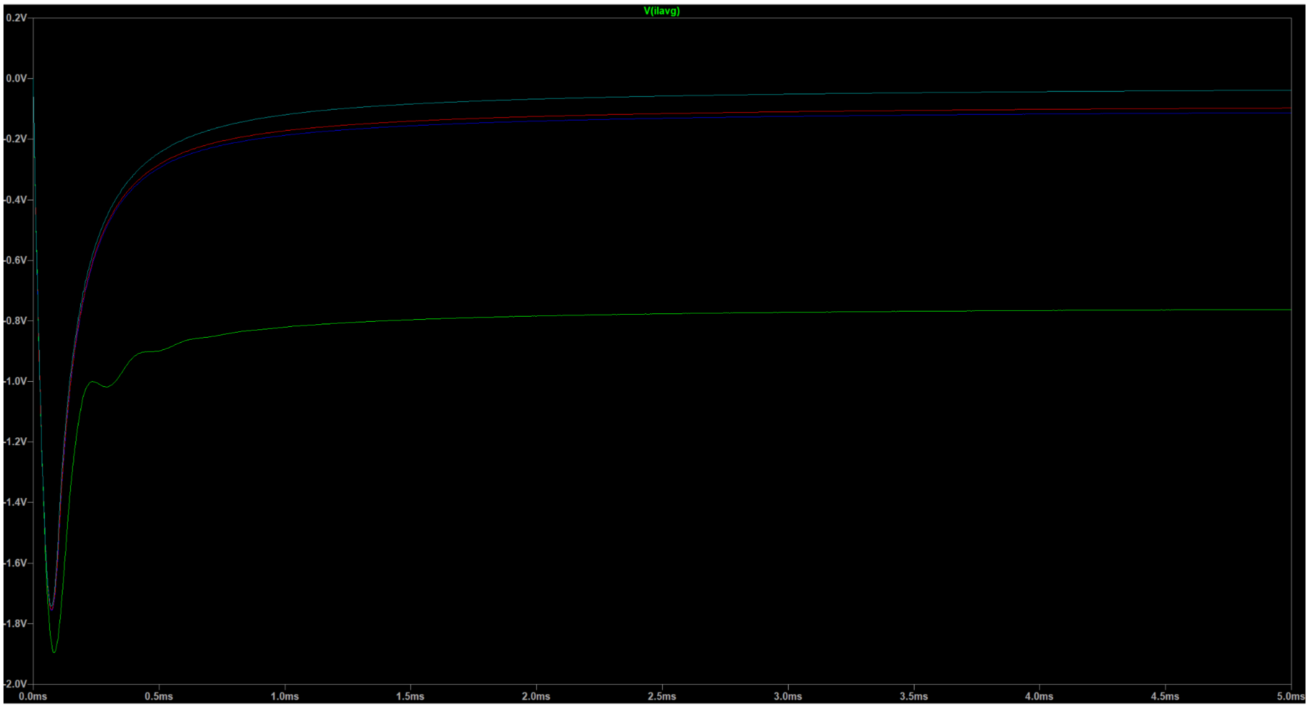
Vout


V(vout1)
15V
14V
13V
12V
11V
10V
9V
8V
7V
6V
5V
4V
3V
2V
1V
0V
0.0ms
0.5ms
1.0ms
1.5ms
2.0ms
2.5ms
3.0ms
3.5ms
4.0ms
4.5ms
5.0ms

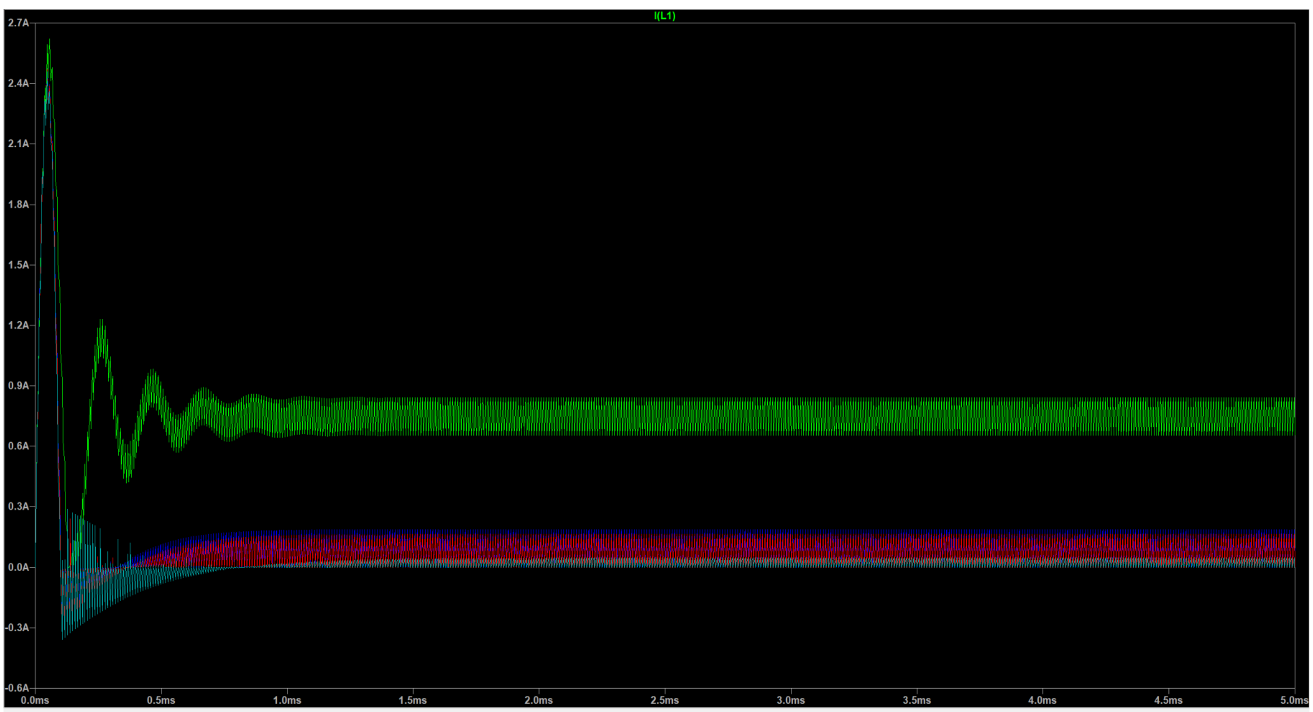
Vavg


V(vavg)
12V
11V
10V
9V
8V
7V
6V
5V
4V
3V
2V
1V
0V
0.0ms
0.5ms
1.0ms
1.5ms
2.0ms
2.5ms
3.0ms
3.5ms
4.0ms
4.5ms
5.0ms

Vilavg


V(ilavg)
0.2V
0.0V
-0.2V
-0.4V
-0.6V
-0.8V
-1.0V
-1.2V
-1.4V
-1.6V
-1.8V
-2.0V
0.0ms
0.5ms
1.0ms
1.5ms
2.0ms
2.5ms
3.0ms
3.5ms
4.0ms
4.5ms
5.0ms

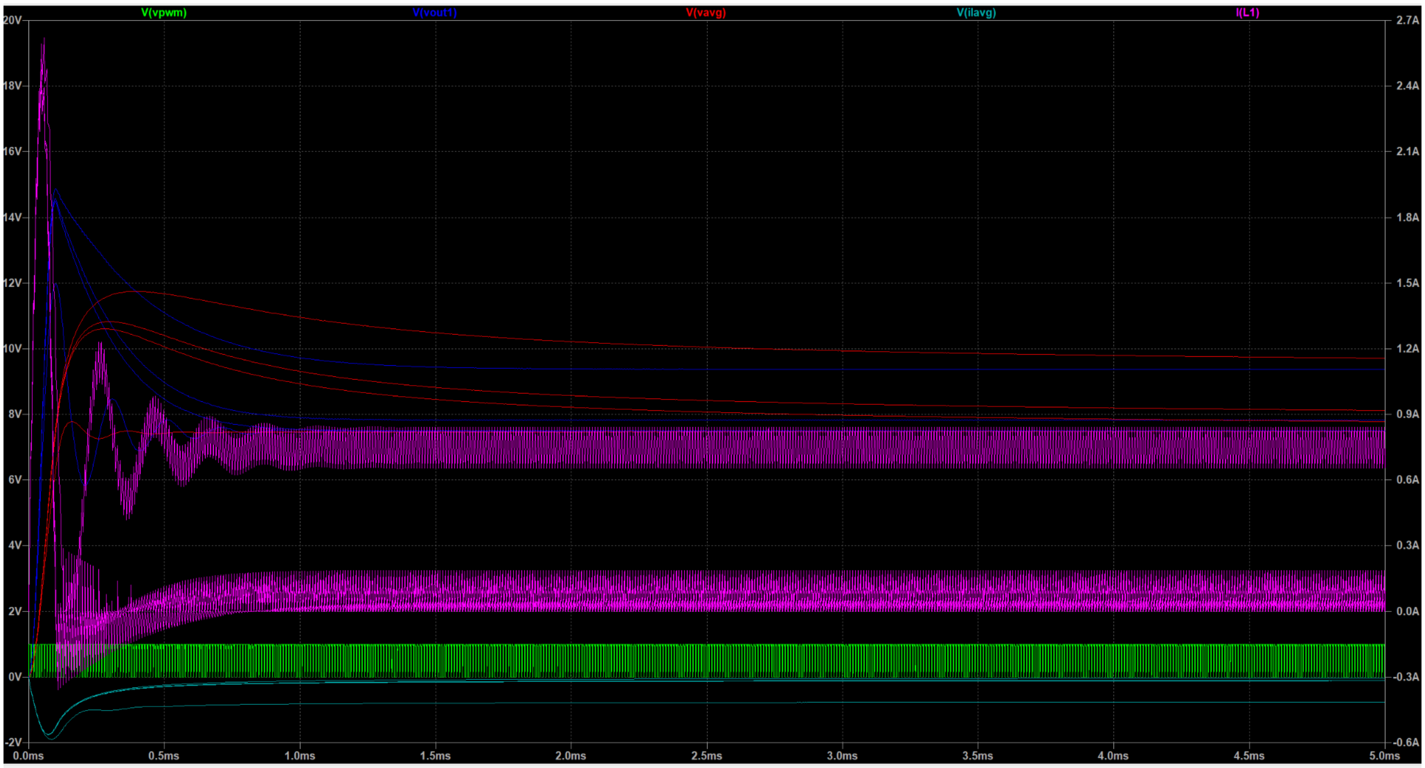
Ilavg


I(L1)
2.7A
2.4A
2.1A
1.8A
1.5A
1.2A
0.9A
0.6A
0.3A
0.0A
-0.3A
-0.6A
0.0ms
0.5ms
1.0ms
1.5ms
2.0ms
2.5ms
3.0ms
3.5ms
4.0ms
4.5ms
5.0ms

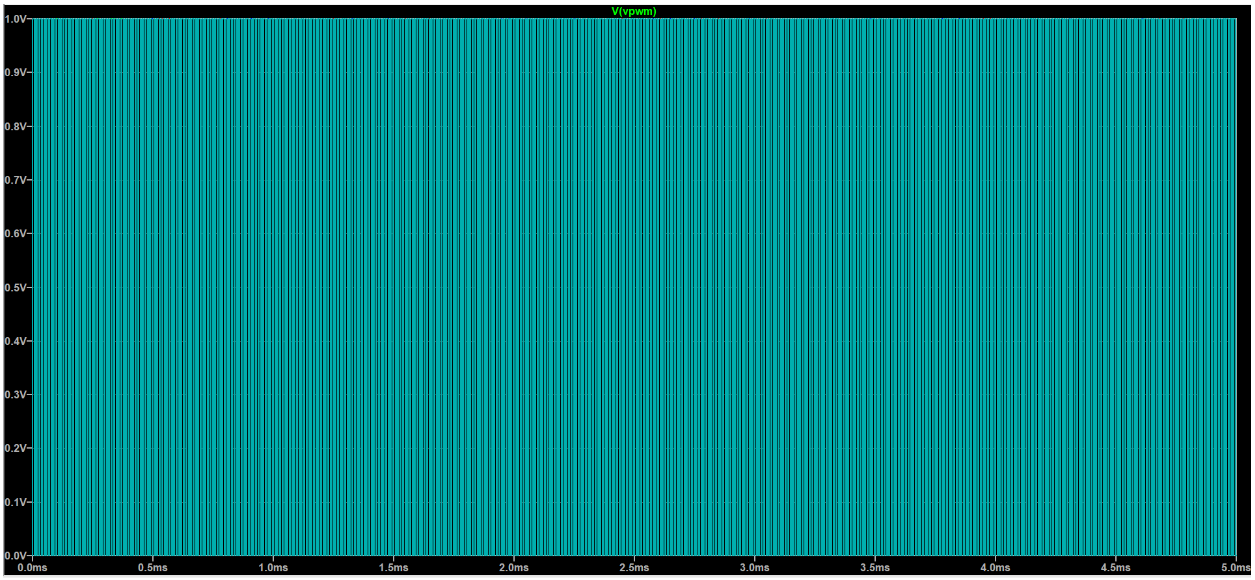
4)
D = 0.75
f = 100 kHz
Lmin = 100 uH

$R_{L,\,CRIT}$ = Lmin * 2f / (1-D) = 80 Ω
Here we can see that we calculated $R_{L,\,Crit}$ using Eq(1.5) which is the lowest possible value for L

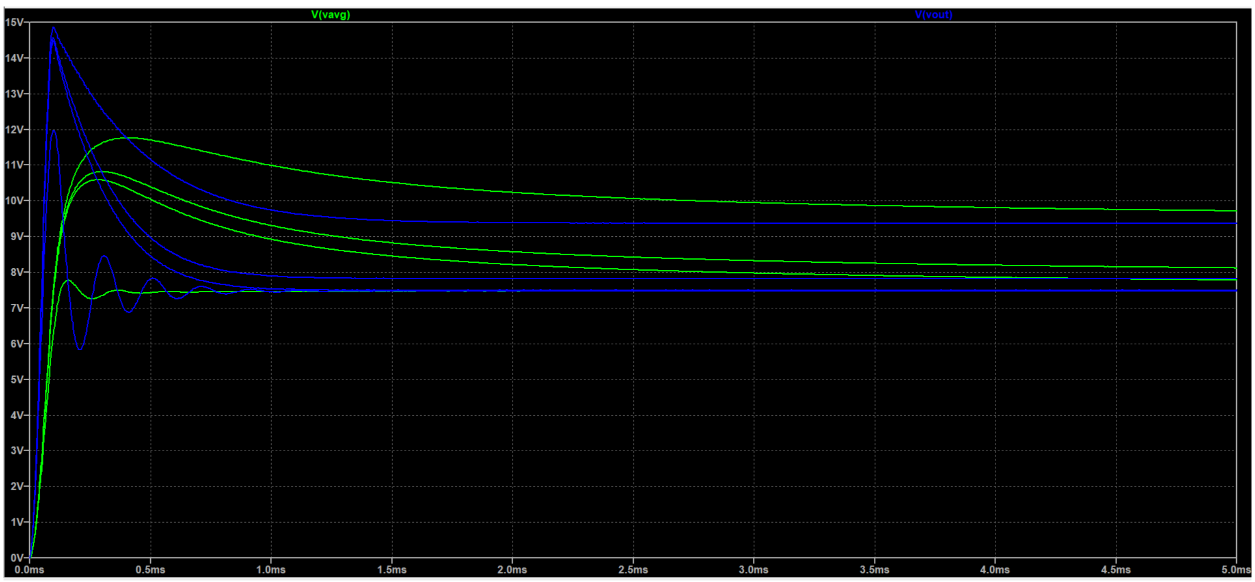
6)



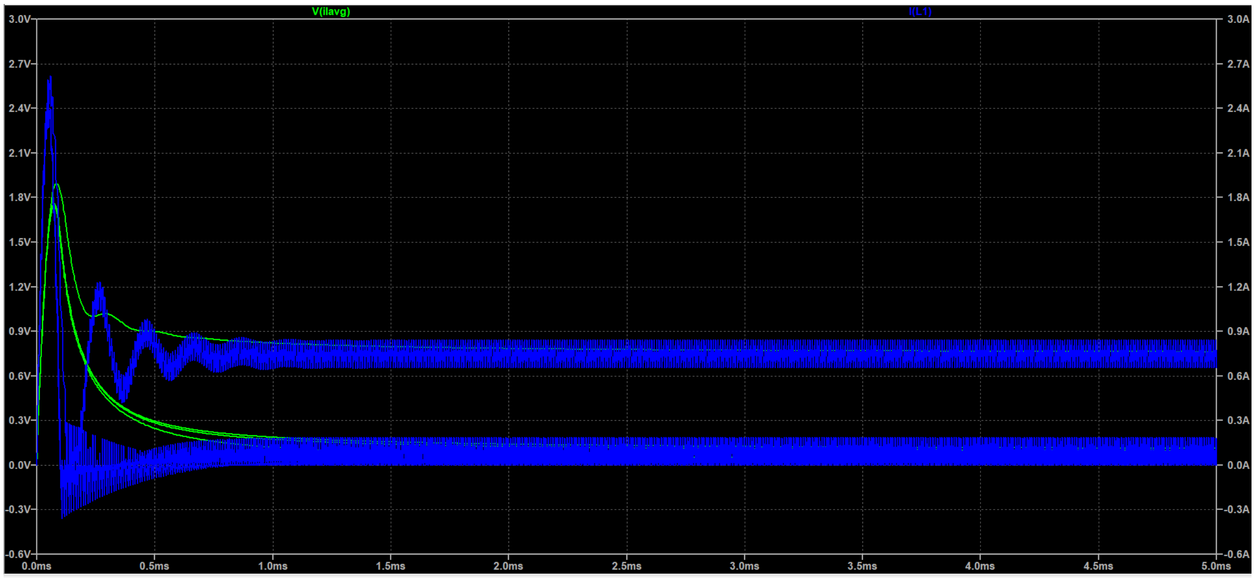
Vpwm



Vout, Vavg

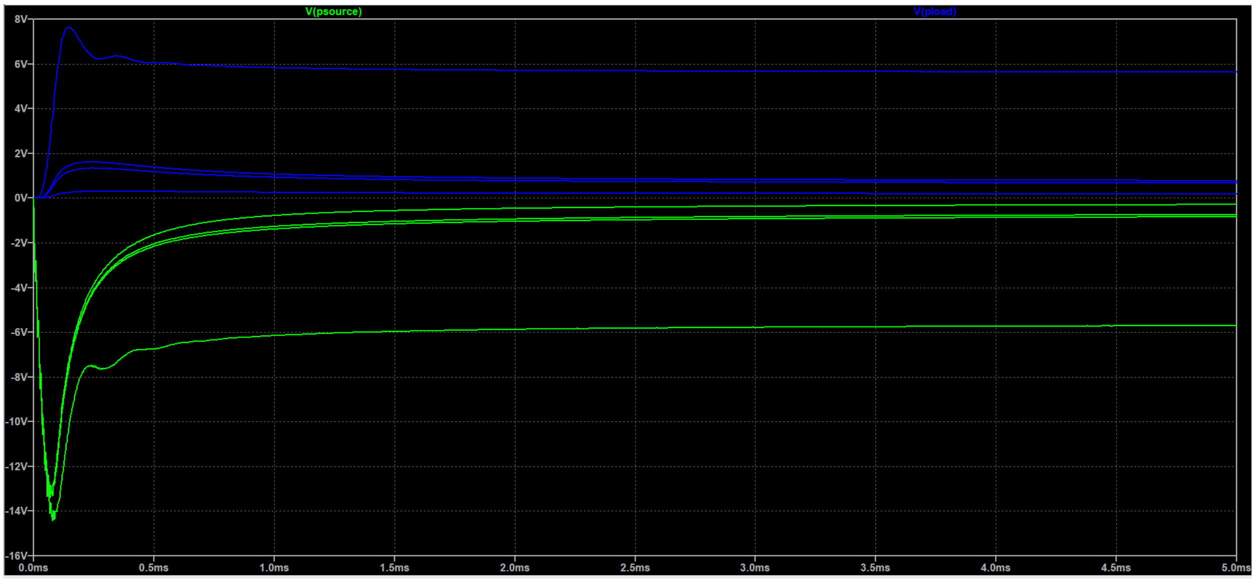


L1, ILavg



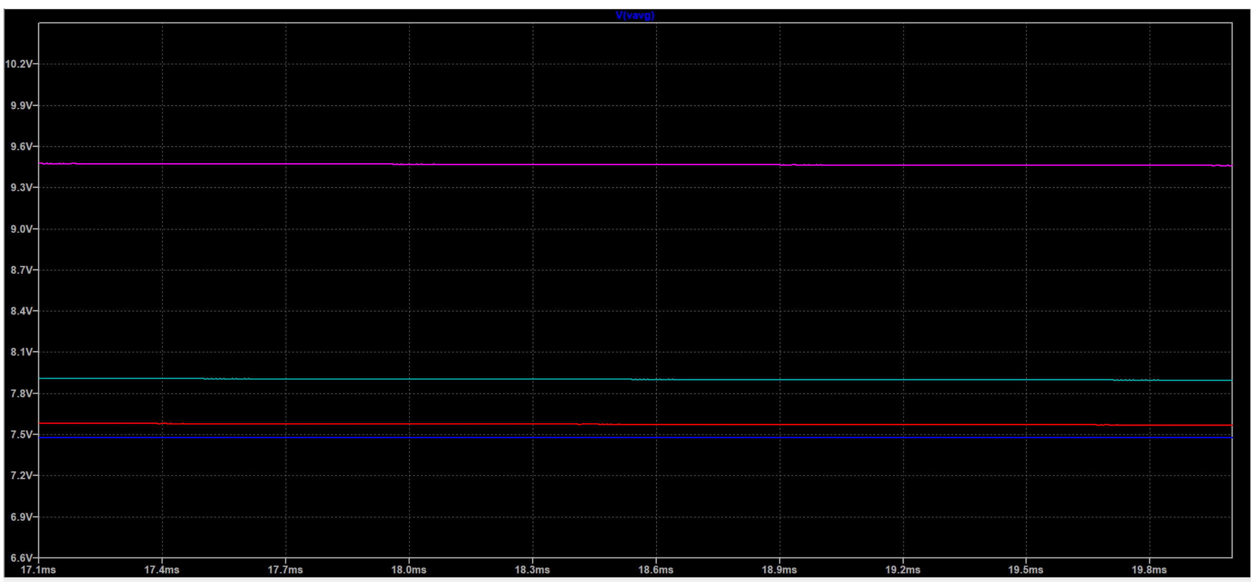
V(ilavg)
I(L1)
3.0V
2.7V
2.4V
2.1V
1.8V
1.5V
1.2V
0.9V
0.6V
0.3V
0.0V
-0.3V
-0.6V
3.0A
2.7A
2.4A
2.1A
1.8A
1.5A
1.2A
0.9A
0.6A
0.3A
0.0A
-0.3A
-0.6A
0.0ms
0.5ms
1.0ms
1.5ms
2.0ms
2.5ms
3.0ms
3.5ms
4.0ms
4.5ms
5.0ms

VPsource, VPload



8)

Experimental



At R_L = 10 Ω, V_o = 7.47 V

At R_L = 80 Ω, V_o = 7.57 V

At R_L = 100 Ω, V_o = 7.89 V

At R_L = 500 Ω, V_o = 9.46 V

Theoretical

(1.1) $$V_o = DV_d$$

(1.9) $$V_o = V_d \frac{2D}{D + \sqrt{D^2 + 8\frac{Lf}{R}}}$$

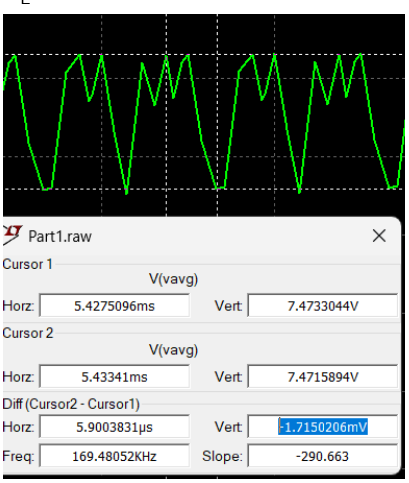
At R_L = 10 Ω (CCM)
$V_o = DV_d$ = 0.75 * 10V = 7.5 V

At R_L = 80 Ω (edge of CCM and DCM)
$V_o = DV_d$ = 0.75 * 10V = 7.5 V
$V_o = V_d$ ((2D) / (D + sqrt(D^2 + (8Lf/R)))
= 10*((2*0.75) / (0.75 + sqrt(0.75^2 + ((8*100*10^-6*100*10^3)/80)))) = 7.5 V

At R_L = 100 Ω (DCM)
$V_o = V_d$ ((2D) / (D + sqrt(D^2 + (8Lf/R)))
= 10*((2*0.75) / (0.75 + sqrt(0.75^2 + ((8*100*10^-6*100*10^3)/80)))) = 7.82 V

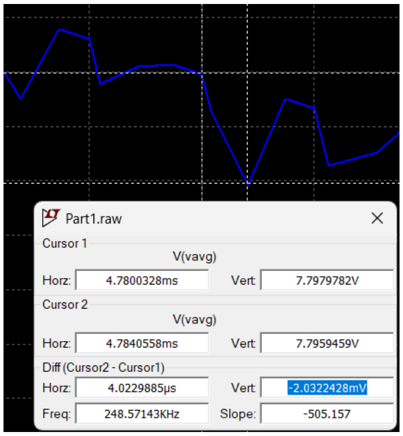
At R_L = 500 Ω (DCM)
$V_o = V_d$ ((2D) / (D + sqrt(D^2 + (8Lf/R)))
= 10*((2*0.75) / (0.75 + sqrt(0.75^2 + ((8*100*10^-6*100*10^3)/80)))) = 9.375 V

9)
Experimental
R_L = 10 Ω



$\Delta V_o / V_o$ = 1.72 mV

R_L = 80 Ω



$\Delta V_o / V_o$ = 2.03 mV

Theoretical

$$(1.7) \qquad \frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2}$$

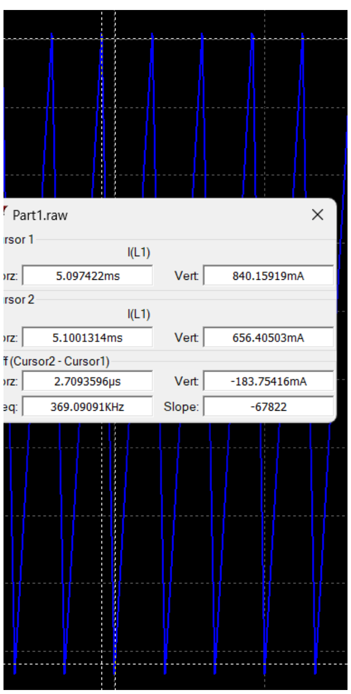
Note: at CCM all V_o = 7.5 V

$\Delta V_o/V_0$ = (V_o)(1 - D)/(8LCf^2) = (1-0.75) / (8*100*10^-6*10*10^-6*(100*10^3)^2) = 0.003125 V = 3.13 mV

10)

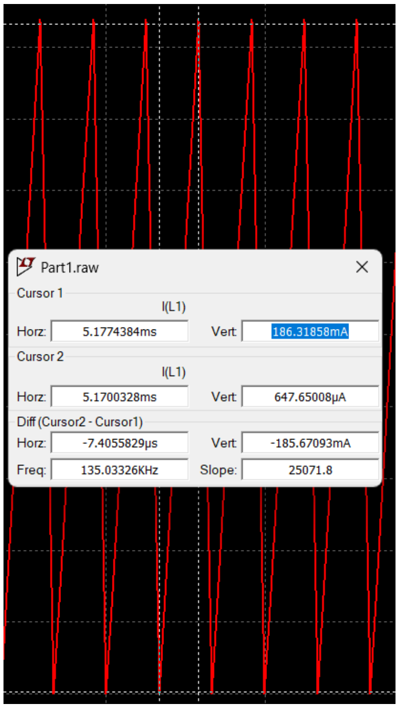
Experimental

R_L = 10 Ω



Δi_L = 183.75 mA

R_L = 80 Ω



Δi_L = 185.67 mA

Theoretical

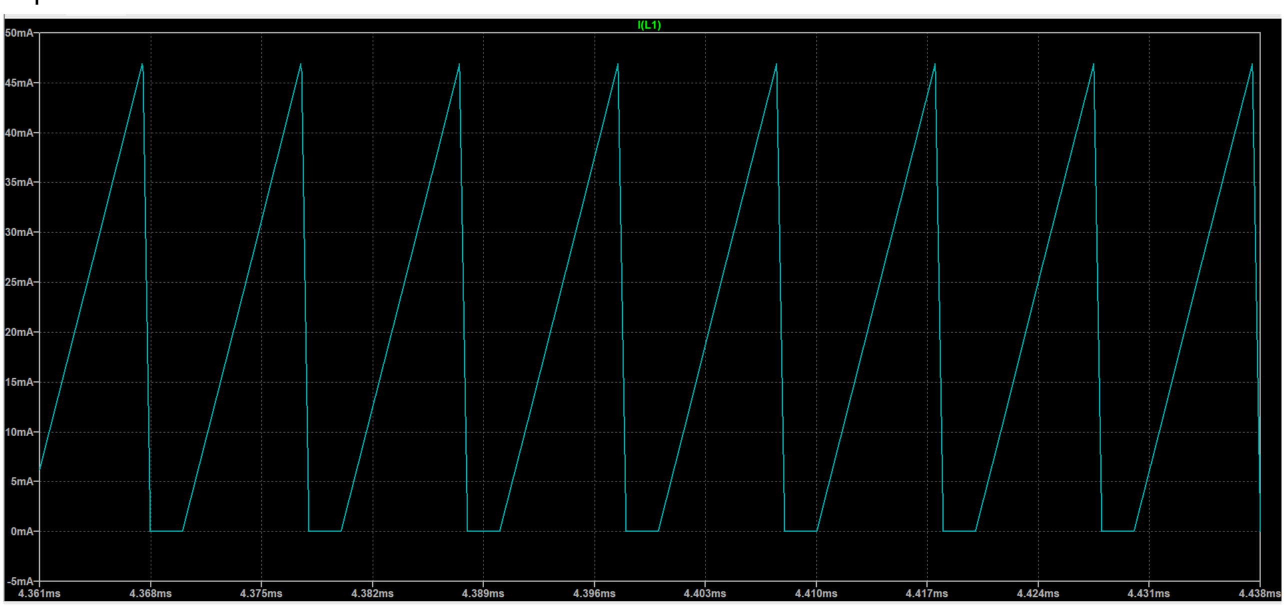
Δi_L is Independent of R in CCM

$\Delta i_L = (V_o/L) * ((1-D)/f)$ = (7.5/(100*10^-6)) * ((1-0.75) / (100*10^3) = 0.1875 = 187.5 mA

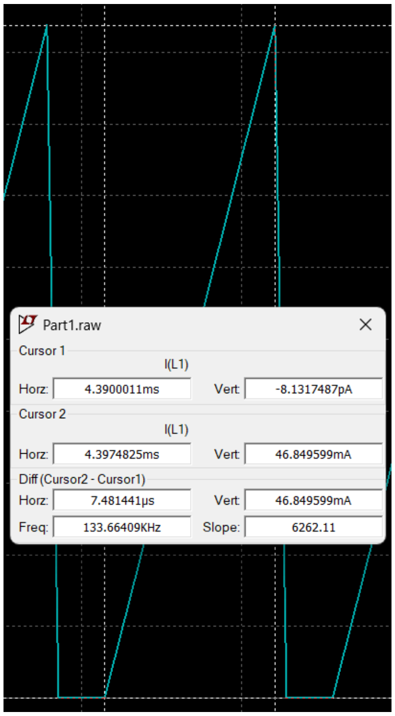
Here we can see that both values are very similar to one another

11)

Experimental

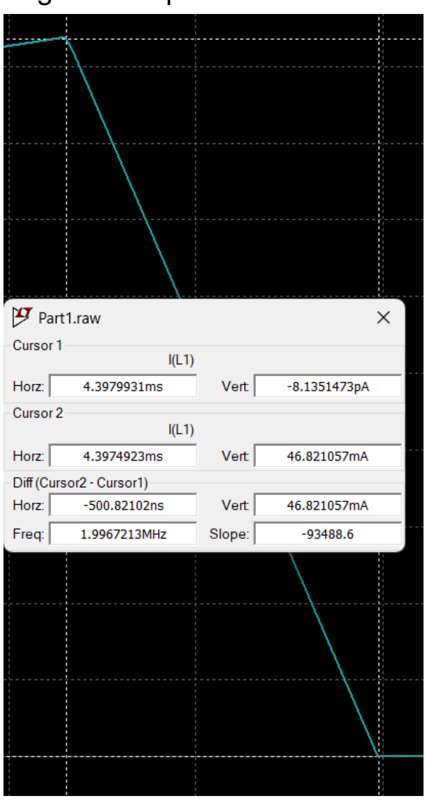


Positive Slope



Slope = 46.85 mA / 7.48 us = 6263.37

Negative Slope



Slope = 46.82 mA / 500.82 ns = -93488.68

Theoretical:
Positive Slope
Slope = $(V_d - V_o) / L_1$ = (10 - 7.5) / (100*10^-6) = 25000
Negative Slope
Slope = -Vo / L_1 = -7.5 / (100*10^-6) = -75000

Here we can see that the slope values are very different but also it is important to note that they are all equally very large. They seem to follow the same trend of the how the positive slope is always smaller than the negative slope. These differences could be due to the manual way of measuring resulting in big changes in the slope
12)
Experimental
At R_L = 10 Ω (CCM)
I_{max} = 841.895 mA
I_{min} = 653.424 mA

At R_L = 80 Ω (edge of CCM and DCM)
I_{max} = 187.630 mA
I_{min} = 0 mA

Theoretical

$$(1.2) \qquad I_{\max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

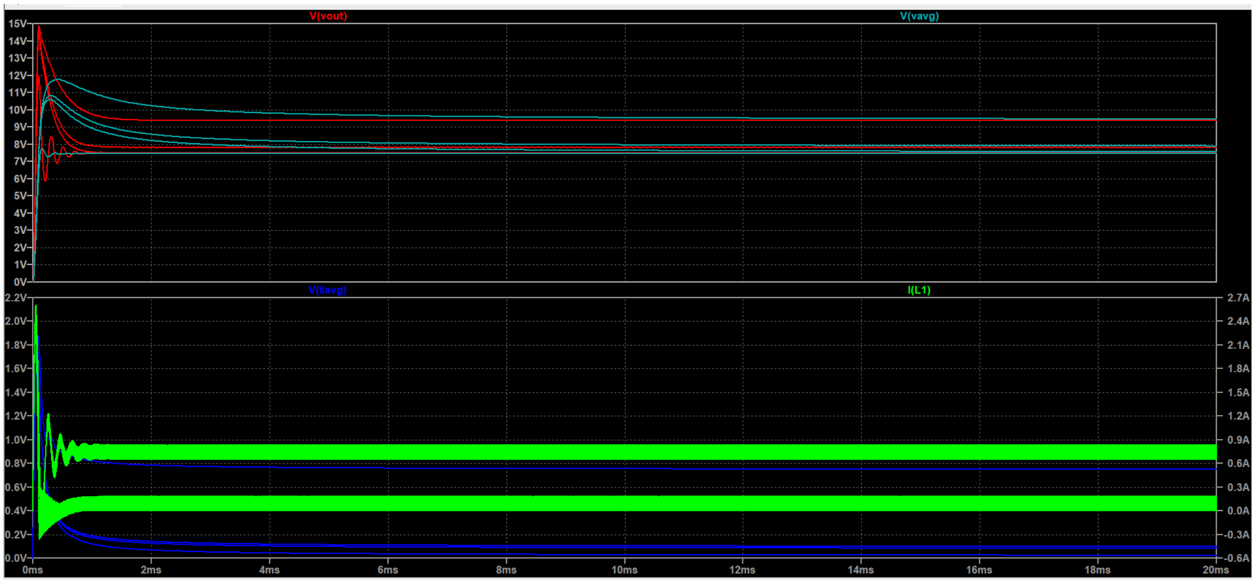
$$(1.3) \qquad I_{\min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

At R_L = 10 Ω (CCM)
I_{max} = =7.5*((1/10) + ((1-0.75)/(2*100*10^-6*100*10^3))) = 0.84 A = 840 mA
I_{min} = =7.5*((1/10) - ((1-0.75)/(2*100*10^-6*100*10^3))) = 0.66 A = 660 mA

At R_L = 80 Ω (edge of CCM and DCM)
I_{max} = 7.5*((1/80) + ((1-0.75)/(2*100*10^-6*100*10^3))) = 0.19 A = 190 mA
I_{min} = =7.5*((1/80) - ((1-0.75)/(2*100*10^-6*100*10^3))) = 0 A

We can see that both values agree with each other

13)
IL1, ILavg, Vout, Vavg



14)

$$\eta = \frac{\text{power consumed by the load}}{\text{power generated}} \times 100\% = \left|\frac{P_{out}}{P_{in}}\right| \times 100\% \qquad (1.10)$$

At R_L = 10 Ω (CCM)
n = (5.62 W / 5.60 W) * 100 = 100.36%

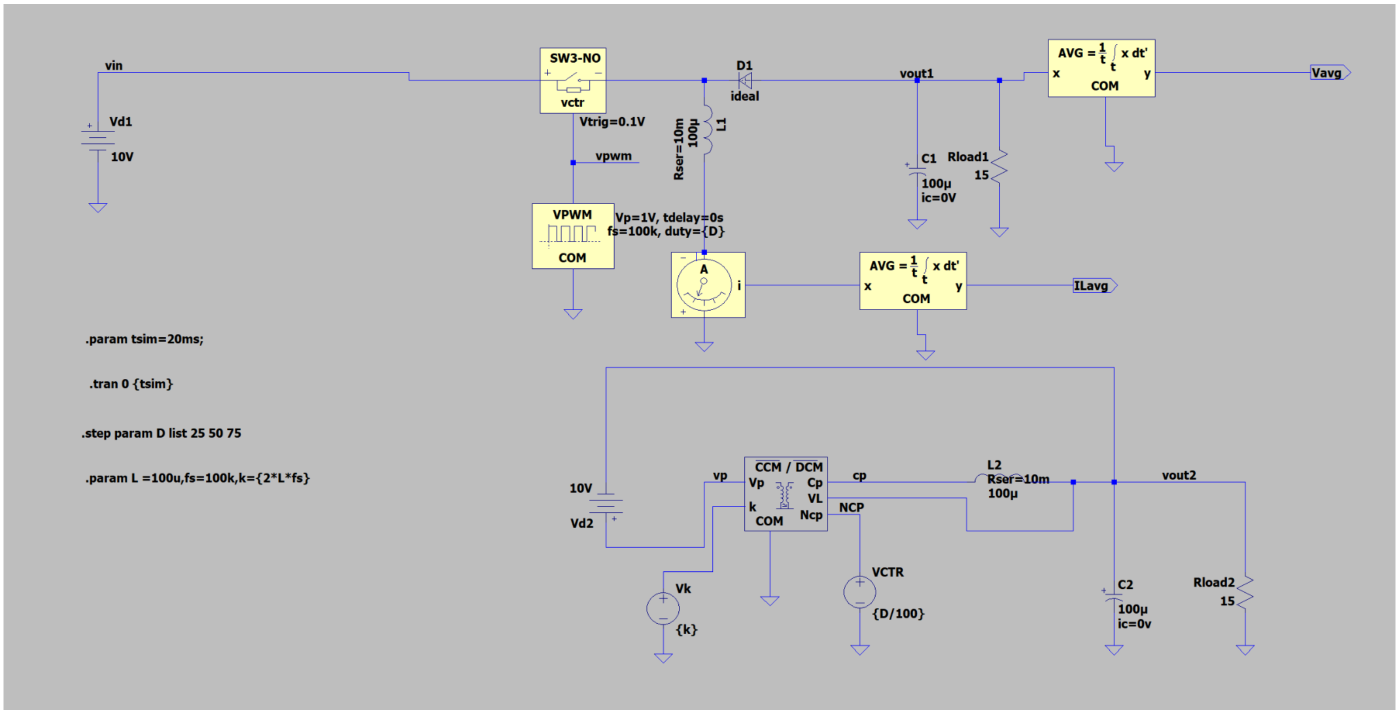
At R_L = 80 Ω (edge of CCM and DCM)
n = (721.60 mW / 737.64 mW) * 100 = 97.83%

At R_L = 100 Ω (DCM)
n = (628.62 mW / 646.87 mW) * 100 = 97.18%

At R_L = 500 Ω (DCM)
n = (179.92 mW / 205.09 mW) * 100 = 87.73%

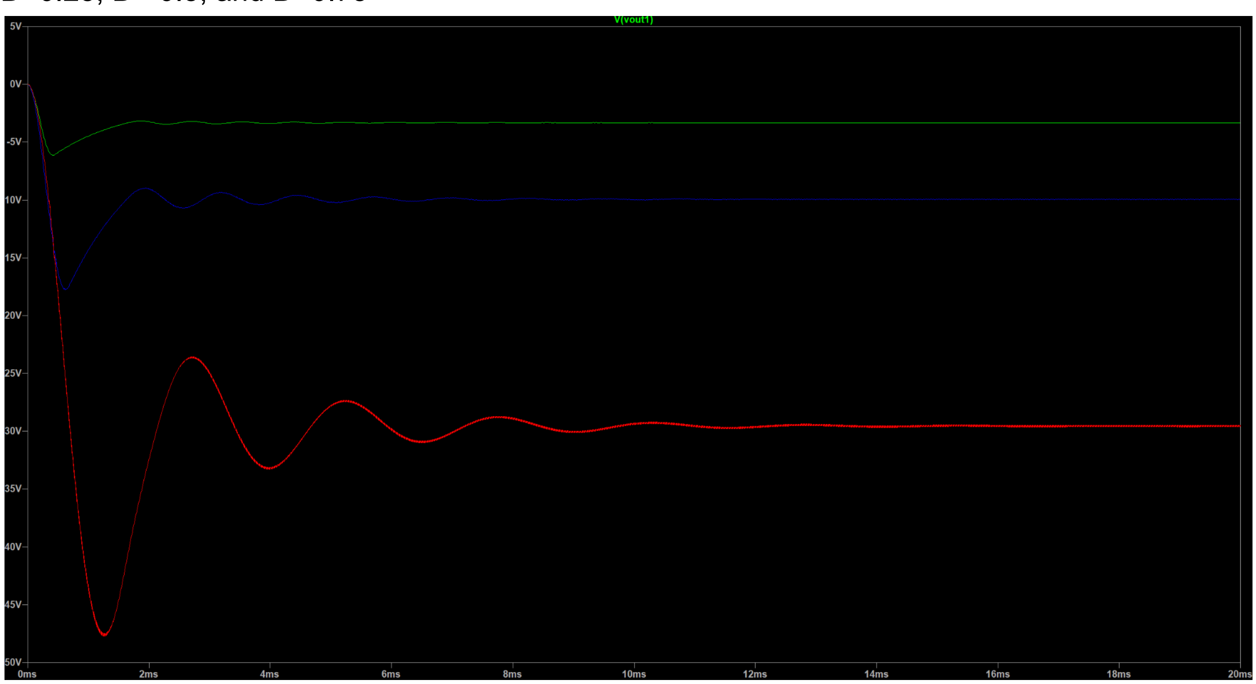
2.2

The schematic:

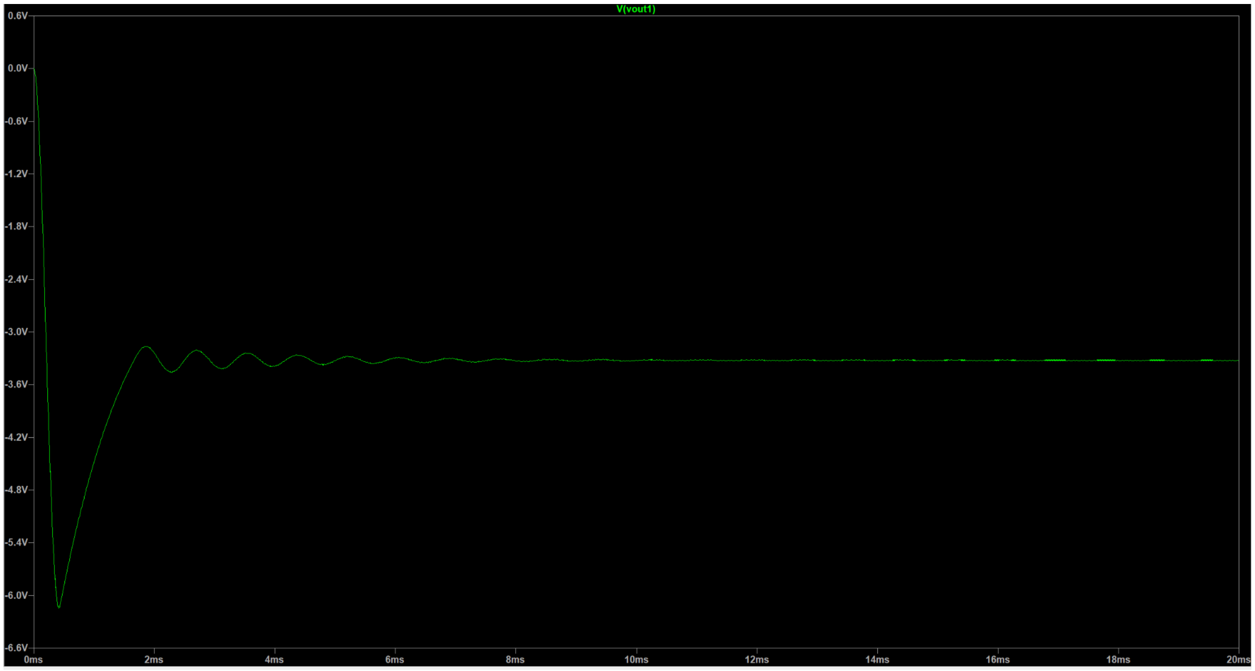


3)

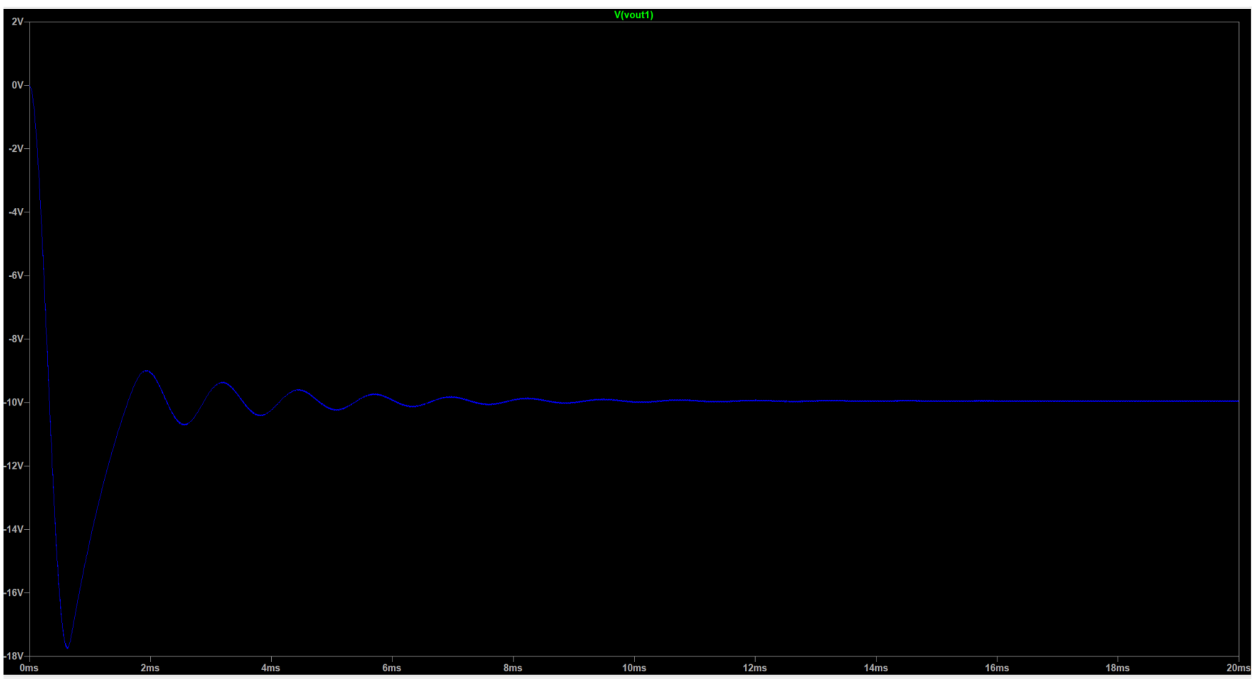
D=0.25, D= 0.5, and D=0.75



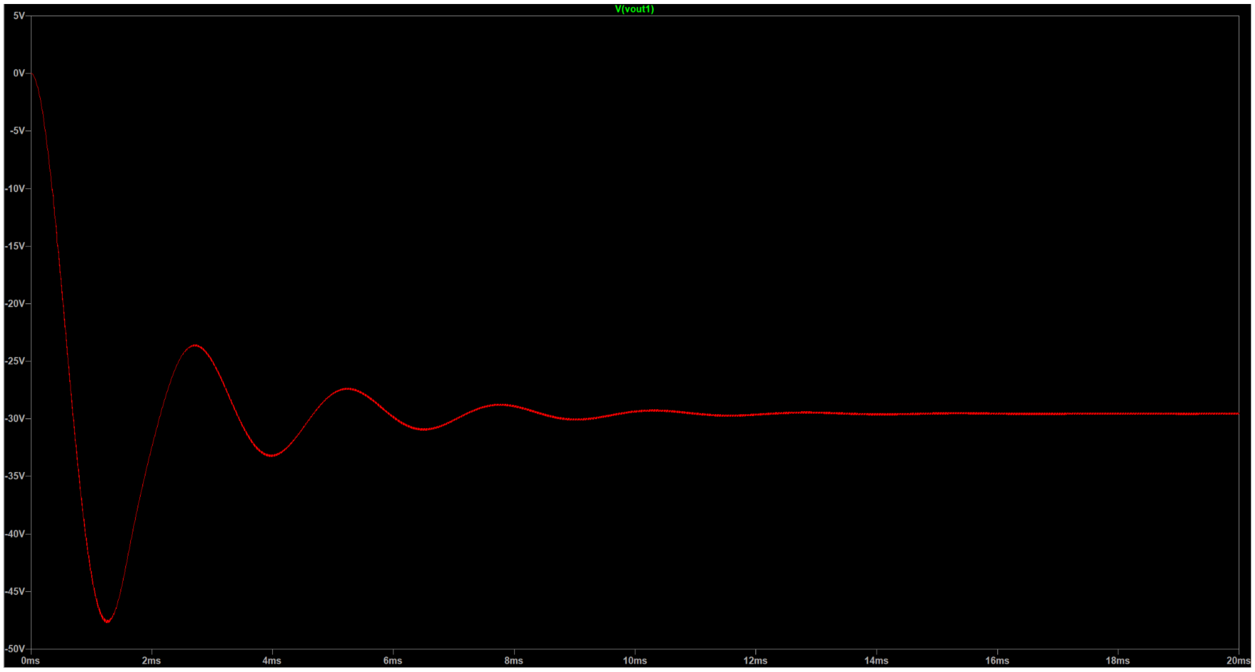
When step Duty cycle =25:



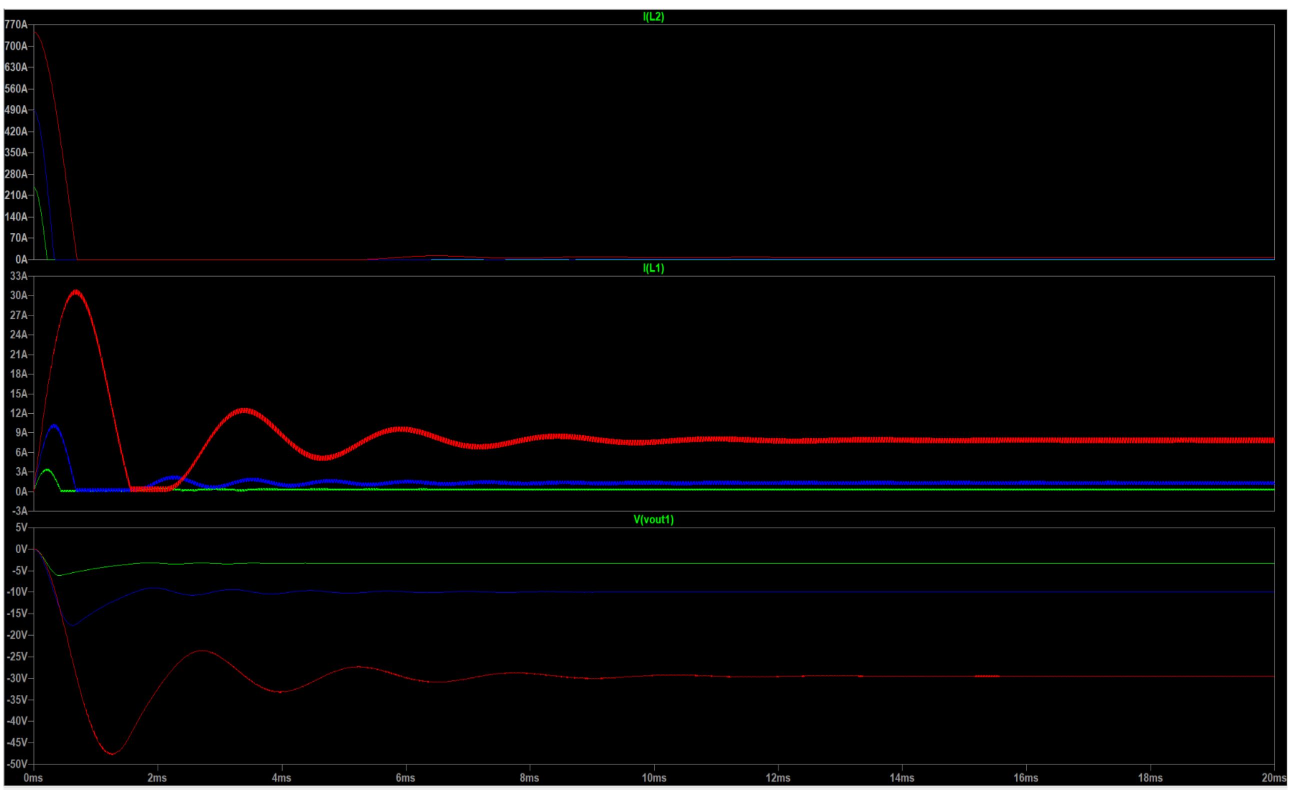
When step Duty Cycle =50:



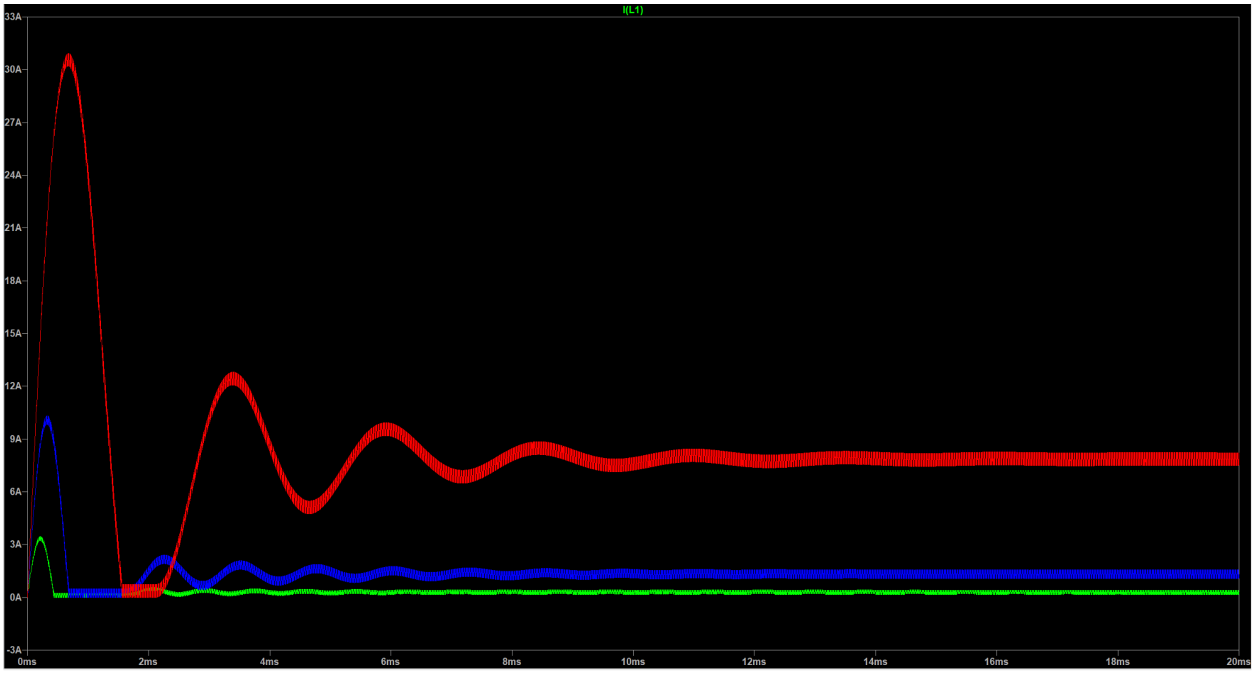
When step Duty Cylce = 75:



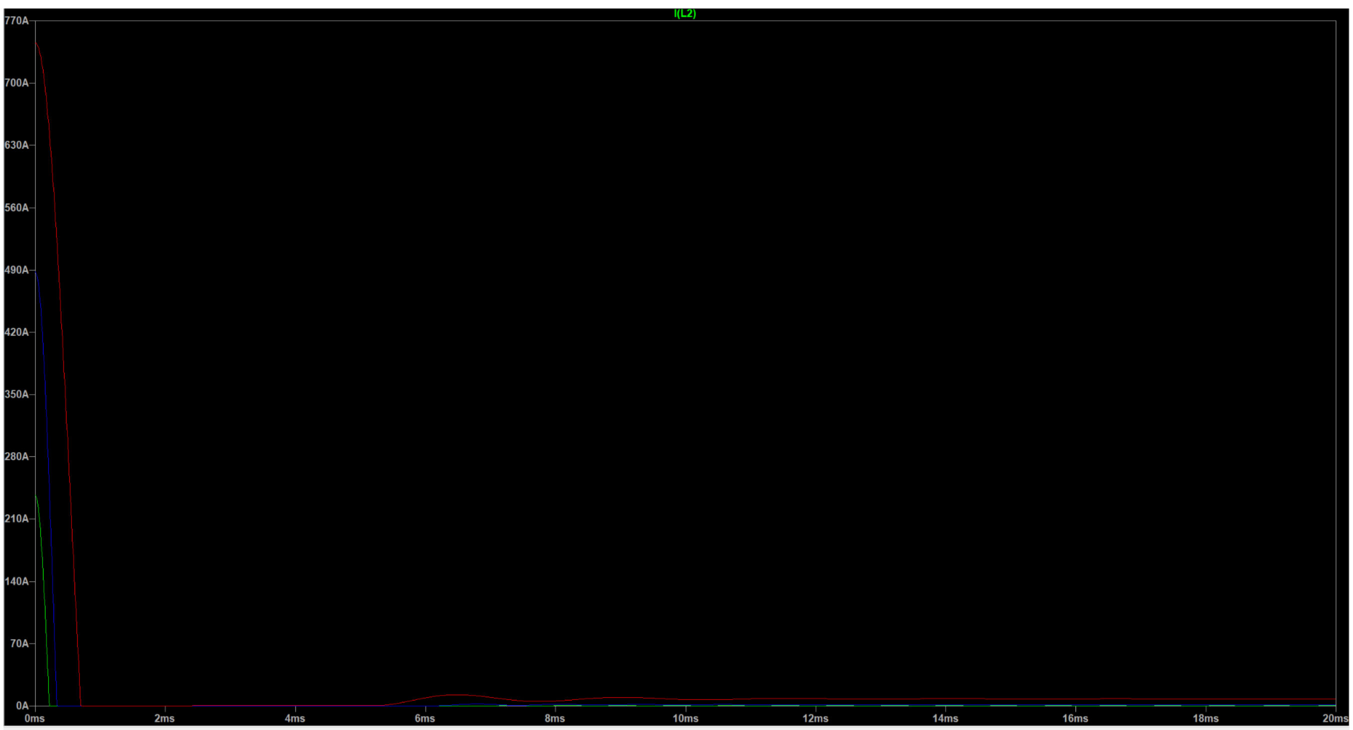
IL1, IL2, VOUT1



IL1



IL2:

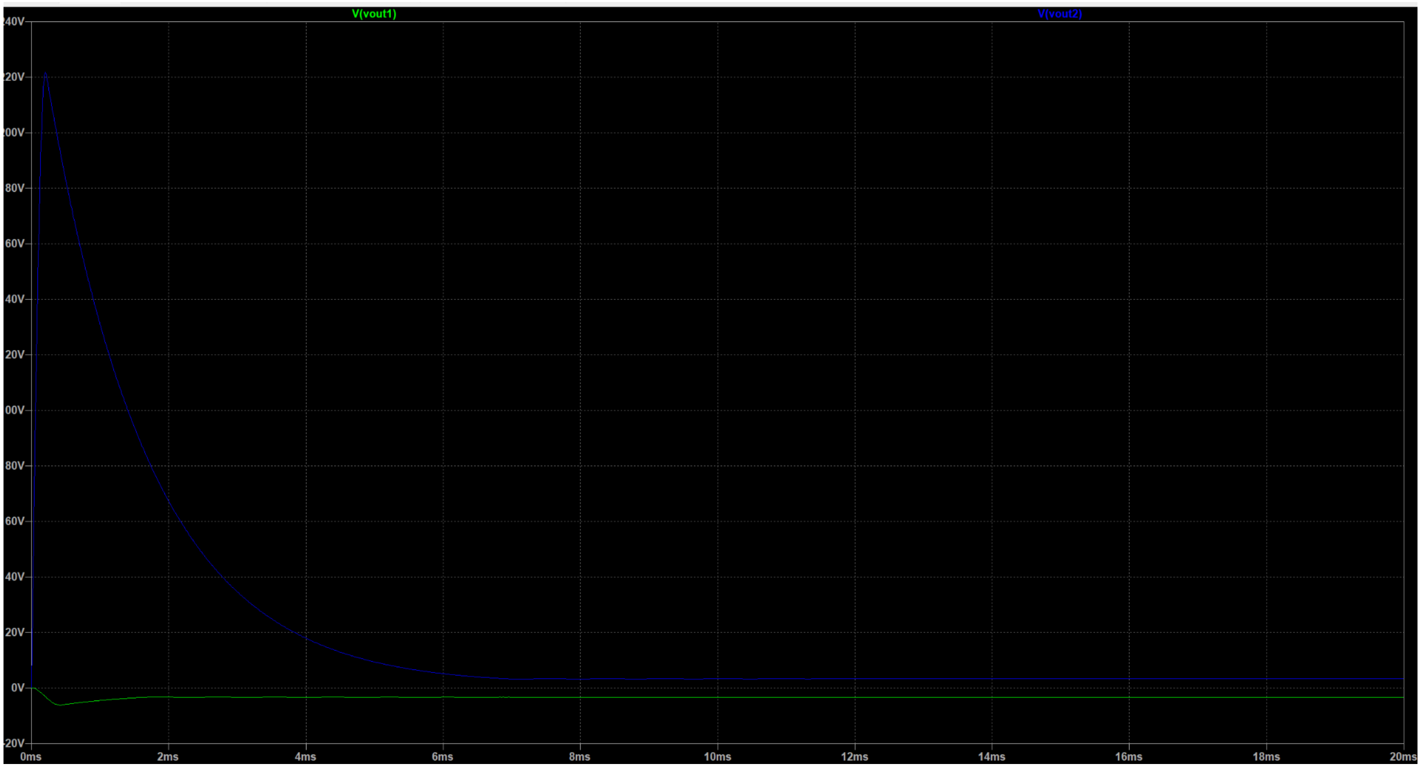


5)

if D > 0.5, the output voltage is greater than the input voltage (stepped-up), and if D < 0.5, the output voltage is less than the input voltage (stepped-down).

$$V_o = -V_d \frac{D}{1-D}$$

For step when D=25



Vout1= -3V

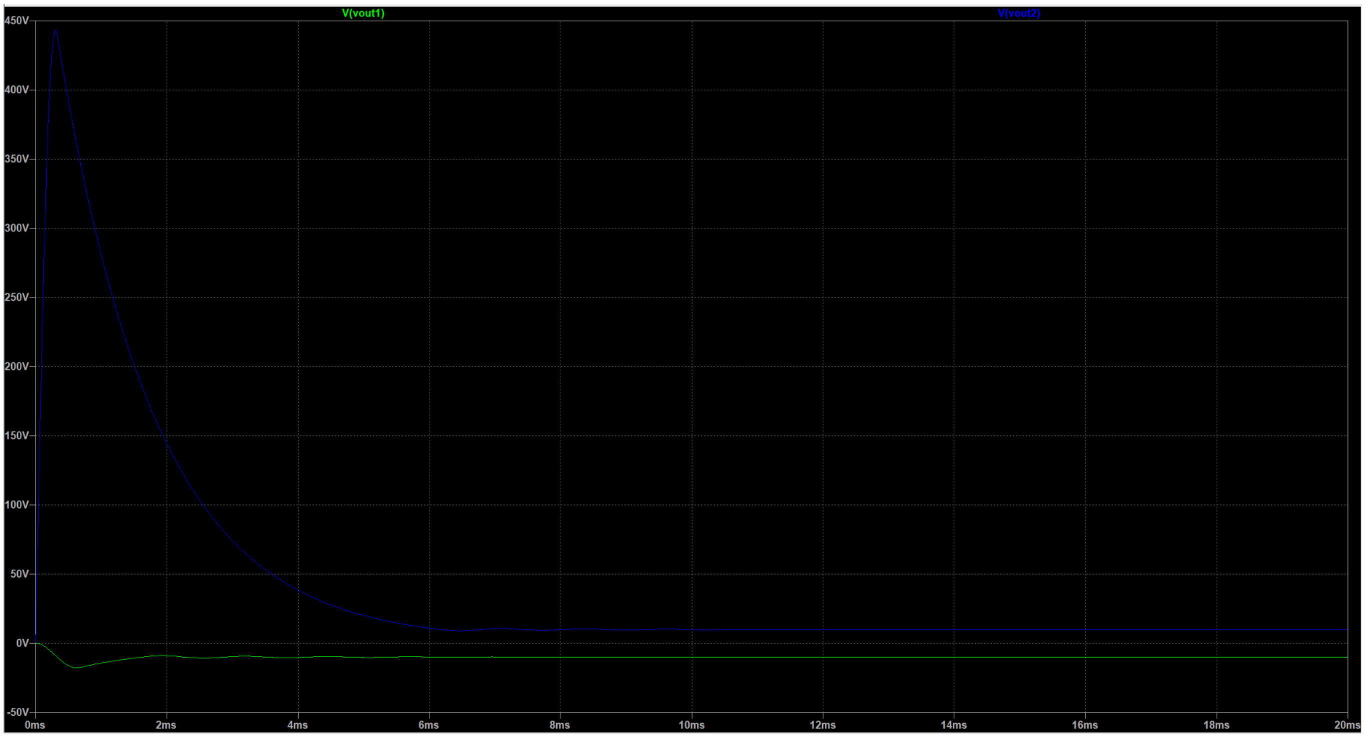
Based on the experiment, Vout= -3 V, Vd=10V

-3=-10(D/1-D) -> D=0.23 -> **Step down voltage**

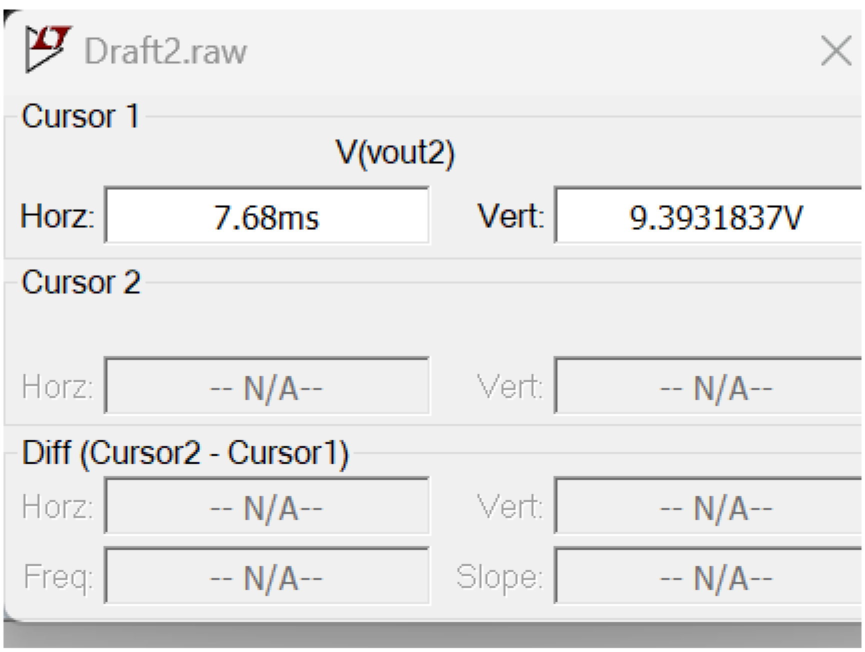
Vout 2= 3.196

3.196= -10(D/1-D) -> D= -0.46-> **step down voltage**

For step when D=50

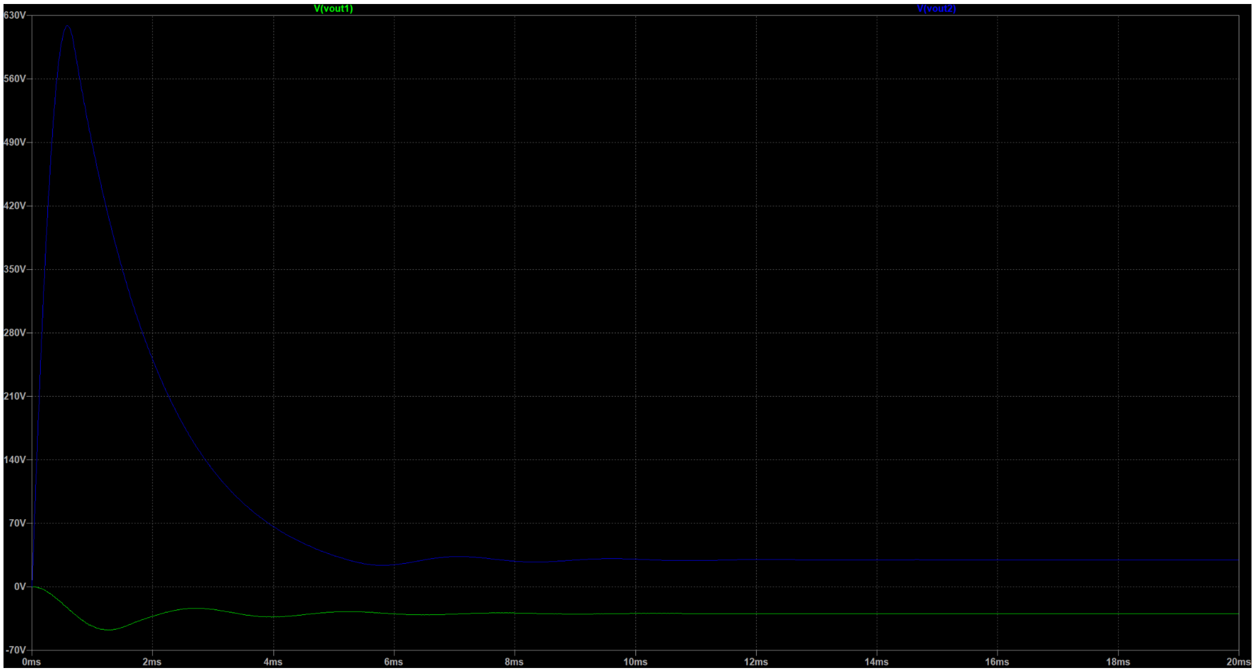


Vout1= -10V
Based on the experiment, Vout= -10 V, Vd=10V
-10=-10(D/1-D) -> D=0.5 -> **Step up voltage**
Vout 2= 9.4V

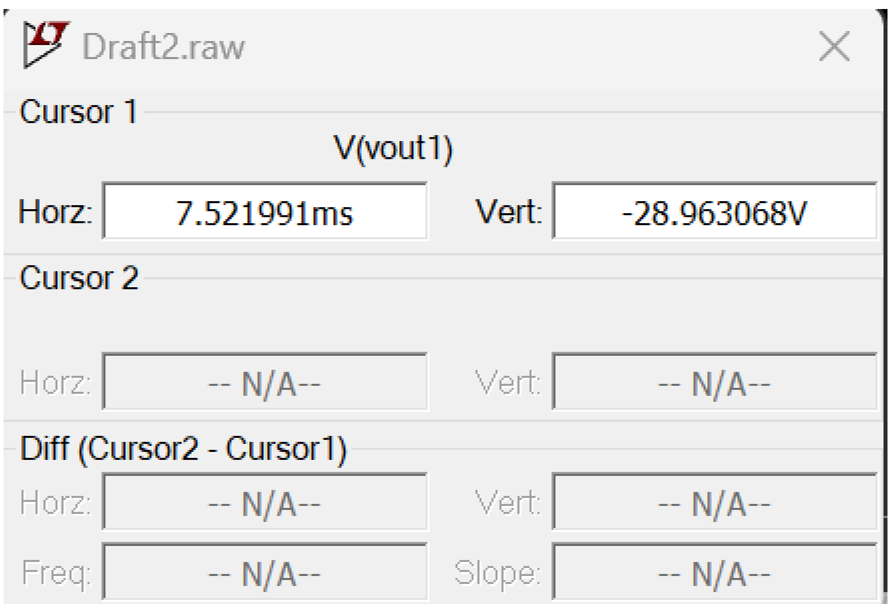


9.2= -10(D/1-D) -> D= -11.5-> **step down voltage**

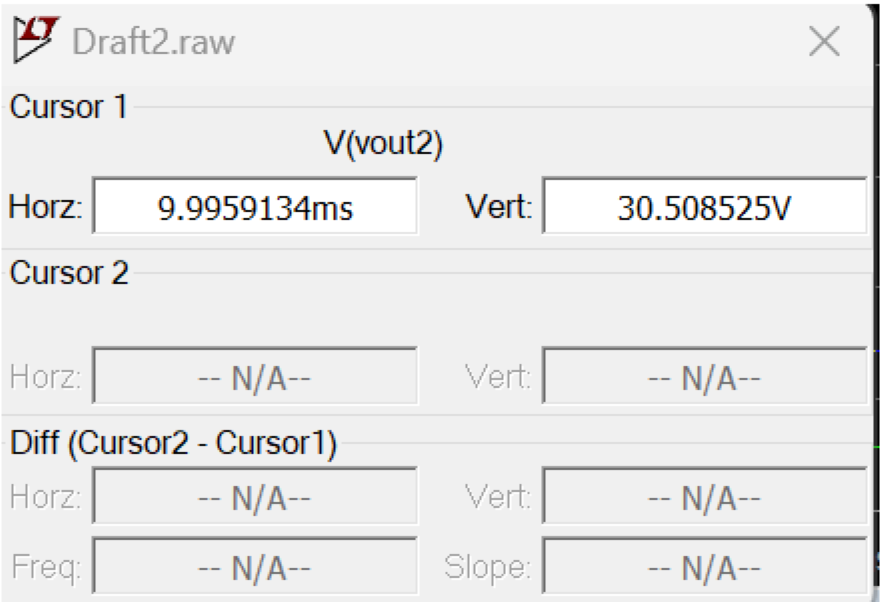
For step when D=75



Vout1= -28.96

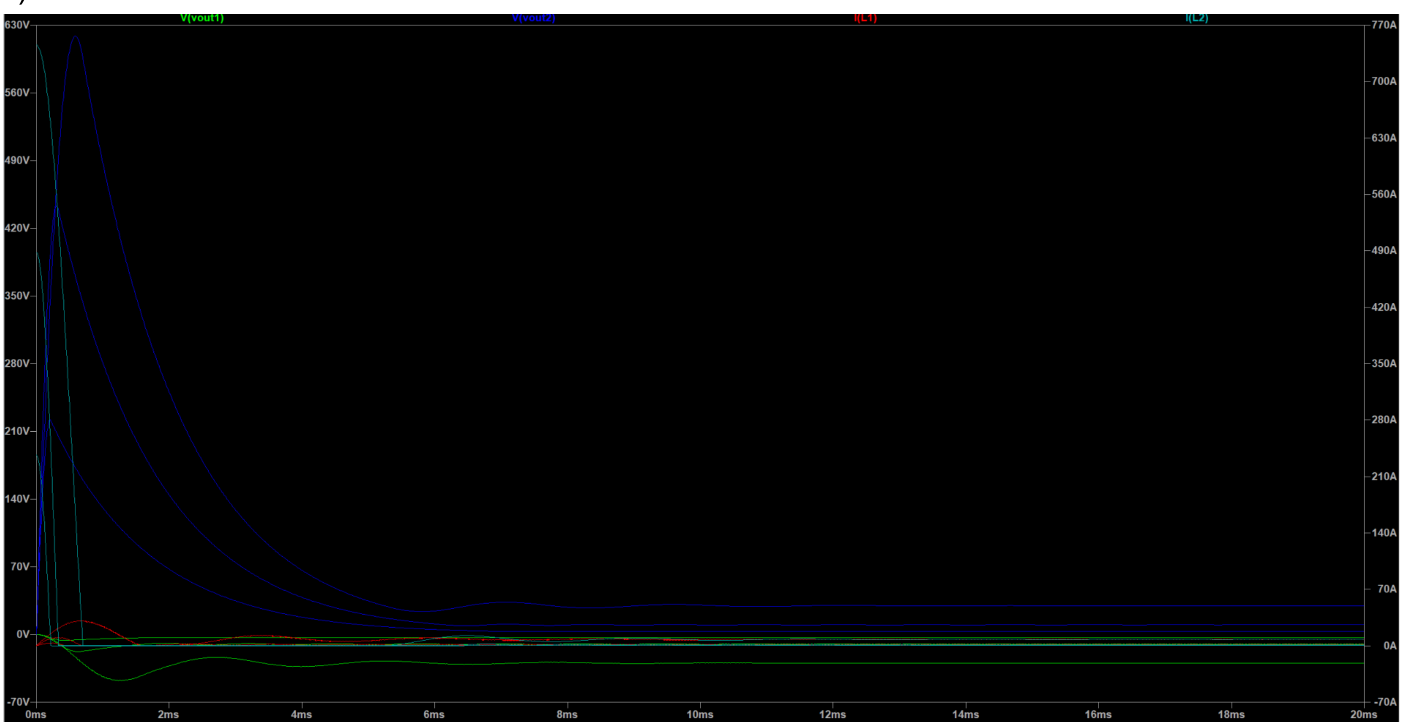


Based on the experiment, Vout= -28.96 V, Vd=10V
-28.96=-10(D/1-D) -> D=0.74 -> **Step up voltage**
Vout 2= 30.51



30.51= -10(D/1-D) -> D= 1.49-> **step up voltage**

6)



Theory:

6)

$$(2.1) \qquad V_o = -V_d \frac{D}{1-D}$$

$$(2.3) \qquad I_L = \frac{V_d D}{R(1-D)^2}$$

When D=25
Vo= -10(25/1-25) = 10.42

When D=50
Vo= -10(50/1-50) = 10.20

When D=75
Vo= -10(75/1-75) = 10.135

Finding IL

When D= 25

IL= VdD/R(1-D)^2 = 10 * 25/ (15 * (1-25)^2) = 0.029 A

When D=50

IL= VdD/R(1-D)^2 = 10 * 50/ (15 * (1-50)^2) = 0.014 A

When D=75

IL= VdD/R(1-D)^2 = 10 * 75/ (15 * (1-75)^2) = 0.009 A

7)

When D= 25:

Iripple = IL * Vout/Vin = 0.029 A * (10.42/ 10)= 0.03 (A)

When D= 50:

Iripple = IL * Vout/Vin = 0.014 A * (10.20/ 10)= 0.02958 (A)

When D= 75:

Iripple = IL * Vout/Vin = 0.009 A * (10.135 / 10)= 9.1215 * 10^-3 (A)

8) Comparision between using the switching and average dynamic models:
Overall, the difference between

Switching model:

+Captures detailed transient responses, including all high-frequency components and oscillations. This level of detail is crucial for designing and testing the control system's reaction to rapid changes.

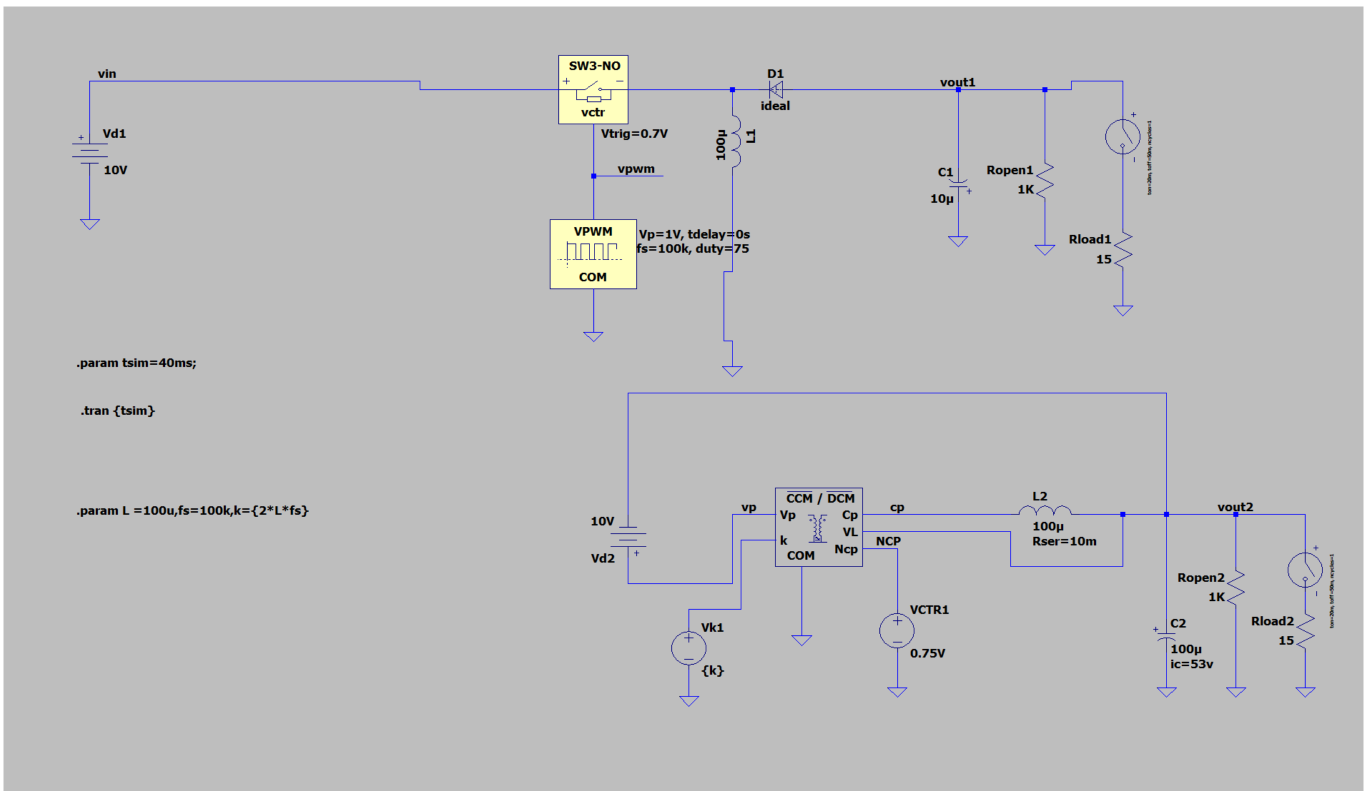
+Essential for precise, detailed analysis, such as studying switch stress, transient responses, and optimizing component-level efficiency.

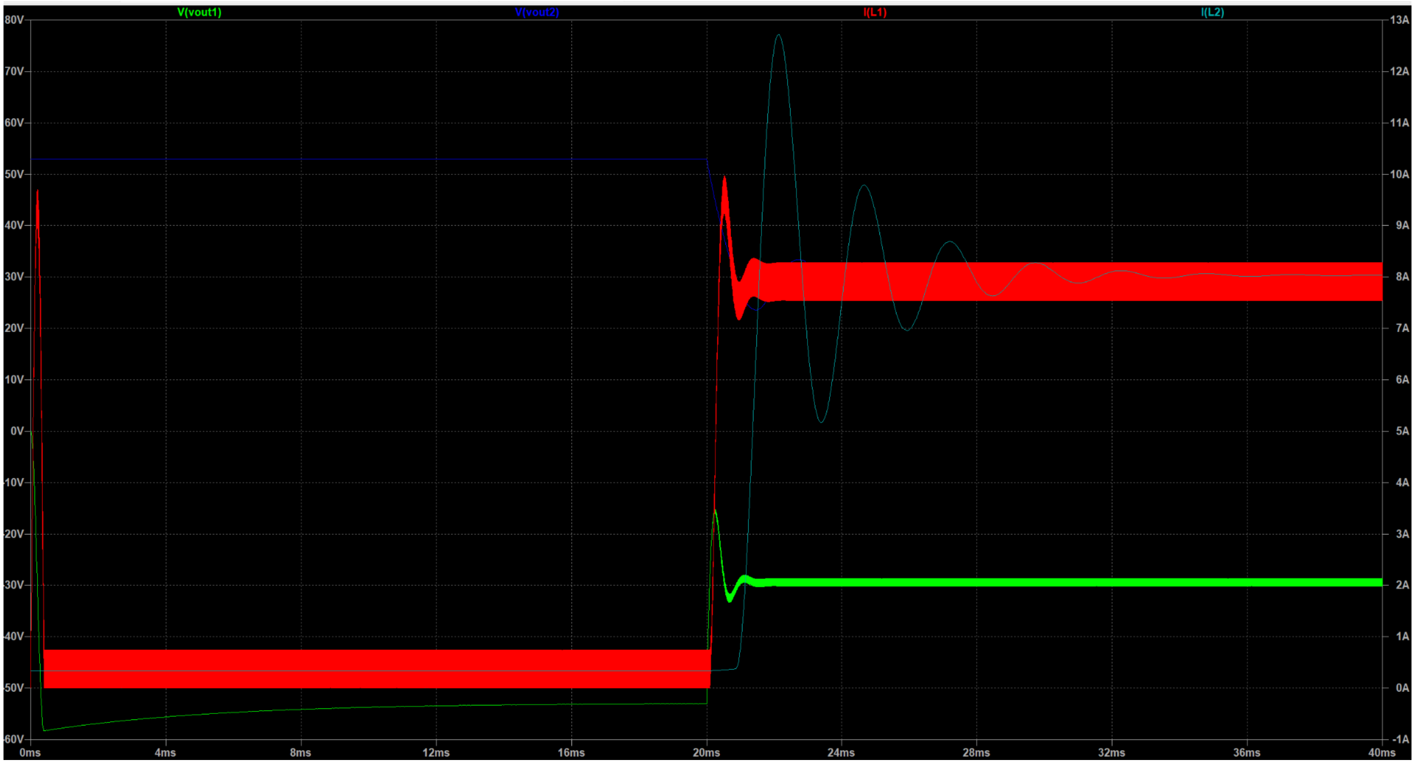
Average Dynamic Model:

+Great for designing systems and developing control strategies. It provides quick insights into overall behavior and stability without the heavy computation required for detailed switching simulations.

PART 3:

Schematic:

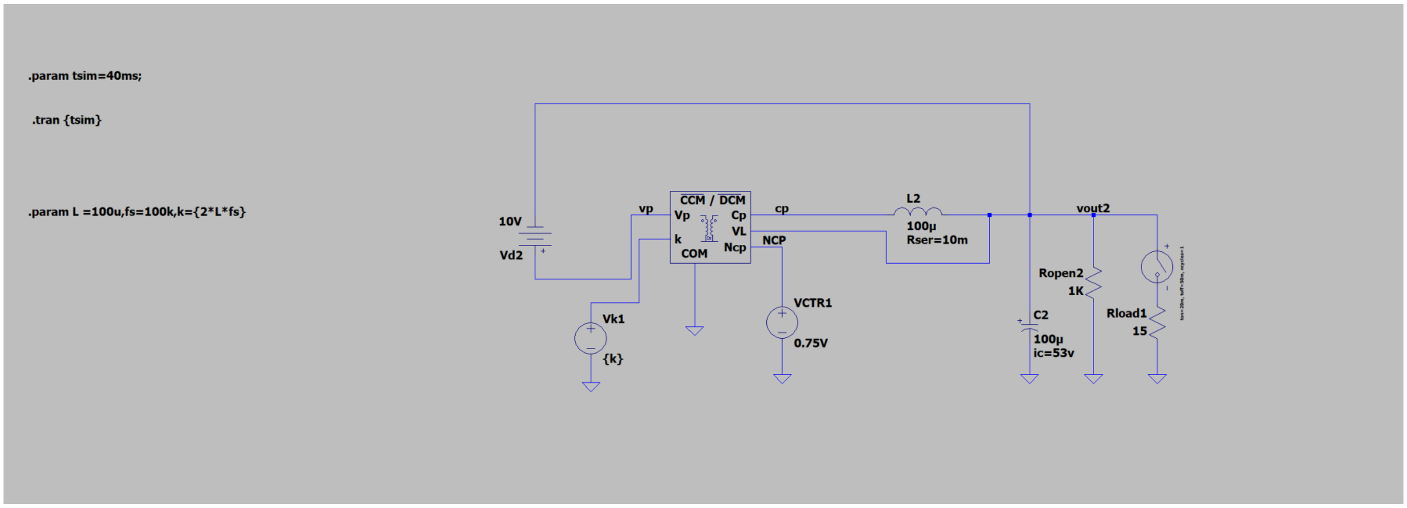


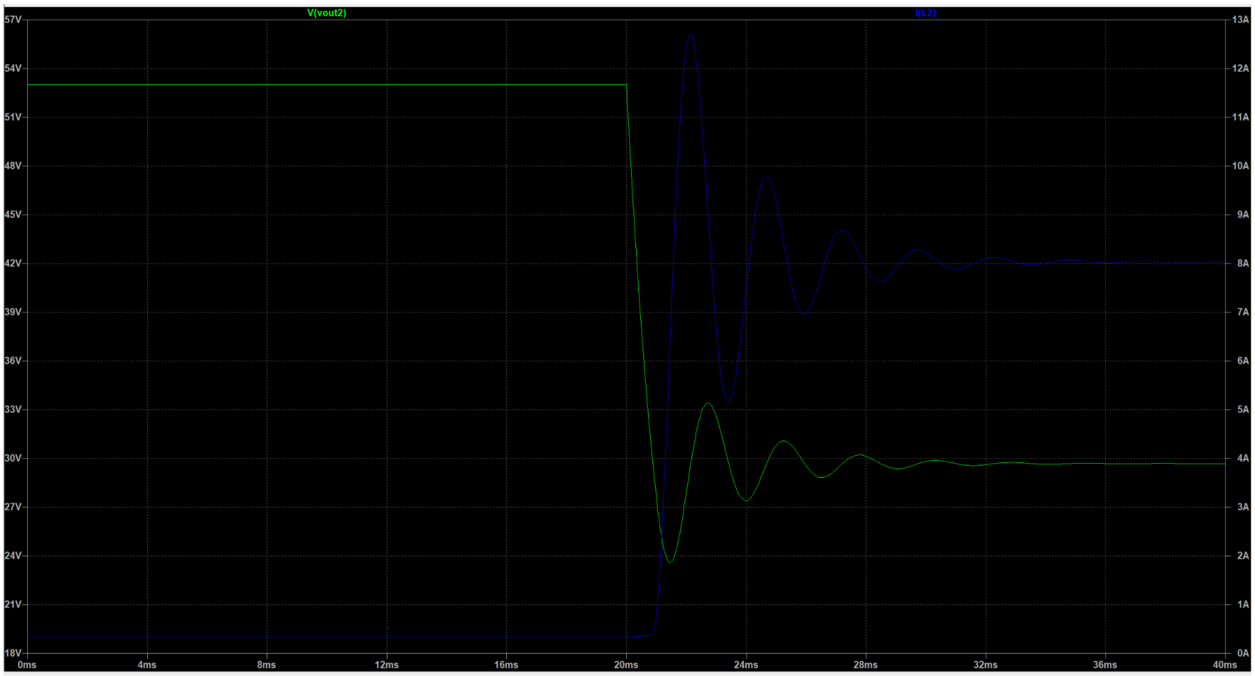


6)

The problem why the output are not stablelized because the transient time, the overshoot current from the inductance IL1 and IL2. For the transient time, IL1 and IL2 oscillates rapidly when the load is changed. For the current overshoot, the current is higher than the steady state current. Because of that, the output is not stablized.

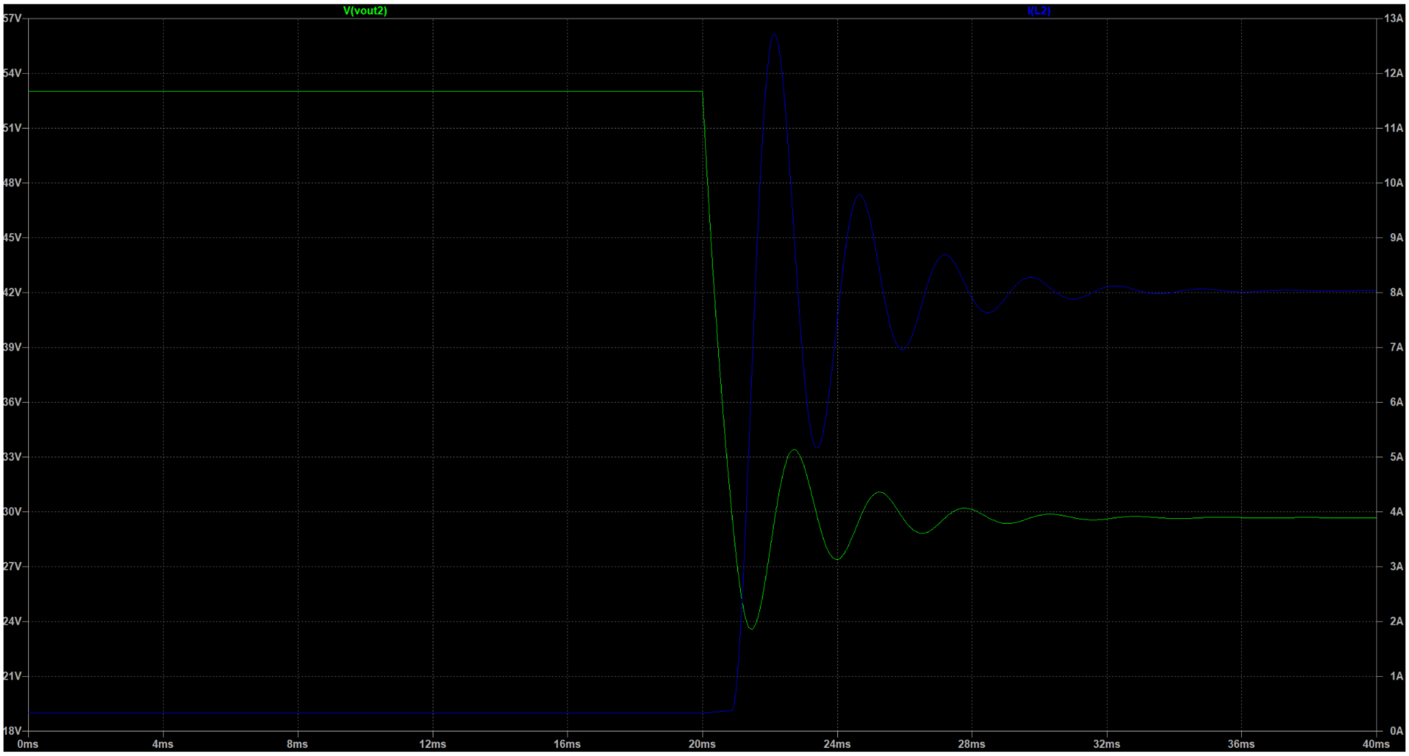
7)
After deleting the switching model:


.param tsim=40ms;
.tran {tsim}
.param L =100u,fs=100k,k={2*L*fs}
10V
Vd2
vp
CCM / DCM
Vp
k
COM
Cp
VL
Ncp
cp
NCP
L2
100µ
Rser=10m
vout2
Ropen2
1K
C2
100µ
ic=53v
Rload1
15
Vk1
{k}
VCTR1
0.75V

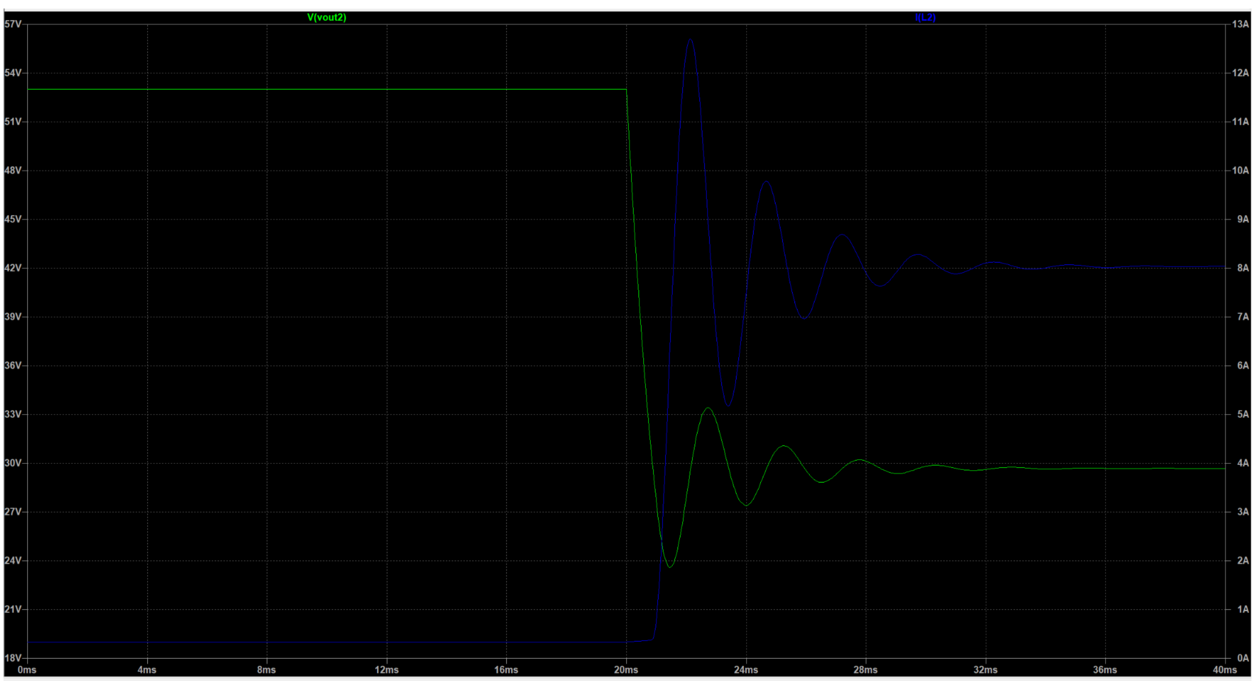

V(vout2)
I(L2)
57V
54V
51V
48V
45V
42V
39V
36V
33V
30V
27V
24V
21V
18V
13A
12A
11A
10A
9A
8A
7A
6A
5A
4A
3A
2A
1A
0A
0ms
4ms
8ms
12ms
16ms
20ms
24ms
28ms
32ms
36ms
40ms

8)

With portion containing the switch model:



Without portion containing the switch model



The time it takes for obtaining the average switching model has similar time it takes with portion containing the switch model.

Practice Problem Encounters:

A problem we encountered is that during part 1, we mistakenly used 10V for V_d. We realized it after we had finished all of our calculations. For part 1, we used resistor values 10, 80, 100, and 500 rather than the given 10, 50, 80, and 500 which resulted in our answers differing from our peers. We did do all the same calculations and processes so our resulting conclusions stayed the same.

Conclusion:

We successfully understood the principles of operation of buck and buck-boost DC-DC converters. We learned how discontinuous current mode (DCM) affects converters. We now know how to design DC-DC converters that operate in continuous current mode (CCM). We learned how the non-linear switching operation can be replaced by the linear average CCM-DCM modeling, and its advantages and disadvantages. We understand the need for system feedback control in the design/development of switch-mode regulators. Even though some of our variable values were wrong we still ended up with the same conclusions due to our calculations and processes being the same.