Pricing European down-and-out call options: an application to Vietnamese financial market

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Motivation

- In Vietnam, derivatives market has just started officially very recently, in August, 2017, with its first products: futures contracts on VN30 index.
- Although the derivatives market is very new to Vietnamese investors, it is expected to strongly develop soon and will be one of the main pillars of Vietnamese financial market.
- Great interest of investors to financial derivatives market: trading volume on futures contracts on VN30 index is increasing significantly over last few months, and is expected to increase with an even faster rate.

Motivation

- One important type of financial derivatives products is options. In Vietnam, covered call options will be traded soon, as planed by the Vietnamese government.
- Understanding clearly the pricing formulas for different types of options is thus very urgent for investors investing in Vietnamese financial market.

Aim of this research

- We first derive the Merton (1973)'s formula of European down-and-out call options under the Black-Scholes framework, using a probabilistic approach.
- We then show in detail how to apply the formula to price European down-and-out call options written on Vietnamese stocks, say FPT stock.
- Our case-study research can serve as a good start for future researches on Vietnamese option market, which is going to develop soon.

Pricing problem of European down-and-out call options

- At time t, an asset has price S_t. An investor speculates that the asset price will always greater than level $B < S_t$ and especially at time $T > t$ the asset price would be greater than $K > B$. An investor pays an amount C_0 to buy a European down-and-out call, which gives the investor a right now to buy the asset at time $T > t$ at price K.
- If the asset price touches the level $B < S_0$ during the period $[t, T)$ then the right is ceased to exist.
- If $S_u > B$, $\forall u \in [t, T)$, then at time T, the investor receive payoff $\max(S_T - K, 0)$.
- What is the fair value of C_0 under the Black-Scholes framework?

Solution procedure

- Under the Black-Scholes framework, the asset price (paying no dividend) is governed by a Geometric Browninan motion:

$$dS_t = rS_t dt + \sigma S_t dZ_t \tag{1}$$

  where $\{S_t : 0 \leq t \leq T\}$ is the stock price process, $\{Z_t : 0 \leq t \leq T\}$ is a standard Brownian motion with respect to the risk-neutral probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{Q})$. Here T, r, σ, which are positive constants, represent for the expiry time, the risk-free interest rate and the volatility rate, respectively.
- We can express: $S_T = S_t e^{\left(r - \frac{1}{2}\sigma^2\right)(T-t) + \sigma W^{\mathbb{Q}}_{T-t}} = S_t e^{\sigma \widehat{W}_{T-t}}$, where $\widehat{W}_{T-t} = \nu(T-t) + W^{\mathbb{Q}}_{T-t}$ and $\nu = \dfrac{1}{\sigma}(r - \dfrac{1}{2}\sigma^2)$.

Solution procedure

- From the Feynman-Kac theorem (Shreve, 2004, p. 268), the price of the European down-and-out call option $C_{d/o}(S_t, t)$ can be computed from the formula:

$$C_{d/o}(S_t, t) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[\max\{S_T - K, 0\}\mathcal{I}_{\min\limits_{t \leq u \leq T} S_u > B}|\mathcal{F}_t],$$

where $\mathbb{E}^{\mathbb{Q}}$ represents the expectation under the risk neutral measure $\mathbb{Q}$, and $\mathcal{I}_{\min\limits_{t \leq u \leq T} S_u > B}$ is the indicator of the set $\{S_u > B\}$.

Solution procedure

- The down-and-out call option price at time t is

$$
\begin{aligned}
&C_{d/o}(S_t, t; K, B, T) \\
&= e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\left[(S_t e^{\sigma \widehat{W}_{T-t}} - K)\mathcal{I}_{\{m_{T-t} > \frac{1}{\sigma}\log\left(\frac{B}{S_t}\right), \widehat{W}_{T-t} > \frac{1}{\sigma}\log\left(\frac{K}{S_t}\right)\}} \middle| \mathcal{F}_t\right] \\
&= e^{-r(T-t)}\int_{\frac{1}{\sigma}\log\left(\frac{K}{S_t}\right)}^{\infty}\int_{\frac{1}{\sigma}\log\left(\frac{B}{S_t}\right)}^{\infty}(S_t e^{\sigma x} - K) f^{\mathbb{Q}}_{m_{T-t}, \widehat{W}_{T-t}}(a, x)\, da\, dx
\end{aligned}
$$

where $f^{\mathbb{Q}}_{m_u, \widehat{W}_u}(m, \omega)$ is the joint probability density function of $(m_u, \widehat{W}_u)$ as given as in Chin et al. (2014, p. 212):

$$
f^{\mathbb{Q}}_{m_u, \widehat{W}_u}(a, x) = \begin{cases} \dfrac{2(x-2a)}{u\sqrt{2\pi u}}\exp(\nu x - \dfrac{1}{2}\nu^2 u - \dfrac{(2a-x)^2}{2u}), & a < 0, x \geq a, \\ 0, & \text{otherwise.} \end{cases}
$$

Pricing formula

$$C_{d/o}\left(S_t, t; K, B, T\right) = C_{bs}\left(S_t, t; K, T\right) - \left(\frac{S_t}{B}\right)^{\lambda} C_{bs}\left(\frac{B^2}{S_t}, t; K, T\right)$$

where $C_{bs}(S_t, t; K, T) = S_t \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2)$, with
$\Phi(x) = \int_{-\infty}^{x} e^{-y^2/2} dy,\ d_1 = \dfrac{\log(S_t/K) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$,
$d_2 = d_1 - \sigma\sqrt{T-t}$ and $\lambda = 1 - \dfrac{2r}{\sigma^2}$.

Pricing problem of a European down-and-out call option written on FPT stock

An investor agrees to pay now an amount C_0 to buy a European down-and-out call option with the underlying FPT stock, which is now priced at $42,750$ VND per share. This option entitles the investor a right, but not an obligation, to buy a FPT share with price $45,000$ VND, at the end of the next 6 months. However, this right will be ceased if the FPT price touches the level $38,000$ VND at any time during the next 6 months. Suppose the risk-free interest rate is 7% and the volatility of the FPT stock is 22.06%. According to the Black-Scholes framework, what should be the fair value for C_0?

Import data into R

In this example, we import the data of FPT stock.

```
setwd("/Users/tantoankhoa10/Downloads/forecast")
df <- read.csv(file="excel_fpt.csv",
    head=T,stringsAsFactors = F)
date <- as.Date(as.character(df$X.DTYYYYMMDD.),
    format="%Y%m%d")
df <- cbind(date,df)
df <- df[,-2]
df <- df[order(df$date),]
names(df) <- paste(c("date", "Open", "High", "Low",
    "Close", "Volume"))
df<-subset(df,df$date >= "2006-12-12" & df$date <=
    "2018-11-16")
library(xts)
df <- xts(df[,2:6],order.by = df[,1])
```

Compute the volatility of the stock

Compute the historical volatility and use it as a parameter for the option

```
price<-df[,4]
price$Ret<-diff(log(price$Close)) ## compute the log
    return
price<-price[-1,] ## remove the NA term
hist.vol<-sd(price$Ret)*sqrt(252)
vol <- hist.vol
```

The volatility of FPT stock is 32.5%

R code for computing the option price

```
BS_call <- function (S0,K,T,r,sigma){
d1 <- (log(S0/K)+(r+0.5*sigma^2)*T)/(sigma*sqrt(T))
d2 <- d1- sigma*sqrt(T)
opt.val <- S0*pnorm(d1)-K*exp(-r*T)*pnorm(d2)
return(opt.val)}
down_out_call <- function (S0,K, T, r, sigma,B){
lamda <- 1-2*r/sigma^2
opt.val <- BS_call(S0,K,T,r,sigma)
-(S0/B)^lamda*BS_call(B^2/S0,K,T,r,sigma)
return(opt.val)}
down_out_call(42750,45000, 0.5, 0.07, 0.325, 38000)
```

As a result, we obtain the European down-and-out call option price as $3,018.038$ (VND).

Thank You For Your Attention

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