# Pricing European down-and-out call options: an application to Vietnamese financial derivatives market

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# Outline

- On August 10, 2017, the VN30-Index futures contract were officially traded in the Vietnam market
- Options is a important type of financial derivatives products
- Barrier options are very popular
- Deriving the pricing formula using probabilistic approach

# Option price



Premium

**Fair premium?**

# Pricing option procedure

Option price at expiry is known from definition

$$C_{d/o}(S_T, T) = \max\{S_T - K, 0\}\mathcal{I}_{\min\limits_{t\leq u\leq T} S_u > B} \tag{1}$$

The stock price $S_t$ follows a Geometric Brownian motion:

$$dS_t = rS_t dt + \sigma S_t dZ_t$$

where $\{S_t : 0 \leq t \leq T\}$ is the stock price process, $\{Z_t : 0 \leq t \leq T\}$ is a standard Brownian motion. $T, r, \sigma$, which are positive constants, represent for the expiry time, the risk-free interest rate and the volatility rate, respectively. $\mathcal{I}_{\min\limits_{t\leq u\leq T} S_u > B}$ is the indicator of the set $\{S_u > B\}$.

# Pricing option procedure

By solving SDE,

$$S_T = S_t e^{\left(r-\frac{1}{2}\sigma^2\right)(T-t)+\sigma W^{\mathbb{Q}}_{T-t}} = S_t e^{\sigma \widehat{W}_{T-t}} \tag{2}$$

where $\widehat{W}_{T-t} = \nu(T-t) + W^{\mathbb{Q}}_{T-t}$ and $\nu = \frac{1}{\sigma}(r - \frac{1}{2}\sigma^2)$. By writing $m_{T-t} = \min_{t\le u\le T} \widehat{W}_{u-t}$. Therefore, $\min_{t\le u\le T} S_u = S_t e^{\sigma m_{T-t}}$
As a result, the payoff can be expressed as:

$$\begin{aligned} C_{d/o}(S_T, T) &= \max\{S_t e^{\sigma \widehat{W}_{T-t}} - K, 0\}\mathcal{I}_{\{S_t e^{\sigma m_{T-t}} > B,\}} \\ &= (S_t e^{\sigma \widehat{W}_{T-t}} - K)\mathcal{I}_{\{m_{T-t} > \frac{1}{\sigma}\log\left(\frac{B}{S_t}\right), \widehat{W}_{T-t} > \frac{1}{\sigma}\log\left(\frac{K}{S_t}\right)\}} \end{aligned}$$

# Pricing option procedure

The down-and-out call option price at time $t$ is

$$
\begin{aligned}
C_{d/o}(S_t, t; K, B, T) &= e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\left[C_{d/o}(S_T, T)|\mathcal{F}_t\right] \\
&= e^{-r(T-t)}\int_{\frac{1}{\sigma}\log\left(\frac{K}{S_t}\right)}^{\infty}\int_{\frac{1}{\sigma}\log\left(\frac{B}{S_t}\right)}^{\infty}(S_t e^{\sigma x}-K) f^{\mathbb{Q}}_{m_{T-t},\widehat{W}_{T-t}}(a,x)\,da\,dx \\
&= C_{bs}\left(S_t, t; K, T\right) - \left(\frac{S_t}{B}\right)^{\lambda} C_{bs}\left(\frac{B^2}{S_t}, t; K, T\right) \qquad (3)
\end{aligned}
$$

where $C_{bs}(S_t, t; K, T) = S_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$, with

$d_1 = \dfrac{\log(S_t/K) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$ and $\lambda = 1 - \dfrac{2r}{\sigma^2}$.

# Apply the formula on Vietnamese stock, FPT

The closing prices of FPT stock from December 12, 2006 to November 16, 2018 are used. Free source: `https://www.cophieu68.vn/export.php`.

A European call option written on FPT stock with the starting date on November 16, 2018 and expiring in the next 6 months. The parameters is given:

| | | |
|---|---|---|
| $S_0$ | Stock price at inception | $42,750$ VND |
| $K$ | Strike price | $45,000$ VND |
| $B$ | Down and out barrier price | $38,000$ VND |
| $r$ | Annual risk-free interest rate | 7% |
| $\sigma$ | Annual volatility | 32.5% |
| $T$ | Option expiration (in years) | 0.5 |

Table: Option parameters

# Apply the formula on Vietnamese stock, FPT

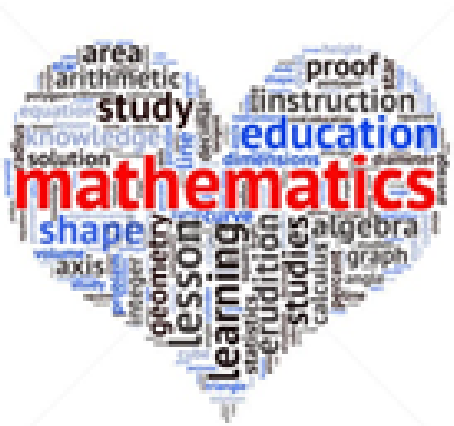
We compute the value of the European down-and-out call option by using the following R code:

```
BS_call <- function (S0,K,T,r,sigma){
d1 <- (log(S0/K)+(r+0.5*sigma^2)*T)/(sigma*sqrt(T))
d2 <- d1- sigma*sqrt(T)
opt.val <- S0*pnorm(d1)-K*exp(-r*T)*pnorm(d2)
return(opt.val)}
down_out_call <- function (S0,K, T, r, sigma,B){
lamda <- 1-2*r/sigma^2
opt.val <- BS_call(S0,K,T,r,sigma)-(S0/B)^lamda*BS_call(B^2/S0,K,T,r,sigma)
return(opt.val)}
down_out_call(42750,45000, 0.5, 0.07, 0.325, 38000)
```

- The final result as $3,018.038$ (VND).
- Compare with the corresponding vanilla call price as 3601.607 (VND) $\Rightarrow$The barrier option is cheaper 16.2%.

## Conclusion

- The pricing formula of European down-and-out call options is derived by using a probabilistic approach
- The application of this formula written on FPT stock
- The R code is used as technique to compute the option price
- This research will be on the pricing formula of other exotic options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Thank You!