Pricing European barrier options with rebates

Department of Mathematics
International University

Author: Ta Thi Phuong Dung
Advisor: Dr. Le Nhat Tan

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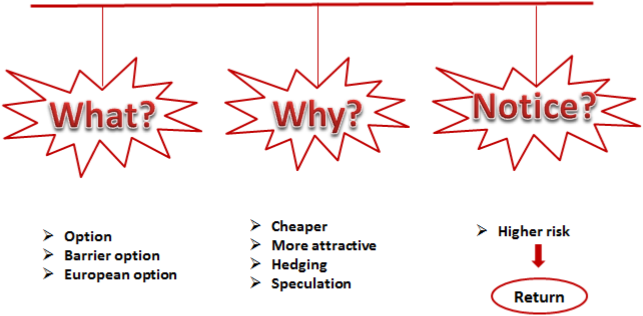
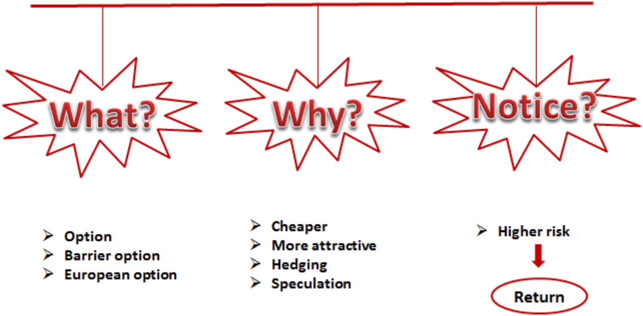
Outline


What?
Why?
Notice?
Option
Barrier option
European option
Cheaper
More attractive
Hedging
Speculation
Higher risk
Return

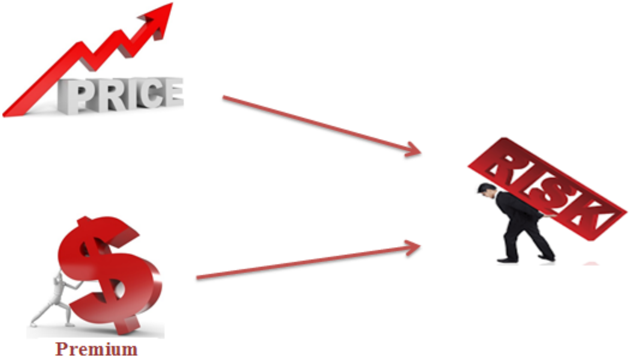


- On August 10, 2017, the VN30-Index futures contract were officially traded in the Vietnam market
- Next April, Coverall call option is expected to open.
- Pricing options is very urgent

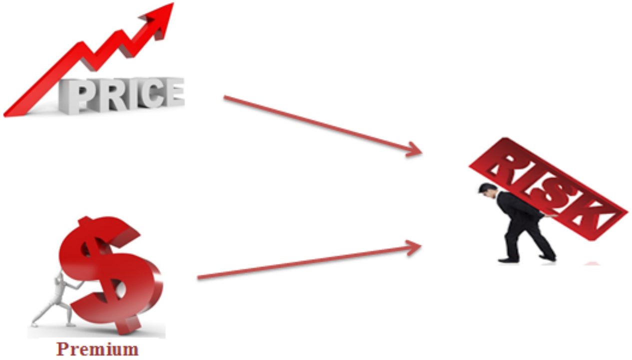
$\Rightarrow$ Using the probabilistic approach for formulating the pricing models

European barrier options with rebates

Option price



Option price



Fair premium?

Pricing option procedure

Option price at expiry is known from definition

$$C_{d/o}(S_T) = \max\{S_T - K, 0\}\mathcal{H}_{\{min_{t \le u \le T} S_u > B\}} \tag{1}$$

The stock price S_t follows a geometric Brownian motion with the following SDE

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

where μ is the drift parameter and σ is the volatility parameter.

Risk-neutral measure

$$\frac{dS_t}{S_t} = rdt + \sigma dW_t$$

Pricing option procedure

By solving SDE,

$$S_t = S_0 e^{\left(r-\frac{1}{2}\sigma^2\right)t+\sigma W_t^{\mathbb{Q}}}$$

Let's consider times from t to T for $t < T$, we obtain

$$\begin{aligned} S_T &= S_t e^{\left(r-\frac{1}{2}\sigma^2\right)(T-t)+\sigma W_{T-t}^{\mathbb{Q}}} \\ &= S_t e^{\sigma \widehat{W}_{T-t}} \end{aligned}$$

where $\widehat{W}_{T-t} = \nu(T-t) + W_{T-t}^{\mathbb{Q}}$ and $\nu = \dfrac{1}{\sigma}(r - \dfrac{1}{2}\sigma^2)$. By writing

$$m_{T-t} = min_{t \leq u \leq T} \widehat{W}_{u-t}$$

Therefore,

$$\min_{t \leq u \leq T} S_u = S_t e^{\sigma m_{T-t}}$$

Pricing option procedure

and we can rewrite the payoff as

$$C_{d/o}(S_T) = (S_t e^{\sigma \widehat{W}_{T-t}} - K)\mathcal{H}_{\{m_{T-t} > \frac{1}{\sigma}\log\left(\frac{B}{S_t}\right), \widehat{W}_{T-t} > \frac{1}{\sigma} log\left(\frac{K}{S_t}\right)\}}$$

$$C_{d/o}(S_t) = e^{-r(T-t)} \int_{\omega=\frac{1}{\sigma} log\left(\frac{K}{S_t}\right)}^{\infty} \int_{m=\frac{1}{\sigma} log\left(\frac{B}{S_t}\right)}^{m=\omega} (S_t e^{\sigma\omega} - K) f^{\mathbb{Q}}_{m,\widehat{W}}(m,\omega) dm d\omega$$

$$= C_{bs}(S_t, t; K, T) - \left(\frac{S_t}{B}\right)^{2\lambda} C_{bs}(\frac{B^2}{S_t}, t; K, T)$$

where $\lambda = \frac{1}{2}\left(1 - \frac{r}{\frac{1}{2}\sigma^2}\right)$ and

$$C_{bs}(S_t, t; K, T) = S_t N(d_1) - K e^{-r(T-t)} N(d2)$$

$$d_1 = \frac{log(S_t/K) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

Pricing option procedure

$$C_{bs}(\frac{B^2}{S_t}, t; K, T) = \frac{B^2}{S_t}N(d_3) - Ke^{-r(T-t)}N(d4)$$

$$d_3 = \frac{log(B^2/(S_t K)) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \quad d_4 = d_3 - \sigma\sqrt{T-t}$$

The expected present value of the rebate is given by

$$\text{Rebates value} = R\int_0^T e^{-ru}Q(u; B)du$$

$$= R\left[\left(\frac{B}{S}\right)^{\alpha_+}\Phi\left(-\frac{\ln\frac{B}{S} + \beta T}{\sigma\sqrt{T}}\right) + \left(\frac{B}{S}\right)^{\alpha_-}\Phi\left(-\frac{\ln\frac{B}{S} - \beta T}{\sigma\sqrt{T}}\right)\right]$$

where

$$\beta = \sqrt{\left(r - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}, \quad \alpha_\pm = \frac{r - \frac{\sigma^2}{2} \pm B}{\sigma^2}$$

Pricing option procedure

The final result

$$C^R_{d/o}(S_t, t; K, B, T) = C_{bs}(S_t, t; K, T) - \left(\frac{S_t}{B}\right)^{2\lambda} C_{bs}(\frac{B^2}{S_t}, t; K, T)$$
$$+R\left[\left(\frac{B}{S}\right)^{\alpha_+} \Phi\left(-\frac{\ln\frac{B}{S} + \beta T}{\sigma\sqrt{T}}\right) + \left(\frac{B}{S}\right)^{\alpha_-} \Phi\left(-\frac{\ln\frac{B}{S} - \beta T}{\sigma\sqrt{T}}\right)\right]$$

Under Black-Scholes model's assumption

Pricing European barrier call option with reabtes on FPT stock.

Source: `http://www.cophieu68.vn/historyprice.php?id=fpt`

Daily return $u_i = \ln \dfrac{S_i}{S_{i-1}}$ for $i = 0, 1, \ldots, n$

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Testing for normal distribution

Using graphical methods: Q-Q plot. Our result

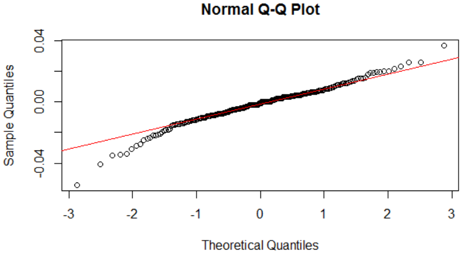

Figure: The distribution of FPT stock

Applied

Parameters is given by

S_0	Stock price at time zero	59.8 VND
K	Strike price	62 VND
σ	Annual volitility	24%
r	Annual riskless rate	3%
T	Option expiration (in years)	0.5

Table: FPT stock

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Table: FPT stock

Substitute the above data into the final result, we obtain

$$C^R_{d/o}(S_t, t; K, B, T) = 9.02944$$

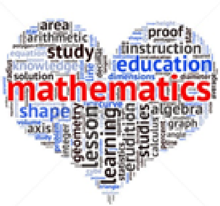
This means that a European barrier down-and-out call option with rebates under FPT stock has price of 9.03 VND.

Conclusion

- Mathematical techniques: Probabilistic approach
- Be essential to Vietnam market, especially Derivatives

Limitations

- Following Black-Scholes model's assupmtion


area
arithmetic
study
knowledge
solution
proof
instruction
education
mathematics
shape
axis
geometry
lesson
learning
erudition
studies
algebra
graph


Thank You!