Precision and Recall for Time Series

VJAI – Tho Phan – 06/2019

https://arxiv.org/pdf/1803.03639.pdf

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- Introduction
- Contribution
- Precision and recall for ranges
- Experimental study
- Discussion

Introduction

- Scope
  - Algorithm type: anomaly detection
  - Data: Time Series

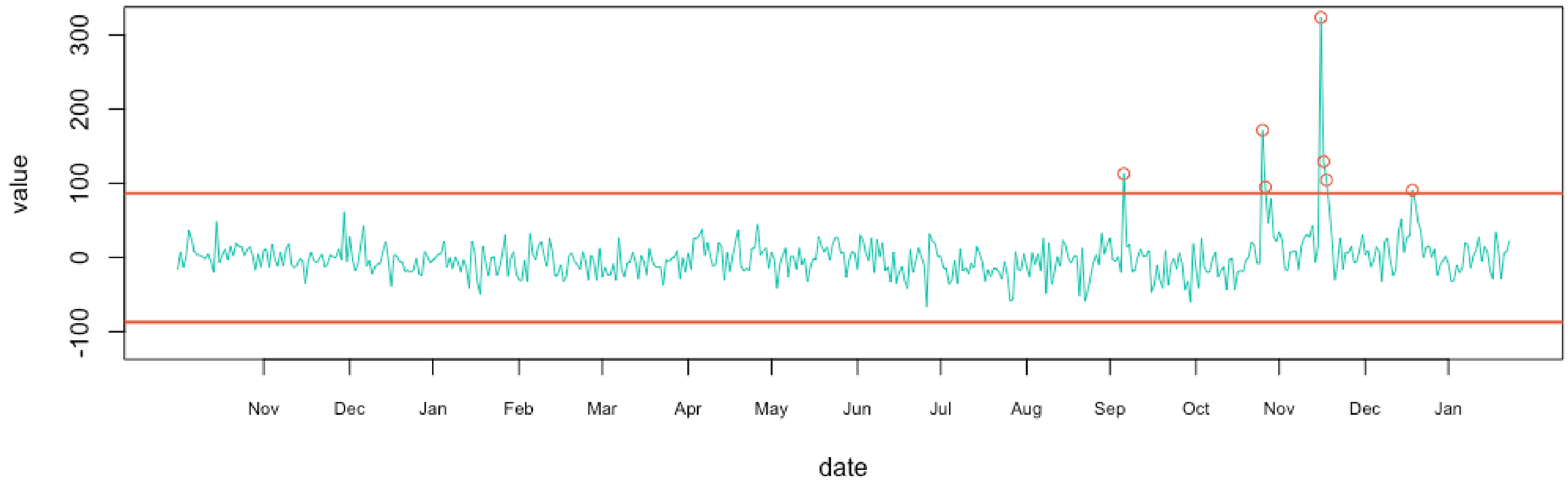


Figure 1: Anomaly detection in time series data

Introduction

- Problem
  - Precision
  - Recall

T/F ← Real condition == Predicted condition
P/N ← Predicted condition

		Real condition	
		True	False
Predicted condition	Positive	TP	FP
	Negative	FN	TN

Precision = $\frac{TP}{TP+FP}$

all predicted anomalies

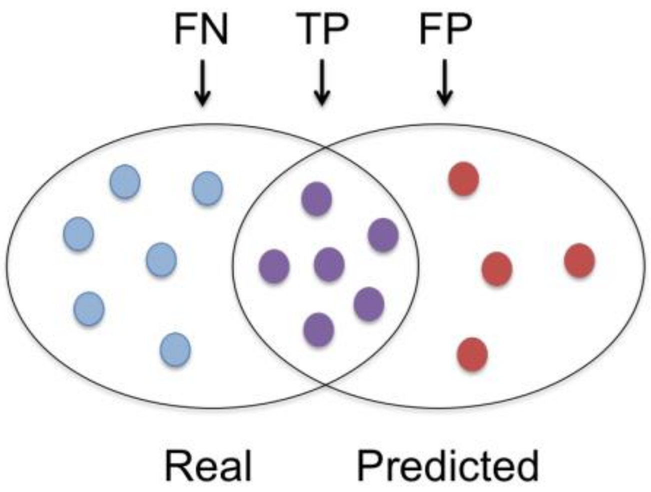
Recall = $\frac{TP}{TP+FN}$

all real anomalies

F_1-*Score* = H_mean(Precision, Recall)

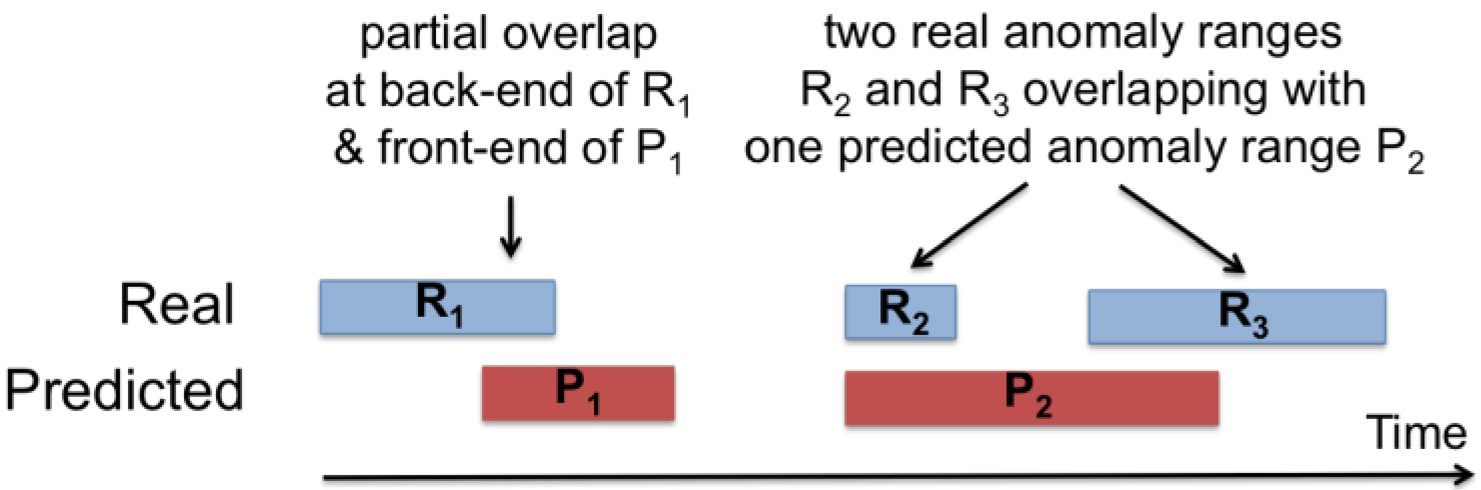
Introduction

- Problem
  - Precision
  - Recall



(a) Precision = 0.6, Recall = 0.5

Point-based data



Range-based data

Introduction

Problem

Point-based data	Range-based data
Independent points	Ordered collection of point
-	Position of the overlaps
-	Size of the partial overlaps

Design goals

Expressive

Flexible

Extensible

Contribution

Redefine *Precision* and *Recall* to encompass range-based anomalies by introducing a customizable mathematical model.

Note:

- It is not a new idea, it extends other classical counterparts (Numeta,…)
- It subsumes the classical one

Precision and recall for ranges

Notations

Notation	Description
R, R_i	R, P → real anomaly set, predicted set
P, P_j	Subscript i, j → the i^{th}, j^{th} element in set
N, N_r, N_p	N → number of points, r and p → R, P
α	Relative weight of existence reward
$\Upsilon()$, $\omega()$, $\delta()$	Just 3 functions of overlap cardinality, overlap size and positional bias

Precision and recall for ranges

Range-based recall

$$Recall_T(R, P) = \frac{\sum_{i=1}^{N_r} Recall_T(R_i, P)}{N_r}$$

$$Recall_T(R_i, P) = \alpha \times ExistenceReward(R_i, P) + (1 - \alpha) \times OverlapReward(R_i, P)$$

$$ExistenceReward(R_i, P) = \begin{cases} 1, & if \sum_{j=1}^{N_p} \left|R_i \cap P_j\right| \geq 1 \\ 0, & otherwise \end{cases}$$

Precision and recall for ranges

Range-based recall

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$$OverlapReward(R_i,P) = CardinalityFactor(R_i,P) \times \sum_{j=1}^{N_p} \omega(R_i, R_i \cap P_j, \delta)$$

$$CardinalityFactor(R_i,P) = \begin{cases} 1, if\, R_i\ overlaps\ with\ at\ most\ one P_j \ \in P \\ \gamma(R_i,P), \qquad otherwise \end{cases}$$

Precision and recall for ranges

Range-based recall

```
function δ(i, AnomalyLength)          ▷ Flat bias
    return 1
function δ(i, AnomalyLength)    ▷ Front-end bias
    return AnomalyLength - i + 1
function δ(i, AnomalyLength)     ▷ Back-end bias
    return i
function δ(i, AnomalyLength)        ▷ Middle bias
    if i ≤ AnomalyLength/2 then
        return i
    else
        return AnomalyLength - i + 1
```

(b) Positional bias

```
function ω(AnomalyRange, OverlapSet, δ)
    MyValue ← 0
    MaxValue ← 0
    AnomalyLength ← length(AnomalyRange)
    for i ← 1, AnomalyLength do
        Bias ← δ(i, AnomalyLength)
        MaxValue ← MaxValue + Bias
        if AnomalyRange[i] in OverlapSet then
            MyValue ← MyValue + Bias
    return MyValue/MaxValue
```

(a) Overlap size

$$Recall_T(R, P) = \frac{\sum_{i=1}^{N_r} Recall_T(R_i, P)}{N_r}$$

$$Recall_T(R_i, P) = \alpha \times ExistenceReward(R_i, P) + (1 - \alpha) \times OverlapReward(R_i, P)$$

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Precision and recall for ranges

Range-based precision

$$Recall_T(R,P) = \frac{\sum_{i=1}^{N_r} Recall_T(R_i,P)}{N_r}$$

$$Recall_T(R_i,P) = \alpha \times ExistenceReward(R_i,P) + (1-\alpha) \times OverlapReward(R_i,P)$$

$$ExistenceReward(R_i,P) = \begin{cases} 1, & if \sum_{j=1}^{N_p} |R_i \cap P_j| \geq 1 \\ 0, & otherwise \end{cases}$$

$$Precision_T(R,P) = \frac{\sum_{i=1}^{N_p} Precision_T(R,P_i)}{N_p}$$

$$Precision_T(R,P_i) = CardinalityFactor(P_i,R) \times \sum_{j=1}^{N_r} \omega(P_i, P_i \cap R_j, \delta)$$

$$CardinalityFactor(P_i,R) = \begin{cases} 1, if\, P_i\ overlaps\ with\ at\ most\ one\ R_j \in R \\ \gamma(P_i,R), \quad otherwise \end{cases}$$

Precision and recall for ranges

Note

$$0 \leq \alpha \leq 1$$
$$0 \leq \gamma() \leq 1$$

$\gamma() \rightarrow$ 1/f(x) i.e. 1/x (typical)

$$\alpha, \gamma(), \omega(), \delta() \ are\ tunable$$

$$Recall_T(R_i, P) = \alpha \times ExistenceReward(R_i, P) + (1 - \alpha) \times OverlapReward(R_i, P)$$

$$CardinalityFactor(R_i, P) = \begin{cases} 1, if\ R_i\ overlaps\ with\ at\ most\ one P_j \in P \\ \gamma(R_i, P), & otherwise \end{cases}$$

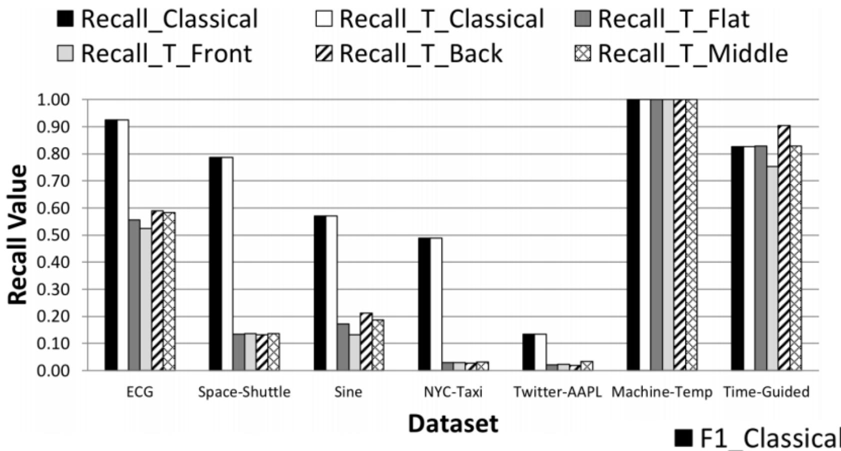
One more thing, if

$R_i \in R$ and $P_j \in P$ are unit-size range
α = 0, ϒ() = 1, δ() is flat, ω() is the same as above

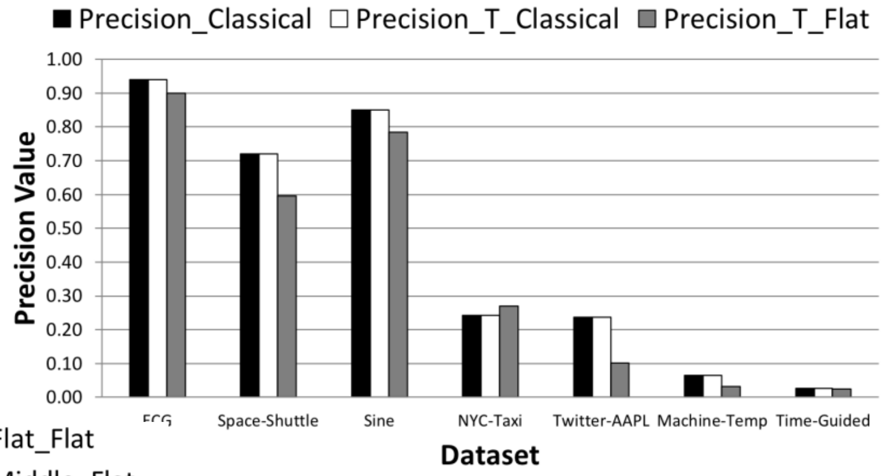
$$Recall_T \equiv Recall = \frac{TP}{TP + FN}$$
$$Precision_T \equiv Precision = \frac{TP}{TP + FP}$$

Experimental study

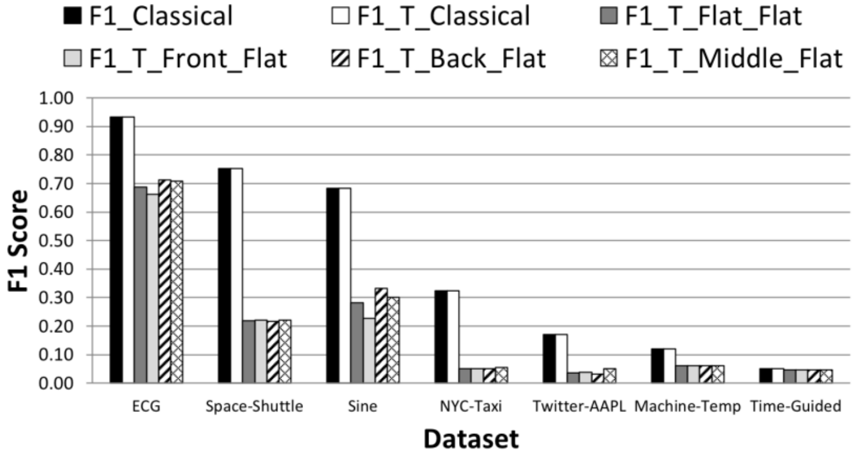
Comparison to the classical point-based model



(a) Recall



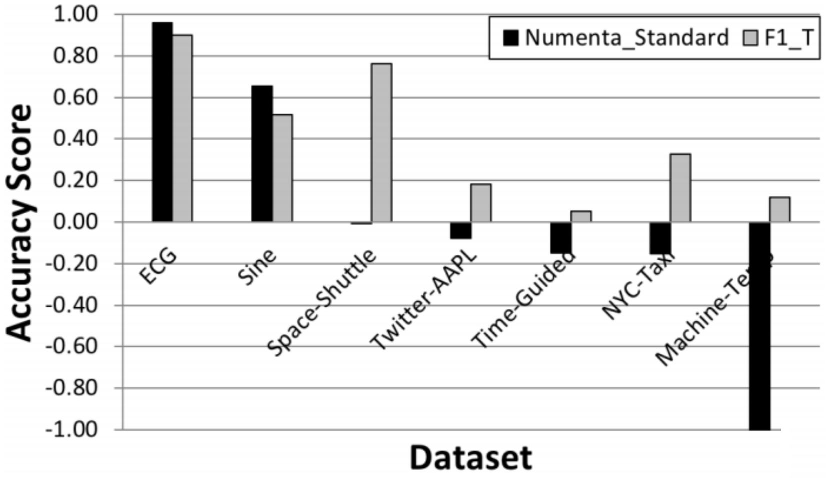
(b) Precision



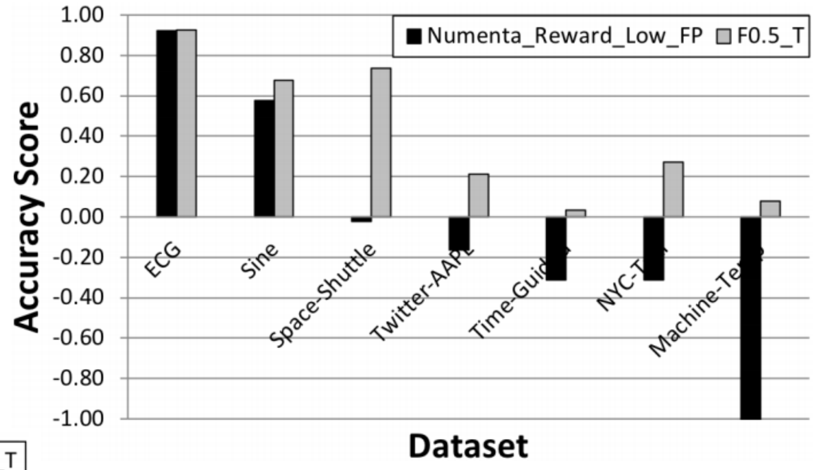
(c) F_1-Score

Experimental study

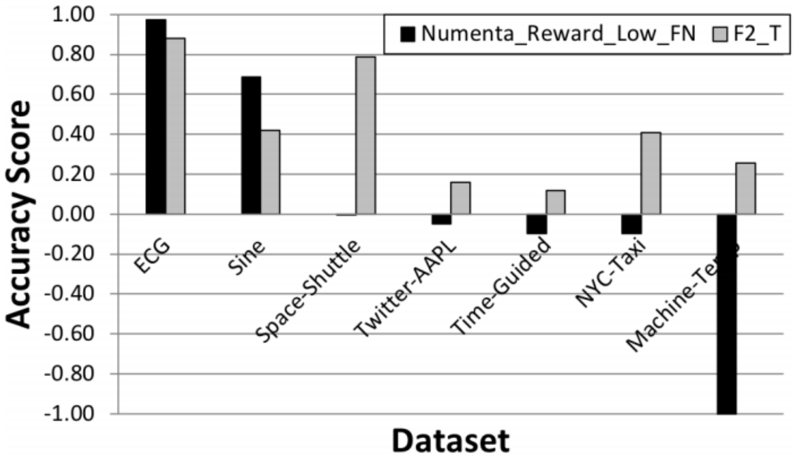
Comparison to the Numenta Anomaly Benchmark (NAB) scoring model



(a) Numenta Standard



(b) Numenta Reward-Low-FP



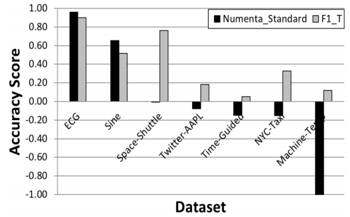
(c) Numenta Reward-Low-FN

Experimental study

Comparison to the Numenta Anomaly Benchmark (NAB) scoring model

Sensitivity to positional bias

	Numenta_standard	F1_T_Front_Flat	F1_T_Back_Flat
Front-Predicted	**0.67**	**0.42**	**0.11**
Back-Predicted	**0.63**	**0.11**	**0.42**



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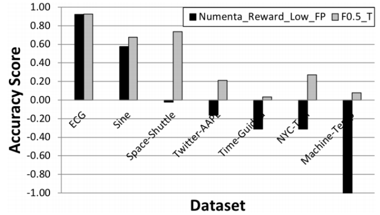
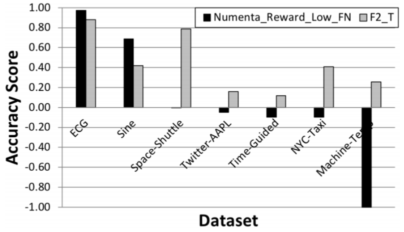


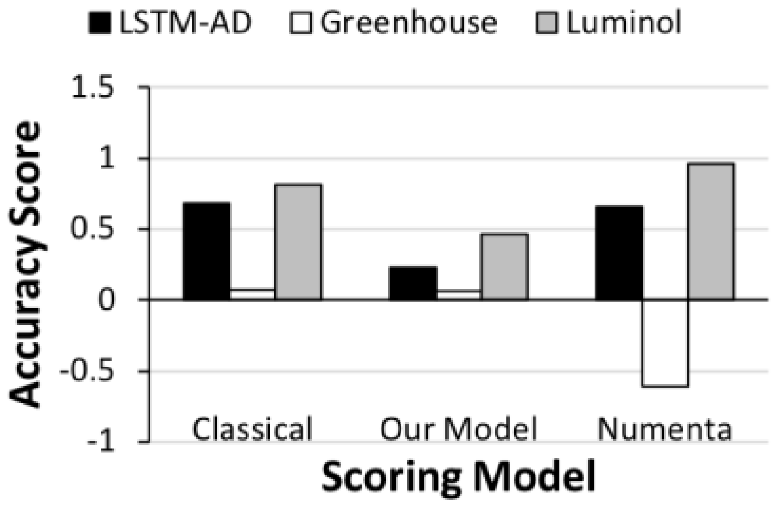
(b) Numenta Reward-Low-FP



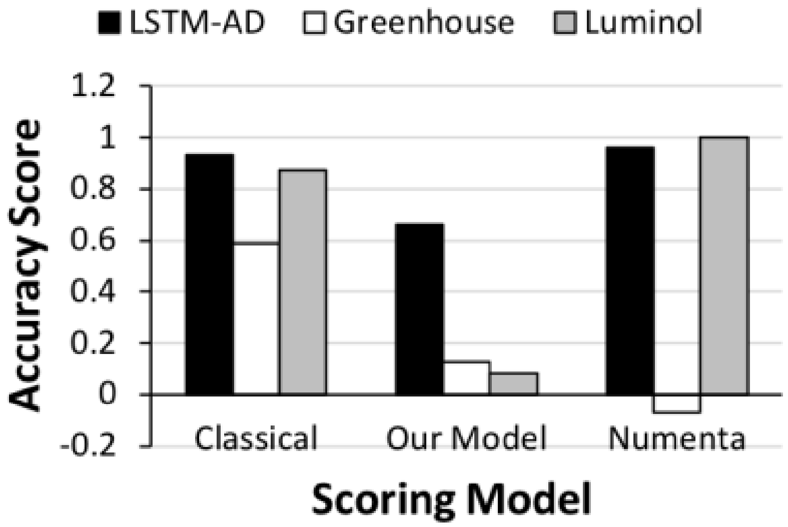
(c) Numenta Reward-Low-FN

Experimental study

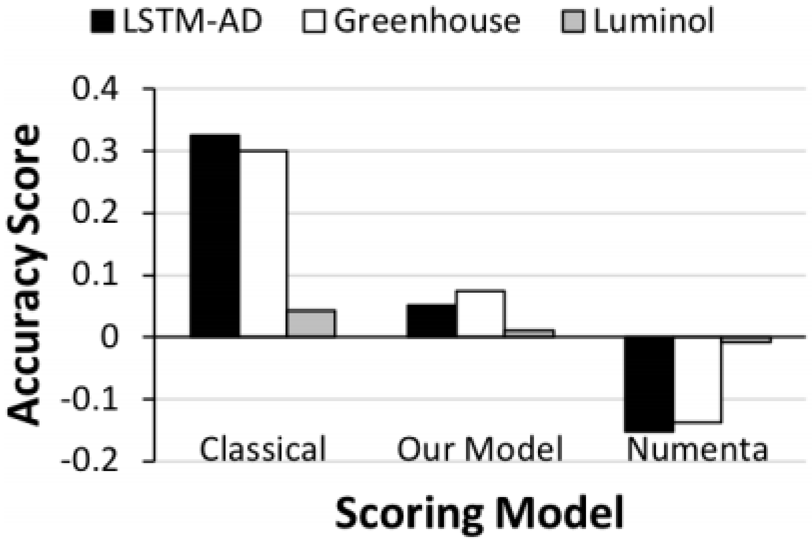
Evaluating multiple anomaly detectors



(a) Sine dataset



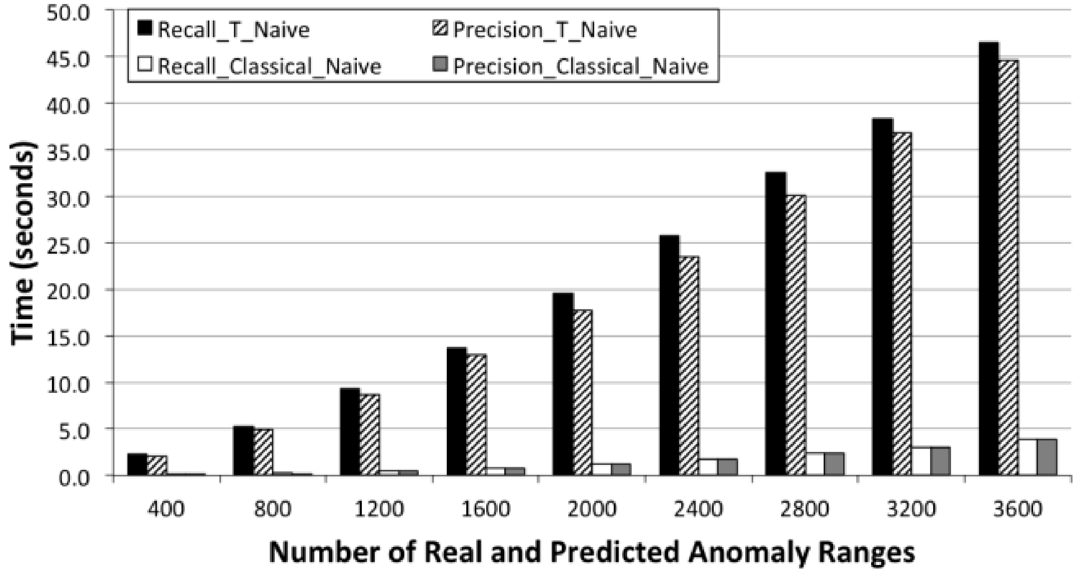
(b) ECG dataset



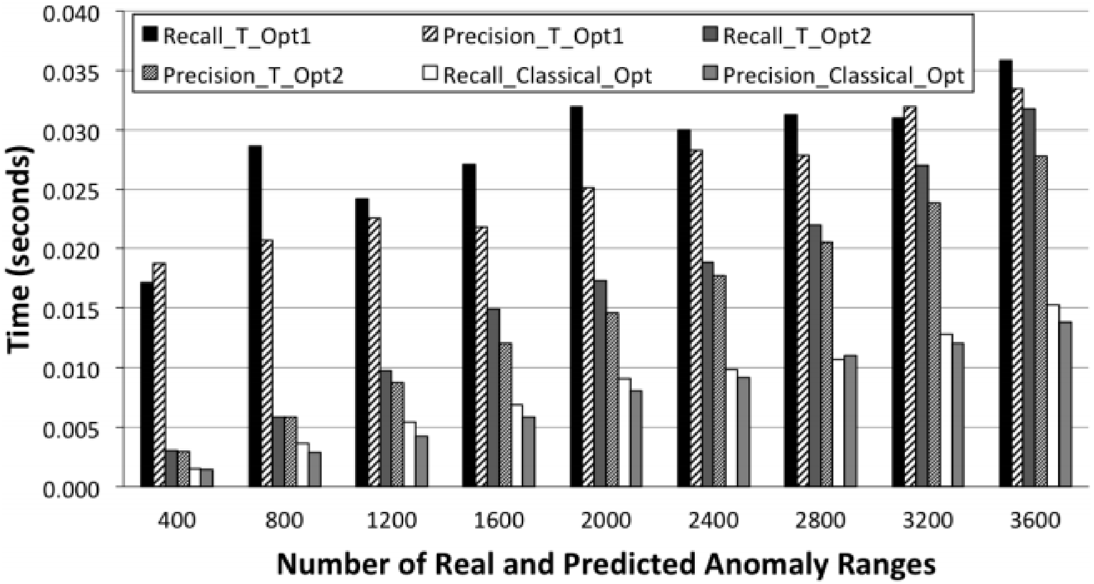
(c) NYC-Taxi dataset

Experimental study

Cost analysis



Naïve



Optimized

Discussion

Redefine *Precision* and *Recall* to encompass range-based anomalies by introducing a customizable mathematical model.

1. How do we extend above idea?
    1. other data types
    2. other domains
2. Is possible to improve the performance of cost analysis?
    1. What kind of optimizations do you want to add?
3. How do we apply above idea to our project?
4. Can we start doing research and writing more papers from above idea?
5. Can we combine with other ideas to make somethings new?

Thank you!