A High Resolution Algorithm Based on Chirp Z-Transform for FMCW Radar

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***Abstract*-Frequency modulation continuous wave (FMCW) radar has the merit of simple structure, small volume, high resolution in space and real-time character. So FMCW radar can detect and orientate the targets on the ground no mater how far they are. However, the measure precision is affected by the FFT method used in the FMCW radar system. In this paper, a new method for increasing the range and velocity measure precision by the Chirp Z-transform in the FMCW radar is proposed. Simulation results show that the proposed method can increase both the calculation efficiency and measure precision.**

I. INTRODUCTION

The theory of frequency modulation continuous wave (FMCW) radar is well known [1], [2]. A FMCW radar transmits series of millimeter waveforms through the antenna and receives the signal reflected by targets. The principle of FMCW radar is shown in Fig.1. The received and the transmitted waveforms have the same shape except for the delay, and the frequency difference between them is the mixer output. The beat signal is called the intermediate frequency (IF).

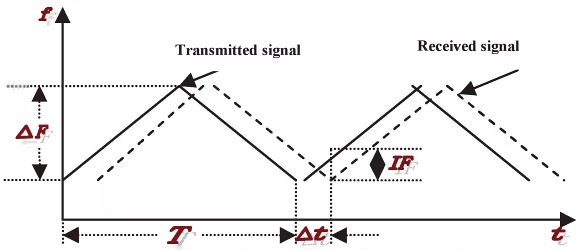

Fig. 1. The principle of FMCW radar

According to the Fig.1, we can know

$$\frac{\Delta t}{IF} = \frac{T/2}{\Delta F} \tag{1}$$

$$\Delta t = 2R/c \tag{2}$$

where Δt is the time delay, T is the frequency modulation periodicity, ΔF is the FM bandwidth and $c = 3\times10^8$ m/s.

And according to (1) and (2), we can get

$$R = \frac{cT}{4\Delta F} IF \tag{3}$$

If the target is moving, the Doppler shift can cause a range error, and the range and velocity can be written as

$$\begin{cases} v = \dfrac{c}{4 f_0}\left(f_{do} - f_{up}\right) \\ R = \dfrac{cT}{8\Delta F}\left(f_{do} + f_{up}\right) \end{cases} \tag{4}$$

where f_{do} is the down beat frequency and f_{up} is the up beat frequency. And we usually take the FFT algorithm to get the max point of the frequency spectrum, namely, we get the down beat frequency and up beat frequency using FFT.

II. THE PRINCIPLE OF CHIRP Z-TRANSFORM METHOD

In practical application, it is impossible to get the entire continuous signal. Digital signal processing techniques deal with the sampled signal, and then the receive signal can be written as the up time series $x_{up}(n)$ and the down time series $x_{do}(n)$. Range and velocity are calculated using the max point of frequency spectrum according to (4). But it is obvious that there must be difference between theoretical data and measured data if we take the FFT method. However, if we want to reduce this difference, we can increase the points of FFT. The increasing points of FFT can increase frequency resolution, however, there will increase the computational complexity. To solve this problem, a new method is proposed in this paper.

As we all known, a fast Fourier transform (FFT) is an efficient algorithm to compute the discrete Fourier transform (DFT). The discrete Fourier transform (DFT) is a specific kind of discrete transform, used in Fourier analysis. The DFT usually requires an input sequences is discrete, such inputs are often created by sampling a continuous sequences. The discrete input sequences must also be a finite set of uniformly spaced time-samples of some signal. And the transformed sequences also are a finite set of uniformly spaced samples of the continuous DTFT and it has the same length as the input sequences. This is to say, DFT compute z-transform at a number of points uniformly spaced on the unit circle in the z-plane. And the frequency resolution is $2\pi / N$. But the Chirp Z-transform can compute a more general transform based on the Z-transform, not have to uniformly or on the unit circle.

The Z-transform of $x(n)$ can be written as [3]-[6]

$$X(z_k) = \sum_{n=0}^{N-1} x(n) z^{-n} \quad ,0 \le n \le N-1 \tag{5}$$

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Let the sample point z_k of z-plane be

$$z_k = AW^{-k} \quad k = 0,1,\ldots,M-1 \tag{6}$$

where M is the point number of the desired spectrum, and

$$A = A_0 e^{j\theta_0}\ ,\ W = w_0 e^{-j\varphi_0} \tag{7}$$

Substituting (6) into (7) yields

$$z_k = AW^{-k} = A_0 e^{j\theta_0} w_0^{-k} e^{jk\varphi_0} \tag{8}$$

where A_0 and w_0 are any positive real data, θ_0 is the starting angle and it is arbitrary, φ_0 is the angle between z_k and z_{k-1}.

Substituting (8) into (5), the Chirp Z-transform of N-point time series $x(n)$ can be written as

$$X(z_k) = \sum_{n=0}^{N-1} x(n) A^{-n} W^{nk} \quad k = 0,1,\ldots,M-1 \tag{9}$$

Due to the Chirp Z-transform is more flexible than the FFT, thus it has additional freedoms. First, the number of time samples does not equal the number of samples of Z-transform. Second, the angular spacing φ_0 is also arbitrary.

Consequently, according to the principle of Chirp Z-transform (9), the proposed method can be summarized in the following steps. FFT method is taken to the beat signal to find the main lobe of the spectrum, and then Chirp Z-transform is used to find peak frequency in the adjacent area.

III. ANALYSIS OF SIMULATION RESULTS

This section uses numerical examples to demonstrate the superior performance of proposed Chirp Z-transform method over that of the FFT method.

A. *Example 1*

The beat signal, for this example, is chosen as $y = \sin(2\pi f_0 t) + \sin(2\pi f_1 t) + \sin(2\pi f_2 t)$, where f_0=999.2 Hz, f_1=1000 Hz, f_2=1001Hz. Assume that the sample frequency is 4000Hz and the simulation time is 5s. To increase the frequency resolution, the M-point narrowband signal in the frequency dominant can be concentrated to a certain frequency range.

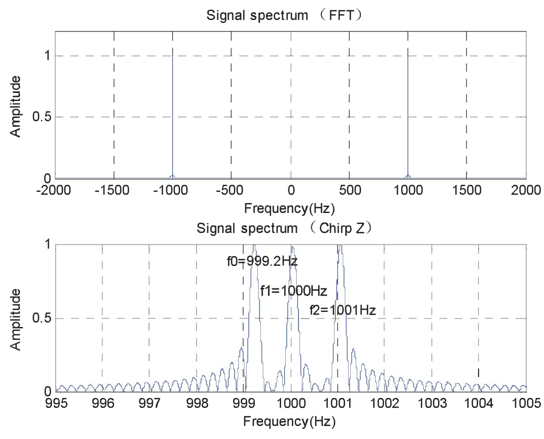

Fig.2. Signal spectrums of FFT method and Chirp Z method

Fig.2 shows that the signal spectrums using FFT method and Chirp Z method. It can be noticed from the Fig.2 that the targets can't distinguish one from another by FFT method. However, the calculated frequency can be easily distinguished by Chirp Z method, and all center frequency is also accurate.

B. *Example 2*

In this example, assume the perfect beat signal without noise and clutter. The frequency transmitted by FMCW radar is 35GHz. the FM bandwidth is 60MHz. the frequency modulation periodicity is 10ms, and the sample frequency is 0.5MHz and 0.1MHz. Assume the range and the velocity are 4500m, 200m/s, respectively. Therefore, the beat frequency is 0.23MHz. And the beat frequency is 4.4 kHz when the range and the velocity is 50m, 10m/s, respectively. The simulation results are shown as Table Ⅰ and Table Ⅱ. Note that the error between theoretical data and real measured data is fined by Chirp Z method.

Table Ⅰ The simulation results with the sample frequency 0.5MHz

Theoretical data		Real measured data							
		FFT				Chirp Z			
Range (m)	Velocity (m/s)	Range (m)	Error (m)	Velocity (m/s)	Error (m/s)	Range (m)	Error (m)	Velocity (m/s)	Error (m/s)
4125	158	4128.75	3.75	157.929	0.071	4126.34	1.34	157.944	0.056
3925	148	3928.75	3.7500	148.071	0.071	3926.25	1.25	148.071	0.071
3567	128.9	3570	3.000	129	0.1000	3567.83	0.8271	128.945	0.045
2589.75	35.8	2593.75	4.000	35.7857	0.0143	2591.25	1.5	35.7857	0.0143
2112	98.5	2115	3.00	98.5714	0.0714	2112.62	0.6172	98.5522	0.0522
1879.9	98.7	1882.5	2.6	98.5714	0.1286	1880.58	0.68	98.6694	0.0306
1098	56.2	1100	2.0000	56.1429	0.0571	1098.23	0.2251	56.2111	0.0111
1019.8	36.7	1022.5	2.700	36.8571	0.1571	1020.44	0.6443	36.7818	0.0818
876.54	23.4	878.75	2.2100	23.3571	0.0429	876.721	0.1812	23.4044	0.0044
612.09	12.63	613.75	1.6600	12.6429	0.0129	612.217	0.1268	12.6328	0.0028
475.9	200	478.75	2.85	199.929	0.071	476.453	0.553	199.962	0.074
189.7	10.98	192.5	2.8000	11.1429	0.1629	190.342	0.6418	11.0851	0.1051
109.7	6.98	112.5	2.8000	6.85714	0.1229	110.227	0.5271	6.89523	0.0848
35.67	3.78	37.5	1.8300	3.85714	0.0771	35.6812	0.0112	3.78055	0.00055
10.98	2.98	12.5	1.5200	3.0000	0.0200	10.9863	0.0063	2.98075	0.00075
5.89	2.5	8.75	2.86	2.35714	0.14286	5.90576	0.01576	2.50488	0.00488

Fig.3 to Fig.6 show the trends of errors varied with ranges and velocities. The measure precision by Chirp Z is about as three times as FFT method in the long range, and the measure precision by Chirp Z is about as much as ten times by FFT in the short range.

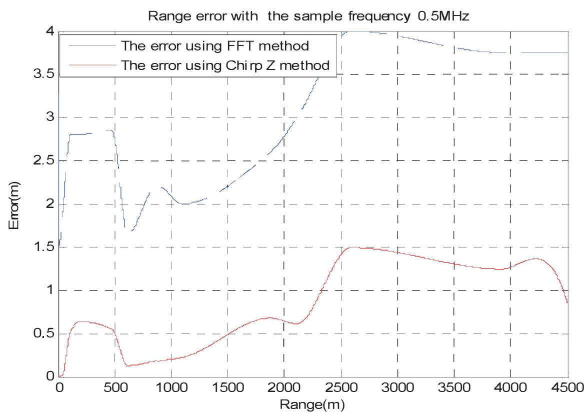

Fig.3. The trend of error varied with range

Table II The simulation results with the sample frequency 0.1MHz

Theoretical data		Real measured data							
		FFT				Chirp Z			
Range (m)	Velocity (m/s)	Range (m)	Error (m)	Velocity (m/s)	Error (m/s)	Range (m)	Error (m)	Velocity (m/s)	Error (m/s)
48.95	9.89	48.75	0.2000	10.0714	0.1814	49.0527	0.1027	9.90988	0.0199
40.56	8.34	41.25	0.6900	8.35714	0.0171	40.647	0.0870	8.35672	0.0167
34.34	7.12	33.75	0.5900	7.07143	0.0486	34.4141	0.0741	7.13421	0.0142
30.9	6.72	31.25	0.3500	6.64286	0.0771	30.9668	0.0668	6.73326	0.0133
23.67	5.67	23.75	0.0800	5.78571	0.1157	23.7231	0.0531	5.6815	0.0115
18.23	5.09	17.5	0.7300	5.14286	0.0529	18.2715	0.0415	5.10017	0.0102
10.21	4.44	10.00	0.2100	4.28571	0.1543	10.2344	0.0244	4.44894	0.0089
9.09	3.98	8.75	0.3400	4.07143	0.0914	9.11377	0.0238	3.98814	0.0081
7.99	3.57	8.75	0.7600	3.64286	0.0729	8.01025	0.0203	3.57715	0.0071
6.56	3.02	7.5	0.9400	3.00	0.0200	6.57715	0.0171	3.02595	0.0059
4.67	1.98	3.75	0.9200	1.92857	0.0514	4.68506	0.0151	1.98424	0.0042
3.06	1.06	3.75	0.6900	1.07143	0.0114	3.07129	0.0113	1.06222	0.0022
1.99	0.99	1.25	0.7400	1.07143	0.0814	1.99707	0.0071	0.991908	0.0019
1.05	0.56	1.25	0.2000	0.642857	0.0829	1.05713	0.0071	0.561244	0.0012
0.58	0.16	1.25	0.6700	0.214286	0.0543	0.585938	0.0059	0.160714	0.000714
0.19	0.5	0	0.1900	0.428571	0.0714	0.195313	0.0053	0.501395	0.0014

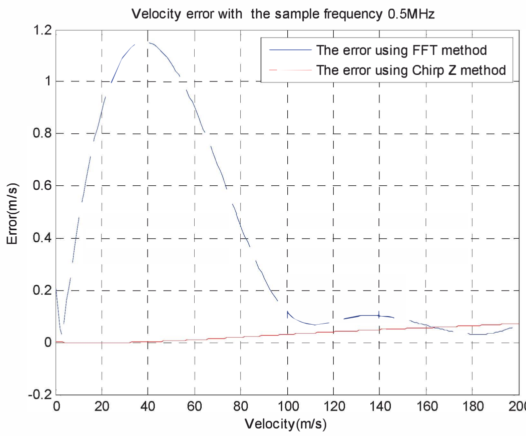

Fig.4. The trend of error varied with velocity

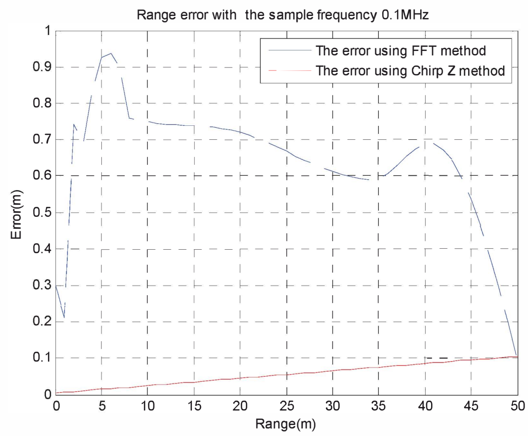

Fig.5. The trend of error varied with range

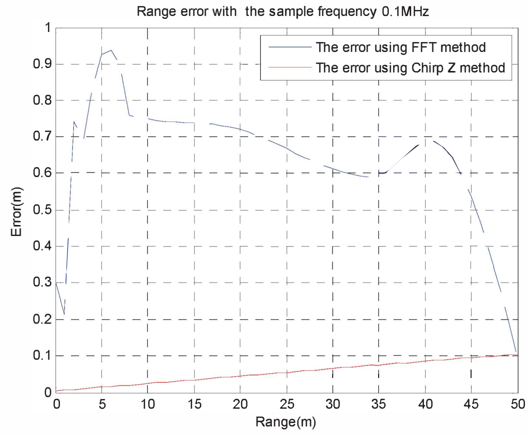

Fig.6. The trend of error varied with velocity

IV. CONCLUSION

A new method of increasing the range and velocity measure precision by the Chirp Z transform for the FMCW radar is proposed in this paper. Simulation results show that the proposed method can increase both the calculation efficiency and measure precision.

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