Autonomous and Mobile Robotics

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Motion Planning 3

Artificial Potential Fields

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on-line planning

- autonomous robots must be able to plan on line, i.e, using partial workspace information collected during the motion via the robot sensors
- incremental workspace information may be integrated in a map and used in a sense-plan-move paradigm (deliberative navigation)
- alternatively, incremental workspace information may be used to plan motions following a memoryless stimulus-response paradigm (reactive navigation)

artificial potential fields

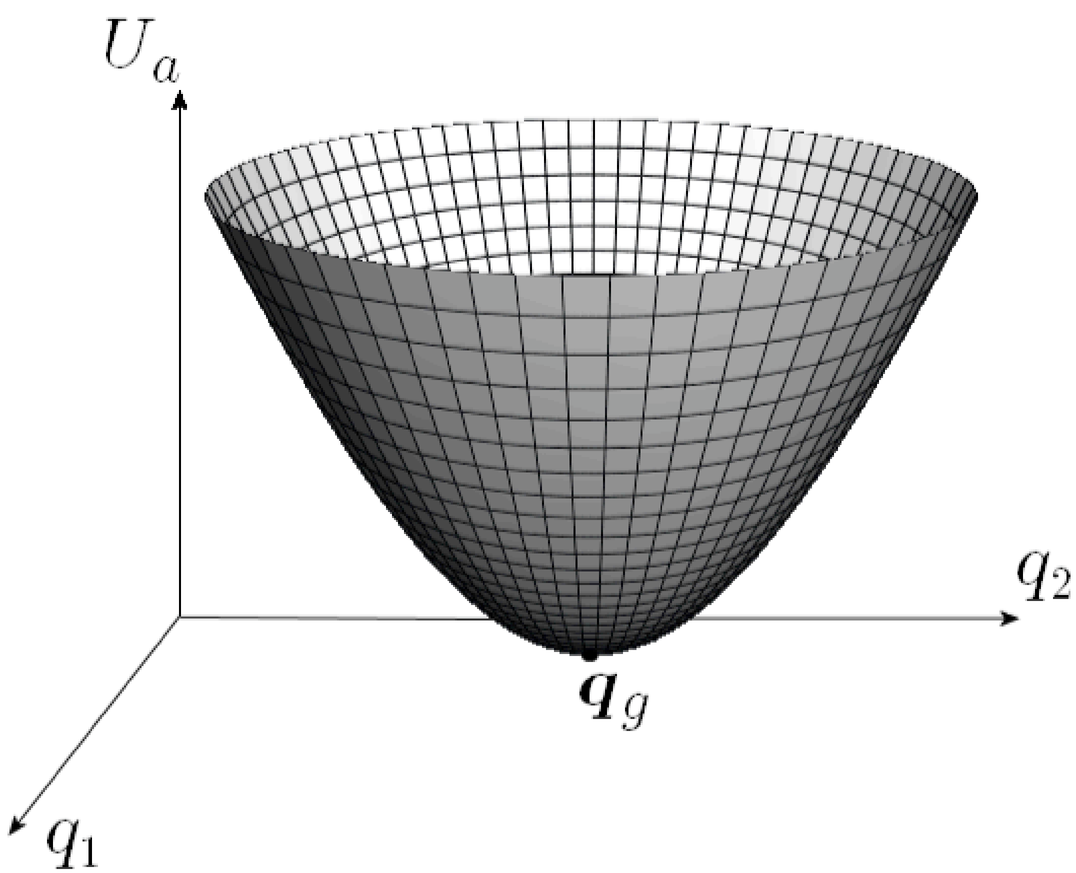
- idea: build potential fields in $\mathcal{C}$ so that the point that represents the robot is attracted by the goal $\boldsymbol{q}_g$ and repelled by the $\mathcal{C}$-obstacle region $\mathcal{CO}$

- the total potential U is the sum of an attractive and a repulsive potential, whose negative gradient $-\nabla U(\boldsymbol{q})$ indicates the most promising local direction of motion

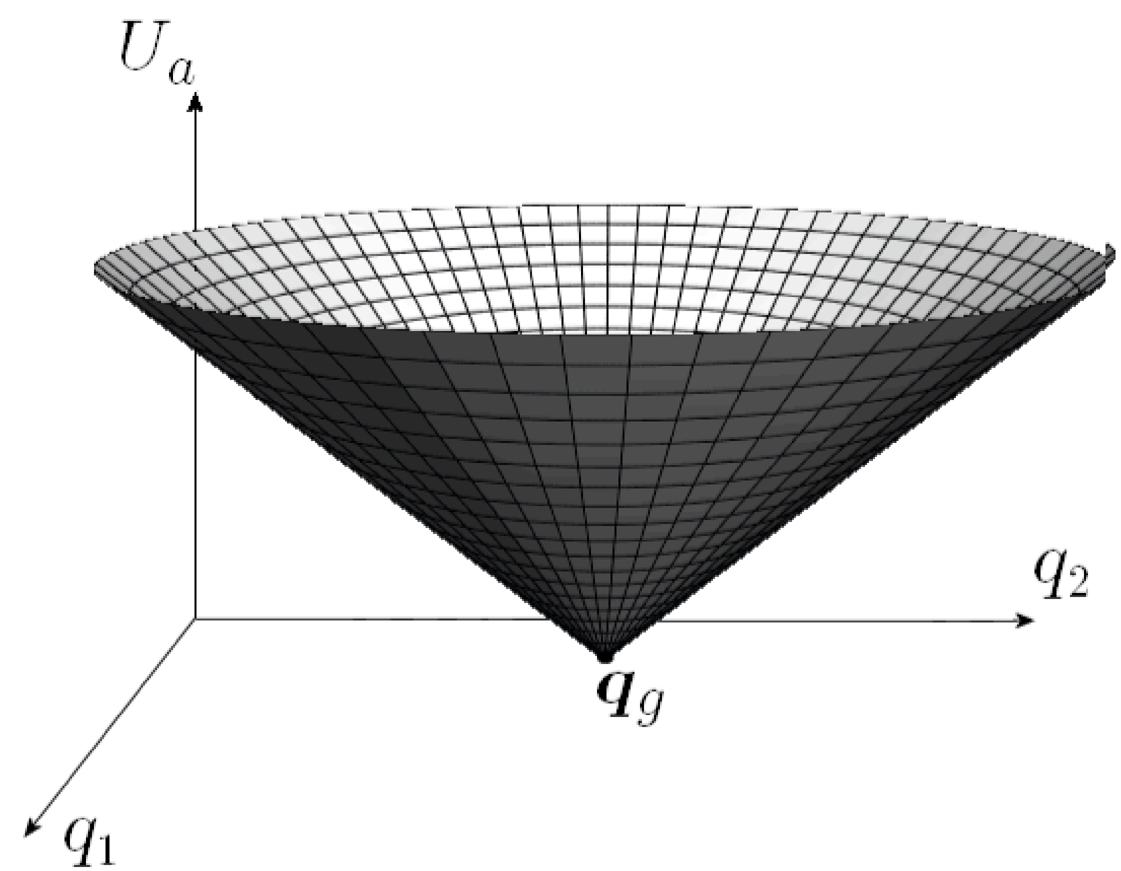
- the chosen metric in $\mathcal{C}$ plays a role

attractive potential

- objective: to guide the robot to the goal $\boldsymbol{q}_g$
- two possibilities; e.g., in $\mathcal{C} = \mathrm{R}^2$



paraboloidal



conical

- paraboloidal: let $\boldsymbol{e} = \boldsymbol{q}_g - \boldsymbol{q}$ and choose $k_a > 0$

$$U_{a1}(\boldsymbol{q}) = \frac{1}{2} k_a \boldsymbol{e}^T(\boldsymbol{q})\boldsymbol{e}(\boldsymbol{q}) = \frac{1}{2} k_a \|\boldsymbol{e}(\boldsymbol{q})\|^2$$

- the resulting attractive force is linear in $\boldsymbol{e}$

$$\boldsymbol{f}_{a1}(\boldsymbol{q}) = -\nabla U_{a1}(\boldsymbol{q}) = k_a \boldsymbol{e}(\boldsymbol{q})$$

- conical:

$$U_{a2}(\boldsymbol{q}) = k_a \|\boldsymbol{e}(\boldsymbol{q})\|$$

- the resulting attractive force is constant

$$\boldsymbol{f}_{a2}(\boldsymbol{q}) = -\nabla U_{a2}(\boldsymbol{q}) = k_a \frac{\boldsymbol{e}(\boldsymbol{q})}{\|\boldsymbol{e}(\boldsymbol{q})\|}$$

- $\boldsymbol{f}_{a1}$ behaves better than $\boldsymbol{f}_{a2}$ in the vicinity of $\boldsymbol{q}_g$ but increases indefinitely with $\boldsymbol{e}$

- a convenient solution is to combine the two profiles: conical away from $\boldsymbol{q}_g$ and paraboloidal close to $\boldsymbol{q}_g$

$$U_a(\boldsymbol{q}) = \begin{cases} \frac{1}{2} k_a \|\boldsymbol{e}(\boldsymbol{q})\|^2 & \text{if} \quad \|\boldsymbol{e}(\boldsymbol{q})\| \leq \rho \\ k_b \|\boldsymbol{e}(\boldsymbol{q})\| & \text{if} \quad \|\boldsymbol{e}(\boldsymbol{q})\| > \rho \end{cases}$$

continuity of $\boldsymbol{f}_a$ at the transition requires

$$k_a \boldsymbol{e}(\boldsymbol{q}) = k_b \frac{\boldsymbol{e}(\boldsymbol{q})}{\|\boldsymbol{e}(\boldsymbol{q})\|} \qquad \text{for} \quad \|\boldsymbol{e}(\boldsymbol{q})\| = \rho$$

i.e., $k_b = \rho k_a$

repulsive potential

- objective: keep the robot away from $\mathcal{CO}$
- assume that $\mathcal{CO}$ has been partitioned in advance in convex components $\mathcal{CO}_i$
- for each $\mathcal{CO}_i$ define a repulsive field

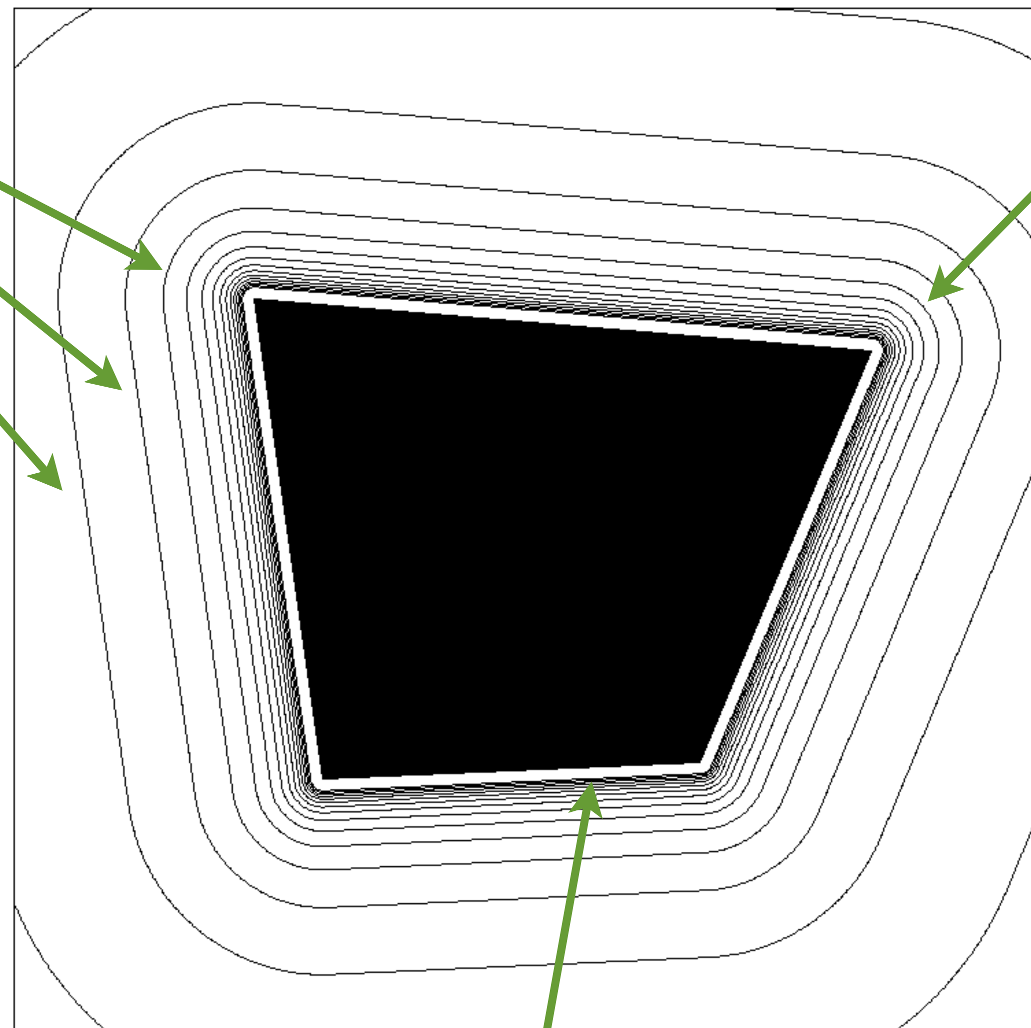
$$U_{r,i}(\boldsymbol{q}) = \begin{cases} \dfrac{k_{r,i}}{\gamma}\left(\dfrac{1}{\eta_i(\boldsymbol{q})} - \dfrac{1}{\eta_{0,i}}\right)^{\gamma} & \text{if } \eta_i(\boldsymbol{q}) \le \eta_{0,i} \\ 0 & \text{if } \eta_i(\boldsymbol{q}) > \eta_{0,i} \end{cases}$$

where $k_{r,i} > 0$; $\gamma = 2, 3, \ldots$; $\eta_{0,i}$ is the range of influence of $\mathcal{CO}_i$; and $\eta_i(\boldsymbol{q})$ is the clearance

$$\eta_i(\boldsymbol{q}) = \min_{\boldsymbol{q}' \in \mathcal{CO}_i} \|\boldsymbol{q} - \boldsymbol{q}'\|$$

equipotential contours

the higher γ, the steepest the slope



$U_{r,i}$ goes to ∞ at the boundary of $\mathcal{CO}_i$

- the resulting repulsive force is

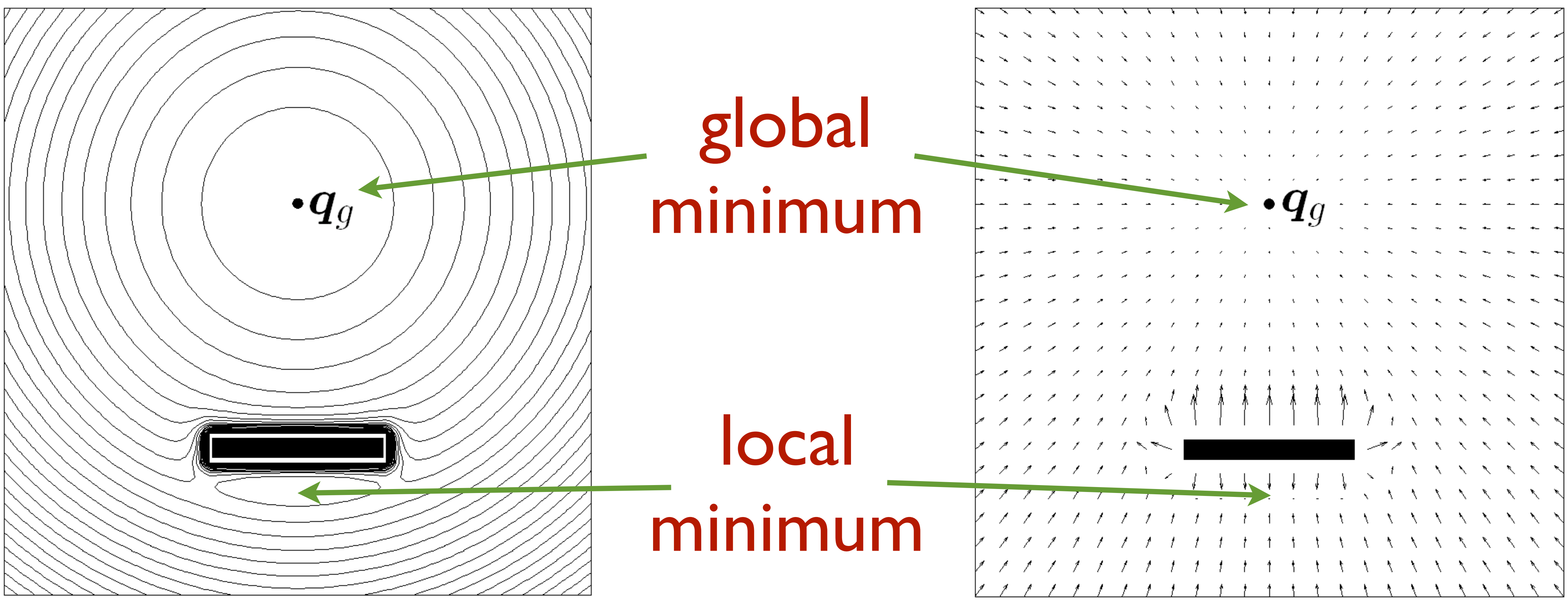
$$\boldsymbol{f}_{r,i}(\boldsymbol{q}) = -\nabla U_{r,i}(\boldsymbol{q}) = \begin{cases} \dfrac{k_{r,i}}{\eta_i^2(\boldsymbol{q})} \left(\dfrac{1}{\eta_i(\boldsymbol{q})} - \dfrac{1}{\eta_{0,i}} \right)^{\gamma-1} \nabla \eta_i(\boldsymbol{q}) & \text{if } \eta_i(\boldsymbol{q}) \leq \eta_{0,i} \\ 0 & \text{if } \eta_i(\boldsymbol{q}) > \eta_{0,i} \end{cases}$$

- $\boldsymbol{f}_{r,i}$ is orthogonal to the equipotential contour passing through $\boldsymbol{q}$ and points away from the obstacle
- $\boldsymbol{f}_{r,i}$ is continuous everywhere thanks to the convex decomposition of $\mathcal{CO}$
- aggregate repulsive potential of $\mathcal{CO}$

$$U_r(\boldsymbol{q}) = \sum_{i=1}^{p} U_{r,i}(\boldsymbol{q})$$

total potential

- superposition: $U_t(\boldsymbol{q}) = U_a(\boldsymbol{q}) + U_r(\boldsymbol{q})$
- force field: $\boldsymbol{f}_t(\boldsymbol{q}) = -\nabla U_t(\boldsymbol{q}) = \boldsymbol{f}_a(\boldsymbol{q}) + \sum_{i=1}^{p} \boldsymbol{f}_{r,i}(\boldsymbol{q})$



planning techniques

- three techniques for planning on the basis of f_t

1. consider f_t as generalized forces: $\boldsymbol{\tau} = \boldsymbol{f}_t(\boldsymbol{q})$

   the effect on the robot is filtered by its dynamics (generalized accelerations are scaled)

2. consider f_t as generalized accelerations: $\ddot{\boldsymbol{q}} = \boldsymbol{f}_t(\boldsymbol{q})$

   the effect on the robot is independent on its dynamics (generalized forces are scaled)

3. consider f_t as generalized velocities: $\dot{\boldsymbol{q}} = \boldsymbol{f}_t(\boldsymbol{q})$

   the effect on the robot is independent on its dynamics (generalized forces are scaled)

- technique 1 generates smoother movements, while technique 3 is quicker (irrespective of robot dynamics) to realize motion corrections; technique 2 gives intermediate results

- strictly speaking, only technique 3 guarantees (in the absence of local minima) asymptotic stability of $\boldsymbol{q}_g$; velocity damping is necessary to achieve the same with techniques 1 and 2

- off-line planning

  paths in $\mathcal{C}$ are generated by numerical integration of the dynamic model (if technique 1), of $\ddot{q} = \boldsymbol{f}_t(\boldsymbol{q})$ (if technique 2), of $\dot{q} = \boldsymbol{f}_t(\boldsymbol{q})$ (if technique 3)

  the most popular choice is 3 and in particular

$$\boldsymbol{q}_{k+1} = \boldsymbol{q}_k + T\boldsymbol{f}_t(\boldsymbol{q}_k)$$

  i.e., the algorithm of steepest descent

- on-line planning (is actually feedback!)

  technique 1 directly provides control inputs, technique 2 too (via inverse dynamics), technique 3 provides reference velocities for low-level control loops

  the most popular choice is 3

local minima: a complication

- if a planned path enters the basin of attraction of a local minimum $\boldsymbol{q}_m$ of U_t, it will reach $\boldsymbol{q}_m$ and stop there, because $\boldsymbol{f}_t\left(\boldsymbol{q}_m\right) = -\nabla U_t(\boldsymbol{q}_m) = 0$; whereas saddle points are not an issue
- repulsive fields generally create local minima, hence motion planning based on artificial potential fields is not complete (the path may not reach $\boldsymbol{q}_g$ even if a solution exists)
- workarounds exist but keep in mind that artificial potential fields are mainly used for on-line motion planning, where completeness may not be required

workaround no. 1: best-first algorithm

- build a discretized representation (by defect) of $\mathcal{C}_{\text{free}}$ using a regular grid, and associate to each free cell of the grid the value of U_t at its centroid

- build a tree T rooted at $\boldsymbol{q}_s$: at each iteration, select the leaf of T with the minimum value of U_t and add as children its adjacent free cells that are not in T

- planning stops when $\boldsymbol{q}_g$ is reached (success) or no further cells can be added to T (failure)

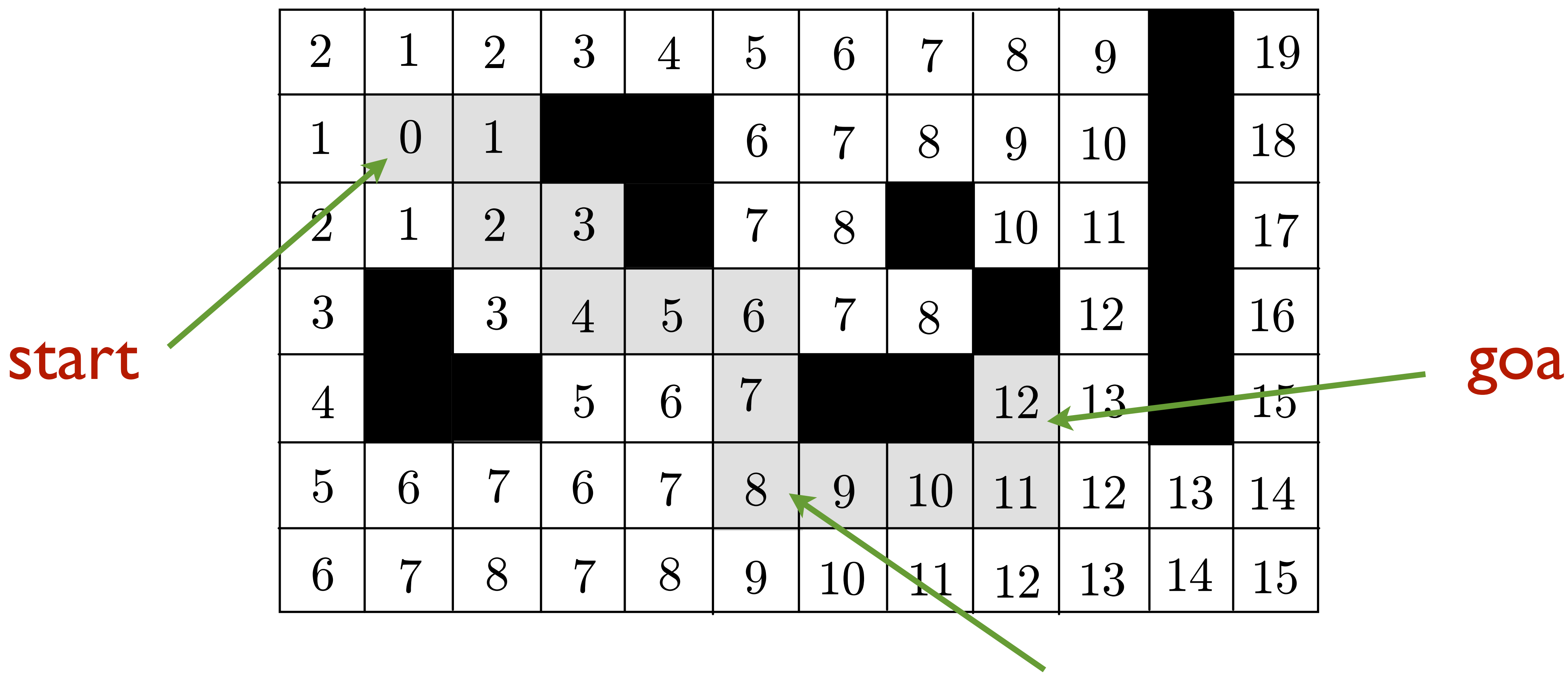
- in case of success, build a solution path by tracing back the arcs from $\boldsymbol{q}_g$ to $\boldsymbol{q}_s$

- best-first evolves as a grid-discretized version of steepest descent until a local minimum is met
- at a local minimum, best-first will "fill" its basin of attraction until it finds a way out
- the best-first algorithm is resolution complete
- its complexity is exponential in the dimension of $\mathcal{C}$, hence it is only applicable in low-dimensional spaces
- efficiency improves if random walks are alternated with basin-filling iterations (randomized best-first)

workaround no. 2: navigation functions

- paths generated by the best-first algorithm are not efficient (local minima are not avoided)
- a different approach: build navigation functions, i.e., potentials without local minima
- if the $\mathcal{C}$-obstacles are star-shaped, one can map $\mathcal{CO}$ to a collection of spheres via a diffeomorphism, build a potential in transformed space and map it back to $\mathcal{C}$
- another possibility is to define the potential as an harmonic function (solution of Laplace's equation)
- all these techniques require complete knowledge of the environment: only suitable for off-line planning

- easy to build: numerical navigation function
- with $\mathcal{C}_{\text{free}}$ represented as a gridmap, assign 0 to start cell, 1 to cells adjacent to the 0-cell, 2 to unvisited cells adjacent to 1-cells, ... (wavefront expansion)



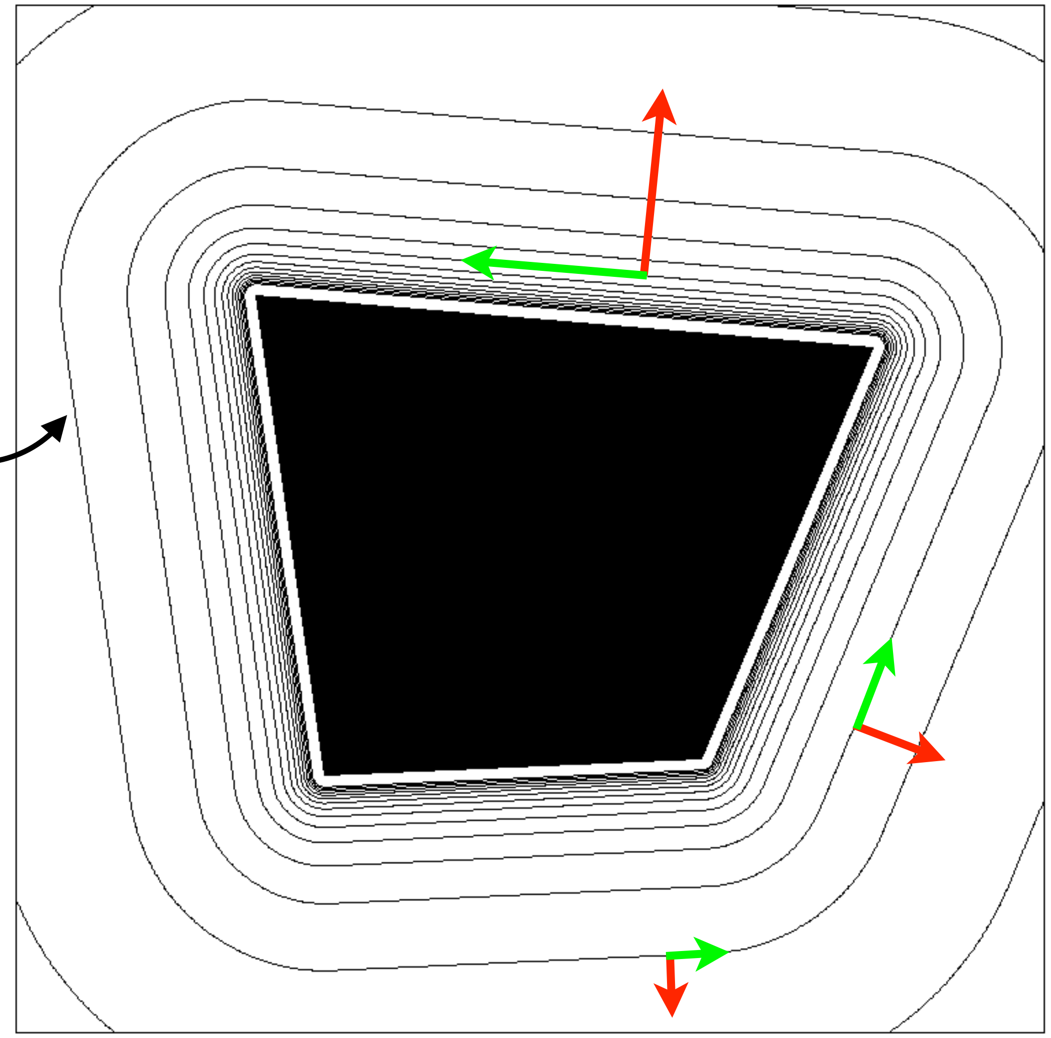
solution path:
steepest descent from the goal

workaround no. 3: vortex fields

- an alternative to navigation functions in which one directly assigns a force field (rather than a potential)
- the idea is to replace the repulsive action (which is responsible for appearance of local minima) with an action forcing the robot to go around the $\mathcal{C}$-obstacle
- e.g., assume $\mathcal{C} = \mathrm{R}^2$ and define the vortex field for $\mathcal{CO}_i$ as

$$\boldsymbol{f}_v = \pm \begin{pmatrix} \dfrac{\partial U_{r,i}}{\partial y} \\ -\dfrac{\partial U_{r,i}}{\partial x} \end{pmatrix}$$

i.e., a vector which is tangent (rather than normal) to the equipotential contours equipotential contours



f_r: repulsive

vs.

f_v: vortex

- the intensity of the two fields is the same, only the direction changes
- if $\mathcal{CO}_i$ is convex, the vortex sense (CW or CCW) can be always chosen in such a way that the total field (attractive+vortex) has no local minima

- in particular, the vortex sense (CW or CCW) should be chosen depending on the entrance point of the robot in the area of influence of the $\mathcal{C}$-obstacle

- vortex relaxation must performed so as to avoid indefinite orbiting around the obstacle

- both these procedures can be easily performed at runtime based on local sensor measurements

- complete knowledge of the environment is not required: also suitable for on-line planning

artificial potentials for wheeled robots

- since WMRs are typically described by kinematic models, artificial potential fields for these robots are used at the velocity level

- however, robots subject to nonholonomic constraints violate the free-flying assumption

- as a consequence, the artificial force f_t cannot be directly imposed as a generalized velocity $\dot{q}$

- a possible approach: use f_t to generate a feasible $\dot{q}$ via pseudoinversion

- the kinematic model of a WMR is expressed as

$$\dot{q} = \boldsymbol{G}(\boldsymbol{q})\boldsymbol{u}$$

- since $\boldsymbol{G}$ is $n\times m$, with $n>m$, it is in general impossible to compute $\boldsymbol{u}$ so as to realize exactly a desired $\dot{\boldsymbol{q}}_{\mathrm{des}}$

- however, a least-squares solution can be used

$$\boldsymbol{u} = \boldsymbol{G}^{\dagger}(\boldsymbol{q})\dot{\boldsymbol{q}}_{\mathrm{des}} = \boldsymbol{G}^{\dagger}(\boldsymbol{q})\boldsymbol{f}_t$$

where

$$\boldsymbol{G}^{\dagger}(\boldsymbol{q}) = (\boldsymbol{G}^T(\boldsymbol{q})\boldsymbol{G}(\boldsymbol{q}))^{-1}\boldsymbol{G}^T(\boldsymbol{q})$$

- as an application, consider the case of a unicycle robot moving in a planar workspace; we have

$$G(q) = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \;\Rightarrow\; G^{\dagger}(q) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

the least-squares solution corresponding to an artificial force $\boldsymbol{f}_t = (f_{t,x} \;\; f_{t,y} \;\; f_{t,\theta})^T$ is then

$$\begin{aligned} v &= f_{t,x}\cos\theta + f_{t,y}\sin\theta \\ \omega &= f_{t,\theta} \end{aligned}$$

v may be interpreted as the orthogonal projection of the cartesian force $(f_{t,x} \;\; f_{t,y})^T$ on the sagittal axis

- assume that the unicycle robot has a circular shape, so that its orientation is irrelevant for collision

- one may build artificial potentials in a reduced $\mathcal{C}' = \mathrm{R}^2$ with $\mathcal{C}'$-obstacles simply obtained by growing the workspace obstacles by the robot radius

- in $\mathcal{C}'$, the attractive field pulls the robot towards (x_g, y_g) while repulsive fields push it away from $\mathcal{C}'$-obstacle boundaries (segments and arcs of circle)

- since $\mathcal{C}'$ does not contain the orientation, the total field will not include a component $f_{t,\theta}$

- this degree of freedom can be exploited by letting

$$\omega = f_{t,\theta} = k_\theta \left(\text{atan2}(f_{t,y}, f_{t,x}) - \theta\right)$$

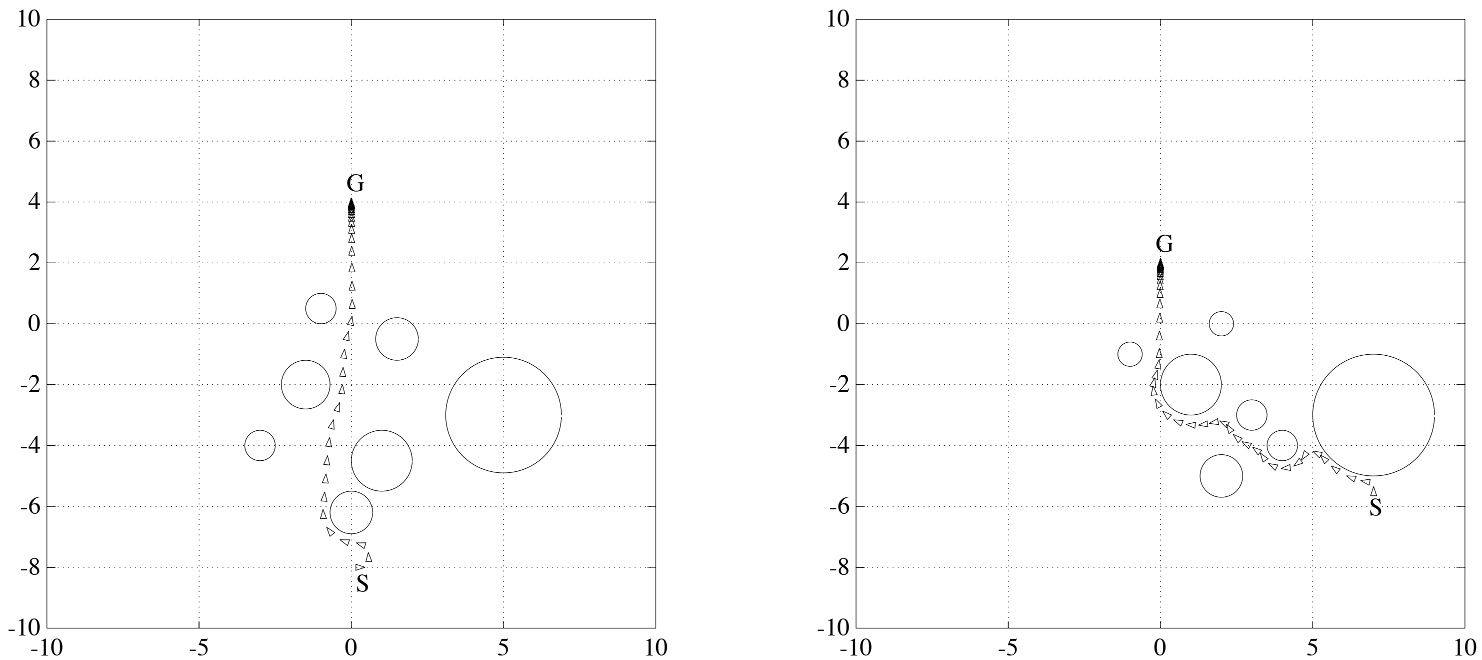
whose rationale is to force the unicycle to align with the total field, so that $\boldsymbol{f}_t$ can be better reproduced

- overall, a feedback control scheme is obtained where v and ω are computed in real time from $\boldsymbol{f}_t$
- assume w.l.o.g. $(x_g, y_g) = (0,0)$; close to the goal, where $\boldsymbol{f}_t = \boldsymbol{f}_a$, the controls become

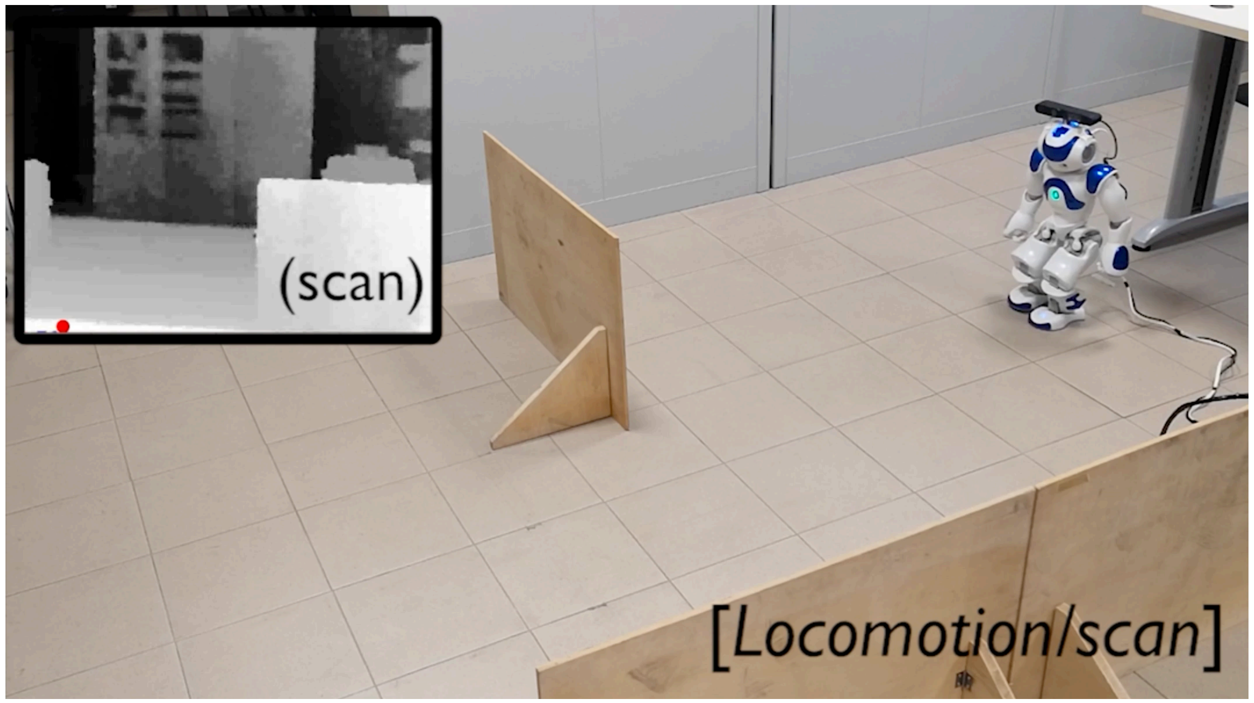
$$v = -k_a (x \cos\theta + y \sin\theta)$$
$$\omega = k_\theta \left(\text{Atan2}(-y, -x) - \theta\right)$$

i.e., a cartesian regulator! (see slides Wheeled Mobile Robots 5)

- results on unicycle (using vortex fields)



- can be applied to robots moving unicycle-like



motion planning for robot manipulators

- complexity of motion planning is high, because the configuration space has dimension typically ≥ 4
- try to reduce dimensionality: e.g., in 6-dof robots, replace the wrist with the total volume it can sweep (a conservative approximation)
- both the construction and the shape of $\mathcal{CO}$ are complicated by the presence of revolute joints
- off-line planning: probabilistic methods are the best choice (although collision checking is heavy)
- on-line planning: adaptation of artificial potential fields

artificial potentials for robot manipulators

- to avoid the computation of $\mathcal{CO}$ and the "curse of dimensionality", the potential is built in $\mathcal{W}$ (rather than in $\mathcal{C}$) and acts on a set of control points $\boldsymbol{p}_1,\ldots,\boldsymbol{p}_P$ distributed on the robot body

- in general, control points include one point per link ($\boldsymbol{p}_1,\ldots,\boldsymbol{p}_{P-1}$) and the end-effector (to which the goal is typically assigned) as $\boldsymbol{p}_P$

- the attractive potential U_a acts on $\boldsymbol{p}_P$ only, while the repulsive potential U_r acts on the whole set $\boldsymbol{p}_1,\ldots,\boldsymbol{p}_P$; hence, $\boldsymbol{p}_P$ is subject to the total $U_t = U_a + U_r$

- two techniques for planning with control points:

1. impose to the robot joints the generalized forces resulting from the combined action of force fields

$$\boldsymbol{\tau} = -\sum_{i=1}^{P-1} \boldsymbol{J}_i^T(\boldsymbol{q})\nabla U_r(\boldsymbol{p}_i) - \boldsymbol{J}_P^T(\boldsymbol{q})\nabla U_t(\boldsymbol{p}_P)$$

   where $\boldsymbol{J}_i(\boldsymbol{q}), i = 1,\dots,P,$ is the Jacobian matrix of the direct kinematics function associated to $\boldsymbol{p}_i(\boldsymbol{q})$

2. use the above expression as reference velocities to be fed to the low-level control loops

$$\dot{\boldsymbol{q}} = -\sum_{i=1}^{P-1} \boldsymbol{J}_i^T(\boldsymbol{q})\nabla U_r(\boldsymbol{p}_i) - \boldsymbol{J}_P^T(\boldsymbol{q})\nabla U_t(\boldsymbol{p}_P)$$

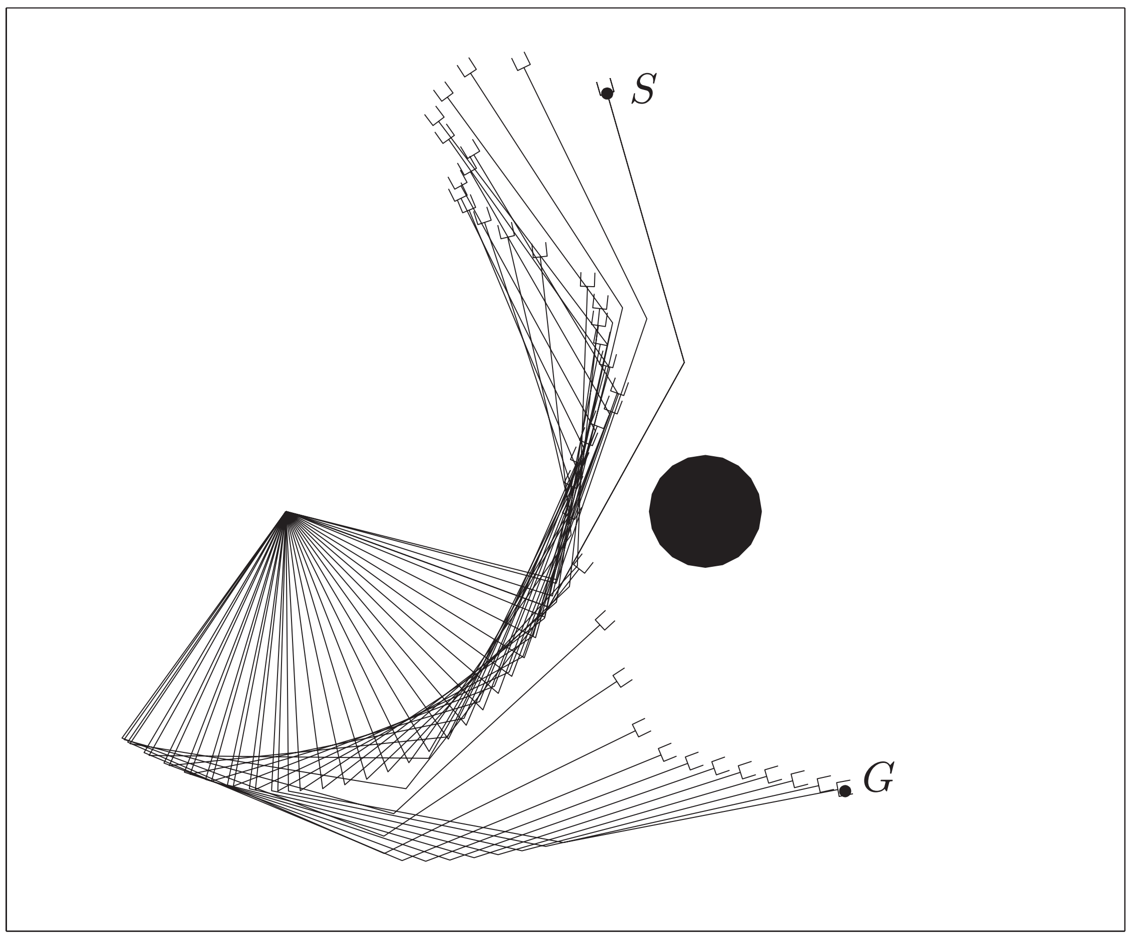
- technique 2 is actually a gradient-based minimization step in $\mathcal{C}$ of a combined potential in $\mathcal{W}$; in fact

$$\nabla_{\boldsymbol{q}} U(\boldsymbol{p}_i) = \left(\frac{\partial U(\boldsymbol{p}_i(\boldsymbol{q}))}{\partial \boldsymbol{q}}\right)^T = \left(\frac{\partial U(\boldsymbol{p}_i)}{\partial \boldsymbol{p}_i}\frac{\partial \boldsymbol{p}_i}{\partial \boldsymbol{q}}\right)^T = \boldsymbol{J}_i^T(\boldsymbol{q})\nabla U(\boldsymbol{p}_i)$$

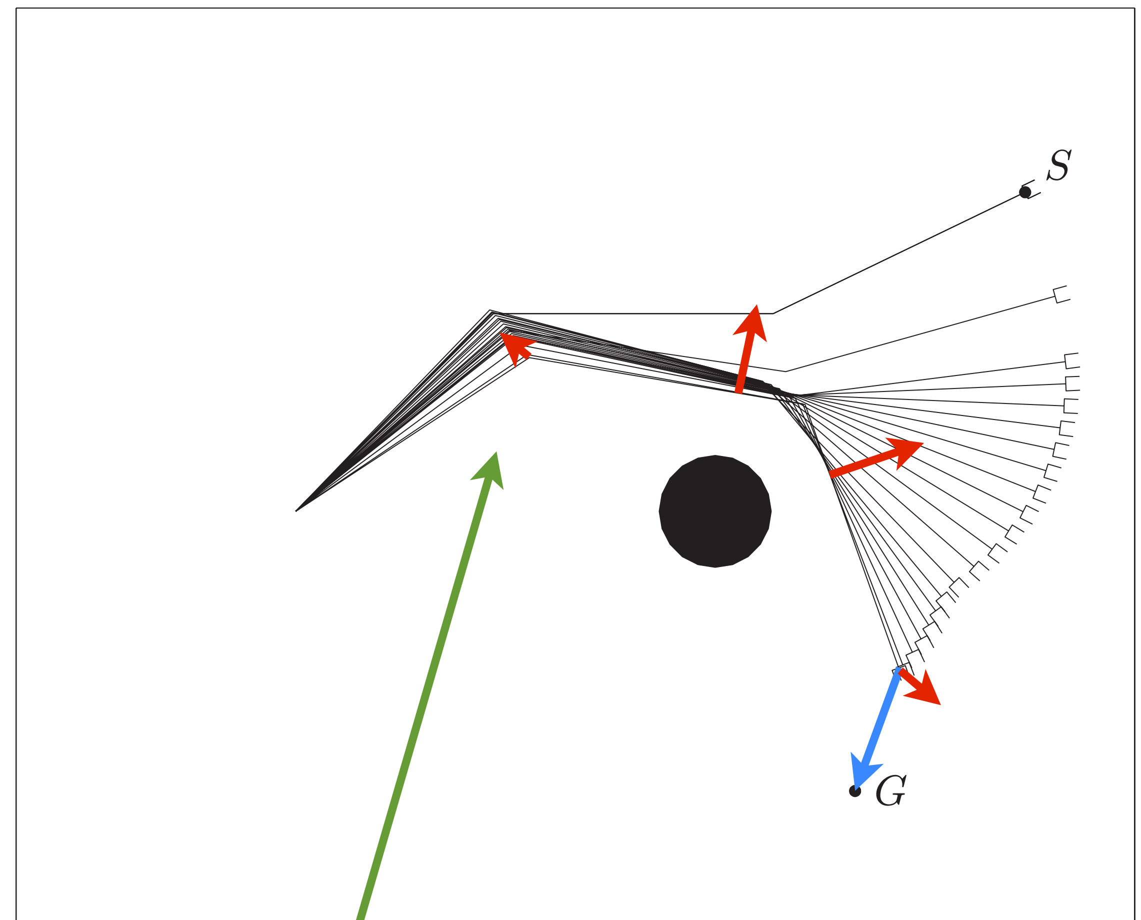
- technique 1 generates smoother movements, while technique 2 is quicker (irrespective of robot dynamics) to realize motion corrections

- both can stop at force equilibria, where the various forces balance each other even if the total potential U_t is not at a local minimum; hence, this method should be used in conjunction with a best-first algorithm

success
(with vortex field
and folding heuristic for sense)



failure
(with repulsive field)



a force equilibrium
between attractive and repulsive forces