Staring Contest

	1	2	3	4	5
a	32	17	51	52	21

Task: Determine $a = (a_1, \ldots, a_n)$ (distinct integers), from queries of the form $q(i, j) = \min(a_i, a_j)$ with $i \neq j$. Minimize #queries.

Note: max a can never be determined, so one underestimate is allowed.

Note: Problem is non-adaptive (a fixed from beginning).

First solution: Perform all $\binom{n}{2}$ many comparisons:

for $i \in \{1, \ldots, n-1\}$
  for $j \in \{i+1, \ldots, n\}$
    $Q_{ij} = Q_{ji} = q(i, j)$

Q	1	2	3	4	5
1		17	32	32	21
2	17		17	17	17
3	32	17		51	21
4	32	17	51		21
5	21	17	21	21	

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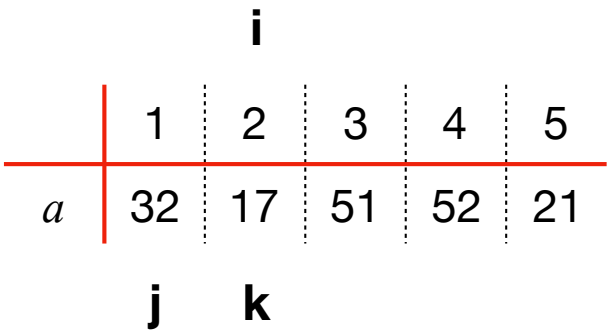
#queries ≤ 3N:

Invariant: Maintain j, k such that a_j, a_k maximal among $(a_1, \ldots, a_i)$.

Increment i, update j, k according to $q(i,j)$, $q(i,k)$, $q(j,k)$



#queries ≤ 2N:

Observation: One of the three queries is redundant

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Randomised *algorithm*:

Idea: Choose "next index" i randomly, not left to right.

As before, maintain $m_{jk} = \min(a_j, a_k)$

Query $m_{ik} = \min(a_k, a_i)$

🙂: If $\min(a_k, a_i) = m_{jk}$ we've learned a_k and can proceed to the next i

☹️: If $\min(a_k, a_i) > m_{jk}$ we must also query $m_{jk} = \min(a_j, a_k)$

How often does ☹️ happen?

Exactly if a_i is largest or next-largest among $(a_1, \ldots, a_i)$.

Pr (☹️ in round i) $= 2/i$

E[# ☹️ in round i] $= 2/i$

$$\text{E[\# ☹️ in any round]} = \frac{2}{3} + \frac{2}{4} + \frac{2}{5} + \cdots + \frac{2}{n} \sim 2 \ln n$$

Variance (thanks to team Latvia):

$$\Pr[\,|X - EX|\,] \leq 14 \ln N] \leq \frac{22}{196 \ln N}$$