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Minequake

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Observations

An optimal solution traverses every tunnel at most twice. Then, only the choice of starting hall matters. (proof later)

Subtask 4 DFS from every hall. $\mathcal{O}(n^2)$

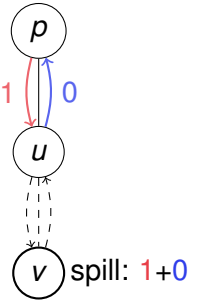
Claim: Only starting hall matters

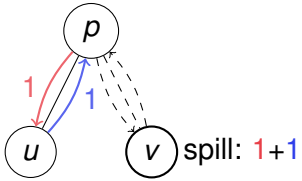
Root the tree at the starting hall. The total spill is *at least* $\sum_{u<v} s(u,v)$ where

$$s(u,v) = \begin{cases} 0 & u \text{ or } v \text{ is the root} \\ 1 & u, v \text{ are ancestors/descendants} \\ 2 & u, v \text{ are incomparable} \end{cases}$$

If every tunnel is visited at most twice, the total spill is *exactly* $\sum_{u<v} s(u,v)$.

Let $p = \text{parent}(u)$.
Consider spill at v when traversing $e = \{p, u\}$.





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Subtask 5 Root the tree arbitrarily. Compute $\mathsf{ans}[\mathit{root}]$ and subtree sizes with DFS. Then

$$\mathsf{ans}[u] = \mathsf{ans}[\mathsf{parent}(u)] + 2 \cdot \mathsf{size}[u] - n.$$