



Daniel Rutschmann AKA dacin21

# Tycho

$b$  distance to base     $n$  hiding spots     $p$  pulse period     $d$  pulse damage

**Subtask 1** Try waiting  $0, 1, \dots, p - 1$  seconds. The  $i$ -th hiding spot helps only when waiting  $a[i] \bmod p$  seconds.  $\mathcal{O}(p + n)$

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**Subtask 3** Dijkstra/DP over states (position, time until next pulse).  $\mathcal{O}(bp)$

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### Observations

Only wait at hiding spots. Wait until right after the next pulse.

**Subtask 2** Try all  $2^n$  subsets of hiding spots.  $\mathcal{O}(2^n \cdot n)$

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## Dynamic Programming

DP[ $i$ ]: minimum damage taken after waiting at  $i$ -th hiding spot.

$$\begin{aligned}
 \text{DP}[i] &= \min_{j < i} \left( \text{DP}[j] + \underbrace{\left\lceil \frac{a[i] - a[j]}{p} \right\rceil \cdot p}_{\text{environment}} + \underbrace{\left\lceil \frac{a[i] - a[j]}{p} \right\rceil \cdot d - d}_{\text{radiation pulses}} \right) \\
 &= \min_{j < i} \left( \text{DP}[j] + \left\lceil \frac{a[i] - a[j]}{p} \right\rceil \cdot (p + d) - d \right)
 \end{aligned}$$

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**Subtask 4** DP in  $\mathcal{O}(n^2)$ .

**Subtask 5** For each  $a[j] \bmod p$ , check only the latest  $j$ .  $\mathcal{O}(np)$

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$$\left\lceil \frac{a[i] - a[j]}{p} \right\rceil = \left\lfloor \frac{a[i]}{p} \right\rfloor - \left\lfloor \frac{a[j]}{p} \right\rfloor + \begin{cases} 1 & a[i] \bmod p > a[j] \bmod p \\ 0 & \text{otherwise} \end{cases}$$

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$$\underbrace{DP[i] - \left\lfloor \frac{a[i]}{p} \right\rfloor \cdot (p + d)}_{\text{depends on } i} = \min_{j < i} \left( \underbrace{DP[j] - \left\lfloor \frac{a[j]}{p} \right\rfloor \cdot (p + d)}_{\text{depends on } j} + \underbrace{\begin{cases} p + d & \dots \\ 0 & \dots \end{cases}}_{\text{range query}} \right) - d$$

→ Range min-query on  $[0, a[i] \bmod p)$  and  $[a[i] \bmod p, p)$ .



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**Subtask 6** Min-segment tree over  $a[j] \bmod p$ .  $\mathcal{O}(p + n \log p)$ .

**Subtask 7** Coordinate Compression / Implicit segment tree.  $\mathcal{O}(n \log p)$