COMP 182: Algorithmic Thinking
30 January 2014

1. What is the effect in the time required to solve a problem when you double the size of the input from n to $2n$, assuming that the number of milliseconds the algorithm uses to solve the problem with input size n is each of the following functions:

   (a) $\log \log n$

   (b) $\log n$

   (c) $100n$

   (d) $n \log n$

   (e) n^2

   (f) n^3

   (g) 2^n

   Express your answer in the simplest form possible, either as a ratio or a difference. Your answer may be a function of n or a constant.

2. Determine whether each of the functions $\log(n+1)$ and $\log(n^2+1)$ is $O(\log n)$.

3. Determine whether each of the following functions is $O(n^2)$.

   (a) $17n + 11$

   (b) $n \log n$

   (c) 2^n

   (d) $n^2 + 1000$

4. Arrange the functions $\sqrt{n}$, $1000 \log n$, $n \log n$, $2n!$, 2^n, 3^n, and $n^2/1000000$ in a list so that each function is big-O of the next function.

5. Big-O, Θ, and Ω notation can be extended to functions in more than one variable. Define these three notations formally.

6. Show that $(n^2 + nm + n \log m)^3$ is $O(n^6 m^3)$.

7. Show that if $c > d > 0$, then n^d is $O(n^c)$ but n^c is not $O(n^d)$.