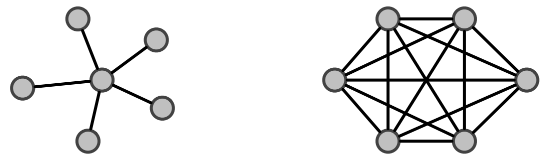
COMP 182: Algorithmic Thinking
16 January 2014

A network topology specifies how computers, printers, and other devices are connected over a network. The figure below illustrates two common topologies of networks: the star and the fully connected mesh.



star **fully connected mesh**

Two notations that are useful in writing formal definitions are:

- $\forall$ (pronounced "for all"). For example, the notation "$\forall v \in V$" means "for every node v in the set V of nodes", and "$\forall e \in E$" means "for every edge e in the set E of edges". More generally, "$\forall a \in A$" means "for every element a in the set A." The notation "$\forall v \in V, degree(v) = 1$" means "every node v in V has degree 1."
- $\exists$ (pronounced "exists" or "there exists"). For example, the notation "$\exists v \in V$" means "there exists a node v in V". The notation "$\exists v \in V, degree(v) = 0$" means "there exists a node v in V that is isolated."

1 Formal definitions

Let $g = (V, E)$ be a graph on n nodes ($n \geq 4$), where $V = \{0, 1, \ldots, n-1\}$. Define formally what it means for g to be a star and to be a fully connected mesh (that is, give two definitions).

2 Problem formulations

Formulate two graph-theoretic, decision problems that correspond to checking whether a network topology is a star or a fully connected mesh (again, assume $n \geq 4$).

3 Brute-force algorithms

Write the pseudo-code of two algorithms **IsStar** and **IsFullyConnectedMesh** for solving the two problems you defined in the previous section (assume the graph is given by its adjacency matrix and that the number of nodes is ≥ 4).

4 Efficiency

For each of your two algorithms, which of the following terms most closely captures the number of steps that your algorithm takes as a function of the number of nodes n in the graph:

$$1000 \quad \log n \quad n \quad n^2 \quad n^3 \quad n^4 \quad 2^n \quad n!$$