Lab 5 - Bayesian Networks – Artificial Intelligence

Teacher: Stephan Schiffel

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In this lab we will look at some basic Bayesian Networks and you will solve two problems using a Bayesian Network simulator. You can work together in groups (up to 3 students), but you may learn more if you do it alone.

Time Estimate

1.5 hour in addition to the time spend in the lab class assuming you attended the lectures on Bayesian Networks.

Material

The material on Canvas contains a Bayesian Network simulator that you can use. It is also available online here using Java Web Start.

The application has two tabs "Create" and "Solve". In "Create" you can create and modify a Bayesian network, in "Solve" you can compute arbitrary probabilities using the network. Suppose you have a network with nodes A and B, where B depends on A. To create the network, you need to:

- create two nodes and give them the names A and B;

- create an arc pointing from A to B;

- modify the probability tables of both nodes and set the probabilities $P(A)$, $P(B|A)$ and $P(B|\neg A)$

Suppose you want to compute $P(\neg A|\neg B)$, that is, you want to know the probability for A being false after having observed B being false. Then you need to:

- Go to the "Solve" tab;

- click on "Make Observation", then on node B and select "F" for false;

- click on "Query" and then on node A (select Brief query mode if asked).

- You should now see the probabilities for $P(A)$ and $P(\neg A)$ under the condition that B = false.

Laws of Probabilities

The following laws of probabilities might be helpful for developing formulas:

- $P(\neg A) = 1 - P(A)$

- Bayes' Law: $P(A \wedge B) = P(A|B) * P(B)$

- Marginalization/Summing out: $P(A) = \sum_b P(A \wedge b)(= P(A \wedge B) + P(A \wedge \neg B))$

- Independence 1: $P(A|B, C) = P(A|C)$, if A and B are independent given C.

- Independence 2: $P(A \wedge B|C) = P(A|C) * P(B|C)$, if A and B are independent given C. (This follows from Independence 1 and Bayes Law.)

All those laws allow arbitrary additional conditions as long as all probabilities have the condition, for example:

- Bayes' Law: $P(A \wedge B|C) = P(A|B, C) * P(B|C)$

- Marginalization/Summing out: $P(A|C) = \sum_b P(A \wedge b|C)$

Tasks

1. (34 points) Smelly Doors

   You are writing a program to control a non-player character (NPC) in a game. The NPC is in a building full of doors. Behind each door, there could be either a reward (e.g. health-points) or a monster which the NPC must fight with (losing health-points). The room can also be empty. Once the NPC opens a door, he must fight the monster behind it if any. However, before opening a door the NPC can stick its nose in the keyhole (it cannot look through it) and smell the air inside the room. The air will smell either bad or not. In summary:

   - The NPC should seek reward but avoid monsters;
   - The NPC doesn't know what's behind a door in advance, but
   - it can check whether the room smells bad or not and use that information as an indicator.

   1.1 (20 points) Design a very simple Bayesian network (2 nodes) usable by the NPC to decide when in front of a door, how likely it is that there is a monster in the room. Start by inventing reasonable probabilities for the relation between the contents of the room and its smell. Use **causal relationships** between the propositions to decide on the direction of the arcs in the network.

   Suppose the NPC is standing in front of a room and notices that the air smells bad. Using your Bayesian Network, answer the question: How likely is it that the room contains a monster?

   Write down the Bayesian Network (nodes, connections, probability tables) and the answer to the question.

   1.2 (14 points) Argue how, instead of using made-up probabilities, the NPC can learn those probabilities as he opens doors and dynamically update the Bayesian network to become smarter. Explain what the agent should observe and how the probabilities can be estimated from the observations.

2. (66 points) The Pirate Treasure

You are a seasoned tomb raider and have spent the last week rummaging through an old pirate cove full of treasure. So far you have opened 100 chests and of those, 50 have in fact contained treasure! Out of these 50, 40 were trapped and you sustained some painful damage from opening them. Out of these 40 trapped chests with treasure, 28 were also locked. Now, of the 10 untrapped chests with treasure, three were locked. One would think that only chests with treasure would be trapped, but these pirates were truly nasty, they also put traps on chests with no treasure. Of the 50 chests containing no treasure, 20 were trapped! You forgot how many of the chests without treasure were locked, but you believe that the ratios were similar to the ones with treasure.

You have now discovered a new chest that you haven't seen before. When you take a careful look, you notice that it is locked. What is the chance that this chest will contain treasure? What is the chance that it will be trapped? You are not feeling so good after all the previous traps, so will it be worth opening this chest if your life is on the line?

2.1 (36 points) Construct a Bayesian Network that helps you answer these questions. Write down the propositions (nodes), the connections between them and the probability table for each node. You should be able to do this by essentially just reading the required information from the hints in the text (i.e., without any complicated computations).

2.2 (6 points) What is the probability that a locked chest contains treasure?

2.3 (6 points) What is the probability that a locked chest is trapped?

2.4 (18 points) Develop a formula for computing the probability of a chest containing treasure, given that you observe whether it is locked or not. Write down the steps you took to get to the formula and the final formula. The formula must only use probabilities that are given in the Bayesian Network. Check whether your formula (and/or the Bayesian Network) is correct by comparing the values you compute with the formula with those that you get from the simulator.

Note: The goal for this question is to look at how the computations in the previous questions can be done using the Bayesian Network. To solve this, first write down what you want to compute, then use use Bayes' Law, Marginalization and the conditional independence rules to relate this to the given probabilities.

Handing In

Hand in a PDF document with the answers to all the questions including images of the Bayesian Networks (e.g., screen-shots from the simulator) that show the structure of the network and probability tables for all nodes.