Practice Examination - Math 142, Frank Thorne (thorne@math.sc.edu)

The Exam is Thursday, October 24, 2013

Instructions and Advice:

- There are eight questions, some of which are shorter than others.

- You are welcome to as much scratch paper as you need. Turn in everything you want graded, and throw away everything you do not want graded.

- **Draw pictures where appropriate.** If you have any doubt, then a picture is appropriate.

- Be clear, write neatly, explain what you are doing, and show your work. **This is especially important for earning partial credit** in case your work contains one or more mistakes. Be warned that **work I cannot understand will not receive any credit.**

- 75 minutes is a long time. Don't dilly-dally, but don't rush. **You are strongly advised to take the entire 75 minutes to complete the examination.** If you finish early, you have the opportunity to check your work.

- Please work without books, notes, calculators, or any assistance from others.

- I will be at the front of the room; if you have any questions, feel free to ask me.

GOOD LUCK!

(1) (12 points) Find the volume of sphere of radius 6.

(*Warning:* You will not get credit for remembering the formula and plugging in 6. A correct answer will carry out the calculus computation. You *may* use the fact that a circle of radius r has area πr^2, without further justification.)

Draw a picture illustrating your computation.

(2) (10 points) The region bounded by $y = \frac{1}{9}x^2$, $x = 3$, and $y = 0$ is rotated around the y-axis. Find the volume of the resulting solid.

As part of your answer, sketch the region, the resulting solid, and a typical disk or washer.

(3) (10 points) Explain what it means for a curve to be defined by parametric equations. Give and graph an example. (Your example should be different than other questions on this exam.)

(4) (10 points) Sketch the curve given by the equations $x = 2\cos t$, $y = t - \cos t$ for $0 \le t \le 2\pi$.

(10 points) In addition, find the equation of the tangent line when $t = \pi/3$, and sketch it on your graph.

(5) (10 points) Three graphs are given, along with parametric equations for them (in a different order). Match the graphs with the equations, and give reasons for your choices.

(6) (10 points) Plot the point whose polar coordinates are $(-3, \pi/4)$. In addition, find Cartesian coordinates of this point, and find another set of polar coordinates (r, θ) for the same point with $r > 0$.

(7) (10 points) Sketch the curve with the polar equation $r = 2\cos 2\theta$ for $0 \le \theta \le 2\pi$.

(8 points) Also, find the slope of the tangent line when $\theta = \pi/3$.

(8) (10 points) A graph of the curve $r = 2 + 2\cos\theta$ is provided. Write down a definite integral which represents the area of the curve. In addition, use the graph to give a rough estimate for the value of this integral.
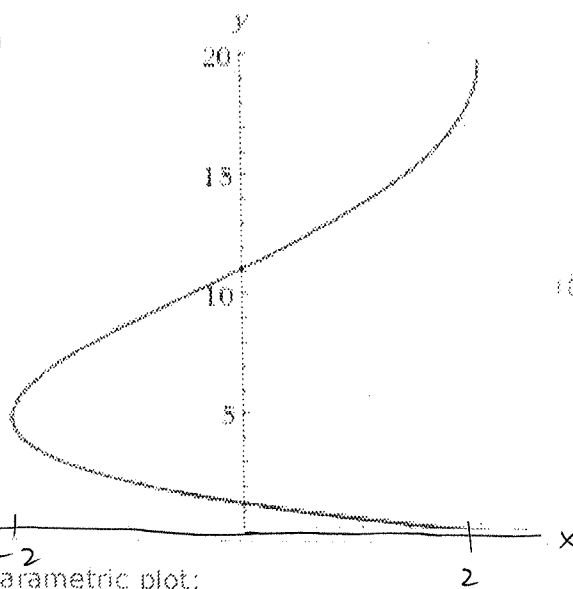
In random order:

1.
$$x = t^3 - t^2$$
$$y = t^2 - t$$

2. $x = e^t$
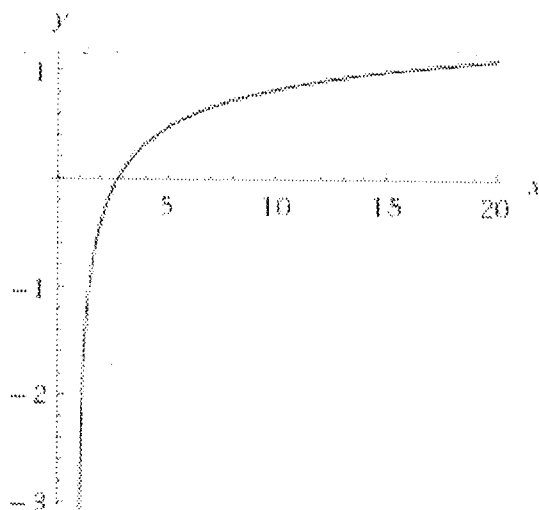$$y = \ln(t)$$

3. $x = 2\cos(t)$
$$y = t^2/2$$

(a)

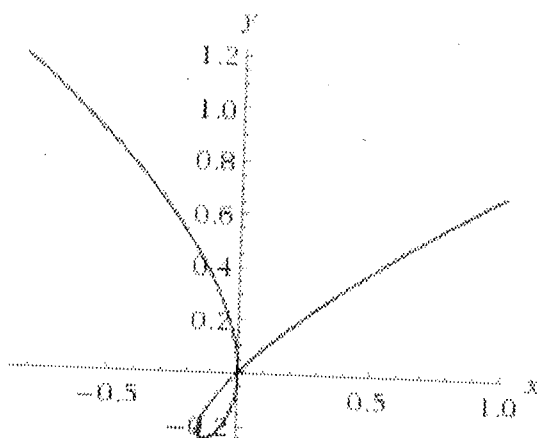Parametric plot:

(t from −1.871 to 6.283)

(b)
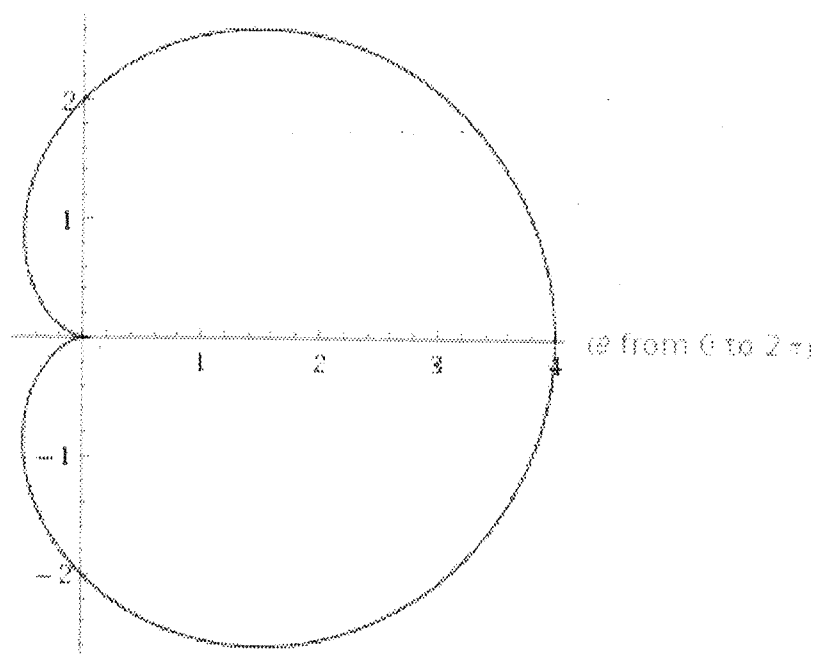
Parametric plot:

(t from −3 to 3)

(c)

Alternate.fproc:

(t from −0.7 to 1.7)

Alternate f....

r = 2 + 2*cos(theta)

Input:

Polar plot:

θ from 0 to 2π

Plots: