Term project - Analytic number theory

Frank Thorne, thornef@webmail.sc.edu

(100 points.) Learn something about analytic number theory, or a related subject; write a paper of 5 pages or more on the subject, and give a 20-minute talk on it in class or in the number theory seminar.

Group submissions are welcome; please write a joint paper of at least 5 pages per author, and each author should speak for 20 minutes (so you can address multiple aspects of a subject).

You are very much welcome to get suggestions from Michael Filaseta, Ogy Trifonov, or Matt Boylan (or from anyone else in the department whose interests overlap with analytic number theory). **Beware:** If you speak on something I don't know well, I will ask a ton of questions.

The topic is up to you. The following are some suggestions.

1. There are **a lot** of topics in Davenport which we skipped or glossed over, and **all of them** are interesting. Pick your favorite.

   If you have the nerve, crack open Iwaniec and Kowalski's book and take your pick of topics. (If you get through a half dozen pages of that, you're doing well.)

2. Learn something about the *pretentious* approach to analytic number theory, being developed by Granville and Soundararajan. (Ask me for a copy of their notes)

3. *Sieve methods* are a powerful and versatile way to bound the number of primes in various sequences. The easiest place is probably the book by Cojocaru and Murty (*An Introduction to Sieve Methods and Their Applications*). A good project would be to explain the Selberg sieve.

   You might crack the covers of *Opera de Cribro* by Friedlander and Iwaniec if you are feeling (extremely) ambitious.

4. *Additive Combinatorics* is a very interesting subject, addressing the problem of finding "additive structure" in sets such as $\mathbb{Z}/N$. An excellent place to start is `http://math.stanford.edu/~ksound/Notes.pdf`; read the introduction and the proof of Roth's theorem.

5. The *circle method* is a useful and versatile tool; it was used by Vinogradov to prove that every large odd integer is the sum of three primes. See Soundararajan's additive combinatorics notes, or the later section of Davenport.

6. Zeta and L-functions can be given an *axiomatic definition* (known as the *Selberg class*). Read about this at the beginning of Chapter 5 of Iwaniec and Kowalski, *Analytic number theory*. Then, skim through the rest of the chapter and see what kind of examples there are, and what generality theorems can be proved in.

7. The *prime number theorem* was given an elementary proof (i.e. no complex analysis) by Erdös and Selberg. Ask me for references (I don't know a good one of the top of my head)

8. Learn about *Chebyshev's bias* (see the paper of Rubinstein and Sarnak) and learn why there are more primes congruent to 3 mod 4 than 1. Also see work of Granville, Martin, on others on prime number races. (Google it!)

9. If you know something about elliptic curves and/or modular forms, learn about their L-functions. (Ask Matt Boylan for references.)

10. There are **many** proofs of the functional equation for the Riemann zeta function. Get a copy of Titchmarsh's book (borrow it from me) and learn a couple of them. (There are, of course, many other goodies in Titchmarsh as well.)

11. Learn about *Maier matrices* and irregularities in the distribution of the primes. Good references are A. Granville, *Unexpected irregularities in the distribution of prime numbers* and K. Soundararajan, *The distribution of prime numbers*.