

(a) $\lim_{x \rightarrow -1^+} f(x) = 1$. True.

As x approaches -1 from the right, $f(x)$ gets closer and closer to 1.

(b) $\lim_{x \rightarrow 0^-} f(x) = 1$. False.

As x approaches 0 from the left, $f(x)$ gets closer to 0, not 1.

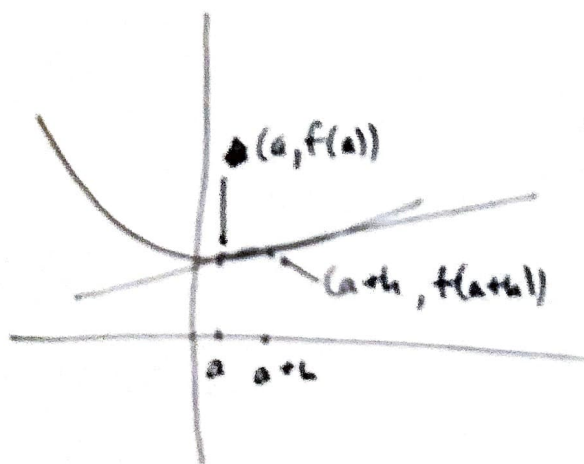
(c) $\lim_{x \rightarrow 0} f(x)$ exists. True.

This limit equals 0, because the function approaches zero from both sides. The value at $x=0$ itself doesn't matter.

(d) $\lim_{x \rightarrow 1} f(x) = 0$. False.

Because the left-side limit is 1, and the right-side limit is 0, and they don't agree.

2 By definition $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$.



Draw the secant line between $(a, f(a))$ and $(a+h, f(a+h))$.

Its slope is

$$\begin{aligned} & \frac{f(a+h) - f(a)}{(a+h) - a} \\ &= \frac{f(a+h) - f(a)}{h} \end{aligned}$$

As we take the point $a+h$ closer and closer to a , the secant line approaches the tangent line, so

$$\frac{f(a+h) - f(a)}{h} \text{ approaches } \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = f'(a).$$

$$3. \quad f(x) = x + \frac{q}{x}.$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h + \frac{q}{x+h}) - (x + \frac{q}{x})}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h + \frac{q}{x+h} - \frac{q}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h + \frac{qx - q(x+h)}{(x+h)x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h + \frac{qx - qx - qh}{(x+h)x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h - \frac{qh}{(x+h)x}}{h}$$

$$= \lim_{h \rightarrow 0} \left(1 - \frac{qh}{(x+h) \cdot x \cdot h} \right)$$

$$= \lim_{h \rightarrow 0} \left(1 - \frac{q}{(x+h) \cdot x} \right) = 1 - \frac{q}{x^2}.$$

The slope of the tangent line at $x = -3$ is equal to $f'(-3) = 1 - \frac{q}{(-3)^2} = 1 - \frac{q}{9} = 0.$