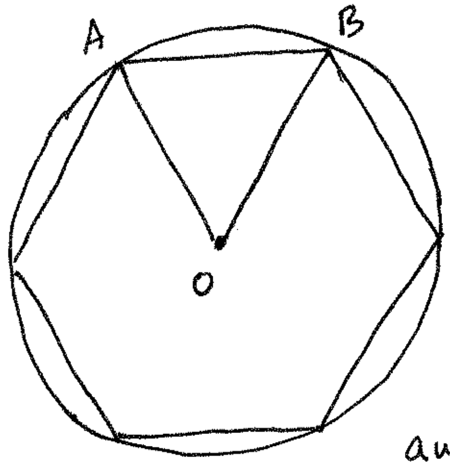
1. We have $\angle P \overset{\circ}{=} \frac{1}{2}(\widehat{CD} - \widehat{AB})$.

We know $\angle P = 30^\circ$ and $\widehat{CD} \overset{\circ}{=} \angle COD = 120^\circ$

so $30^\circ = \frac{1}{2}(120^\circ - \widehat{AB})$

$60^\circ = 120^\circ - \widehat{AB}$ so $\widehat{AB} = 60^\circ$.

2.



We have $\overline{OB} = 1$, $\overline{OA} = 1$.

$\widehat{AB} = 60^\circ$ because the hexagon is regular and $60^\circ = \frac{1}{6} \cdot 360^\circ$.

$\triangle OAB$ is isosceles with base AB.

By the pons asinorum, $\angle BAO = \angle ABO$

and so $2\angle BAO = 180^\circ - \angle AOB$

$= 180^\circ - 60^\circ$

$= 120^\circ$

So $\angle BAO = 60^\circ$ and $\angle ABO = 60^\circ$.

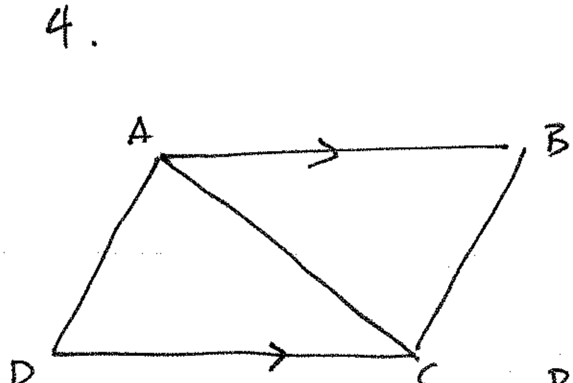
Thus $\triangle AOB$ is equiangular. Thus by the converse of the pons asinorum, $\overline{AB} = \overline{BO} = \overline{AO} = 1$ and so the hexagon has perimeter 6.

3. By the pons asinorum $\angle ABC = \angle ACB$.

Also $\angle BZC = \angle CYB = 90^\circ$, and $BC = BC$.

So $\triangle BZC \cong \triangle CYZ$ by SAA and so $BY = ZC$.

4.



Draw AC. Because $AB \parallel CD$, we have $\angle BAC = \angle ACD$ (they are alternate interior angles).

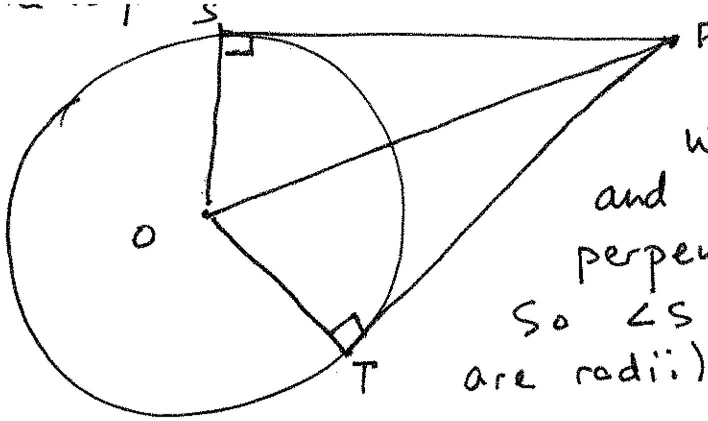
We also have $AC = AC$ and $AB = CD$,

So $\triangle BAC \cong \triangle DCA$ by SAS

and so $\angle DAC = \angle ACB$.

But since AC is a transversal of AD and BC this implies $AD \parallel BC$, so $ABCD$ is a parallelogram as desired.

5.

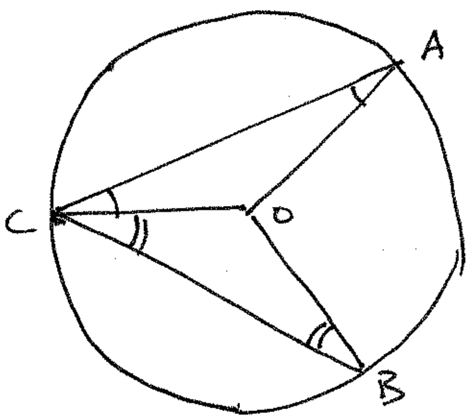


We know ~~angles~~ $PT \perp OT$ and $PS \perp OS$ (tangents are perpendicular to radii).

So $\angle S = \angle T$, $SO = OT$ (both are radii), and $OP = OP$, so by HA $\Delta SOP \cong \Delta TOP$. Therefore $PS = PT$.

Also, $\angle SPO = \angle TPO$ so that both are equal to $\frac{1}{2}\angle SPT$ as desired.

6.



We have $AO = OC = OB$, so by the pons asinorum $\angle OAC = \angle OCA$, $\angle OCB = \angle OBC$.

We have $\angle COA = 180^\circ - 2\angle ACO$

$\angle COB = 180^\circ - 2\angle BCO$

and $\angle COA + \angle COB + \angle AOB = 360^\circ$.

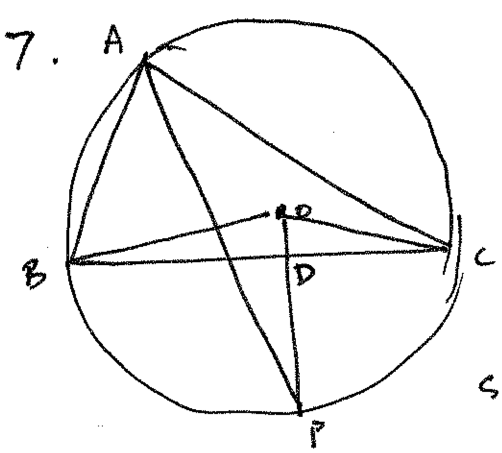
Adding, we get

$$360^\circ - \angle AOB = 360^\circ - 2(\angle ACO + \angle BCO)$$

$$= 360^\circ - 2(\angle ACB)$$

and so $\angle AOB = 2\angle ACB$.

7.



We have $\angle BAP = \angle CAP$ by hypothesis.

$\angle BAP = \frac{1}{2}\overset{\frown}{BP} = \frac{1}{2}\angle BOP$, and

$\angle CAP = \frac{1}{2}\overset{\frown}{CP} = \frac{1}{2}\angle COP$,

so $\angle BOP = \angle COP$.

Also $BO = OC$, so ΔBOC is isosceles, so by the pons asinorum, $\angle OBC = \angle OCB$.

Also $OD = OD$, so by AAS $\Delta OBD \cong \Delta OCD$. Thus $\angle BDO = \angle CDO$, and since they add to 180° each must be 90°.