State High School Mathematics Tournament

University of South Carolina

Tiebreaker – January 25, 2020

Tiebreaker Rules

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- **Solving it exactly within 90 seconds is probably impossible.**
- Try to solve it **approximately**, as accurately as you can, and make an educated guess.
- The answer(s) **closest to the truth** (in either direction) win the tiebreaker.

Tiebreaker Question

How many integers $n \leq 2020$ can be written in the form

$$n = a^3 + b^3 + c^3,$$

where a, b, c are positive integers?

Tiebreaker Answer

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- We must have $1 \leq a, b, c \leq 12$. Start with the number of such triples ($12^3 = 1728$).
- Divide by 6 to get 288, because most triples are counted six times:

$$a^3 + b^3 + c^3 = a^3 + c^3 + b^3 = b^3 + a^3 + c^3 = b^3 + c^3 + a^3 = c^3 + a^3 + b^3 = c^3 + b^3 + a^3$$

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- Subtract a little bit, because a, b, and c can't all be close to 12.
- To get the exact answer, ask a computer.

Tiebreaker 2

How many integers $n \leq 202020$ can be written in the form

$$n = a^3 + b^3 + c^3,$$

where a, b, c are positive integers?

Tiebreaker 2 Answer

Answer. 21581.