

# BOUNDED GAPS BETWEEN PRODUCTS OF PRIMES WITH APPLICATIONS TO ELLIPTIC CURVES AND MODULAR $L$ -FUNCTIONS

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ABSTRACT. In recent work Goldston, Graham, Pintz, and Yıldırım use a variant of the Selberg sieve to prove the existence of small gaps between  $E_2$  numbers; that is, squarefree numbers with exactly two prime factors. We apply their techniques to prove similar bounds for  $E_r$  numbers for any  $r \geq 3$ , where these numbers are required to have all of their prime factors in a set of primes  $\mathcal{P}$ . Our result holds for any  $\mathcal{P}$  of positive density that satisfies a Siegel-Walfisz condition regarding distribution in arithmetic progressions. We also prove a stronger result in the case that  $\mathcal{P}$  satisfies a Bombieri-Vinogradov condition. In addition, using results of Ono and Soundararajan, we give applications to divisibility of class numbers, critical values of  $L$ -functions, and ranks of elliptic curves.

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