## Homework 13 - Math 142, Frank Thorne (thornef@mailbox.sc.edu)

## Due Friday, December 5

- (a) What is a power series?
- (b) Describe an example of a power series which converges for all values of x.
- (c) Describe an example of a power series which converges only for some values of x.
- (d) Are there any power series which converge for no values of x?
- (e) What is the radius of convergence of a power series? How do you find it?
- (f) 11.8, 5-18. However, you do not have to test the endpoints of the interval for convergence.
- (g) What is a Taylor series? Why is the formula for it true?
- (h) Find the Maclaurin (Taylor) series for the following functions. Determine their radii of convergence.
  - $f(x) = x^2$
  - $f(x) = e^x$
  - $f(x) = e^{2x}$
  - $f(x) = \cos(x)$
  - $f(x) = \sin(x)$
  - $f(x) = \cos(4x)$
  - $f(x) = \sin(x^2)$
  - $f(x) = x^3 \sin(x)$ .
  - $f(x) = x + e^x$ .
- (i) Explain why the Taylor series for  $e^x$  gives you a formula for e.
- (j) Compute e, as a fraction or decimal, to fairly good accuracy. Your estimate should plausibly be within  $\frac{1}{10}$ , but you don't need to show this.
- (k) Compute 1/e, as a fraction or decimal, to fairly good accuracy. Your estimate should plausibly be within  $\frac{1}{100}$ , but you don't need to show this.
- (l) Compute  $\sin(1/10)$ , as a fraction or decimal, to fairly good accuracy. Your estimate should plausibly be within  $\frac{1}{100}$ ,
- (m) Compute  $\sqrt{1.1}$ , as a fraction or decimal, to fairly good accuracy. Your estimate should plausibly be within  $\frac{1}{1000}$ , but you don't need to show this.
- (n) Stewart, 11.10, 29, 30.

## Additional problems:

(a) Stewart, 11.8, 19-24, 11.10, 15, 16, 31-36.

Bonus: Use Taylor series to explain why  $e^{ix} = \cos(x) + i\sin(x)$ .

## That's all folks!