State High School Mathematics Tournament

University of South Carolina

January 25, 2020

Tiebreaker Rules

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- Try to solve it **approximately**, as accurately as you can, and make an educated guess.
- The answer(s) **closest to the truth** (in either direction) win the tiebreaker.

Tiebreaker Question

How many integers $n \leq 2020$ can be written in the form

$$n = a^3 + b^3 + c^3,$$

where a, b, c are positive integers?

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- Subtract a little bit, because a, b, and c can't all be close to 12.
- To get the exact answer, ask a computer.