Homework 4 - Analytic number theory

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Due Friday, September 23

1. (5 points) (a) Prove (as was taken for granted in lecture) that if f and g are multiplicative functions, then so is their convolution $f * g$.

   (b) If f is multiplicative and g is not, must $f * g$ be multiplicative? Prove or find a counterexample.

2. (5 points) Prove that $\sum_n d(n)^2 n^{-s} = \frac{\zeta^4(s)}{\zeta(2s)}$.

3. (5 points) Prove that $\sum_n d(n^2) n^{-s} = \frac{\zeta^3(s)}{\zeta(2s)}$.

4. (5 points) (Trick question. Explain.) Write out the character tables for all primitive real characters to the following moduli: 14, 15, 16, 20, 22, 24, 25, 27.

5. (7 points) Write $g(n)$ be the number of primitive (not necessarily real) characters modulo n. Prove an explicit formula for $g(n)$. **No messy computations allowed.**