NOTE: These have NOT been proofread. Please email thorne [at] math.wisc.edu in case you find any errors.

I've given answers with quick solutions... in many cases it is advisable that you show more work.

1. Fall 03, M.C. 1. If F is a linear function, $F(2) = 6$ and $F(-6) = -2$, then what is $F(10)$?

   Answer: Linear function just means $F(x) = mx + b$ for some values of m and b. We plug in the values given to get a system $6 = 2m + b$, $-2 = -6m + b$. We solve the system in the usual way (e.g., subtract the second equation from the first), and obtain $m = 1$ and $b = 4$. Therefore $F(10) = 14$.

2. Fall 03, M.C. 2. If you divide $x^9 + x^8 + x^7 + x^6 + x^5 + x^3 + x^2 + x$ by $x - i$, what is the remainder?

   Answer: Let $f(x)$ denote the polynomial given. We plug in i for x, and determine that $f(i) = i$.. (Remember $i^4 = 1$, so $i^5 = i$, $i^6 = -1$, etc...) Then the Remainder Theorem (see p. 577) says the remainder is just $f(i) = i$.

3. Fall 03, M.C. 9. Solve for x if $\log_9(\frac{\sqrt{3}}{81}) = 9x + 9$. And simplify.

   Answer: Observe that $\sqrt{3} = 3^{1/2} = 9^{1/4}$. Therefore, $\frac{\sqrt{3}}{81} = \frac{9^{1/4}}{9^2} = 9^{1/4}9^{-2} = 9^{-7/4}$. The log cancels the exponent, we have $-7/4 = 9x + 9$. Solving for x we get $-43/36$.

4. Spring 05, M.C. 5. Let $f(x) = 2 - 7e^{x+5}$. What is the domain of $f^{-1}(x)$?

   The inverse function is not too hard to find. We write y in place of x and solve for x in terms of y:

   $$\frac{y-2}{-7} = e^{x+5}$$

   $$x = \ln(\frac{y-2}{-7}) - 5.$$

   To find the inverse function, we just switch x and y:

   $$f^{-1}(x) = \ln(\frac{x-2}{-7}) - 5.$$

We can't take the ln of a negative number. So $x-2<0$ (why?), and we get $(-\infty, 2)$.

5. Spring 05, M.C. 8. Solve for x, if $f(1)=-2$.

$$3+f^{-1}(x-1)=4.$$

Answer: $f^{-1}(x-1)=1$, so using the property of the inverse functions $x-1=-2$. So $x=-1$.

6. Spring 05, 6. If $a_1=1$, $a_2=4$, and $a_n=\frac{a_{n-1}}{2}+a_{n-2}$ for $n\neq 3$, find a_4.

Answer: First plug in a_1 and a_2 to find a_3:

$$a_3=\frac{4}{2}+1=3$$

Now you can figure out what a_4 is:

$$a_4=\frac{3}{2}+2=\frac{7}{2}.$$

7. Fall 05, 7. If $f(x)$ is a second degree polynomial with roots 1 and -1, and it passes through $(3,4)$, find the equation for f.

Remember that finding roots is the same as factoring! We can write $f(x)=a(x-1)(x+1)$, where we don't know what a is. We plug in $x=3$ and $f(x)=4$ to get $4=8a$. So $a=1/2$.

8. Review, 2. If $A=\log_a 2$, $B=\log_a 3$, $C=\log_a 10$, then find:

$\log_a(4/3)=\log_a 4-\log_a 3=\log_a(2^2)-\log_a 3=2\log_a 2-\log_a 3=2A-B.$

$\log_a(\sqrt{12})=\frac{1}{2}\log_a 12=\frac{1}{2}\log_a(2{*}2{*}3)=\frac{1}{2}(\log_a 2+\log_a 2+\log_a 3)=\frac{1}{2}(2A+B).$

$$\log_a \sqrt[7]{125}=\log_a(125^{1/7})=\frac{1}{7}\log_a 125=\frac{1}{7}\log_a 5^3=\frac{3}{7}\log_a 5.$$

Now observe $\log_a 5=\log_a(10/2)=\log_a 10-\log_a 2$, so we get $\frac{3}{7}(A-C)$.

9. Review, 5. This one is a little bit weird... I doubt this will show up on the exam, but maybe I'm wrong. If you take $\log_{10}$ of both sides, use the rules for logs, and simplify, you get

$$10000 \log_{10} 5830 = \log_{10} x + N.$$

Moreover, if $1 < x < 10$, then take log_{10} of this inequality, you get

$$\log_{10} 1 < \log_{10} x < \log_{10} 10$$

which just tells you

$$0 < x < 1.$$

So the easiest way to solve the problem is to plug in $10000 \log_{10} 5830$ into your calculator, and then N is everything before the decimal point, and $\log_{10} x$ is everything after. If you want to use the hint, then write

$$10000 \log_{10} 5830 = 10000 \log_{10}(1000 \cdot 5.83) = 10000(\log_{10} 1000 + \log_{10} 5.83)$$

$$= 10000(3 + \log_{10} 5.83) = 30000 + 10000 \log_{10} 5.83 = 30000 + 7656.7 = 37656.7.$$

So $N = 37656$ and x is about .7. (x is not exactly .7, because $\log_{10} 5.83$ is not exactly .76567.)