Homework 11 - Analytic number theory

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1. (8 points) If $N \not\equiv 0 \mod p$, compute the number of triples $(n_1, n_2, n_3) \in (\mathbb{Z}/p)^3$ such that $n_1 + n_2 + n_3 = N$ (modulo p) and none of the n_i are zero.

   Do the same calculation if $N = 0$.

   In light of these calculations, explain the constant $\mathfrak{S}(N)$ appearing in Vinogradov's three primes theorem.

2. (10 points) Let $\{a_1, a_2, \cdots a_k\}$ be any k-tuple of positive integers. Conjecture how many integers n there are less than x such that the $n + a_i$ are simultaneously prime. Your conjecture should include a qualitative description of when your answer is zero (or finite).

   For additional points, test your conjecture and report the results.