Quiz 5 - Math 544, Frank Thorne (thorne@math.sc.edu)

Monday, October 12, 2015

1. What does it mean for a set of vectors S to be linearly dependent? You may give the definition, or answer using any of the 'if and only if' results from lecture or the book.

2. Determine whether each of the following subsets of $\mathbb{R}^2$ is linearly dependent or not.

$$S_1 := \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \end{bmatrix} \right\}$$

$$S_2 := \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \end{bmatrix} \right\}$$

Quiz 5.

S is linearly dependent, if equivalently

(1) There is $\vec{v} \in$ ~~Span~~ S with $\mathrm{Span}(S - \{\vec{v}\}) = \mathrm{Span}(S)$.

(2) There is a solution to an equation

$$a_1 \vec{v_1} + a_2 \vec{v_2} + \cdots + a_n \vec{v_n} = 0$$

with the $\vec{v}_i$ all distinct elements of S and not all a_i zero.

(3) Some $\vec{v} \in S$ is a linear combination of other elements of S.

(2) S_1: Linearly dependent, because

$$\begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

S_2: Linearly independent.

Suppose $a_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + a_2 \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

We have $\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 2 & 3 & 0 \end{array}\right] \xrightarrow{\text{sub } 2R1 \text{ from } R2} \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 7 & 0 \end{array}\right]$

mul R2 by 1/7 $\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 1 & 0 \end{array}\right]$

add 7·R2 to R1 $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$

So $a_1 = a_2 = 0$.

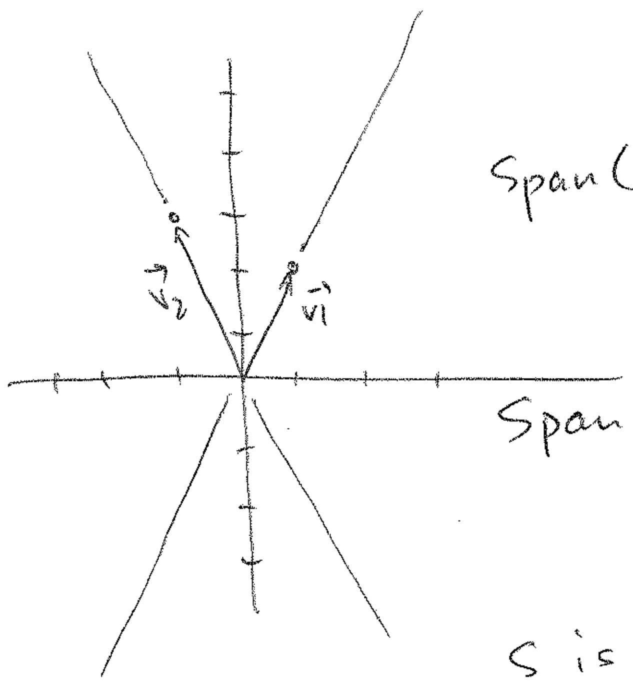
So the set is linearly independent.

HW 5. oops! this wasn't assigned

4.1, A1. We can take b, c, d as free variables so that $V = \left\{ \begin{bmatrix} -b-c-d & b \\ c & d \end{bmatrix} : b, c, d \in \mathbb{R} \right\}$

and so $V = \mathrm{Span}\left(\left\{ \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \right)$.

4.2, A1.



$\mathrm{Span}(S) = \left\{ a\begin{bmatrix} 1 \\ 2 \end{bmatrix} + b\begin{bmatrix} -1 \\ 3 \end{bmatrix} : a, b \in \mathbb{R} \right\}$

$= \mathbb{R}^2$

the entire plane.

$\mathrm{Span}(S - \{\vec{v_1}\}) = \mathrm{Span}(\{\vec{v_2}\})$

$= \left\{ b\begin{bmatrix} -1 \\ 3 \end{bmatrix} : b \in \mathbb{R} \right\}$

the line through $\vec{v_2}$.

$\mathrm{Span}(S - \{\vec{v_2}\}) = \mathrm{Span}(\{\vec{v_1}\})$

$= \left\{ a\begin{bmatrix} 1 \\ 2 \end{bmatrix} : a \in \mathbb{R} \right\}$

the line through $\vec{v_1}$.

S is linearly independent because if you remove any vector, the resulting subspace which is spanned is smaller.

4.1 A8. Show $\mathrm{Span}\left(\left\{\begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}\right\}\right) = \mathbb{R}^3$.

Can we solve $r\begin{bmatrix}0\\1\\1\end{bmatrix} + s\begin{bmatrix}1\\1\\1\end{bmatrix} + t\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

for <u>arbitrary</u> $a, b, c \in \mathbb{R}$?

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & a \\ 1 & 1 & 0 & b \\ 1 & 1 & 1 & c \end{array}\right] \xrightarrow{\text{Sub R1 from R3}} \left[\begin{array}{ccc|c} 0 & 1 & 1 & a \\ 1 & 1 & 0 & b \\ 1 & 0 & 0 & c-a \end{array}\right]$$

Sub R3 from R2 $\left[\begin{array}{ccc|c} 0 & 1 & 1 & a \\ 0 & 1 & 0 & b-c+a \\ 1 & 0 & 0 & c-a \end{array}\right]$

Sub R2 from R1 $\left[\begin{array}{ccc|c} 0 & 0 & 1 & c-b \\ 0 & 1 & 0 & b-c+a \\ 1 & 0 & 0 & c-a \end{array}\right]$

Switch R1, R3 $\left[\begin{array}{ccc|c} 1 & 0 & 0 & c-a \\ 0 & 1 & 0 & b-c+a \\ 0 & 0 & 1 & c-b \end{array}\right]$

So we can find r, s, t no matter what a, b, c are.

So the conclusion follows.