Research Statement: Frank Thorne

I work in number theory. My primary research interests are in **analytic number theory** and **the enumeration of number fields, class groups and related objects**, and especially in questions at the intersection of these topics.

In my best-known work (joint with Taniguchi; see the included articles), I proved that the number of cubic fields K with $|\mathrm{Disc}(K)| < X$ is

$$N_3(X) = \frac{1}{3\zeta(3)}X + \frac{4(1+\sqrt{3})\zeta(1/3)}{5\Gamma(2/3)^3\zeta(5/3)}X^{5/6} + O(X^{7/9+\epsilon}). \tag{1}$$

In this paper and two follow-ups (one of them unfinished, and joint with Bhargava), we obtained a variety of improvements and generalizations: to 3-torsion in class groups of quadratic fields, to S_3-sextic fields, to discriminants in arithmetic progressions, to cubic extensions of an arbitrary base number field, and more. The unfinished work also improves the error term to $O(X^{2/3+\epsilon})$.

To prove this, we applied properties of the *Shintani zeta function* associated to the space of binary cubic forms. We further developed both (a) the intrinsic theory of these zeta functions, and (b) the analytic number theory tools needed to subsequently extract results such as (1).

Meanwhile, Bhargava, Shankar, and Tsimerman gave an independent proof of (essentially) (1), using the geometry of numbers instead of zeta functions. Careful study has revealed a close relation between these two approaches, which I will further explore.

I am also working on an algebraic approach, using Kummer theoretic techniques pioneered by Cohen. I first used these to prove that the Shintani zeta function cannot be written as a finite sum of Euler products, and this led to follow-up papers with Cohen and Rubinstein-Salzedo. These papers were pursued for their own interest, and also for the light they shed on questions originating from the zeta function approach.

My research goals for the near future, to be pursued in collaboration, are as follows:

- To further develop the connections between the approaches to counting fields described above, as well a 'Heegner point' approach due to Hough, and an algebro-geometric approach due to Zhao. To this end I have organized a series of collaborative workshops at the American Institute of Mathematics involving six of the mathematicians named above.

- To further develop the theory of Shintani's zeta function, with an eye towards applications and open problems. There is a zeta function related to quartic fields, but the existing theory (due mostly to Yukie) is quite complicated, and one important goal is to simplify it, and in a generalizable fashion. (Bhargava informs me that a related process was also indispensible in his own work on quartic and quintic fields.) Taniguchi has made interesting first steps on this, and I will seek to contribute to this developing theory.

- To further study the relationship between Shintani's zeta function and classical and emerging techniques in analytic number theory. For example, can we say anything about the multiplicities of field discriminants, other than what is "well known"? Such questions seem difficult, but so far have seen rather limited attention.

I have begun a collaboration with Pierce, which seems likely to eventually yield interesting results along these lines. I will also strike up collaborations with other analytic number theorists (who may be developing or applying related analytic techniques, and not know about Shintani zeta functions) if opportunities arise.