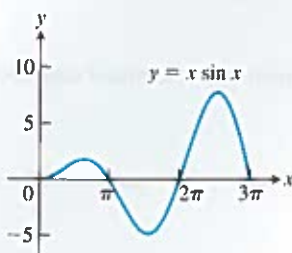**Theory and Examples**

53. Finding area Find the area of the region enclosed by the curve $y = x \sin x$ and the x-axis (see the accompanying figure) for

a. $0 \le x \le \pi$.

b. $\pi \le x \le 2\pi$.

c. $2\pi \le x \le 3\pi$.

d. What pattern do you see here? What is the area between the curve and the x-axis for $n\pi \le x \le (n + 1)\pi$, n an arbitrary nonnegative integer? Give reasons for your answer.
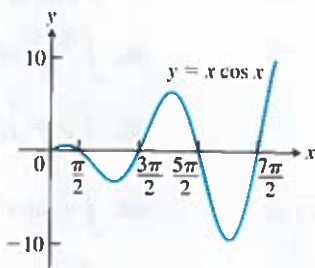

54. Finding area Find the area of the region enclosed by the curve $y = x \cos x$ and the x-axis (see the accompanying figure) for

a. $\pi/2 \le x \le 3\pi/2$.

b. $3\pi/2 \le x \le 5\pi/2$.

c. $5\pi/2 \le x \le 7\pi/2$.

d. What pattern do you see? What is the area between the curve and the x-axis for

$$\left(\frac{2n - 1}{2}\right)\pi \le x \le \left(\frac{2n + 1}{2}\right)\pi,$$

n an arbitrary positive integer? Give reasons for your answer.

55. Finding volume Find the volume of the solid generated by revolving the region in the first quadrant bounded by the coordinate axes, the curve $y = e^x$, and the line $x = \ln 2$ about the line $x = \ln 2$.

56. Finding volume Find the volume of the solid generated by revolving the region in the first quadrant bounded by the coordinate axes, the curve $y = e^{-x}$, and the line $x = 1$

a. about the y-axis.

b. about the line $x = 1$.

57. Finding volume Find the volume of the solid generated by revolving the region in the first quadrant bounded by the coordinate axes and the curve $y = \cos x$, $0 \le x \le \pi/2$, about

a. the y-axis.

b. the line $x = \pi/2$.

58. Finding volume Find the volume of the solid generated by revolving the region bounded by the x-axis and the curve $y = x \sin x$, $0 \le x \le \pi$, about

a. the y-axis.

b. the line $x = \pi$.

(See Exercise 53 for a graph.)

59. Consider the region bounded by the graphs of $y = \ln x$ and $x = e$.

a. Find the area of the region.

b. Find the volume of the solid formed by revolving this about the x-axis.

c. Find the volume of the solid formed by revolving this about the line $x = -2$.

d. Find the centroid of the region.

60. Consider the region bounded by the graphs of $y = \tan^{-1} x$ and $x = 1$.

a. Find the area of the region.

b. Find the volume of the solid formed by revolving this about the y-axis.

61. Average value A retarding force, symbolized by the da the accompanying figure, slows the motion of the weighted so that the mass's position at time t is

$$y = 2e^{-t} \cos t, \qquad t \ge 0.$$

Find the average value of y over the interval $0 \le t \le 2\pi$.
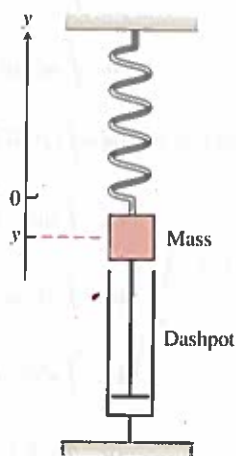
62. Average value In a mass-spring-dashpot system like the Exercise 61, the mass's position at time t is

$$y = 4e^{-t}(\sin t - \cos t), \qquad t \ge 0.$$

Find the average value of y over the interval $0 \le t \le 2\pi$.

Reduction Formulas

In Exercises 63–67, use integration by parts to establish the re formula.

63. $\int x^n \cos x \, dx = x^n \sin x - n \int x^{n-1} \sin x \, dx$

64. $\int x^n \sin x \, dx = -x^n \cos x + n \int x^{n-1} \cos x \, dx$