Examination 3 - Math 141, Frank Thorne (thornef@mailbox.sc.edu)

Friday, October 18, 2016

Please work without books, notes, calculators, or any assistance from others. If you have any questions, feel free to ask me. Please do your work on separate paper; you should staple this sheet to your work (put this on top) and turn in everything together.

Each problem is worth 16 points, except #3 which is worth 20 points.

Please remember: a complete answer must include a picture when relevant, and this picture should be labeled if appropriate.

(1) What is a definite integral? Explain thoroughly and draw a picture.

(2) Find

$$\lim_{x \to \infty} (1 + 2x)^{1/(2 \ln x)}.$$

(3) A 216 m^2 rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

(4) Solve the initial value problem

$$\frac{ds}{dt} = \cos t + \sin t, \qquad s(\pi) = 1.$$

(5) Estimate the area under the graph of the function $f(x) = x^3$ between $x = 0$ and $x = 1$ using (a) a lower sum with four rectangles of equal width, and (b) an upper sum with four rectangles of equal width.

(6) Graph the function $f(t) = t^2 - t$ on the interval $[-2, 1]$ and find its average value over this interval.

#1. Also on 2015 exam, see there for solution.

Rubric:

Area under curve: 4

Signed:
- (w/ explanation) 3 (an explanation in words or by picture is okay)
- (w/o explanation) 2

Picture:
- Illustrating boxes: 4
- Not: 3

Limit of area of rectangles:
- Precise definition and explanation: 5
- One of these two: 3

2. $\lim\limits_{x \to \infty} (1+2x)^{1/(2\ln x)}$:

Its natural logarithm is $\lim\limits_{x \to \infty} \frac{1}{2\ln x} \cdot \ln(1+2x)$

This is $\frac{\infty}{\infty}$ so you use L'Hôpital again.

Using L'Hôpital, the derivative of the top and bottom gives

$$\lim_{x \to \infty} \frac{\ln(1+2x)}{2\ln x} = \lim_{x \to \infty} \frac{\frac{2}{1+2x}}{\frac{2}{x}} = \lim_{x \to \infty} \frac{2x}{2(1+2x)}$$

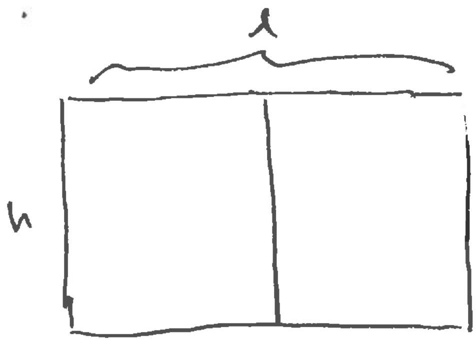
$$= \lim_{x \to \infty} \frac{x}{1+2x}$$

Either by comparing leading terms, or L'Hôpital again

$$= \frac{1}{2}$$

So $\lim\limits_{x \to \infty} (1+2x)^{1/(2\ln x)} = e^{1/2}$.

3.



Picture accurate and labeled : (+6)

Let h = height of fence
(3 pieces of fence in this direction)
l = length of fence
(2 in this direction)

The total amount of fence used is

$$P = 3h + 2l.$$

The area is $216 = h \cdot l$.

So $l = \frac{216}{h}$.

$$P = 3h + 2 \cdot \frac{216}{h} = 3h + \frac{432}{h}.$$

$$\frac{dP}{dh} = 3 - \frac{432}{h^2}.$$

Set this $= 0$: $3 = \frac{432}{h^2}$

$$h^2 = \frac{432}{3} = 144$$

$$\boxed{h = 12 \text{ m}}$$

Solve for h or l: +9.

$$\boxed{l = \frac{216}{12} = 18 \text{ m}}$$

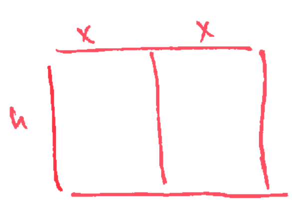
Units : +1

Solve for both : +2

+2

Total amount of fence: $3 \cdot 12 + 2 \cdot 18 = \boxed{72 \text{ m}}$

Note: You can do something like this instead:



This also leads to a correct answer.

4. If $\dfrac{ds}{dt} = \cos t + \sin t$

then $s = \sin t - \cos t + C$.

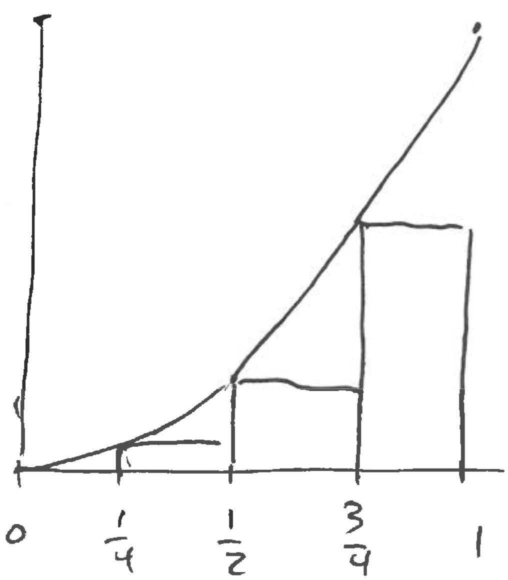
We know $1 = s(\pi) = \sin(\pi) - \cos(\pi) + C$

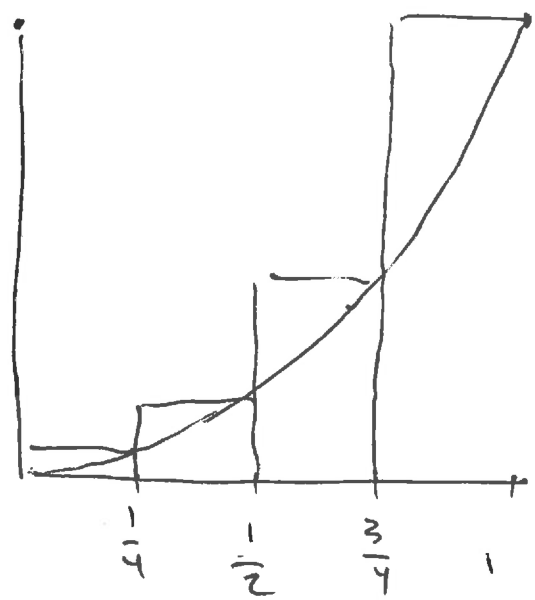
$= 0 - (-1) + C$ so $C = 0$.

So $s = \sin t - \cos t$.

Your solution is not correct if you don't write down $+C$ and solve for $C = 0$, unless you otherwise show it's correct.

5.





Lower estimate:

$$0 + \frac{1}{4}\cdot\left(\frac{1}{4}\right)^3 + \frac{1}{4}\cdot\left(\frac{1}{2}\right)^3 + \frac{1}{4}\cdot\left(\frac{3}{4}\right)^3 = \frac{1}{4}\left(\frac{1}{64} + \frac{8}{64} + \frac{27}{64}\right)$$

$$= \frac{1}{4}\cdot\frac{36}{64} = \frac{9}{64}$$

Upper:

$$\frac{1}{4}\cdot\left(\frac{1}{4}\right)^3 + \frac{1}{4}\cdot\left(\frac{1}{2}\right)^3 + \frac{1}{4}\cdot\left(\frac{3}{4}\right)^3 + \frac{1}{4}\cdot 1 = \frac{1}{4}\left(\frac{1}{64} + \frac{8}{64} + \frac{27}{64} + 1\right)$$

$$= \frac{9}{64} + \frac{1}{4} = \frac{25}{64}.$$

#6. Please see 2015 exam.