Quantifier Examples

January 19, 2024

Abraham Lincoln

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Formalize and negate each part.

Extra Credit Question From Hell

Weierstrass's theorem

Theorem (Weierstrass)

There exists a real-valued function which is continuous everywhere and differentiable nowhere.

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  exists.
- We say that $f(x)$ is continuous everywhere if, for all $a \in \mathbb{R}$, $f(x)$ is continuous at $x = a$. Analogously for 'differentiable everywhere'.

The extra credit problem

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- Formalize, and then negate Weierstrass's theorem in predicate logic. Your predicates should only involve inequalities and should not use any calculus terminology.