

## Homework 6a - Analytic number theory

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**Due Friday, October 7**

**Some challenge problems. If you prefer complex analysis review or practice, please ignore this assignment and work on Homework 6b instead.**

1. (10 points) Find all the automorphs of  $x^2 + xy + y^2$ . Observe that they form a group.  
Write down an explicit, *natural* bijection between these matrices and the sixth roots of unity. No points for just showing that one of your matrix has exact order 6! Explain how, and why, it corresponds to a root of unity.
2. (12 points) Prove that for any discriminant  $d > 0$ , there are infinitely many integer solutions to Pell's equation  $x^2 - dy^2 = 4$ . One path to a solution follows the steps in Andrew Granville's notes; see the bottom of p. 13 in <http://www.dms.umontreal.ca/~andrew/Courses/Chapter4.pdf>.
3. Granville's notes are an interesting starting point for a term project. One natural project (if you know some algebraic number theory) is to explain the correspondence between binary quadratic forms and quadratic fields in much greater detail.