(a) 15x+21y=51 is equivalent to 5x + 7y = 17.

By inspection we see that x=2, y=1 is a solution. Then all solutions are given by

X = 2 - 7 +for all integers +. Y= 1+5+

(b) We have 3/15x + 21y = 3(5x + 7y) and 3+52 so 15x + 21y = 52 has no solutions.

(2)

(a) ||x = 22 (mod 31)

Equivolent to

X = 2 (mod 31) since (11,31) = 1.

(b) 11x = 22 (wod 33).

Here when dividing by II we have to divide the modeles as well.

X = 2 (mod 3).

(c) 11 x = 21 (mod 33).

No solutions.

(Mod 11), the left side is 0 the right side is 21 (mod 11) =10 (mod 11) and 0 \$ 10 (mod 11).

(3) least residue of 72020 (mad 11).

We know 7 = 1 (mod 11) by Fermot's Little Thm.

So 72020 = (710)202 = 1202 = 1 (wod 11)

(4) Solve X= 4 (mod 9) and X= 7 (mod 11). Since (9,11) = 1 the onsuer is a unique congruence ( wod 11 ).

The quick and dirty colotion's x = 4 (mod 9): 4, 13, 22, 31, 40, 49,

X=7(wod 11): 7,18,29,40,51,...

we see 40 in both lists so [x = 40 (wed 99)]

(5) Prove (k, u+k)=d iff (k,n)=d.

Suppose that a is a common divisor of k and n+k
Then al(n+k)-k=n. So a is a common divisor of

Conversely, suppose that bis a common divisor of 1c ord n. Then b| k+ n too, so b is a common divisor of k ord n+k.

so the set of common divisors of I and n+k is the same aso the set of common divisors of the ord n. So both sets have the same greatest element,

(6) let p be an odd prime. we know (p-1)! =-1 (wod p). what is (p-2)! (mod p)?

Notice that (p-1)! = (p-2)!(p-1). So (p-2)! (p-1) = -1 (mod p)

(p-2)! (-1) = -1 (mod p).

We concel the - I from the both sider. (This is okey eince (-1,p)=1.) We get (p-2)! = 1 (mad p).

(7) Suppose (a,m)=1.

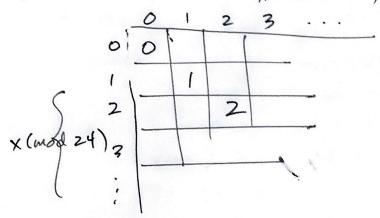
Prove that a, 2a, ... (m-1) a represent different residue classes (mod m).

Suppose to the contrary that ra = sa (wed m), where  $1 \le r < s \le m - 1$ . Then we have

(B-E) a (mod m) with 1≤5-r≤m-2.

we have m/a(s-r), and since (m,a)=1 we have mls-r. But this is impossible if

185-rem, a contradiction.



(a) The box I (mod 24), O (mod 30), among many others, will be empty.

If  $x \equiv 1 \pmod{24}$  then x is even ord if  $y \equiv 0 \pmod{30}$  then  $y \equiv 0 \pmod{30}$ .

(b) what numbers between 0 and 719 appear in the same hox as 0?

Those numbers which are divisible by both 24 and 30. We have

24=23.3

and by urique factorization, a number is dilisible by both if and only if it is dilisible by 23.3.5 = 120.

So the miltiples of 120: 0, 120, 240, 360, 480, 600. (c) what numbers appear in the same box as 171 and 291?

Suppose X = 171 (mod 24) X = 171 (mod 30)

Then X-17/20 (mod 24) x-171 =0 (wod 30)

So X-171 has to be a nultiple of 120 by (b),

(d) - (e). 120 boxes have 6 numbers The rest have none.

Given a box (a, b). Then this box contains a number iff x = a (mod 24), X = b (mod 30) is possesolvable. Since we have x = a (wod 6) and x=b (wed 61, we must have a=b (wod 6). So only 120 no boxes (6.720 = 120) con contain book numbers. Furthermon, by the organish in (c), if x and y one in the same box, then x-y is a multiple of 120. So every box which contains a number contains exactly six of then. Therefore, since there are 120.6 = 720 numbers, every box that can contain numbers does.