

Midterm Examination 1 - Math 141, Frank Thorne (thorne@math.sc.edu)

Thursday, February 9, 2012

Please work without books, notes, calculators, or any assistance from others.

1. (3 points each) For each sentence below, say whether it is logically equivalent to the sentence

If it's snowing, the temperature must be at or below freezing.

(No explanation for your answers is required.)

- (a) If the temperature is above freezing, then it is not snowing.
- (b) It only snows if the temperature is at or below freezing.
- (c) If it is not snowing, then the temperature must be above freezing.
- (d) If the temperature is at or below freezing, then it is snowing.

2. (3 points each) For each sentence below, say whether it is logically equivalent to the sentence

For all animals A , if A has claws then A is dangerous.

(No explanation for your answers is required.)

- (a) Any animal that is dangerous has claws.
- (b) Among all the animals which claws, some are dangerous.
- (c) Every animal with claws is dangerous.
- (d) Some of all the dangerous animals have claws.

3. (3 points each) (For this question, no explanation for your answers is required. Recall that \in means "is an element of" and \subseteq means "is a subset of". Be careful!)

- (a) Is $\{y\} \in \{x, y, z\}$?
- (b) Is $\{y\} \in \{x, \{y\}, \{\{z\}\}\}$?
- (c) Is $\emptyset \subseteq \{42, 1119, \{\{\emptyset\}\}\}$?
- (d) How many elements are in the set $\{a, \{b, c\}\}$?

4. (4 points each) Formally rewrite each of the following statements using appropriate truth variables, predicates, and quantifiers. Specify the domain for all your variables.

For example, an answer may look like $\forall x \in \mathbb{Z} \ (P(x) \rightarrow Q(x))$ where you should say what P and Q are.

- (a) If it walks like a duck and it quacks like a duck, then it is a duck.
- (b) The sum of any two even integers is even.

5. (5 points each) (For this question, no explanation for your answers is required.) Write negations of the following statements.

- (a) If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.
- (b) There is a computer program that gives the correct answer to every question that is posed to it. (Your answer should include the phrase "does not give the correct answer".)

6. (10 points) Determine (with proof) whether the following statement forms are logically equivalent.

$$p \rightarrow (q \rightarrow r), \quad (p \rightarrow q) \rightarrow r.$$

7. (6 points each) Formalize the arguments below using symbols, and identify whether each is valid or invalid. Explain your answer.

You may explain using truth tables, informally in English, or by citing the logical rule of inference used by name. For at least one argument, please explain using truth tables.

- (a) If Frank studied hard for the test, then he got a good score. Frank did not get a good score. Therefore, Frank did not study hard for the test.
- (b) If there are as many rational numbers as there are irrational numbers, then the set of all irrational numbers is infinite. The set of all irrational numbers is infinite. Therefore, there are as many rational numbers as there are irrational numbers.

8. (6 points each) Indicate which of these statements are true and which are false. Explain your answers. (\mathbb{Z}^+ indicates the positive integers.)

- (a) $\forall x \in \mathbb{Z}^+, \exists y \in \mathbb{Z}^+$ such that $x = y + 6$.
- (b) $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}, x = y + 2$.

9. (6 points) Give an example (with explanation) of an argument which is valid but not sound.

10. (6 points) What does it mean to say that

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} \neq 0?$$