3) FTC part 1: $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$

FTC part 1 states that the derivative and definite integral are inverse operations.

FTC PART 2: $\int_a^b f(x)\,dx = F(b) - F(a)$

10

where F is an antiderivative of f

This is important because it gives us an easy way to calculate integrals.

---

4) $s = \frac{\sin t}{1-\cos t}$

$\frac{\sin t}{1-\cos t}$ $\frac{\cos t}{\sin t}$

$\frac{ds}{dt} = \frac{(1-\cos t)\cos t - \sin^2 t}{(1-\cos t)^2} = \frac{\cos t - \cos^2 t - \sin^2 t}{1 - 2\cos t + \cos^2 t}$

8

$(1-\cos t)(1-\cos t)$ $1-2\cos t+\cos^2 t$

$\cos^2 t + \sin^2 t = 1$

$-\cos^2 t - \sin^2 t = -1$

$\boxed{\frac{ds}{dt} = \frac{\cos(t) - 1}{1-2\cos t+\cos^2 t}}$

---

5) $g(x) = \frac{\tan(3x)}{(x+7)^4}$

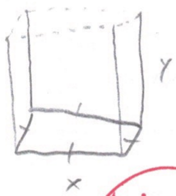
$\frac{\tan(3x)}{(x+7)^4}$ $\frac{3\sec^2 3x}{4(x+7)^3 \cdot 1}$

$g'(x) = \frac{(3\sec^2 3x)(x+7)^4 - (4(x+7)^3)(\tan(3x))}{(x+7)^8}$

10

$\boxed{g'(x) = \frac{(3\sec^2(3x))(x+7) - 4\tan(3x)}{(x+7)^5}}$

8)



$500 ft^3 = V = x^2 y$ $\qquad Y = \frac{\quad}{x^2}$

let x = base length.
let y = height

$A = x^2 + 4xy$

$A = x^2 + 4x\left(\frac{500}{x^2}\right) = x^2 + \frac{2000}{x}$

+10

$\frac{dA}{dx} = 2x - \frac{2000}{x^2}$ $\quad$ Set $= 0$

$\frac{2000}{x^2} = 2x$

$1000 = x^3$

$(10^2)5 = 500$

$Y = \frac{500}{(10)^2} = \frac{500}{100} = 5$

$\boxed{\begin{matrix} X = 10ft \\ Y = 5ft \end{matrix}}$

Weight was taken into account by minimizing surface area, meaning that the amount of steel used was minimized. in turn, decreases the weight.

9) $\int_1^8 \frac{2x^{1/3} + 2 - x - x^{2/3}}{x^{1/3}} dx = \int_1^8 \frac{2x^{1/3}}{x^{1/3}} + \frac{2}{x^{1/3}} - \frac{x}{x^{1/3}} - \frac{x^{2/3}}{x^{1/3}} dx$

$= \int_1^8 2 + \frac{2}{x^{1/3}} - x^{2/3} - x^{1/3} dx = \int_1^8 2 + 2x^{-1/3} - x^{2/3} - x^{1/3} dx$

$= \left[2x + \frac{2x^{2/3}}{\frac{2}{3}} - \frac{x^{5/3}}{\frac{5}{3}} - \frac{x^{4/3}}{\frac{4}{3}}\right]_1^8 = \left[2x + 3x^{2/3} - \frac{3}{5}x^{5/3} - \frac{3}{4}x^{4/3}\right]_1^8$

+10

$\left(16 + 3(8)^{2/3} - \frac{3}{5}(8)^{5/3} - \frac{3}{4}(8)^{4/3}\right) - \left(2 + 3(1)^{2/3} - \frac{3}{5}(1)^{5/3} - \frac{3}{4}(1)^{4/3}\right)$

$\boxed{14 + 3(8)^{2/3} - \frac{3}{5}(8)^{5/3} - \frac{3}{4}(8)^{4/3} - 3(1)^{2/3} + \frac{3}{5}(1)^{5/3} + \frac{3}{4}(1)^{4/3}}$

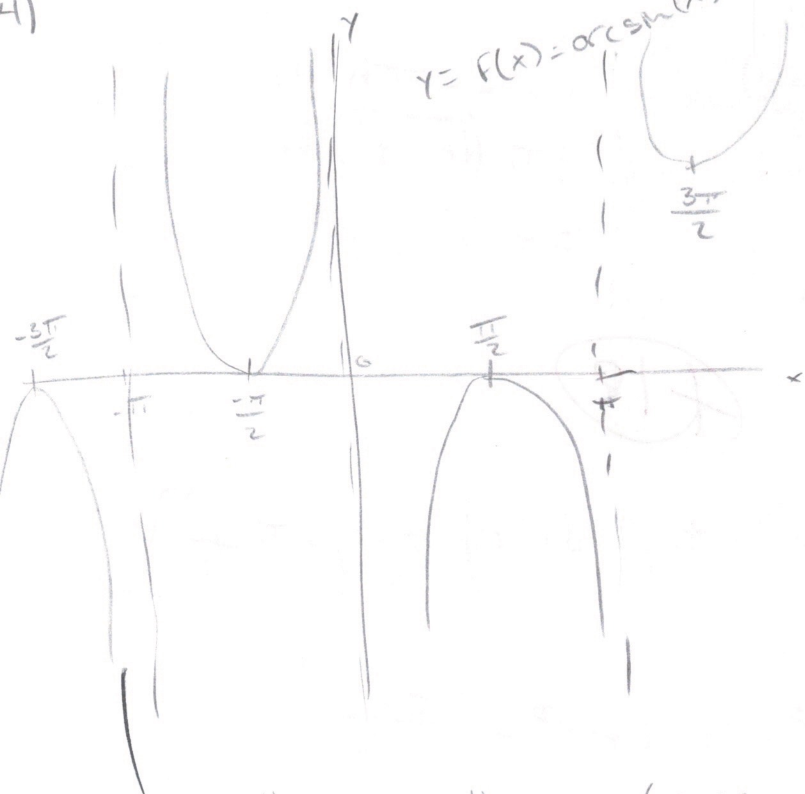
10) $\int \frac{1}{\sqrt{t}} \cos(\sqrt{t} + 3) dt$ $\qquad u = \sqrt{t} + 3$ $\quad \frac{du}{dt} = \frac{-1}{2\sqrt{t}}$ $\qquad -2du = \frac{1}{\sqrt{t}} dt$

$-2\int \cos u \, du = -2 \sin u + c = \cancel{-}2 \sin(\sqrt{t} + 3) + c$

$+2\sin(\sqrt{t} + 3)$

+9

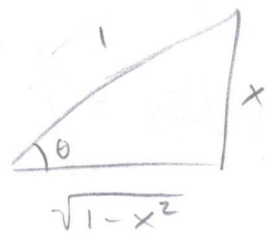
4)



Domain: $(k\pi, (k+1)\pi)$

range: $(-\infty, 0) \cup (0, \infty)$

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$



using pythagoreans theorem $(a^2+b^2=c^2)$ and setting the triangle to ~~where~~ $\sin\theta = x \to$ ( ). We can determine that the missing side of the triangle is $\sqrt{1-x^2}$.

---

15) $\lim\limits_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x} = \frac{\ln(e^0-1)}{\ln 0} = \frac{0}{0}$ L'Hôpital

$\lim\limits_{x\to 0^+} \frac{\frac{e^x}{e^x-1}}{\frac{1}{x}} \lim\limits_{x\to 0^+} \frac{e^x}{e^x-1} \cdot \frac{x}{1} = \lim\limits_{x\to 0^+} \frac{xe^x}{e^x-1} = \frac{0}{0}$ L'Hôpital

+10

$\lim\limits_{x\to 0^+} \frac{e^x+xe^x}{e^x} = \lim\limits_{x\to 0} 1+x = 1+0 = \boxed{1}$