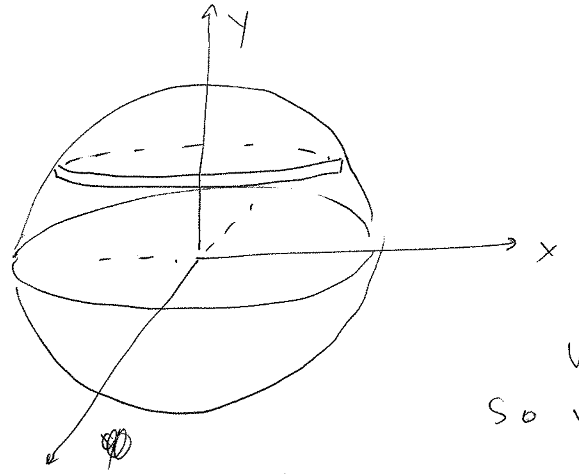
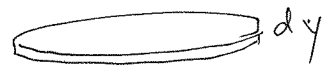
1. Find the volume of a sphere of radius 6.





Typical Slice:

Volume is $\pi x^2 \, dy$.

We know that $x^2 + y^2 = 36$,

So volume of the slice is $\pi(36 - y^2)\,dy$.

The volume of the sphere is

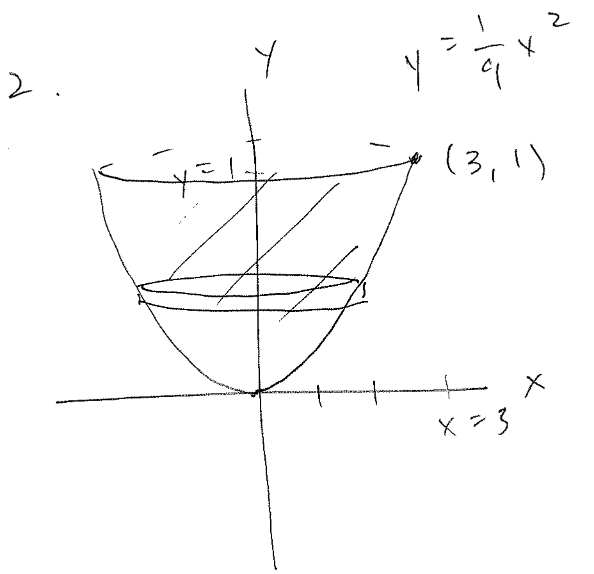
$$\int_{-6}^{6} \pi(36 - y^2)\,dy = \pi\left[36y - \frac{y^3}{3}\right]_{-6}^{6}$$

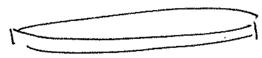
$$= \pi\left[\left(216 - \frac{216}{3}\right) - \left(-216 - \frac{-216}{3}\right)\right]$$

$$= \pi\left[216\left(1 - \frac{1}{3} + 1 - \frac{1}{3}\right)\right]$$

$$= \pi \cdot 216 \cdot \frac{4}{3} = 288\pi.$$

2.





Typical slice:

Volume is $\pi \cdot x^2 \, dy$

$= \pi \cdot \frac{9y}{\cancel{9}} \, dy$.

The volume of the solid is

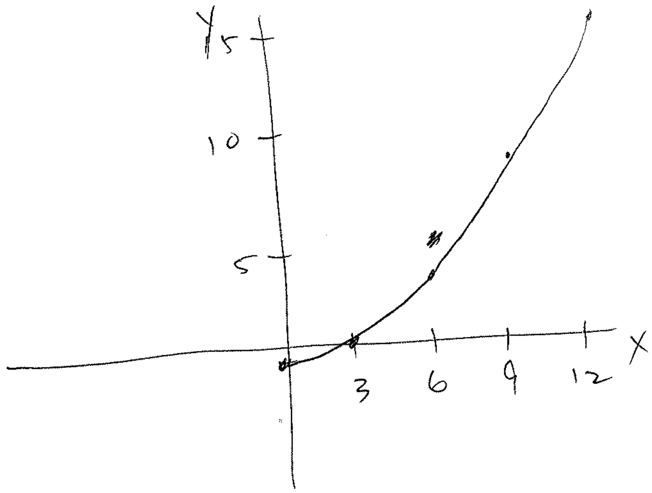
$$\int_0^1 \frac{9\pi y}{\cancel{9}}\,dy = \frac{9}{2}\frac{\pi y^2}{\cancel{18}}\Big]_0^1 = \cancel{\frac{\pi}{18}} \cdot \frac{9\pi}{2}.$$

3. For a curve to ~~depend~~ be defined by parametric equations means that both x and y depend on another variable t (which we think of as time).

For example, if $x = 3t$ and $y = t^2 - 1$ for $0 \le t \le 4$

then the following points are on the graph:

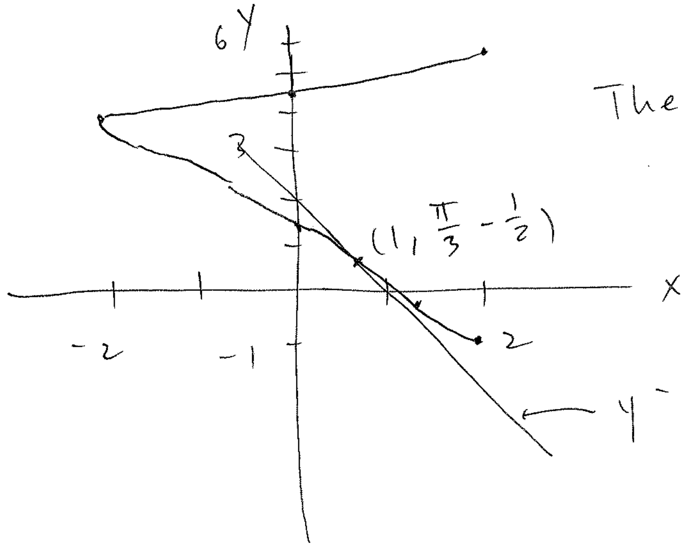
t	0	1	2	3	4
x	0	3	6	9	12
y	-1	0	3	8	15



we think of the graph as representing the motion of a particle from $(0,-1)$ $(t=0)$ to $(12, 15)$ $(t=4)$.

4. Some points:

t	0	$\pi/4$	$\pi/2$	π	$3\pi/2$	2π
x	2	$\sqrt{2}$	0	-2	0	2
y	-1	$\frac{\pi}{4} - \frac{\sqrt{2}}{2}$	$\frac{\pi}{2}$	$\pi + 1$	$\frac{3\pi}{2}$	$2\pi - 1$



The graph looks roughly like this.

$y - \left(\frac{\pi}{3} - \frac{1}{2}\right) = \left(\frac{-\sqrt{3}}{3} - \frac{1}{2}\right)(x - 1).$

4 (cont.)

We have $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{1+\sin t}{-2\sin t}$. When $t = \pi/3$, this is

$$\frac{1+\frac{\sqrt{3}}{2}}{-2\frac{\sqrt{3}}{2}} = \frac{-1-\frac{\sqrt{3}}{2}}{\sqrt{3}} = \frac{-\sqrt{3}-\frac{3}{2}}{3} = -\frac{\sqrt{3}}{3} - \frac{1}{2}.$$

Also $(x,y) = \left(1, \frac{\pi}{3} - \frac{1}{2}\right)$

and so the tangent line has equation

$$y - \left(\frac{\pi}{3} - \frac{1}{2}\right) = \left(-\frac{\sqrt{3}}{3} - \frac{1}{2}\right)(x-1).$$

6. 1 - (c)

2 - (b)

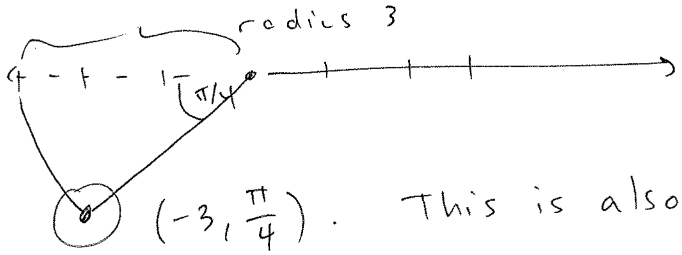
3 - (a).

1 must be (c) because it contains the point $(0,0)$.

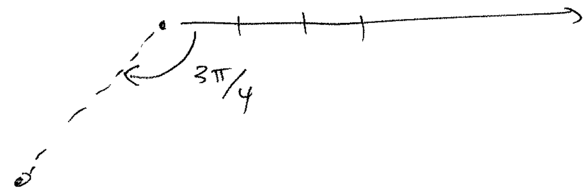
2 must be (b) because it is the only graph with $x > 0$ always.

3 must be (a) because it contains $(2,0)$, or by process of elimination.

6.



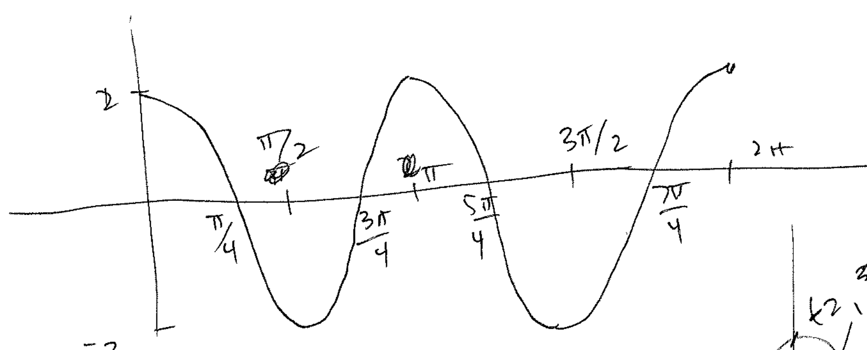
$\left(-3, \frac{\pi}{4}\right)$. This is also $\left(3, -\frac{3\pi}{4}\right)$:



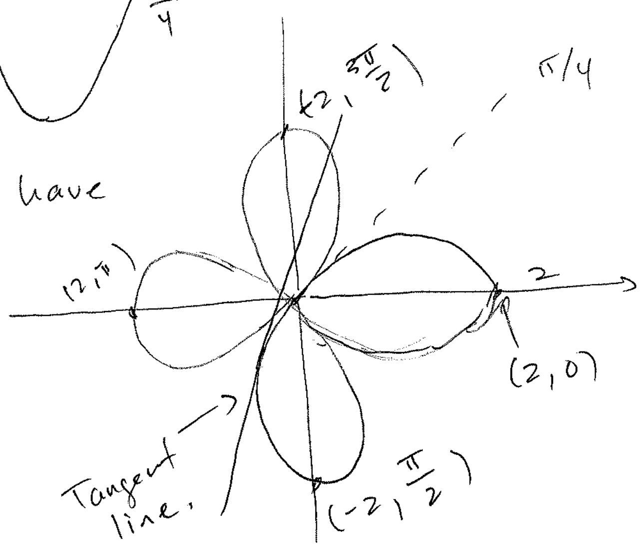
The Cartesian coordinates are $\left(-3\cos\frac{\pi}{4}, -3\sin\frac{\pi}{4}\right)$

$= \left(\frac{-3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)$.

7. In Cartesian coordinates we have $r = 2\cos 2\theta$



So in polar coordinates we have



We have $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{\frac{d}{d\theta}(r \sin\theta)}{\frac{d}{d\theta}(r\cos\theta)}$

$$= \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}.$$

Here, $\dfrac{dr}{d\theta} = -4\sin 4\theta.$

If $\theta = \frac{\pi}{3}$ then $\sin\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{1}{2}$

$r = 2\cos\left(\frac{2\pi}{3}\right) = -1$

$\dfrac{dr}{d\theta} = -4\sin\left(\frac{4\pi}{3}\right) = -2\sqrt{3}$

So $\dfrac{dy}{dx} = \cancel{\dfrac{\frac{\sqrt{3}}{2} - 2\sqrt{3}\cdot\frac{1}{2}}{(-1)\cdot\frac{1}{2} - 2\sqrt{3}\cdot\left(\frac{\sqrt{3}}{2}\right)}} \quad \dfrac{(-1)\left(\frac{1}{2}\right) - 2\sqrt{3}\cdot\frac{\sqrt{3}}{2}}{1\cdot\frac{\sqrt{3}}{2} + (-2\sqrt{3})\cdot\frac{1}{2}}$

$$= \frac{-\frac{1}{2} - 3}{\frac{\sqrt{3}}{2} - \sqrt{3}} = \frac{-\frac{7}{2}}{-\frac{\sqrt{3}}{2}}$$

$$= \frac{7}{\sqrt{3}}.$$

8. The area is given by

$$\int_a^b \frac{1}{2} r^2 \, d\theta = \int_0^{2\pi} \frac{1}{2} (2 + 2\cos\theta)^2 \, d\theta .$$

The integrand varies between 0 and 8, so $2\pi \cdot 8 \approx 16\pi$ seems possibly reasonable

The graph looks like a "pinched" circle of radius about $\frac{5}{2}$, so this yields an approximation of $\pi \cdot \frac{25}{4}$ which is a lot lower!

The actual area is 6π.