Brian Smit

138.5

1) The derivative of a function $f(x)$ at the point $x=a$ is the slope of the tangent line at that point. We can use a limit as $h \to 0$, h being the distance between 2 points.

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

We use this limit because it is the change in y divided by the change in x, the definition of slope.

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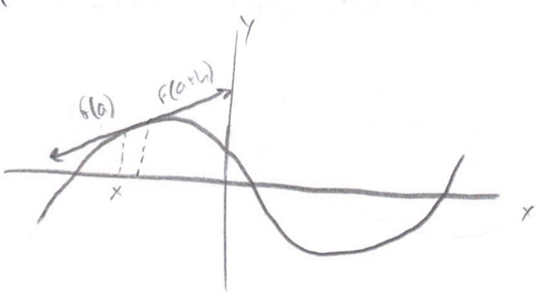

as $h \to 0$ the slope between the two points approaches the slope of 1 point.

2) The definite integral of a function $f(x)$ from $x=a$ to $x=b$ is signed area under the graph from $x=a$ to $x=b$.

we write $\int_a^b f(x)\,dx$.

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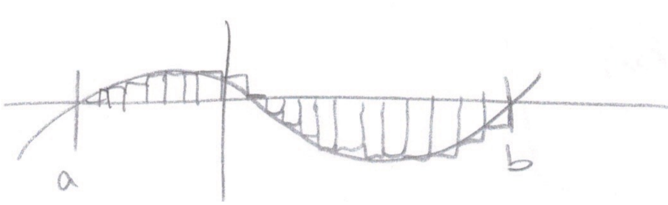
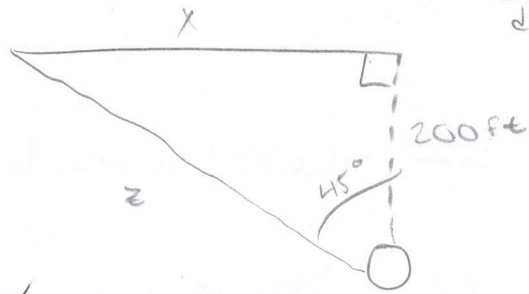
We can approximate this area using rectangles with width Δx. Taking the limit of the sum of all rectangles as $x \to 0$ approximates the area under the curve

$$\lim_{\Delta x \to 0} f(a)\cdot\Delta x + f(a+\Delta x)\cdot\Delta x + f(a+2\Delta x)\cdot\Delta x \ldots + f(b-\Delta x)\cdot\Delta x + f(b)\Delta x$$



6)



$\frac{dx}{dt} = ?$

$\frac{1}{6}\,\frac{\text{revolution}}{\text{min}} = \frac{d\theta}{dt}$

$\theta = 45^\circ \cdot \frac{\pi}{180}$

$\theta = \frac{45\pi}{180}$

$\theta = \frac{9\pi}{36}$

$\theta = \frac{\pi}{4}$

$\tan\left(\frac{\pi}{4}\right) = \frac{x}{200}$

$200\tan\left(\frac{\pi}{4}\right) = x$

$200\sec^2\left(\frac{\pi}{4}\right) = \frac{dx}{dt}$

$200(2) = \frac{dx}{dt}$

$\boxed{400\ \text{ft/min} = \frac{dx}{dt}}$

$\frac{\pi}{4}\ \cancel{2\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)}$

$\sec^2 = \frac{1}{\cos^2}$

$\sec^2\left(\frac{\pi}{4}\right) = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \left(\frac{2}{\sqrt{2}}\right)^2 = \frac{4}{2} = 2$

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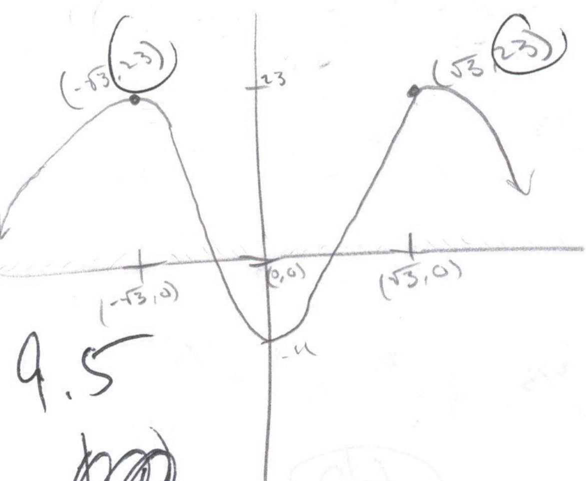
7) $f(x) = -x^4 + 6x^2 - 4 = x^2(6 - x^2) - 4$



9.5

$f'(x) = -4x^3 + 12x$

Set = 0

$0 = x(-4x^2 + 12)$ $\quad x = 0$

$4x^2 = 12 \quad x^2 = 3$

$x = \pm\sqrt{3}$

crit points $\boxed{x = 0, \pm\sqrt{3}}$

+	$-\sqrt{3}$	−	0	+	$\sqrt{3}$	−	$f'(x)$

$f'(2) = -32 + 24 = -$

$f'(1) = +$

$f'(-1) = 4 - 12 = -$

$f'(-2) = 32 - 12 = +$

inc: $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

dec: $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

local max: $x = \pm\sqrt{3}$

local min: $x = 0$

abs max: DNE, 2 local max of same y value

abs min: DNE, graph continues in downward direction

Concave up: $(-1, 1)$

concave down: $(-\infty, -1) \cup (1, \infty)$

$f''(x) = -12x^2 + 12$

Set = 0

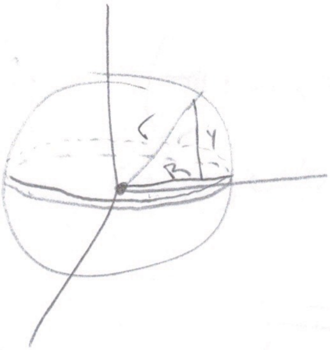
$12x^2 = 12$

$x^2 = 1$

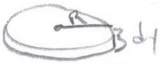
$x = \pm 1$

−	-1	+	1	−	$f''(x)$

11)



typical slice:



volume of 1 slice $= \pi R^2 \, dy$

$= \pi \left(\sqrt{r^2 - y^2}\right)^2 dy$

$= \pi (r^2 - y^2) \, dy$

$R^2 + y^2 = r^2$

$R = \sqrt{r^2 - y^2}$

+10

$V_{sphere} = \int_{-r}^{r} \pi (r^2 - y^2) \, dy = \int_{-r}^{r} \pi r^2 - \pi y^2 \, dy = \left[\pi r^2 y - \pi \frac{y^3}{3} \right]_{-r}^{r}$

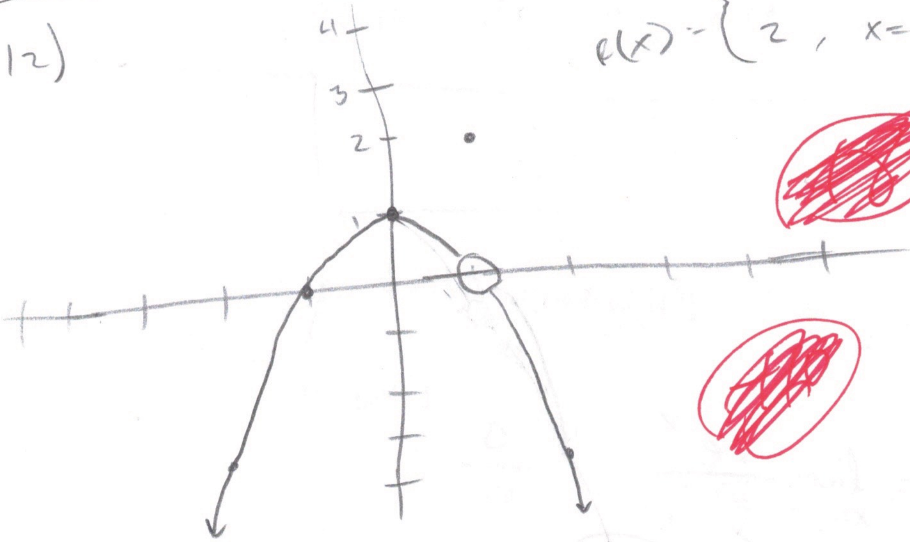
$= \pi r^3 - \frac{\pi r^3}{3} - \left(-\pi r^3 + \frac{\pi r^3}{3}\right) = 2\pi r^3 - \frac{2\pi r^3}{3}$

$= \frac{6\pi r^3}{3} - \frac{2\pi r^3}{3} = \frac{4}{3}\pi r^3$

12)

$f(x) = \begin{cases} 1 - x^2, & x \neq 1 \\ 2, & x = 1 \end{cases}$

+9



$\lim_{x \to 1^+} f(x) = 0$

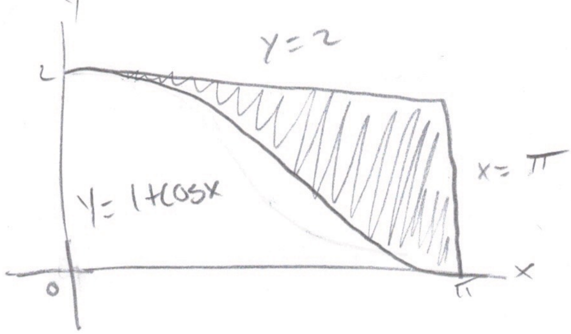
$\lim_{x \to 1^-} f(x) = 0$

$\lim_{x \to 1} f(x) = 0$

13) $\lim_{x \to -3} (x^2 - 13) = (-3)^2 - 13 = 9 - 13 = -4$

+10

16)



$A_{Box} = 2\pi$

$2\pi - \pi = \boxed{\pi}$

$A_{curve} = \int_0^{\pi} 1 + \cos x \, dx$

$= \left[x + \sin x \right]_0^{\pi} = \pi + \cancel{0 - 0 + 0}$

$= \pi$

+10