

State High School Mathematics Tournament

University of South Carolina

Tiebreaker – April 22, 2023

Tiebreaker Rules

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- ▶ The answer(s) **closest to the truth** (in either direction) win the tiebreaker.

Tiebreaker Question

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What is $p(9)$?

Tiebreaker Answer

Answer. 30.

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By the Hardy-Ramanujan formula

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} A_k(n) \sqrt{k} \frac{d}{dn} \left(\frac{\sinh \left(\frac{\pi}{k} \sqrt{\frac{2}{3} \left(n - \frac{1}{24} \right)} \right)}{\sqrt{\frac{2}{3} \left(n - \frac{1}{24} \right)}} \right),$$

Tiebreaker 2

How many integers $n \leq 202020$ can be written in the form

$$n = a^3 + b^3 + c^3,$$

where a, b, c are positive integers?

Tiebreaker 2 Answer

Answer. 21581.