UNIVERSITY OF SOUTH CAROLINA

Frank H. Thorne

Thursday, November 29, 2012

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I write to overwhelmingly support **John Willis**'s application to the mathematics Ph.D. program at the University of Texas.

John is currently enrolled in our Ph.D. program at the University of South Carolina, and it is with some sadness that I write: he has tremendous potential, he loves talking about math, he is very sociable and friendly, he loves talking about math, and he is *easily* our strongest student in number theory. I don't want to see him leave. But it has become obvious that we cannot provide him the opportunities he deserves.

In John's first year at USC (also mine), he took my graduate course in analytic number theory. My course was ambitious; I taught out of Davenport's *Analytic Number Theory*, as opposed to gentler introductions to the subject which had been used at USC in the past. His weekly problem sets were outstanding: his exposition was thorough and clear, and he tackled difficult problems which were not attempted by anyone else in the class.

He also did an excellent term project on Deligne and Serre's work on Galois representations and modular forms, explaining how the Chebotarev Density Theorem implies that for a fixed modular form $f(z) = \sum_n a(n)q^n$ and integer m, almost all of the coefficients $a(n)$ are divisible by m. He thoroughly addressed both the algebraic and the analytic aspects of this result.

Finally, John was very engaged, and consistently asked me thought-provoking questions during and after class. Overall, John's performance in my class was roughly equalled by one other student, and surpassed by nobody.

Although John excelled in my analytic number theory course, he is very much an algebraist at heart, indeed much more so than any of the number theorists in our department. He has been reading everything algebraic he can get his hands on, including: Vakil's notes on algebraic geometry, Serre's *Local Fields*, Silverman's *The Arithmetic of Elliptic Curves*, and a research monograph by Darmon. He has been learning substantial amounts of algebraic number theory, algebraic geometry, group cohomology, and related topics, largely on his own.

His progress has been quite apparent. This was especially visible when he recently gave two hour-long lectures in our number theory seminar on Heegner points, complex multiplication, and the Gross-Zagier formula. The topic is extremely technical (I will omit the details), but he did an excellent job of giving an overview of the subject, and also of presenting a couple of nice proofs where appropriate. Plainly, John is developing fluency in the language and techniques of arithmetic geometry: his talk involved algebraic number theory, class field theory, derived functors, and a diagram chase.

I have also found John an excellent person to talk math with overall. For random questions in number theory outside my immediate research interests and those of my colleagues, I have found John the best person in the department to talk to. If I have some question about tensor products, or Galois cohomology, or what have you, I always ask John if I run into him in the hall. It is not that he will know the answer of the top of his head (usually he doesn't), but he is always interested and eager to talk, and typically comes up with something intelligent to say.

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John has the clear potential to be an outstanding arithmetic geometer, and he is well on his way to knowing more about the subject than anyone here at USC. This is one reason why I am supporting his application so strongly: Michael Filaseta and I are analytic number theorists, and Matt Boylan works in modular forms. While all of us have some interest in arithmetic geometry, none of us is an expert in the topic, and John deserves the chance to work with a legitimate expert.

Still more important than our faculty are our other graduate students. John is a very social mathematician, and he very much likes to talk math with others. He definitely does so here, but he doesn't have any peers who share his interest in learning the heavy machinery of arithmetic geometry. (Most of our graduate students share Michael Filaseta's taste for the more easily accessible side of the subject.) I think John would very much like to form a group of a half-dozen or so graduate students, and go through Hartshorne's *Algebraic Geometry* or Vakil's notes (among other books), and do a large proportion of the exercises. Unfortunately, none of our other current graduate students has the right combination of background, motivation, interest, and ability. John does, and in the company of the right peer group I would expect him to be even more successful than he already is.

I recently finished my Ph.D. at Wisconsin, so I can assert with confidence that I would expect him to succeed there. I'm less familiar with Texas's program, but my understanding is that it has roughly comparable expectations, and so I would certainly expect him to succeed at Texas also. I know Texas number theory graduates Kimberly Hopkins, Adriana Salerno, and Riad Masri, and current Texas postdoc Ekin Ozman (formerly a fellow student at Wisconsin), and I think John compares well with this group. To the extent that I can reasonably judge, I would say that John especially reminds me of Ozman in terms of ability, interests, and motivation.

John Willis is our strongest student in number theory, and I'm not aware of any stronger student at USC in any area of math. He is very patient, curious, and persistent, and he has the demeanor of a marathon runner than a sprinter – which bodes very well for his long term prospects in math. He is the one current USC student whom I could imagine surpassing me and my colleagues, and I recommend him to Texas and other top-notch Ph.D. programs without hesitation.

Sincerely,

Frank Thorne
Assistant Professor of Mathematics