The Mathematics of Game Shows

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1 Introduction

We will begin by watching a few game show clips and seeing a little bit of the math behind them.

1.1 Example: The Price Is Right, Contestants' Row

We begin with the following clip from The Price Is Right:

`https://www.youtube.com/watch?v=TmKP1a03E2g`

Game Description (Contestants' Row - The Price Is Right): *Four contestants are shown an item up for bid. In order, each guesses its price (in whole dollars). You can't use a guess that a previous contestant used. The winner is the contestant who bids the closest to the actual price without going over.*

In this clip, the contestants are shown some scuba equipment, and they bid 750, 875, 500, and 900 in that order. The actual price is $994, and the fourth contestant wins. What can we say about the contestants' strategy?

- As a first step, it is useful to precisely describe the results of the bidding: the first contestant wins if the price is in $[750, 874]$[1]; the second, if the price is in $[875, 899]$; the third, in $[500, 749]$; the fourth, in $[900, \infty)$. If the price is less than $500, then all the bids are cleared and the contestants start over.

  We can see who did well before we learn how much the scuba gear costs. Clearly, the fourth contestant did well. If the gear is worth anything more than $900 (which is plausible), then she wins. The third contestant also did well: he is left with a large range of winning prices – 250 of them to be precise. The second contestant didn't fare well at all: although his bid was close to the actual price, he is left with a very small winning range. This is not his fault: it is a big disadvantage to go early.

[1] Recall that $[a, b]$ is mathematical notation for all the numbers between a and b.

- The next question to ask is: could any of the contestants have done better?

  We begin with the fourth contestant. Here the answer is *yes*, and her strategy is **dominated** by a bid of \$876, which would win in the price range $[876, \infty)$. In other words: *a bid of \$876 would win every time a bid of \$900 would, but not vice versa.* Therefore it is better to instead bid \$876 if she believes the scuba gear is more than \$900.

  Taking this analysis further, we see that there are exactly four bids that make sense: 876, 751, 501, or 1. Note that each of these bids, except for the one-dollar bid, screws over one of her competitors, and this is not an accident: Contestant's Row is a **zero-sum game** – if someone else wins, you lose. If you win, everyone else loses.

- The analysis gets much more subtle if we look at the *third* contestant's options. **Assume that the fourth contestant will play optimally.** (Of course this assumption is very often not true in practice.

  Suppose, for example, that the third contestant believes that the scuba gear costs around \$1000. The previous bids were \$750 and \$875. Should he follow the same reasoning and bid \$876? Maybe, but this exposes him to a devastating bid of \$877.

There is much more to say here, but we go on to a different example.

1.2 Deal or No Deal

Here is a clip of the game show **Deal or No Deal**:

`https://www.youtube.com/watch?v=I3BzYiCSTo8`

The action starts around 4:00.

Game Description (Deal or No Deal): *There are 26 briefcases, each of which contains a variable amount of money from \$0.01 to \$1,000,000, totalling \$3,418,416.01, and averaging \$131477.53. The highest prizes are \$500,000, \$750,000, and \$1,000,000.*

The contestant chooses one briefcase and keeps it. Then, one at a time, the contestant chooses other briefcases to open, and sees how much money is in each (and therefore establishes that these are not the prizes in his/her own briefcase). Periodically, the 'bank' offers to buy the contestant out, and give him/her a fixed amount of money to quit playing. The contestant either accepts or says 'no deal' and continues playing.

The **expected value** of the game is the average amount of money you expect to win. (We'll have much more to say about this.) So, at the beginning, the expected value of the game is \$131477.53, presuming the contestant rejects all the deals. In theory, that means that the contestant should be equally happy to play the game or to receive \$131477.53. (Of course, this might not be true in practice.)

Now let's look at the game after he chooses six briefcases. The twenty remaining contain a total of \$2936366, or an average of \$146818. The expected value has gone up, because the contestant eliminated mostly small prizes and none of the three biggest. If he wants to maximize his expected value (and I repeat that this won't necessarily be the case), then all he has to know is that

$$146818 > 51000$$

and so he keeps playing.

The show keeps going like this. After five more cases are eliminated, he again gets lucky and is left with fifteen cases containing a total of \$2808416, so an average of \$187227. The bank's offer is \$125,000 which he refuses. And it keeps going.

1.3 Jeopardy – Final Jeopardy

Here we see the Final Jeopardy round of the popular show Jeopardy:

`https://www.youtube.com/watch?v=DAsWPOuF4Fk`

Game Description (Jeopardy, Final Round): *Three contestants start with a variable amoung of money (which they earned in the previous two rounds). They are shown a category, and are asked how much they wish to wager on the final round. The contestants make their wagers privately and independently.*

After they make their wagers, the contestants are asked a trivia question. Anyone answering correctly gains the amount of their wager; anyone answering incorrectly loses it.

Perhaps here an English class would be more useful than a math class! This game is difficult to analyze; unlike our two previous examples, the players play *simultaneously* rather than *sequentially*.

In this clip, the contestants start off with \$9,400, \$23,000, and \$11,200 respectively. It transpires that nobody knew who said that *the funeral baked meats did coldly furnish forth the marriage tables*. (Richard II? Really? When in doubt, guess Hamlet.) The contestants big respectively \$1801, \$215, and \$7601.

We will save a thorough analysis for later, but we will make one note now: the second contestant can obviously win. If his bid is less than \$600, he will end up with more than \$22,400.