

Practice Examination - Math 142, Frank Thorne (thorne@math.sc.edu)

The Exam is Thursday, October 24, 2013

Instructions and Advice:

- There are eight questions, some of which are shorter than others.
- You are welcome to as much scratch paper as you need. Turn in everything you want graded, and throw away everything you do not want graded.
- **Draw pictures where appropriate.** If you have any doubt, then a picture is appropriate.
- Be clear, write neatly, explain what you are doing, and show your work. **This is especially important for earning partial credit** in case your work contains one or more mistakes. Be warned that **work I cannot understand will not receive any credit.**
- 75 minutes is a long time. Don't dilly-dally, but don't rush. **You are strongly advised to take the entire 75 minutes to complete the examination.** If you finish early, you have the opportunity to check your work.
- Please work without books, notes, calculators, or any assistance from others.
- I will be at the front of the room; if you have any questions, feel free to ask me.

**GOOD LUCK!**

- (1) (12 points) Find the volume of sphere of radius 6.

(*Warning:* You will not get credit for remembering the formula and plugging in 6. A correct answer will carry out the calculus computation. You *may* use the fact that a circle of radius  $r$  has area  $\pi r^2$ , without further justification.)

Draw a picture illustrating your computation.

- (2) (10 points) The region bounded by  $y = \frac{1}{9}x^2$ ,  $x = 3$ , and  $y = 0$  is rotated around the  $y$ -axis. Find the volume of the resulting solid.

As part of your answer, sketch the region, the resulting solid, and a typical disk or washer.

- (3) (10 points) Explain what it means for a curve to be defined by parametric equations. Give and graph an example. (Your example should be different than other questions on this exam.)

- (4) (10 points) Sketch the curve given by the equations  $x = 2 \cos t$ ,  $y = t - \cos t$  for  $0 \leq t \leq 2\pi$ .

(10 points) In addition, find the equation of the tangent line when  $t = \pi/3$ , and sketch it on your graph.

- (5) (10 points) Three graphs are given, along with parametric equations for them (in a different order). Match the graphs with the equations, and give reasons for your choices.

- (6) (10 points) Plot the point whose polar coordinates are  $(-3, \pi/4)$ . In addition, find Cartesian coordinates of this point, and find another set of polar coordinates  $(r, \theta)$  for the same point with  $r > 0$ .

- (7) (10 points) Sketch the curve with the polar equation  $r = 2 \cos 2\theta$  for  $0 \leq \theta \leq 2\pi$ .

(8 points) Also, find the slope of the tangent line when  $\theta = \pi/3$ .

- (8) (10 points) A graph of the curve  $r = 2 + 2 \cos \theta$  is provided. Write down a definite integral which represents the area of the curve. In addition, use the graph to give a rough estimate for the value of this integral.