### The Mathematics of Game Shows

#### Frank Thorne

June 16, 2025

#### 1 Deal or No Deal

https://www.youtube.com/watch?v=JFQvVeOFreY

- 1. At the beginning, what is the expected value of the contestant's suitcase?
- 2. After the first round, what is the expected value?
- 3. What about the second?
- 4. Do you agree with the contestant's choices?
- 5. How do you think the banker is deciding what to offer?

# 2 The Price Is Right – Contestants' Row

https://www.youtube.com/watch?v=kcfmRgahBss

What do you see?

#### The Coin Stealing Game:

- 1. For two to five players. Determine the order in advance.
- 2. Start with a pile of coins. The first player takes some or all of the coins.
- 3. Each successive player, in turn, takes all or part of (a) the remaining coins, or (b) any one other player's coins.
- 4. Each player's objective is to get as many coins as possible. You're *not* trying to get more coins than the other players you don't care one way or the other how much they get, only how much you get.

#### Questions:

- 1. Would you rather go first, last, or in the middle?
- 2. In a two-player game, what is each player's optimal strategy? If every player plays optimally, how many coins do they get?
- 3. What about a three-player game? Four? Five? n, for any positive integer n?
- 4. What does this have to do with The Price Is Right?

## 3 The Price Is Right – Plinko

https://www.youtube.com/watch?v=GnIEc4s12x8

We will discuss a simplified model for the game.

- 1. How realistic is the model?
- 2. According to the model, how do you compute the probability of landing in each slot?
- 3. Where should the contestant drop the chip?
- 4. If the contestant wins every chip, what is the expected value of the game?
- 5. If the contestant guesses the prices of the small prizes at random, what is the expected value of the game?

Now try this Plinko Simulator:

https://phet.colorado.edu/sims/html/plinko-probability/latest/plinko-probability\_en.html

- 1. Describe the distribution that is taking place.
- 2. (Challenge!) Let n be the number of rows. As  $n \to \infty$ , can you find (with proof!) the limiting distribution of the balls?

### 4 Press Your Luck

Watch the following video, from 14:30 to 20:20.

https://www.youtube.com/watch?v=WltjaxiowW4

- 1. Describe what you see.
- 2. According to your description, compute the approximate probability of this happening by chance.

## 5 Golden Balls – Split or Steal

https://www.youtube.com/watch?v=yM38mRHY150

- 1. What is each player's optimal strategy?
- 2. If both players played their optimal strategy, then why did ... that happen?

You can compare with the classical *Prisoner's Dilemma*:

Two prisoners, A and B, suspected of committing a robbery together, are isolated and urged to confess. Each is concerned only with getting the shortest possible prison sentence for himself; each must decide whether to confess without knowing his partner?s decision. Both prisoners, however, know the consequences of their decisions: (1) if both confess, both go to jail for five years; (2) if neither confesses, both go to jail for one year (for carrying concealed weapons); and (3) if one confesses while the other does not, the confessor goes free (for turning state's evidence) and the silent one goes to jail for 20 years.

# 6 Jeopardy

https://www.youtube.com/watch?v=eeFdPALpSgk

- 1. What do you think of the contestants' choices?
- 2. Consider a simplified model two players, each with \$10,000, and they flip a coin to see whether they win or lose their wager. How much should they wager?
- 3. Now consider the same model, where one player has \$10,000 and another player has \$12,000. What should the players do?
- 4. What about the game of rock, paper, scissors? What is the optimal strategy in that game?

## 7 The Price Is Right – Bonkers

https://www.youtube.com/watch?v=mjGEV8s0Dc4

- 1. What should a good strategy optimize for?
- 2. Can you find the best strategy? (Try starting with two or three digits.)
- 3. How many moves are required in an optimal strategy?
- 4. What other famous math problem has exactly the same solution?

## 8 The Price Is Right – Spelling Bee

https://www.youtube.com/watch?v=Pm-bfQmMROA

- 1. After the contestant has revealed three cards, what is her probability of winning?
- 2. After the contestant has revealed two cards, what is her probability of winning?
- 3. A YouTube comment says:

She should had [sic] gone for the \$4,000 because the chance of winning a car is slim since there are only 11 Cs, 11 As, 6 Rs, and 2 Car Signs.

Another YouTube comment says

Would of [sic] taken the money very hard to get the word car

The price of the car is \$22,975. Explain why, if the contestant's odds of winning are greater than  $\frac{160}{919}$ , she should go for the car.

- 4. Prove that the probability she wins is greater than  $\frac{160}{919}$ . (Try to look for an easy solution!)
- 5. (Challenge!) Compute the exact probability she wins the car.

# 9 (Challenge) The Price Is Right – Three Strikes

https://www.youtube.com/watch?v=AAIU6knD7BA

Come up with a strategy, and estimate the contestant's odds of winning following that strategy.