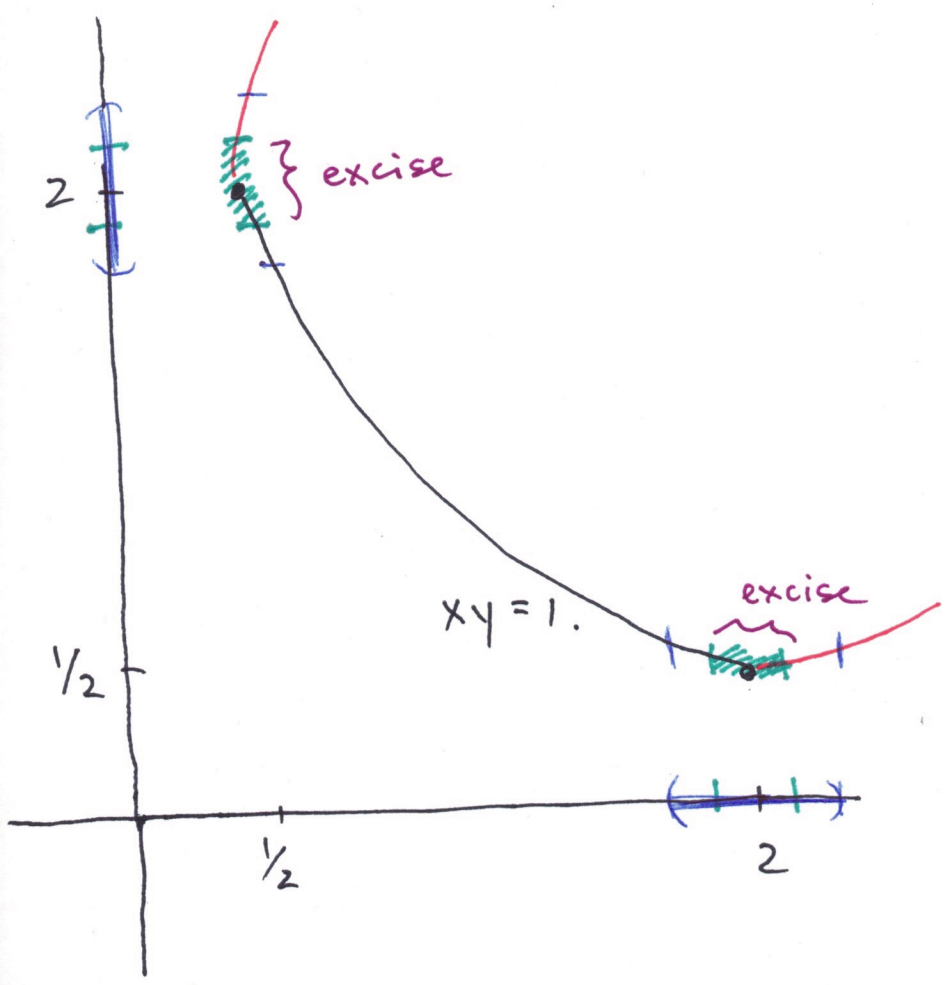

Step 1. $xy = 1$ between $x = 1/2$ and $x = 2$: original function to extend.

Step 2. Extend to a closed curve. Here I used the circle of center $(2, 2)$ and radius $3/2$; exact choice shouldn't matter. Maintain convexity but ignore smoothness.

Step 3. Choose neighborhoods of $x = 2$ and $y = 2$ on which the curve can be regarded as a function of x and y, respectively.
(Here it's clearly possible; we need to prove it always is.)

Step 4. Choose subneighborhoods of those in Step 3, still containing $x = 2$ and $y = 2$.

Step 5. Excise portions of the curve corresponding to Step 4. Use convolution/whitney extension/etc. on the neighborhoods in Step 3 to replace with something smooth and convex.