Rigid-Body Dynamics Project

Rigid-Body Spring Pendulum

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Dr. Mark Sensmeier
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ES204 Dynamics - Fall 2018

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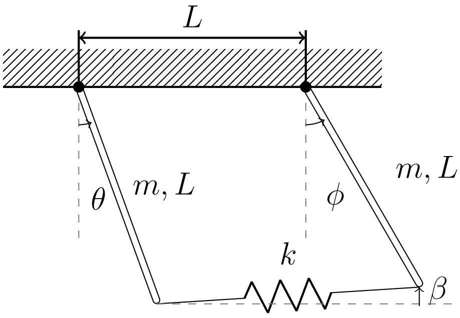
1 Conceptualize the Problem



The pendulumn system consists of two rigid bars attached at their ends by a linear spring.

1.1 Constants and Assumptions

Constants:		Assumptions:
Bar Mass:	$m = 0.25$kg	Frictionless
Bar Length:	$L = 0.5$m	Released from Rest
Gravity:	$g = 9.81 {}^{m}/_{s^2}$	Rigid-Body Dynamics
Linear Spring:		
Spring Coefficient:	$k = 25\ {}^{N}/_{m}$	
Unstretched Length:	L	

We are asked to determine the following:

1. The 6 Equations / 6 Unknowns for the system to solve for the Equations of Motion.

2. Integrate the Equations of Motion using various initial conditions.

   (a) $\theta_o = {}^{\pi}/_{12}\ rad, \quad \phi_o = {}^{\pi}/_{12}\ rad$

   (b) $\theta_o = {}^{-\pi}/_{12}\ rad, \quad \phi_o = {}^{\pi}/_{12}\ rad$

   (c) $\theta_o = {}^{\pi}/_{36}\ rad, \quad \phi_o = {}^{\pi}/_{12}\ rad$

3. Linearize the Equations of Motion assuming small angular positions and velocities (i.e. small angle approximation $\sin(x) \approx x,\ \cos(x) \approx 1$)

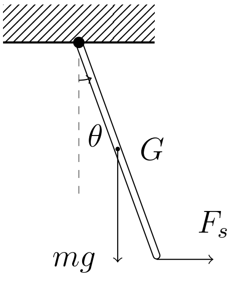
   - Determine the A matrix below.

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \theta \\ \phi \end{bmatrix}$$

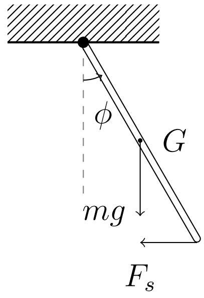
4. Find the natural frequencies of the system and their respective eigenvectors using the eigenvalues and eigenvectors of [A].

5. Using information from (5), solve for the analytical solution to the linearized Equations of Motion and plot them for the initial conditions defined in (2).

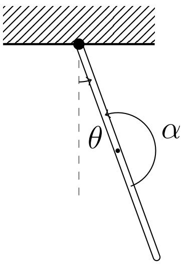
2 Free Body Diagram



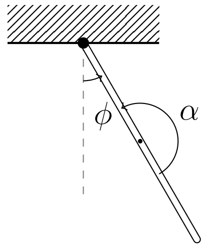
F_s: Force onto bar by the spring
mg: Mass · gravity, weight of the bar
G: Center of gravity of each bar
$\theta,\ \phi$: Angle of bar relative to vertical



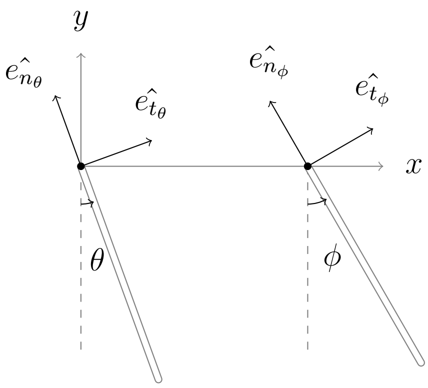
Acceleration Diagram

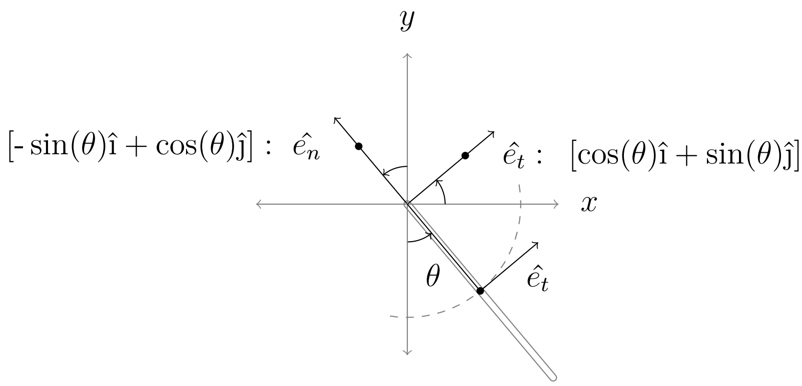


α: $\ddot{\theta},\ \ddot{\phi}$ respectively



3 Coordinate Frame





4 Sum of Forces

$$\sum Stuff \tag{1}$$

Where:

θ:	Position of the left bar.
ϕ:	Position of the right bar.
$\dot{\theta}$:	Angular velocity of the left bar.
$\ddot{\theta}$:	Angular acceleration of the left bar.
$\dot{\phi}$:	Angular velocity of the right bar.
$\ddot{\phi}$:	Angular acceleration of the right bar.
m, L, g:	Are constants; mass, length of each bar, and gravity, respectively

5 Knowns and Unknowns

Knowns:		Unknowns:	
Mass:	m = 0.142kg	String Tension:	T
String Length:	L = 0.5m	Angular Acceleration:	$\ddot{\theta}$
Gravity:	g = 9.81$^m/_{s^2}$		
State Variables:			
Angular Velocity:	$\dot{\theta}$		
Angular Acceleration:	$\ddot{\theta}$		

6 Constraints

7 Solve for the Equations of Motion

8 Solve the Equations of Motion

9 Does it Make Sense?

9.1 Units

9.2 Magnitude

10 Appendix

10.1 Attributions

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Joint Effort

10.2 Analytical Solution

10.3 Numerical Solution