Particle EOM Project

This project will cover the topic of modeling a complex system using a linear 2nd order differential equation.

Step 1) Determine the equation of motion (EOM) for a simple pendulum system. Integrate the EOM for 10 seconds using the following characteristics:

$$m = 5\text{oz} = 0.142\text{kg}$$
$$L = 0.5\text{m}$$
$$g = 9.81\frac{\text{m}}{\text{s}^2}$$

The system starts from rest at various initial angles of release: θ_o = 5°, 10°, 15°, 30°, 60°, and 90°.

Step 2) For each of the starting angles determine the natural frequency, ω_n, of the system. You can use an event to locate the exact times of the peaks and/or valleys. Plot ω_n versus the initial angle, θ_o. If you use small-angle approximation ($\sin(\theta) \approx \theta, \cos(\theta) \approx 1$), then what would the approximate EOM be and what would be the natural frequency of this system? Plot a straight horizontal line across your graph at this natural frequency. $\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n{}^2\theta = 0$

Step 3) It is determined that the drag force is:

$$\bar{F}_{Drag} = \frac{1}{2}\rho C_D A V^2\left(-\hat{V}\right) = -1.65\cdot 10^{-3}\dot{\theta}\left|\dot{\theta}\right|\hat{e}_t$$

Include the drag force in your model, determine a new (more accurate) EOM, and integrate the EOM starting from rest at 15° for 100 seconds.

Step 4) Determine the linear 2nd order differential equation that correctly models the system with drag. Knowing that the underdamped system has the following general solution:

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n{}^2\theta = 0$$

$$\theta(t) = Ae^{st}$$
$$\therefore s = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2} = -\zeta\omega_n \pm i\omega_d$$
$$\theta(t) = e^{-\zeta\omega_n t}\left(A\cos\left(\omega_d t\right) + B\sin\left(\omega_d t\right)\right)$$

Determine the damped natural frequency, ω_d (from the times between peaks or valleys), the decay rate, $\zeta\omega_n$ (from the decay of the peak values), and then the damping ratio, ζ, and natural frequency, ω_n (from ω_d and $\zeta\omega_n$). Confirm your values by plotting the analytical solution, θ *(t)*, versus time on the same graph for your Step 3 and Step 4 EOMs.

The deliverable for this assignment is a published PDF file of your code that using MATLAB markup to included necessary headings, text, pictures, equations, etc. similar to the example file provided on Canvas. It should follow the 9-step process where the points allocated to each step, and what must be included, are in a rubric associated with the assignment on Canvas. A good rule of thumb, explain everything as if someone not currently in Dynamics is reading it and needs to understand it. One sentence explainations or descriptions of figures usually is not good enough for full credit.