Particle Dynamics Project

Vibration Analysis

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By:

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Name 1 Name 2 Name 3

Submitted to Dr. Bradley Wall
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College of Engineering
In Partial Fulfillment
Of the Requirements
Of
ES 204
Dynamics
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1. CONCEPTUALIZE THE PROBLEM
   Introduce the problem including a figure of the dynamical system. Write down and understand what the problem is asking for. Write down all the given information, e.g., the constants of the problem. Determine the number of degrees of freedom the problem has and what coordinate system might best model the objects motion, e.g., Cartesian or Polar coordinates. Write down any assumptions given in the problem statement. These may include no friction, no drag, particle dynamics (no rotation), mass or mass moment of inertia of an object is negligible, or the object is moving at a constant velocity, i.e., the acceleration of the object is zero.

2. FREE-BODY DIAGRAM
3. COORDINATE FRAME
   Draw a Coordinate System that best describes the motion that occurs. There are 3 main choices: Cartesian (x,y), Polar($\hat{e}_n, \hat{e}_t$), and Polar ($\hat{e}_r, \hat{e}_t$). If you have multiple particles or rigid-bodies in the system you may have a different coordinate frame for each. Draw the Free-Body Diagram of the system including all acting forces and/or moments. You must draw the forces assuming positive position deflection, e.g., draw your diagram with a small but positive position deflection, and positive velocity based on your Coordinate System. If friction is included, include a unit vector in the direction of the velocity vector so that if you have negative velocity the direction of the frictional force will change.
   Explain your choice for the positive direction of each coordinate frame chosen. Also introduce/define each force and variable used in the free-body diagram.

4. SUM OF FORCES
   $\sum \bar{F} = m \cdot \bar{a}_{cm}$ where (using dot notation) $\bar{a}_{cm} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} r\dot{\theta}^2 \\ r\ddot{\theta} \end{bmatrix} = \begin{bmatrix} \ddot{r} - r\dot{\theta}^2 \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{bmatrix}$ are the general possibilities for the 3 main Coordinate System choices: Cartesian (x,y), Polar ($\hat{e}_n, \hat{e}_t$), and Polar ($\hat{e}_r, \hat{e}_t$). $\bar{a}_{cm}$ must be relative to a *fixed* point. For rigid body dynamics include
   $\sum M_{z_{cm}} = I_{z_{cm}} \ddot{\theta}$
   You may use $\sum M_{z_O} = I_{z_O} \ddot{\theta}$, but point O must be fixed in space.
   Discuss each equation, reiterating what each variable in the equations represents.

5. KNOWNS AND UNKNOWNS
   Categorize each variable in the equations from Step 4 as a known or unknown. Velocity and position state variables associated with an unknown acceleration are considered known since they can be obtained from the acceleration via integration. Compare the number of unknowns to the number of equations in Step 4. If you have the same number of unknowns as equations proceed to Step 7; no constraint equations are required. If you have more unknowns than equations a type of constraint equation is required in order to solve the problem.

6. CONSTRAINTS
   Include all particle and rigid body constraint equations including friction, length of rope is constant, gears, rolling without slipping, surface, and pin constraints. Discuss the constraint and then develop the mathematical equation that enforces said constraint. A figure detailing the length of rope constraint or the variables being used in a relative velocity equation is required.

7. SOLVE FOR THE EQUATION(S) OF MOTION
Using the equations from Step 4 and Step 6 solve for the accelerations of the body, e.g., $\ddot{x}$ and/or $\ddot{\theta}$. Display and discuss the major algebraic steps. Refer to the appendix for the full algebra.

8. SOLVE THE EQUATION OF MOTION, SOLVE THE PROBLEM
Analytically: $a = \frac{dv}{dt} = v\frac{dv}{dx}$
Display and discuss the major differential equations and/or calculus steps taken to solve the equation(s) of motion and solve the problem. Refer to an appendix with the full analytical solution available. Display and discuss any relevant figures that provide insight into how the dynamical system behaves.

Numerically: MATLAB ode45
Refer to an appendix with the full MATLAB code available. Display and discuss any relevant figures that provide insight into how the dynamical system behaves.

Compare results to the physical system if relevant. Compare the numerical solution to the analytical solution if relevant.

9. DOES IT MAKE SENSE
Check the units of your equation of motion and any other answers. Discuss whether the units of your equations result in the units required of your answers. Check the signs/directions of your answers. Do the signs make physical sense for various signs of initial conditions? Check the magnitude of your answers. Are they on the same order of magnitude of the information given or are they similar to that of a physical system? Check any assumptions, e.g., $f_f \leq \mu_s N$ for rolling with slipping.

APPENDICES:

A. ATTRIBUTIONS
Who did what? List each team member and how they contributed to the project. This can be in a simple table.

B. ANALYTICAL SOLUTION
Scan the full analytical solution into this appendix.

C. NUMERICAL SOLUTION
Publish your MATLAB code so that your code, command window output, and any figures are present in this appendix.