Rigid-Body Dynamics Project

Rigid-Body Spring Pendulum

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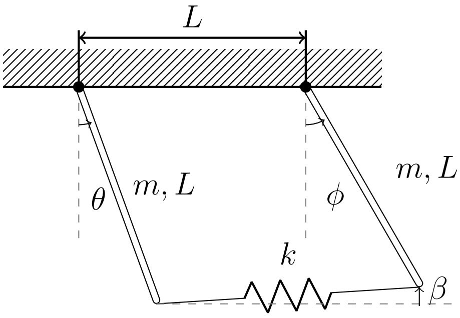
1 Conceptualize the Problem



The pendulumn system consists of two rigid bars attached at their ends by a linear spring.

1.1 Constants and Assumptions

Constants:		Assumptions:
Bar Mass:	$m = 0.25$kg	No Losses
Bar Length:	$L = 0.5$m	Released from Rest
Gravity:	$g = 9.81 m/s^2$	Slender Bars
Linear Spring:		Rigid-Body Dynamics
Spring Coefficient:	$k = 25\ N/m$	Planar
Unstretched Length:	L	

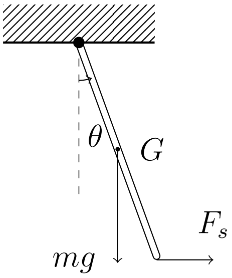
We are asked to determine the following:

1. The 6 Equations / 6 Unknowns for the system to solve for the Equations of Motion.
2. Integrate the Equations of Motion using various initial conditions.
   (a) $\theta_o = \pi/12\ rad, \quad \phi_o = \pi/12\ rad$
   (b) $\theta_o = -\pi/12\ rad, \quad \phi_o = \pi/12\ rad$
   (c) $\theta_o = \pi/36\ rad, \quad \phi_o = \pi/12\ rad$
3. Linearize the Equations of Motion assuming small angular positions and velocities (i.e. small angle approximation $\sin(x) \approx x,\ \cos(x) \approx 1$)
   - Determine the A matrix below.

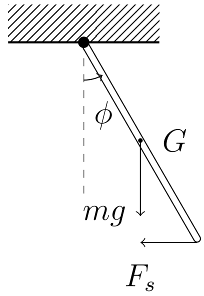
$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \theta \\ \phi \end{bmatrix}$$

4. Find the natural frequencies of the system and their respective eigenvectors using the eigenvalues and eigenvectors of [A].
5. Using information from (5), solve for the analytical solution to the linearized Equations of Motion and plot them for the initial conditions defined in (2).

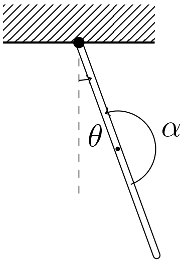
2 Free Body Diagram



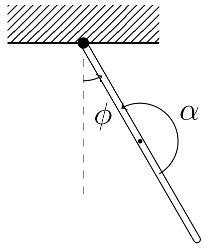
F_s:	Force onto bar by the spring
mg:	Mass · gravity, weight of each bar
G:	Center of gravity of each bar
$\theta,\ \phi$:	Angle of bar relative to vertical
$A_n,\ C_n$:	Reaction forces in the normal direction
$A_t,\ C_t$:	Reaction forces in the tangential direction



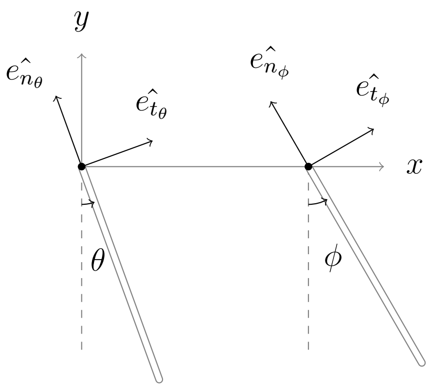
Acceleration Diagram

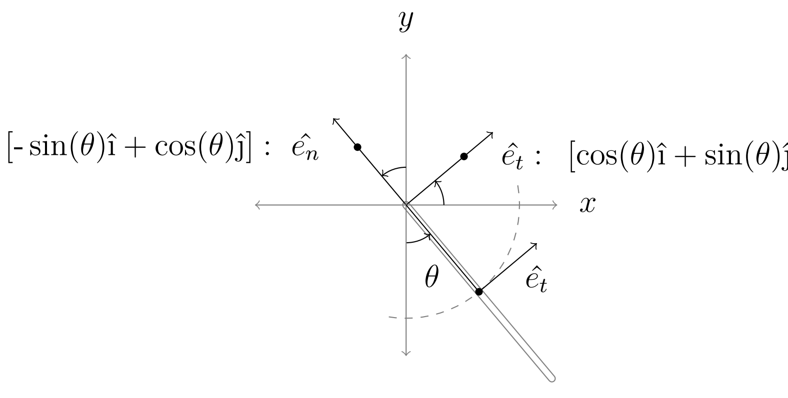


α: $\ddot{\theta},\ \ddot{\phi}$ respectively



3 Coordinate Frame





$\hat{e_n}$: [- $\sin(\theta)\hat{\imath} + \cos(\theta)\hat{\jmath}$]
$\hat{e_t}$: [$\cos(\theta)\hat{\imath} + \sin(\theta)\hat{\jmath}$]

4 Sum of Forces

Using a normal tangential coordinate system we produce the following force equations

$$\sum F_n = ml\dot{\theta}^2 = F_{sn} + A_n - mg\cos\theta \tag{1}$$

Sum of Normal Forces on the left bar (Theta)

$$\sum F_t = ml\dot{\theta} = F_{sn} + A_t - mg\sin\theta \tag{2}$$

Sum of Tangential Forces on the left bar (Theta)

$$\sum F_n = ml\dot{\phi}^2 = F_{sn} + C_n - mg\cos\phi \tag{3}$$

Sum of Normal Forces on the right bar (Phi)

$$\sum F_t = ml\dot{\phi} = F_{sn} + C_t - mg\sin\phi \tag{4}$$

Sum of Tangential Forces on the right bar (Phi)

We also product the following moment equations

$$\sum M_A = -(\frac{l}{2}mg\sin\theta + F_{st}l) = \frac{1}{3}ml^2\dot{\theta} \tag{5}$$

Sum of Moments about A (Theta)

$$\sum M_C = -(\frac{l}{2}mg\sin\phi + F_{st}l) = \frac{1}{3}ml^2\dot{\phi} \tag{6}$$

Sum of Moments about C (Phi)

Where:

θ:	Position of the left bar.
ϕ:	Position of the right bar.
$\dot{\theta}$:	Angular velocity of the left bar.
$\ddot{\theta}$:	Angular acceleration of the left bar.
$\dot{\phi}$:	Angular velocity of the right bar.
$\ddot{\phi}$:	Angular acceleration of the right bar.
A:	Reaction force in the normal or tangential direction at the top of the left bar.
C:	Reaction force in the normal or tangential direction at the top of the right bar.
F_s:	Force due to the spring in either the normal or tangential direction.
m, L, g:	Are constants; mass, length of each bar, and gravity, respectively.

5 Knowns and Unknowns

Knowns:		Unknowns:	
Mass:	$m = 0.25$kg	Reaction Forces:	$A_n,\ A_t$
String Length:	$L = 0.5$m		$C_n,\ C_t$
Gravity:	$g = 9.81 {}^{m}/_{s^2}$	Angular Accelerations:	$\ddot{\theta},\ \ddot{\phi}$
Linear Spring:			
Spring Coefficient:	$k = 25\ {}^{N}/_{m}$		
Unstretched Length:	L		
State Variables:			
Angular Position:	$\theta,\ \phi$		
Angular Velocity:	$\dot{\theta},\ \dot{\phi}$		

6 Constraints

No constraint equations were needed to find a solution to the system.

7 Solve for the Equations of Motion

The equations of motion for this system are (from Eqs. (1-6))

$$\ddot{\theta} = -3\frac{gm\sin\theta - 2kL\cos\theta - 2kL\sin\phi - \theta + \frac{2kL^2\cos\theta}{\sqrt{L^2(\sin\phi - \sin\theta + 1)^2 + L^2(\cos\phi - \cos\theta)}} + \frac{2kL^2\sin\phi - \theta}{\sqrt{L^2(\sin\phi - \sin\theta)^2 + L^2(\cos\phi - \cos\theta)^2 + L^2(\cos\phi - \cos\theta)^2)}}}{2Lm}$$

$$\ddot{\phi} = -3\frac{gm\sin\phi - 2kL\cos\phi - 2kL\sin\phi - \theta + \frac{2kL^2\cos\phi}{\sqrt{L^2(\sin\phi - \sin\theta + 1)^2 + L^2(\cos\phi - \cos\theta)}} + \frac{2kL^2\sin\phi - \theta}{\sqrt{L^2(\sin\phi - \sin\theta)^2 + L^2(\cos\phi - \cos\theta)^2 + L^2(\cos\phi - \cos\theta)^2)}}}{2Lm}$$

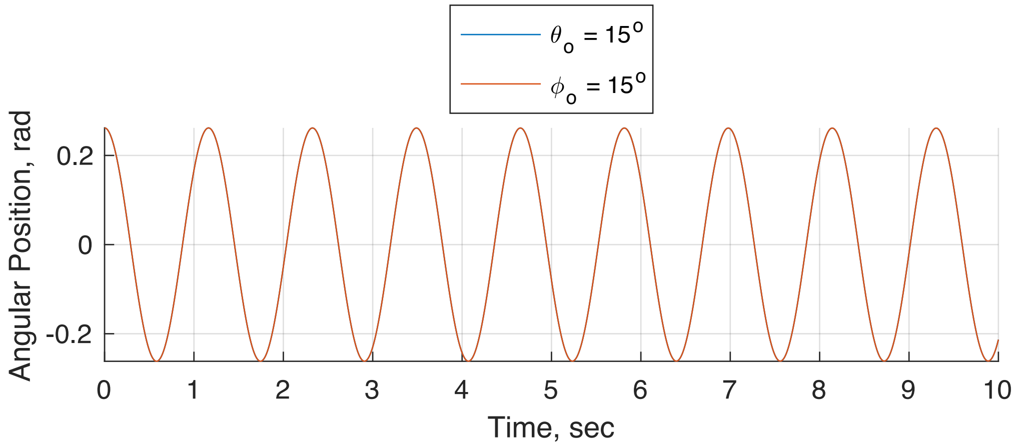


Figure 1: Meow.

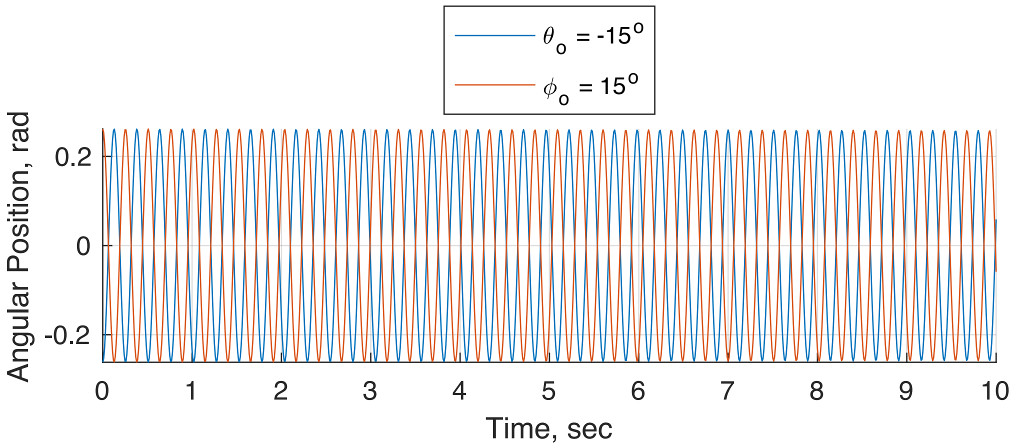


Figure 2: Meow.

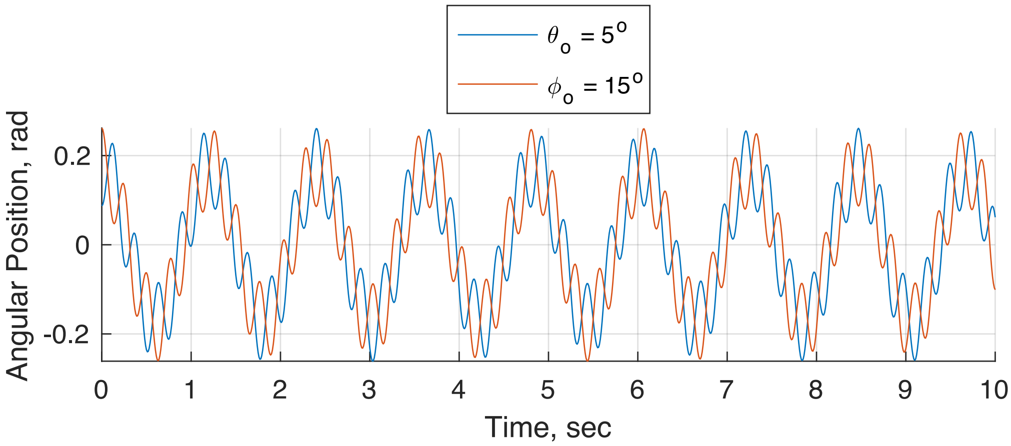


Figure 3: Meow.

8 Solve the Equations of Motion

9 Does it Make Sense?

9.1 Units

9.2 Magnitude

10 Appendix

10.1 Attributions

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Joint Effort

10.2 Analytical Solution

10.3 Numerical Solution