Lagrangian Dynamics Project

Rigid-Body Spring Pendulum

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By:

Jeffrey Chen	Thorne Wolfenbarger	Trey Dufrene

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Dr. Mark Sensmeier
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College of Engineering
Embry-Riddle Aeronautical University
Prescott, AZ

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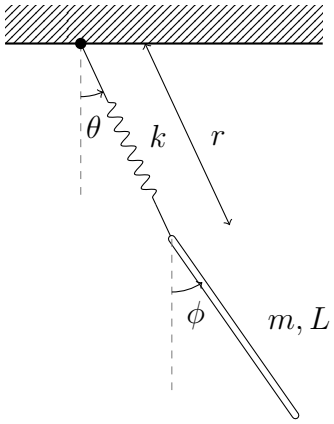
1 Conceptualize the Problem



The pendulum system consists of a rigid bar pinned to the free end of a linear spring, which rotates about its opposite end at a fixed point.

1.1 Constants and Assumptions

Constants:		Assumptions:
Bar Mass:	$m = 1$ kg	No Losses
Bar Length:	$L = 1$ m	Released from Rest
Gravity:	$g = 9.81 {}^{m}/_{s^2}$	Slender Bar
Linear Spring:		Planar
Spring Coefficient:	$k = 25 \ {}^{N}/_{m}$	Rigid-Body Dynamics
Unstretched Length:	$L = 0.5$ m	

We are asked to determine the following:

1. The Equations of Motion for the system via the Lagrangian method.
2. Integrate the Equations of Motion using various initial conditions and plot the behavior of the system for 10 seconds.
   (a) $\theta_o = 0 \ rad, \quad \phi_o = 0 \ rad$
   (b) $\theta_o = {}^{\pi}/_{18} \ rad, \quad \phi_o = {}^{\pi}/_{9} \ rad$
   (c) $\theta_o = {}^{\pi}/_{6} \ rad, \quad \phi_o = {}^{\pi}/_{3} \ rad$
3. Plot the total energy versus time for all 3 cases.
4. Repeat 2. and 3. using a 'RelTol' of 1e-6 and 'AbsTol' 1e-9 for the `ode45` integration tolerances.

2 Free Body Diagram

3 Coordinate Frame

4 Sum of Forces

5 Constraints

6 Solve for the Equations of Motion

7 Solve the Equations of Motion

8 Does it Make Sense?

8.1 Units

Checking with the MATLAB symbolic units tool (from Section 9.3):

8.2 Magnitude

9 Appendix

9.1 Attributions

Jeffrey Chen
Thorne Wolfenbarger
Trey Dufrene
Joint Effort

9.2 Analytical Solution

9.3 Numerical Solution