Lagrangian Dynamics Project

Rigid-Body Spring Pendulum

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Submitted to:
Dr. Mark Sensmeier
In Partial Fulfillment of the Requirements of
ES204 Dynamics – Fall 2018

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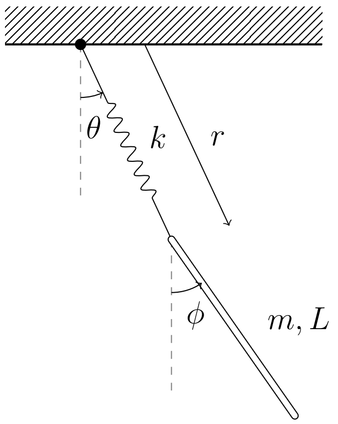
1 Conceptualize the Problem



The pendulum system consists of a rigid bar pinned to the free end of a linear spring, which rotates about its opposite end at a fixed point; there are three degrees of freedom, since the spring and bar each have an individual angular deflection with respect to the vertical, and the radial distance the bar is from the point of rotation due to the variation in the length of the spring.

1.1 Constants and Assumptions

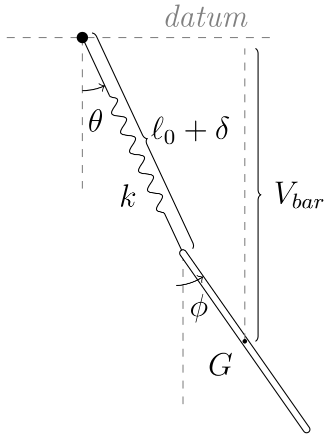
Constants:		Assumptions:
Bar Mass:	$m = 1$ kg	No Losses
Bar Length:	$L = 1$ m	Released from Rest
Gravity:	$g = 9.81 {}^{m}/_{s^2}$	Uniform Slender Bar
Linear Spring:		Planar
Spring Coefficient:	$k = 25\ {}^{N}/_{m}$	Rigid-Body Dynamics
Unstretched Length:	$L = 0.5$ m	

We are asked to determine the following:

1. The Equations of Motion for the system via the Lagrangian method.
2. Integrate the Equations of Motion using various initial conditions and plot the behavior of the system for 10 seconds.
   (a) $\theta_o = 0\ rad, \quad \phi_o = 0\ rad$
   (b) $\theta_o = {}^{\pi}/_{18}\ rad, \quad \phi_o = {}^{\pi}/_{9}\ rad$
   (c) $\theta_o = {}^{\pi}/_{6}\ rad, \quad \phi_o = {}^{\pi}/_{3}\ rad$
3. Plot the total energy versus time for all 3 cases.
4. Repeat 2. and 3. using a '`RelTol`' of 1e-6 and '`AbsTol`' of 1e-9 for the `ode45` integration tolerances.

2 Free Body Diagram

(a) Free Body Diagram

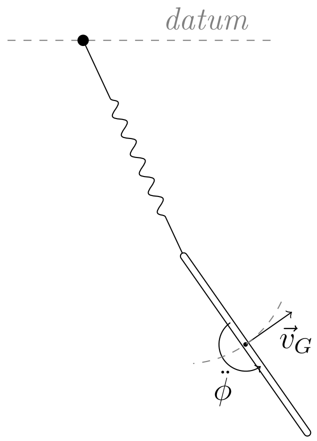


(b) Acceleration Diagram

Figure 1: Acceleration and Free Body Diagrams

G: Center of gravity of the bar
ℓ_0: Spring unstretched length
δ: Spring deflection
k: Spring constant
V_{bar}: Potential energy of bar

G: Center of gravity of the bar


datum
$\vec{v}_G$
$\ddot{\phi}$

3 Coordinate Frame

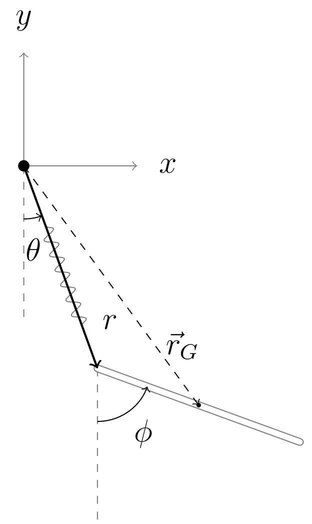


Figure 2: Coordinate Frame

Motion Variables:

θ:	Angle of spring relative to vertical
ϕ:	Angle of bar relative to vertical
r:	Radial length of spring

Supplemental Variables:

$\vec{r}_G$:	Vector to bar center of mass from origin

4 Sum of Forces

5 Constraints

The system is fully constrained, therefore no constraint equations were needed to solve the problem.

6 Solve for the Equations of Motion

Spring

From Figure (2),

$$\vec{r}_G = \left[\ell_s \sin(\theta) + \frac{\ell_b}{2}\sin(\phi)\right]\hat{\imath} + \left[-\ell_s \cos(\theta) - \frac{\ell_b}{2}\cos(\phi)\right]\hat{\jmath}$$

Taking the time derivative,

$$\frac{d}{dt}\vec{r}_G \equiv \dot{\vec{r}}_G = \vec{v}_G$$

$$\vec{v}_G =$$

Kinetic Energy of Spring:

$$T_1 = 0$$

Kinetic Energy of Bar:

$$T_2 = \frac{1}{2}m_b(\vec{v}_G \cdot \vec{v}_G) + \frac{1}{24}m_b\ell_b^2\dot{\phi}^2$$

Total Kinetic Energy:

$$T = T_1 + T_2$$

Potential Energy of Spring:

$$V_1 = \frac{1}{2}k\Big(\ell_s - \ell_0\Big)^2$$

Potential Energy of Bar

$$V_2 = -m_b g\Big(\ell_s \cos(\theta) + \frac{\ell_b}{2}\cos(\phi)\Big)$$

Total Potential Energy

$$V = V_1 + V_2$$

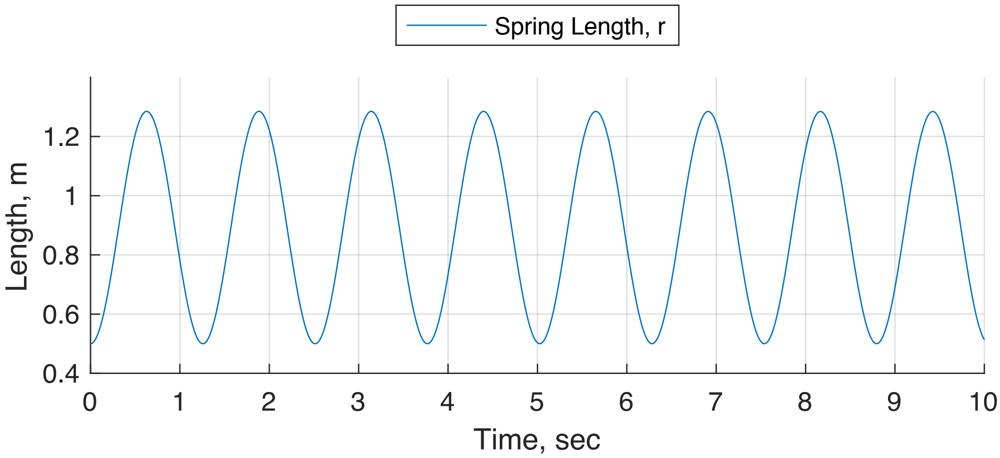
Lagrangian $\mathscr{L} = T - V$.

$$\ddot{\theta} = \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathscr{L}}{\partial \theta} = \Big(Q_\theta\Big)_{\text{non}}$$

$$\ddot{\phi} = \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\phi}}\right) - \frac{\partial \mathscr{L}}{\partial \phi} = \Big(Q_\phi\Big)_{\text{non}}$$

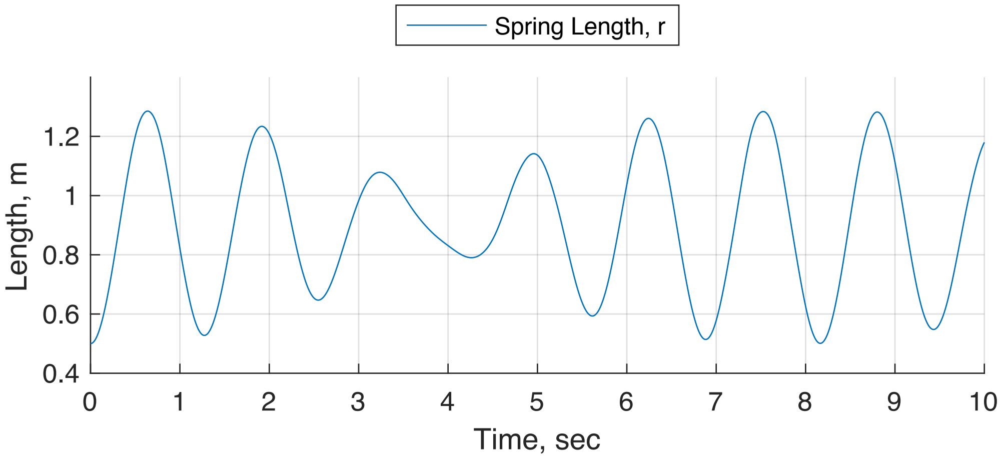
7 Solve the Equations of Motion

Figure 3: Numerical Solution Motion Behavior Plot, ($\theta_o :\ 0,\ \phi_o :\ 0$)



(a) Spring Length vs. Time

Figure 4: Numerical Solution Motion Behavior Plots, ($\theta_o : {}^{\pi}/_{18},\ \phi_o : {}^{\pi}/_{9}$)

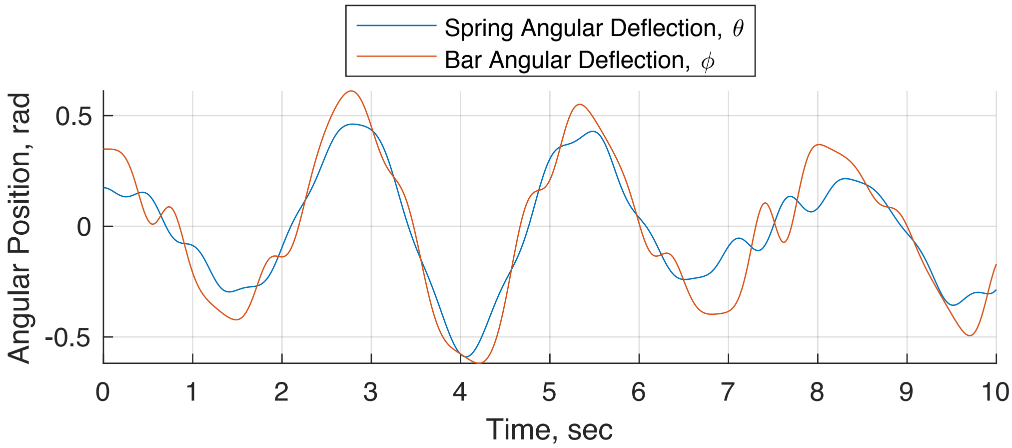


(a) Spring Length vs. Time

8 Does it Make Sense?

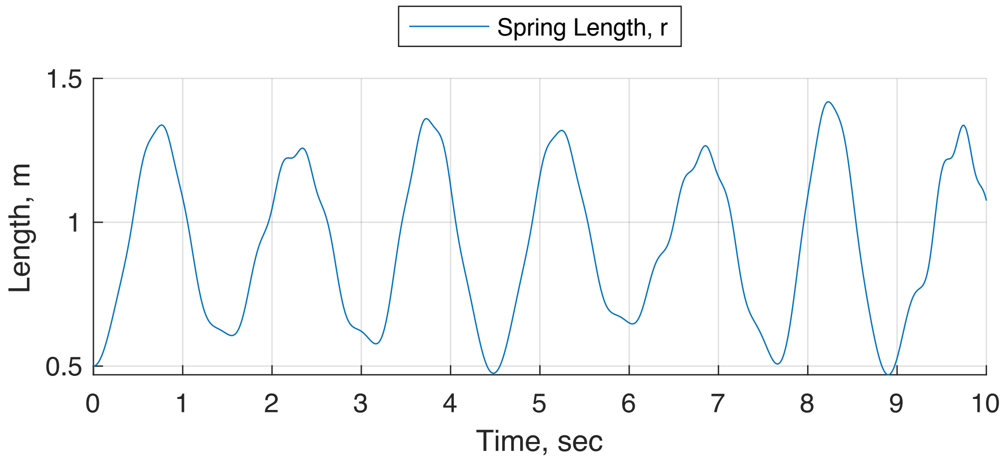
8.1 Units

Checking with the MATLAB symbolic units tool (from Section 9.3):

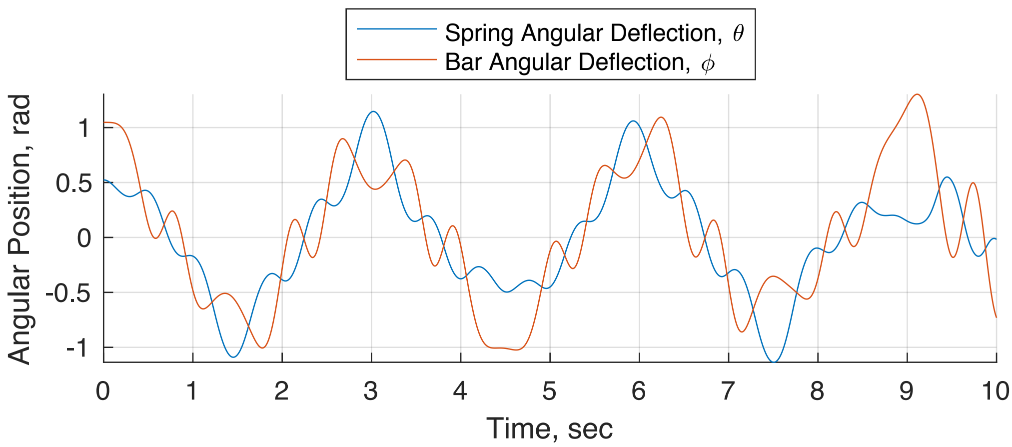


(b) Angular Position vs. Time

Figure 5: Numerical Solution Motion Behavior Plots, ($\theta_o : {}^{\pi}/_{6},\ \phi_o : {}^{\pi}/_{3}$)



(a) Spring Length vs. Time



(b) Angular Position vs. Time

8.2 Magnitude

9 Appendix

9.1 Attributions

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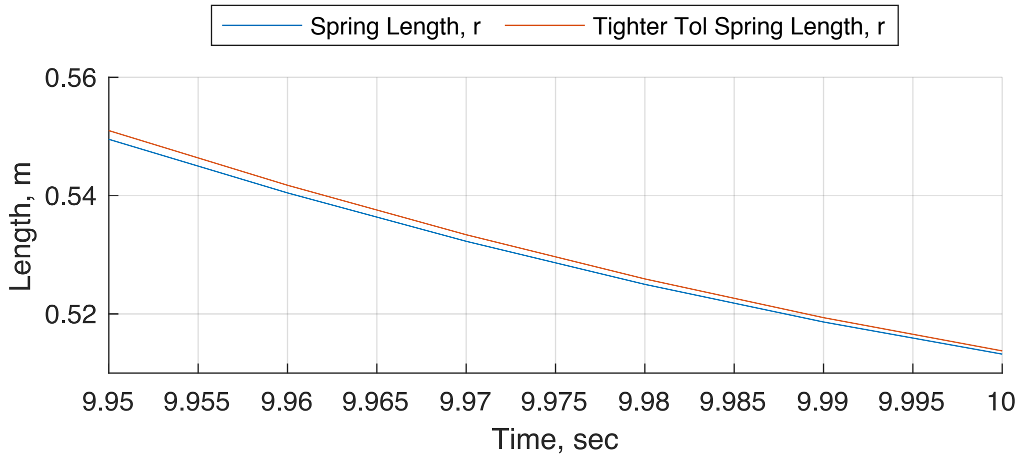
Joint Effort

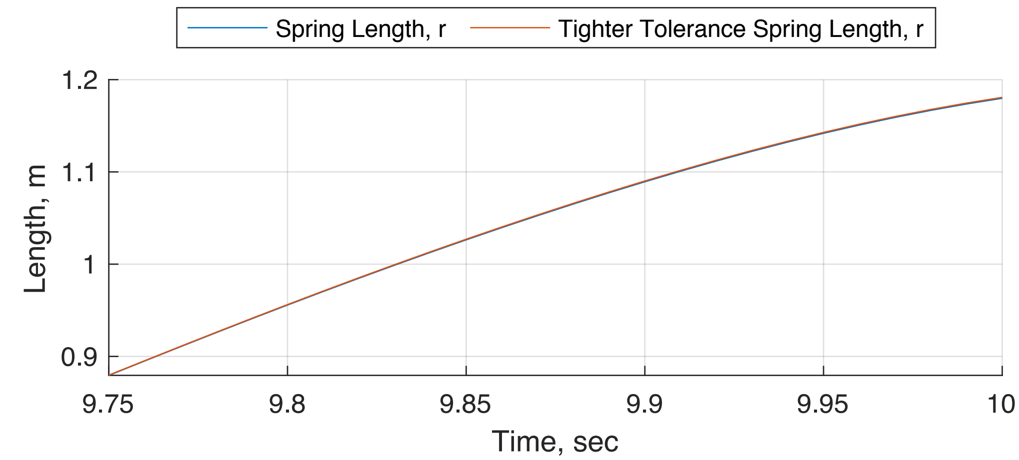
9.2 Analytical Solution

9.3 Numerical Solution

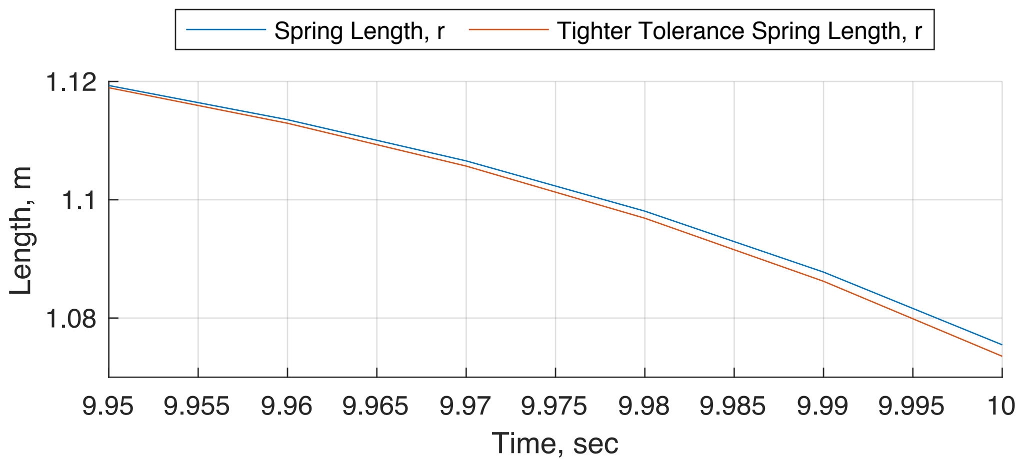
Figure 6: Comparison Plots of ode45 Tolerance Options



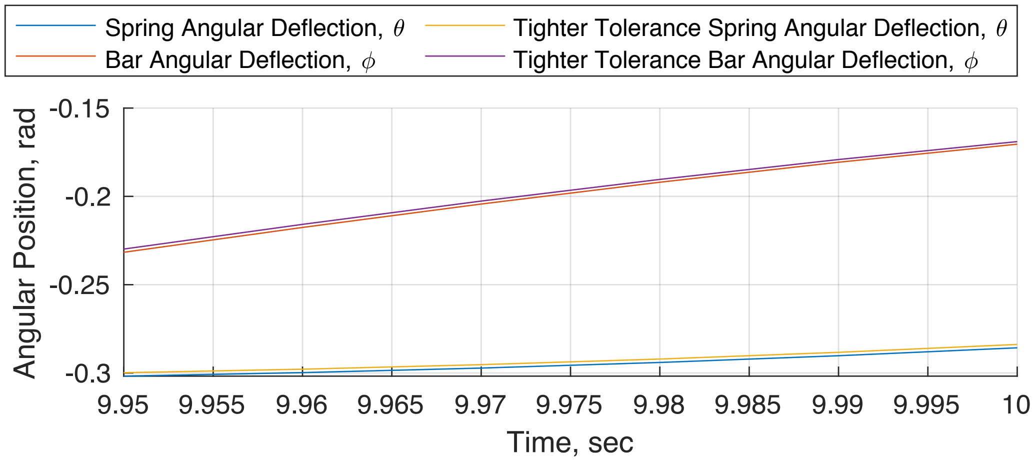
Spring Deflection, (θ_o : 0, ϕ_o : 0)



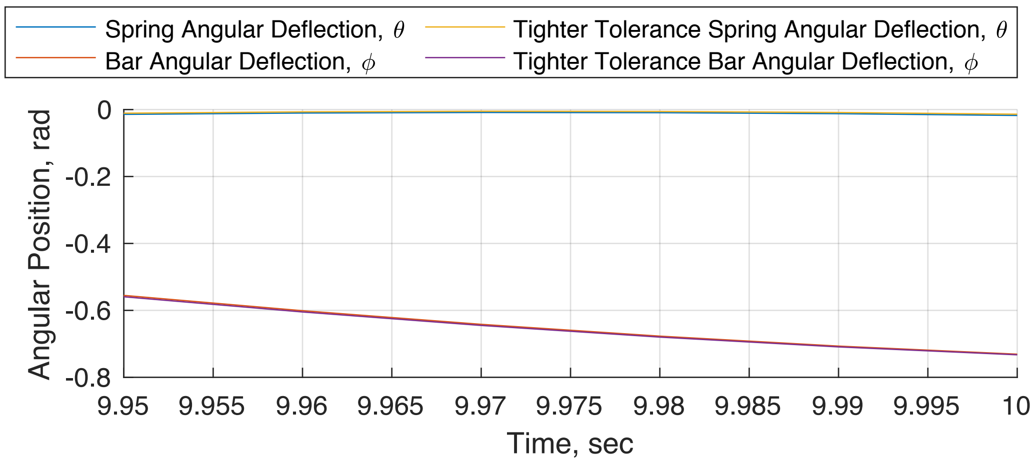
Spring Deflection, ($\theta_o : {}^{\pi}/_{18}$, $\phi_o : {}^{\pi}/_{9}$)



Spring Deflection, ($\theta_o : {}^{\pi}/_{6}$, $\phi_o : {}^{\pi}/_{3}$)



Bar and Spring Angular Deflection, $(\theta_o : {}^{\pi}/_{18},\ \phi_o : {}^{\pi}/_{9})$



Bar and Spring Angular Deflection, $(\theta_o : {}^{\pi}/_{6},\ \phi_o : {}^{\pi}/_{3})$