Lagrangian Dynamics Project

Rigid-Body Spring Pendulum

December 1, 2018

By:

Jeffrey Chen	Thorne Wolfenbarger	Trey Dufrene

Submitted to:
Dr. Mark Sensmeier
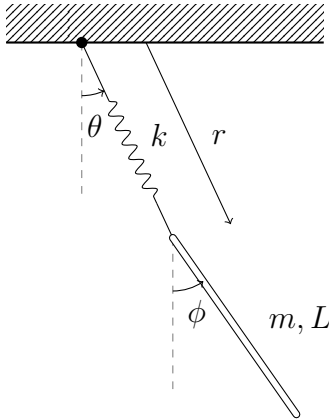In Partial Fulfillment of the Requirements of
ES204 Dynamics – Fall 2018

College of Engineering
Embry-Riddle Aeronautical University
Prescott, AZ

Contents

List of Figures

1   Conceptualize the Problem

The pendulum system consists of a rigid bar pinned to the free end of a linear spring, which rotates about its opposite end at a fixed point; there are three degrees of freedom, since each bar has an individual rotation with respect to the vertical, as well as the radial distance the bar is from the point of rotation.

1.1   Constants and Assumptions

| Constants:		Assumptions:
Bar Mass:	$m = 1$ kg	No Losses
Bar Length:	$L = 1$ m	Released from Rest
Gravity:	$g = 9.81\,m/s^2$	Slender Bar
Linear Spring:		Planar
Spring Coefficient:	$k = 25\ ^N/_m$	Rigid-Body Dynamics
Unstretched Length:	$L = 0.5$ m	

We are asked to determine the following:

1. The Equations of Motion for the system via the Lagrangian method.

2. Integrate the Equations of Motion using various initial conditions and plot the behavior of the system for 10 seconds.

    (a) $\theta_o = 0\ rad,$ $\phi_o = 0\ rad$
    (b) $\theta_o = \pi/18\ rad,$ $\phi_o = \pi/9\ rad$
    (c) $\theta_o = \pi/6\ rad,$ $\phi_o = \pi/3\ rad$

3. Plot the total energy versus time for all 3 cases.

4. Repeat 2. and 3. using a 'RelTol' of 1e-6 and 'AbsTol' of 1e-9 for the ode45 integration tolerances.

2 Free Body Diagram
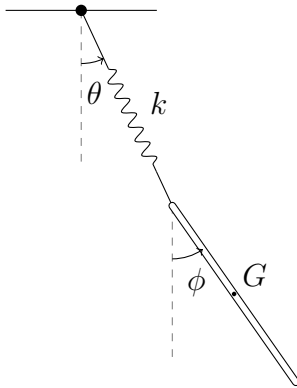
<table>
<tr><td>θ:</td><td>Angle of spring relative to vertical</td></tr>
<tr><td>ϕ:</td><td>Angle of bar relative to vertical</td></tr>
<tr><td>G:</td><td>Center of gravity of the bar</td></tr>
</table>

3 Coordinate Frame
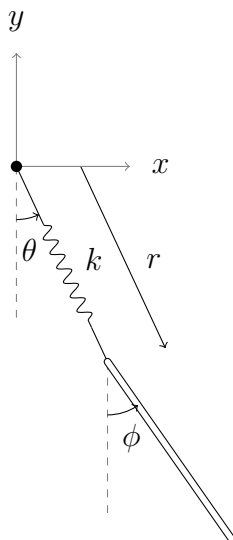
<table>
<tr><td>θ:</td><td>Angle of spring relative to vertical</td></tr>
<tr><td>ϕ:</td><td>Angle of bar relative to vertical</td></tr>
</table>

4 Sum of Forces

5 Constraints

6 Solve for the Equations of Motion

7 Solve the Equations of Motion

8 Does it Make Sense?

8.1 Units

Checking with the MATLAB symbolic units tool (from Section 9.3):

8.2 Magnitude

9 Appendix

9.1 Attributions

Jeffrey Chen
Thorne Wolfenbarger
Trey Dufrene
Joint Effort

9.2   Analytical Solution

9.3 Numerical Solution