Group Exam 1

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Grace Day

JiYoung Hwang

Aaron Scott

Thorne Wolfenbarger

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Professor Kaela Martin
College of Engineering

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Contents

1 Calculate the center of mass with respect to the sun

Using the center of mass equation (Eq. 1) we produced MATLAB code to solve for the system's center of mass with respect to the sun.

$$\vec{r}_{cm} = \frac{\Sigma_{i=1}^{n} m_i \vec{r}_{Sun \to i}}{\Sigma_{i=1}^{n} m_i} \tag{1}$$

```
rCM = (rSunMars * M_Mars + rSunPhobos * M_Phobos + rSunInSight * M_InSight
    )/(M_InSight + M_Phobos + M_Mars + M_Sun);
```

The center of mass with respect to the sun is $(64.1296\hat{\imath} + 23.5509\hat{\jmath} - 0.1080\hat{k})\ km$

2 Find the acceleration of InSight

Find the acceleration of InSight ($\ddot{\vec{r}}_{InSight}$). What is the acceleration relative to?

Using (Eq. 2) and MATLAB we calculated $\ddot{\vec{r}}_{InSight}$

$$\ddot{\vec{r}}_i = -\Sigma_{j=1}^{n} \frac{G m_j}{r_{j \to i}^3} \cdot \vec{r}_{j \to i} \tag{2}$$

```
Accel_InSight = -G * (((M_Sun * rSunInSight) / norm(rSunInSight)^3 )+((
    M_Phobos * rPhobosInSight) / (norm(rPhobosInSight)^3 )) +((M_Mars *
    rMarsInSight)/ (norm(rMarsInSight)^3)));
```

The acceleration of InSight is $(-2.744\hat{\imath} - 1.116\hat{\jmath} + 0.007\hat{k}) \cdot 10^{-6}\ km/s^2$. This acceleration is relative to an inertial point, which is the center of mass of the system. (Calculated in Eq. 1)

3 Write the relative motion differential equation

Write the differential equation that governs the relative motion of InSight relative to Mars.

$$\ddot{\vec{r}}_{Mars \to InSight} = \frac{-G(m_{InSight} + m_{Mars})}{r_{Mars \to InSight}^3} \cdot \vec{r}_{Mars \to InSight} + G \cdot m_{Sun}\left(\frac{\vec{r}_{InSight \to Sun}}{r_{InSight \to Sun}^3} - \frac{\vec{r}_{Mars \to Sun}}{r_{Mars \to Sun}^3}\right) + G \cdot m_{Phobos}\left(\frac{\vec{r}_{InSight \to Phobos}}{r_{InSight \to Phobos}^3} - \frac{\vec{r}_{Mars \to Phobos}}{r_{Mars \to Phobos}^3}\right) \tag{3}$$

4 Determine Accelerations

Determine the dominant, direct, and indirect acceleration (vector) on InSight.

Dominant acceleration on InSight:

$$\frac{-G(m_{InSight} + m_{Mars})}{r^3_{Mars\rightarrow InSight}} \cdot \vec{r}_{Mars\rightarrow InSight} = (3.206\hat{\imath} - 8.66\hat{\jmath} + 0.243\hat{k}) \cdot 10^{-8}\ km/s^2 \tag{4}$$

Direct accelerations on InSight:

Due to the Sun:

$$G \cdot m_{Sun} \cdot \frac{\vec{r}_{InSight\rightarrow Sun}}{r^3_{InSight\rightarrow Sun}} = (-2.776\hat{\imath} - 1.030\hat{\jmath} + 0.005\hat{k}) \cdot 10^{-6}\ km/s^2 \tag{5}$$

Due to Phobos:

$$G \cdot m_{Phobos} \cdot \frac{\vec{r}_{InSight\rightarrow Phobos}}{r^3_{InSight\rightarrow Phobos}} = (0.511\hat{\imath} - 1.401\hat{\jmath} + 0.037\hat{k}) \cdot 10^{-15}\ km/s^2 \tag{6}$$

Indirect accelerations on InSight:

Due to the Sun:

$$-G \cdot m_{Sun} \cdot \frac{\vec{r}_{Mars\rightarrow Sun}}{r^3_{Mars\rightarrow Sun}} = (2.780\hat{\imath} + 1.021\hat{\jmath} - 0.005\hat{k}) \cdot 10^{-6}\ km/s^2 \tag{7}$$

Due to Phobos:

$$-G \cdot m_{Phobos} \cdot \frac{\vec{r}_{Mars\rightarrow Phobos}}{r^3_{Mars\rightarrow Phobos}} = (-0.151\hat{\imath} + 8.089\hat{\jmath} + 0.609\hat{k}) \cdot 10^{-12}\ km/s^2 \tag{8}$$

5 Evaluate Accelerations

For the same relative acceleration (InSight relative to Mars), evaluate the acceleration due to Mars, acceleration due to the Sun, and acceleration due to Phobos.

Using equations 4-8 and MATLAB we solve for the various accelerations induced on InSight by other celestial bodies.

```
Accel_InSight_due_to_Mars = (-G * (M_InSight + M_Mars) / (norm(
    rMarsInSight)^3) * (rMarsInSight));
Accel_InSight_due_to_Sun = (G * (M_Sun * ((rInSightSun / norm(rInSightSun)
    ^3)- (rMarsSun / norm(rMarsSun)^3))));
Accel_InSight_due_to_Phobos = (G * (M_Phobos * ((rInSightPhobos / norm(
    rInSightPhobos)^3)- (rMarsPhobos/norm(rMarsPhobos)^3))));
```

Acceleration of InSight due to Mars is $(3.206\hat{\imath} - 8.666\hat{\jmath} + 0.243\hat{k}) \cdot 10^{-8}\ km/s^2$
Acceleration of InSight due to the Sun is $(3.282\hat{\imath} - 8.935\hat{\jmath} + 0.251\hat{k}) \cdot 10^{-9}\ km/s^2$
Acceleration of InSight due to Phobos is $(-1.508\hat{\imath} + 0.8088\hat{\jmath} + 0.0695\hat{k}) \cdot 10^{-13}\ km/s^2$

6 Is it Reasonable?

At this instant is it reasonable to assume a two-body relative model for motion of the spacecraft with respect to Mars? Why or why not?

It is not reasonable to assume a 2-body relative model for motion of the spacecraft with respect to Mars. The gravitational influence of the Sun on InSight is approximately 10% of the influence of Mars on InSight. If a two-body relative model of motion was assumed between InSight and Mars, 10% of the total accelerative forces would be ignored. This opens up potential for mission disrupting inaccuracies.

Team3_GroupExam1.m

```matlab
%% AE 313 Group Exam 1
% Team 3 - Jiyoung Hwang, Grace Day, Thorne Wolfenbarger, Aaron Scott

% Position Vectors
rSunMars = [198720000, 72978000, -334700]; % km
rMarsPhobos = [174.29, -9319.9, -702.19]; % km
rMarsInSight = [-236120, 638170, -17906]; % km
rSunPhobos = rSunMars + rMarsPhobos; % km
rSunInSight = rSunMars + rMarsInSight; % km
rPhobosInSight = rMarsInSight - rMarsPhobos; % km
rInSightSun = -rSunInSight; % km
rMarsSun = -rSunMars; % km
rInSightPhobos = -rPhobosInSight; % km

% Gravitational Parameters
muInSight = 4.6318e-17; % km^3/s^2
muPhobos = 0.000709; % km^3/s^2
muSun = 132712440000; % km^3/s^2
muMars = 42828; % km^3/s^2
G = 6.67*10^-11; % N*m^2/kg^2

% Masses
M_InSight = (muInSight) / G; % kg
M_Phobos = (muPhobos) / G; % kg
M_Sun = (muSun) / G; % kg
M_Mars = (muMars) / G; % kg

%% Q1 -
% Calculate the center of mass with respect to the Sun

rCM = (rSunMars * M_Mars + rSunPhobos * M_Phobos + rSunInSight * M_InSight ) ...
      / (M_InSight + M_Phobos + M_Mars + M_Sun);

%% Q2 -
% Find the acceleration of InSight. What is the acceleration relative to?

Accel_InSight = -G * (((M_Sun * rSunInSight) / norm(rSunInSight)^3 ) + ...
                     ((M_Phobos * rPhobosInSight) / (norm(rPhobosInSight)^3 )) + ...
                     ((M_Mars * rMarsInSight) / (norm(rMarsInSight)^3)));

% The Acceleration is relative to the center of mass (inertial point)
```

```
%% Q3 -
% Write the differential equation that governs the relative motion of
% InSight relative to Mars.

% Answers in Document

%% Q4 -
% Determine the dominant, direct, and indirect acceleration (vector)
% on InSight
fprintf(['Question 4'])
Accel_InSight_due_to_Mars = (-G * (M_InSight + M_Mars) ...
                            / (norm(rMarsInSight)^3) * (rMarsInSight))

Dir_Accel_InSight_due_to_Sun = (G * (M_Sun * ((rInSightSun / norm(rInSightSun) ^3))))
InDir_Accel_InSight_due_to_Sun = (G * (M_Sun * (-(rMarsSun / norm(rMarsSun)^3))))

Dir_Accel_InSight_due_to_Phobos = (G * (M_Phobos * ((rInSightPhobos / norm( rInSightPhobos)^3))))
InDir_Accel_InSight_due_to_Phobos = (G * (M_Phobos * (-(rMarsPhobos/norm(rMarsPhobos)^3))))
% Answers in Document

%% Q5 -
% For the same relative acceleration (InSight relative to Mars), evaluate
% the acceleration due to Mars, acceleration due to the Sun, and
% acceleration due to Phobos

Accel_InSight_due_to_Mars = (-G * (M_InSight + M_Mars) ...
                            / (norm(rMarsInSight)^3) * (rMarsInSight));

Accel_InSight_due_to_Sun = (G * (M_Sun * ((rInSightSun / norm(rInSightSun)^3) ...
                           - (rMarsSun / norm(rMarsSun)^3))));

Accel_InSight_due_to_Phobos = (G * (M_Phobos * ((rInSightPhobos / norm(rInSightPhobos)^3) ...
                              - (rMarsPhobos/norm(rMarsPhobos)^3))));

%% Q6 -
% At this instant, is it reasonable to assume a two-body relative motion
```

```
% for motion of the spacecraft with respect to Mars? Why or why not?

% It is not reasonable to assume a 2-body relative model for motion
% of the spacecraft with respect to Mars because the gravitational
% influence of the Sun is only 1 order of magnitude less than that of Mars.
% This is a 10% error, which can lead to an unpredictable trajectory and
% landing zone."

fprintf(['\nQuestion 1: rCM = <%.3e, %.3e, %.3e> km\n' ...
         'Question 2: a_InSight = <%.3e, %.3e, %.3e> km/s^2\n' ...
         'Question 5: a_InSight_Due_To_Mars = <%.3e, %.3e, %.3e> km/s^2\n' ...
         '            a_InSight_Due_To_Sun = <%.3e, %.3e, %.3e> km/s^2\n' ...
         '            a_InSight_Due_To_Phobos = <%.3e, %.3e, %.3e> km/s^2\n'], ...
         rCM, Accel_InSight, Accel_InSight_due_to_Mars, ...
         Accel_InSight_due_to_Sun, Accel_InSight_due_to_Phobos
```