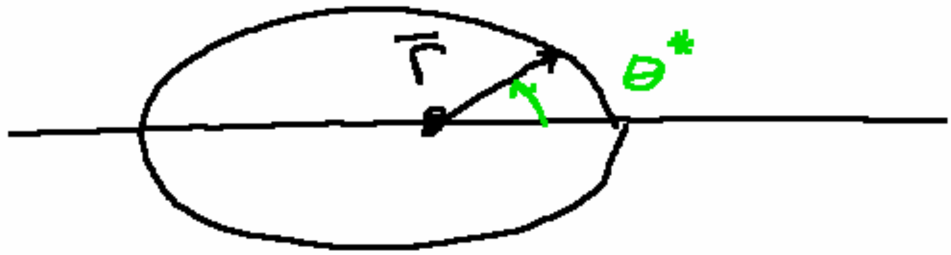
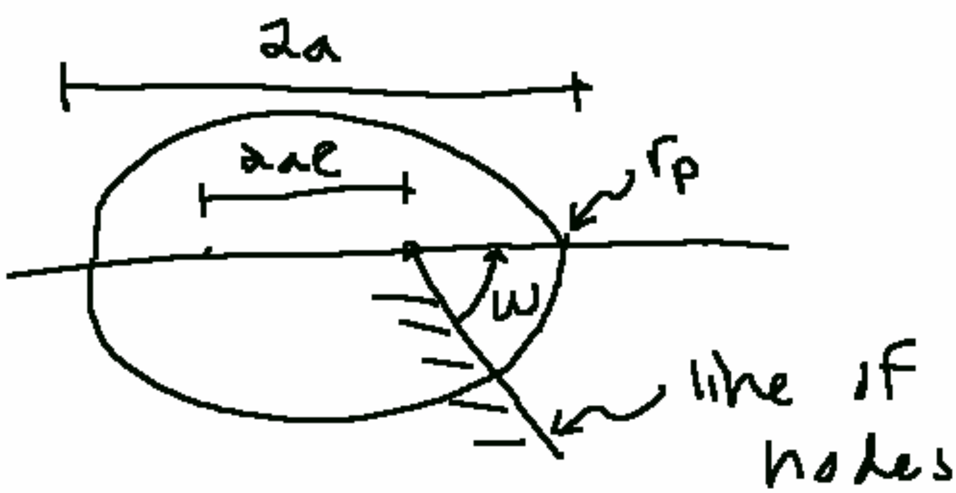
1. Locate s/c in orbit: time $\Leftarrow M, E, \theta^*$

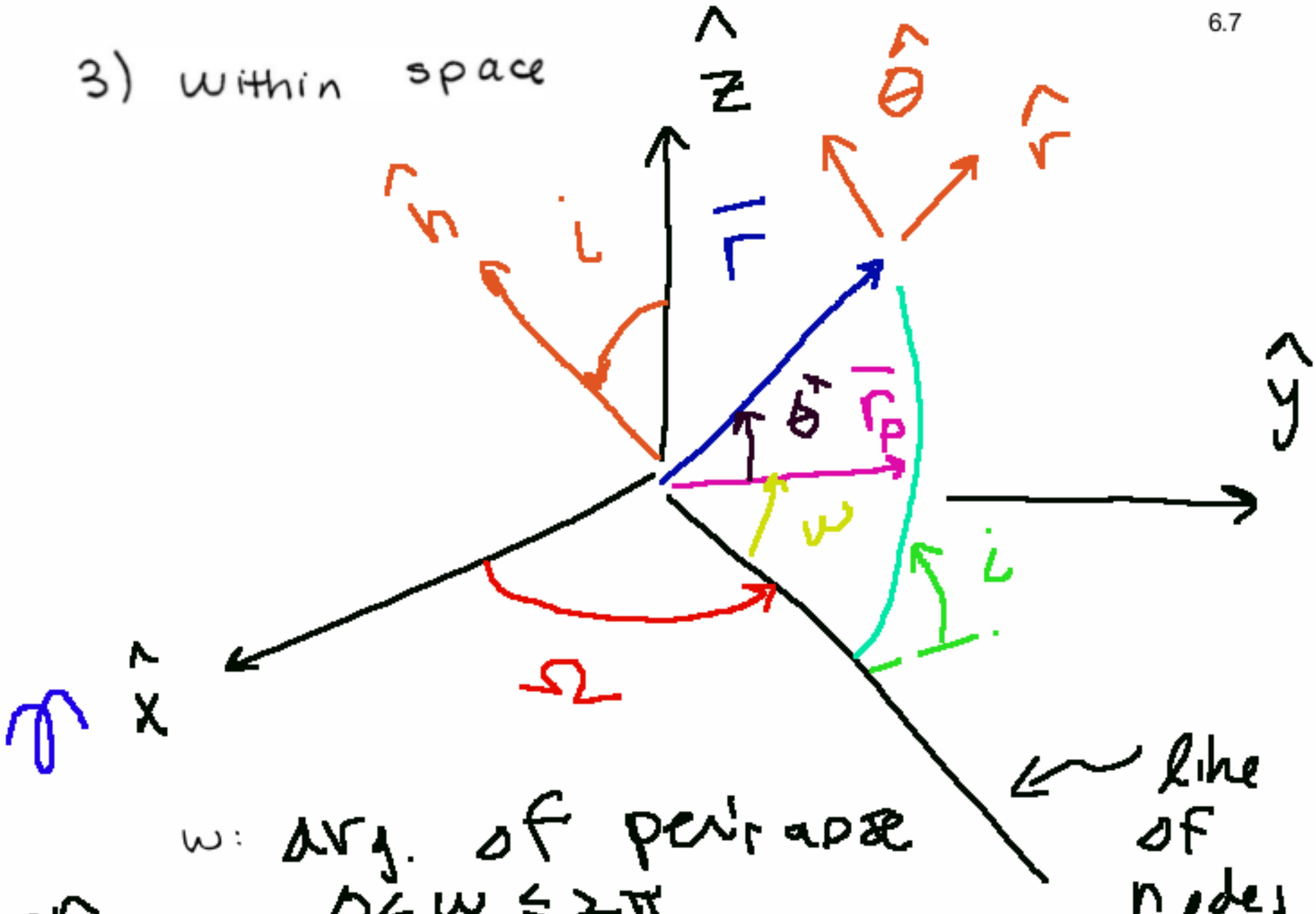


2. within orbit plane: orbit size + shape (a, e)

orbit orientation in orbit plane (ω)



3) within space



ω: arg. of periapse
$0 \le \omega \le 2\pi$

when $i=0$ $\begin{cases} \Omega + \omega = \bar{\omega} : \text{longitude of periapse} \\ \bar{\omega} + \theta^* = L : \text{true longitude at epoch} \end{cases}$

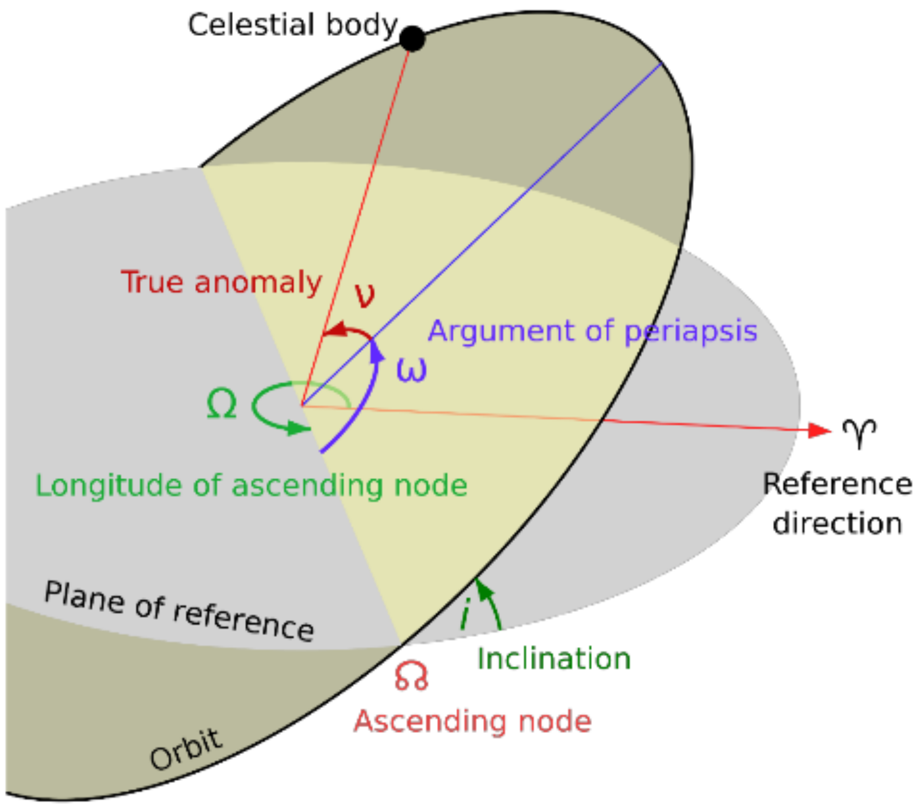
$\bar{r}^{xyz} = A^{xyz}_{r\theta h}\, \bar{r}^{r\theta h}$ $\quad r^{r\theta h} = A^{r\theta h}_{xyz}\, \bar{r}^{xyz}$

To transform use 3-1-3 Euler sequence $\quad A^{r\theta h}_{xyz} = \left(A^{xyz}_{r\theta h}\right)^T$

C: cosine
s: sine

$$A^{xyz}_{r\theta h} = \begin{array}{c|ccc} & \hat{r} & \hat{\theta} & \hat{h} \\ \hline \hat{x} & c\Omega c\theta - s\Omega ci s\theta & -c\Omega s\theta - s\Omega ci c\theta & s\Omega si \\ \hat{y} & s\Omega c\theta + c\Omega ci s\theta & -s\Omega s\theta + c\Omega ci c\theta & -c\Omega si \\ \hat{z} & si s\theta & si c\theta & ci \end{array}$$

$$\hat{z} = si s\theta\, \hat{r} + si c\theta\, \hat{\theta} + ci\, \hat{h}$$


Celestial body
True anomaly
ν
Argument of periapsis
ω
Ω
♈
Reference
direction
Longitude of ascending node
Plane of reference
i
Inclination
☊
Ascending node
Orbit

Example 1:

Given: $\bar{r}_1 = 1.6772\, R_\oplus \hat{x} - 1.6772\, R_\oplus \hat{y} + 2.3719\, R_\oplus \hat{z}$

$\bar{V}_1 = 3.1574\,\hat{x} + 2.4987\,\hat{y} + 0.4658\,\hat{z}$ km/s

Find: $a, e, i, \Omega, \omega, \theta^*$

Shape? → $r_1 = |\bar{r}_1|$ $V_1 = |\bar{V}_1|$

a, e $r_1 = 21394$ km $V_1 = 4.0533$ km/s

Find $\mathcal{E}$. What shape is the orbit?

$$\mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2a} = -10.416\ \text{km}^2/\text{s}^2$$

$\mathcal{E} < 0$ ellipse!

$$\boxed{a = -\frac{\mu}{2\mathcal{E}} = 19{,}134\ \text{km}}$$

$$h = |\bar{r} \times \bar{V}| = \sqrt{\mu p} = \sqrt{\mu a(1-e^2)} = 85567\ \text{km}^2/\text{s}$$

$$\boxed{e = 0.2}$$

Check for collisions, $r_p > r_\oplus$? yes!

Find magnitude of θ_1^*

$$\theta_1^* = \pm\cos^{-1}\left(\frac{p}{er_1} - \frac{1}{e}\right) \pm 135.01^\circ \text{ wait}$$

From rotation matrix

	$\hat{r}$	$\hat{\theta}$	$\hat{h}$
$\hat{x}$	—	—	$s\Omega si$
$\hat{y}$	—	—	$-c\Omega si$
$\hat{z}$	$si\,s\theta$	$si\,c\theta$	ci

Find $\hat{h}$.

$$\hat{h} = \frac{\bar{r}\times\bar{v}}{|\bar{r}\times\bar{v}|} = \underbrace{-\frac{1}{2}\hat{x} + \frac{1}{2}\hat{y} + 0.7071\,\hat{z}}_{\text{check } |\hat{h}|=1}$$

Then

$$\hat{h}\cdot\hat{z} = ci = 0.7071 \Rightarrow i = \pm 45^\circ$$

$$0 \le i \le 180^\circ \qquad \boxed{i = 45^\circ} \qquad \Omega = -135^\circ$$

$$\hat{h}\cdot\hat{y} = -c\Omega si = \frac{1}{2} \quad \{\Omega = \pm 135^\circ$$

$$\hat{h}\cdot\hat{x} = s\Omega si = -\frac{1}{2} \quad \{\Omega = -45, \underbrace{225^\circ}_{-135^\circ}$$

We can obtain the remaining elements from

$$\hat{r}_1 = \frac{\bar{r}_1}{|\bar{r}_1|} = 0.5\,\hat{x} - 0.5\,\hat{y} + 0.7071\,\hat{z}$$

$$\hat{\theta}_1 = \hat{h} \times \hat{r}_1 = 0.7071\,\hat{x} + 0.7071\,\hat{y}$$

What is θ_1?

$$\hat{r}_1 \cdot \hat{z} = \sin i \sin\theta = .7071 \quad \} \theta_1 = 90^\circ$$

$$\hat{\theta}_1 \cdot \hat{z} = \sin i \cos\theta = 0 \quad \} \theta_1 = 90^\circ \text{ or } 270^\circ$$

$$\boxed{\theta_1 = 90^\circ}$$

Back to θ_1^*. recall

$$\bar{V}_1 = \underbrace{(\bar{v}_1 \cdot \hat{r}_1)}_{\dot{r}_1}\hat{r}_1 + (\bar{v}_1 \cdot \hat{\theta}_1)\hat{\theta}_1$$

$\bar{v}_1 \cdot \hat{r}_1 \rightarrow$ if $\dot{r}_1 > 0$ ascending

$\dot{r}_1 < 0$ descending

$\dot{r}_1 = +0.659 \rightarrow > 0$ ascending

$\boxed{\theta^* = 135^\circ}$ since ascending

$\boxed{\nu = \theta_1 - \theta^* = -45^\circ}$