Patched Conics

(passage through local gravity fields)

Ideally, consider all gravitational influences at all time, but can only be done numerically.

Good approximation by considering the transfer in 3 phases (each a 2-body problem)

Consider an Earth-to-Mars mission

Assume planets are circular coplanar orbits about the sun

I:

II:

III:

I: 2-Body Problem Near $\oplus$

- Assume circular parking orbit at Earth (could be any orbit)
- No effect of Sun.
- Transfer "instantaneously" from influence of Earth to influence of Sun.
- To escape Earth, must depart on parabola or hyperbola
- Once escaped, possess exactly correct velocity for transfer orbit about $\odot$.

→ For our trip to Mars, velocity wrt $\odot$ > velocity wrt $\oplus$ ⇒ hyperbola.

Geocentric View

Heliocentric

In geocentric view:

$\Delta\bar{v}_i$ is tangential where s/c jumps from circular orbit to hyperbolic orbit.

After escape, velocity is

In heliocentric view

NOTE:

$\odot$

$\oplus$ S/C

$\bar{r}_{s/c} =$

$\bar{V}_{s/c} =$

$\bar{V}^{+} =$

vector diagram in heliocentric view

$\bar{V}^{+} =$

Use excess velocity to compute extra vector $\Delta\bar{v}_i$ to jump from parking orbit to hyperbola.

Most effective at

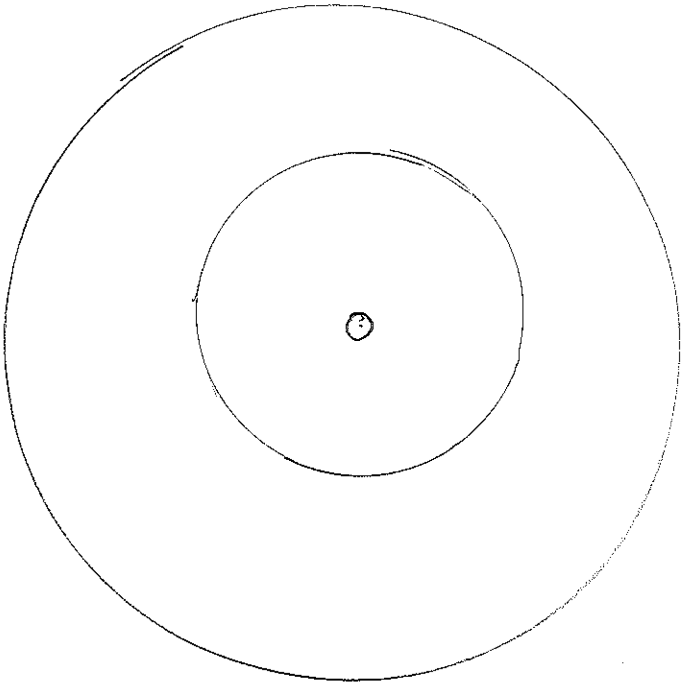
At perigee

$\mathcal{E} =$

$\boxed{\Delta v_i =}$

Δv_i: initial burn required to place s/c on heliocentric ellipse with correct v^+.

II: 2-Body Problem under ☉



- Assume leaving and approaching massless planets
- $\bar{V}^+ = \bar{V}_p$; perihelion on Hohmann transfer ellipse
- $\bar{V}^- = \bar{V}_a$; aphelion on Hohmann transfer ellipse for Mars approach

NOTE:

Patched Conics Example - Departure

Let's say we are going from Earth to Mars. We will leave Earth from LEO with an altitude of 500 km. The heliocentric orbit will leave Earth at periapsis and have a transfer angle of 140°.

What is the ΔV required at Earth to get on the trajectory?

Ideally we would start at Earth capture, but we don't have enough information yet.

Start with interplanetary travel and assume $\oplus$ + ♂ are circular co-planar orbits.

$r_{\oplus} =$

$r_{♂} =$

At Earth, Earth has the following

S/C has the following

vector diagram

Now that we have $v^{+}_{\infty/\oplus}$, we can solve for $\Delta \bar{V}_1$.

III : 2 Body Problem near ♂

- Assume goal is to capture into a circular orbit about ♂ with radius r_f.
- Circular orbit must be defined in a particular direction.
- Since s/c at apohelion on transfer ellipse, s/c will be moving slower than Mars. Must arrive "ahead" of planet and "fall" into its influence.

Use heliocentric velocity to determine the velocity on the hyperbola.

$V_{\infty/♂} =$

Vector Diagram

For circular capture

$\frac{V_{\infty/♂}^2}{2} =$

ΔV_{TOTAL}

Patched Conics Example - capture

For the same Earth-Mars case, let's move to capture around Mars. We will capture into a 200 km circular orbit.

We need to find $V_{\infty/♂}$, so start with heliocentric properties.

At arrival in heliocentric orbit,

Mar's values

Vector diagram

Around Mars

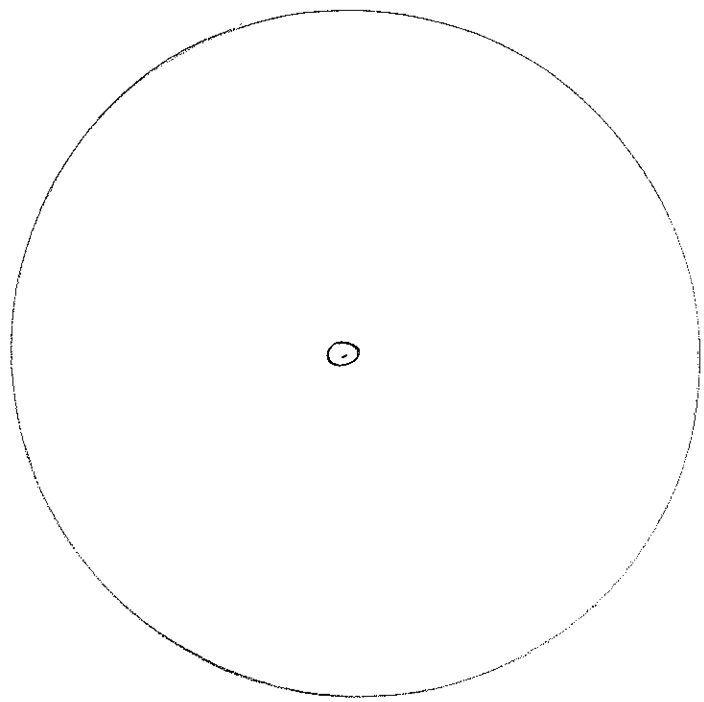
Swing-by / Fly-by / Gravity Assist

Rather than capture into Mars orbit, pass the planet at closest approach, r_f.

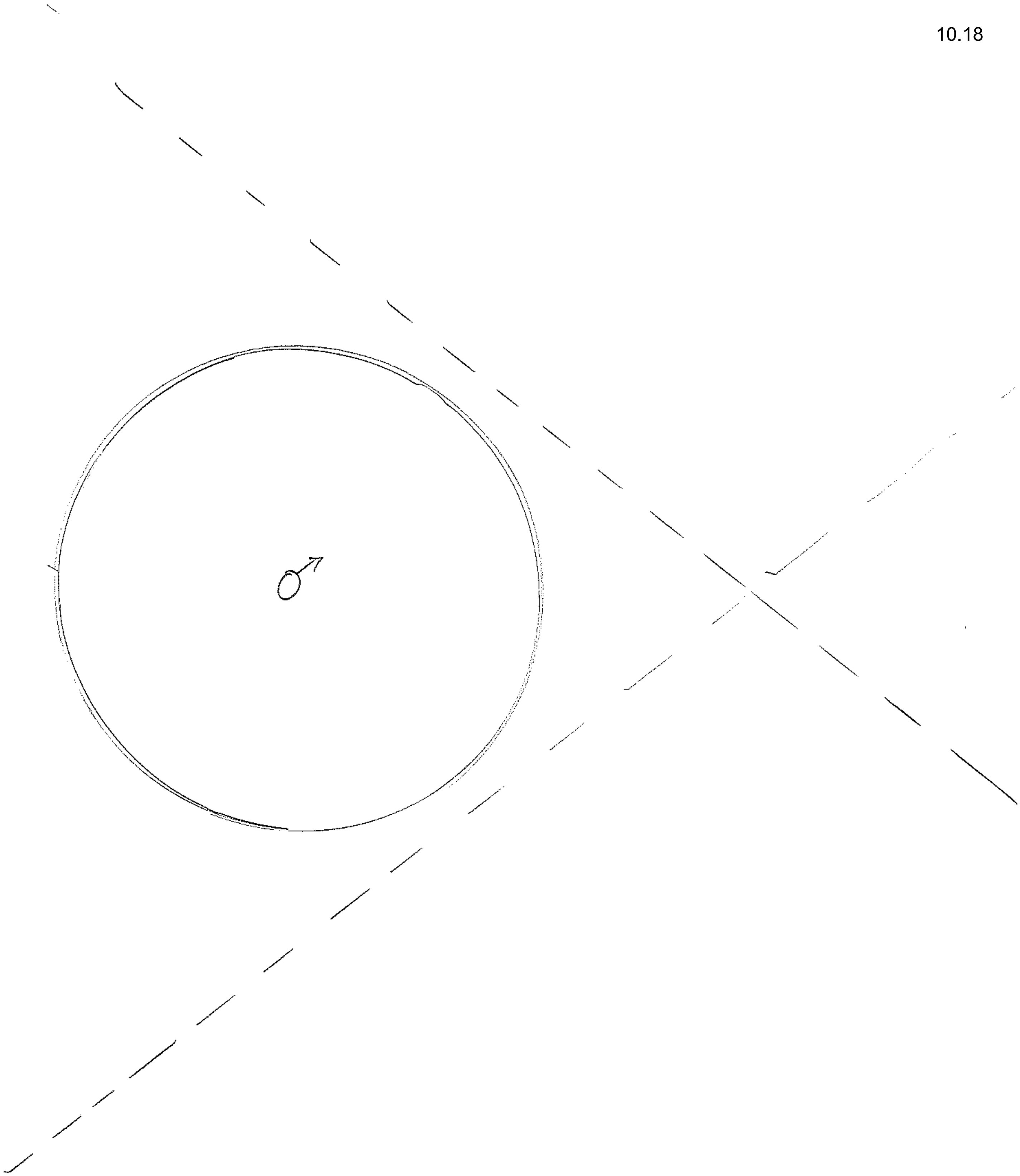
Same approach hyperbola, but

For swing-by, $V^{+}_{\infty/♂}$ same magnitude but different direction.

Vector Diagram



New orbit relative to sun for "free"



A tangential sunside passage will yeild same V^+ but now γ^+ is positive →

Since it's a circular Mars orbit, $\bar{V}_♂$ along local horizon.

If nontangential orbit use "ahead" or "behind"

For our case, $V^+ > V^-$

Patched conic method yeilds pretty accurate thrust requirements

=>

Greatest difficulty is

$\bar{V}_c + \Delta\bar{V} = \bar{V}_p$

$V_\infty^2 =$

Differentiate

For Hohmann to Mars,

Patched Conics Example - Grav. Assist

Instead of performing a burn at Mars to capture, we will be flying-by the planet at an approach altitude of 200 km

Arrival conditions are same

$$r = r_{♂} = 227,920,000 \text{ km}$$

$$v^- = 22.175 \text{ km/s}$$

$$\gamma^- = 10.788°$$

$$V_{♂} = 24.13 \text{ km/s}$$

$$\gamma_{♂} = 0°$$

Let's choose a fly-by that increases the energy. What is Δv_{eq+d}?

$r_p =$

$\mathcal{E} =$

If we move pass distance to 1000 km,

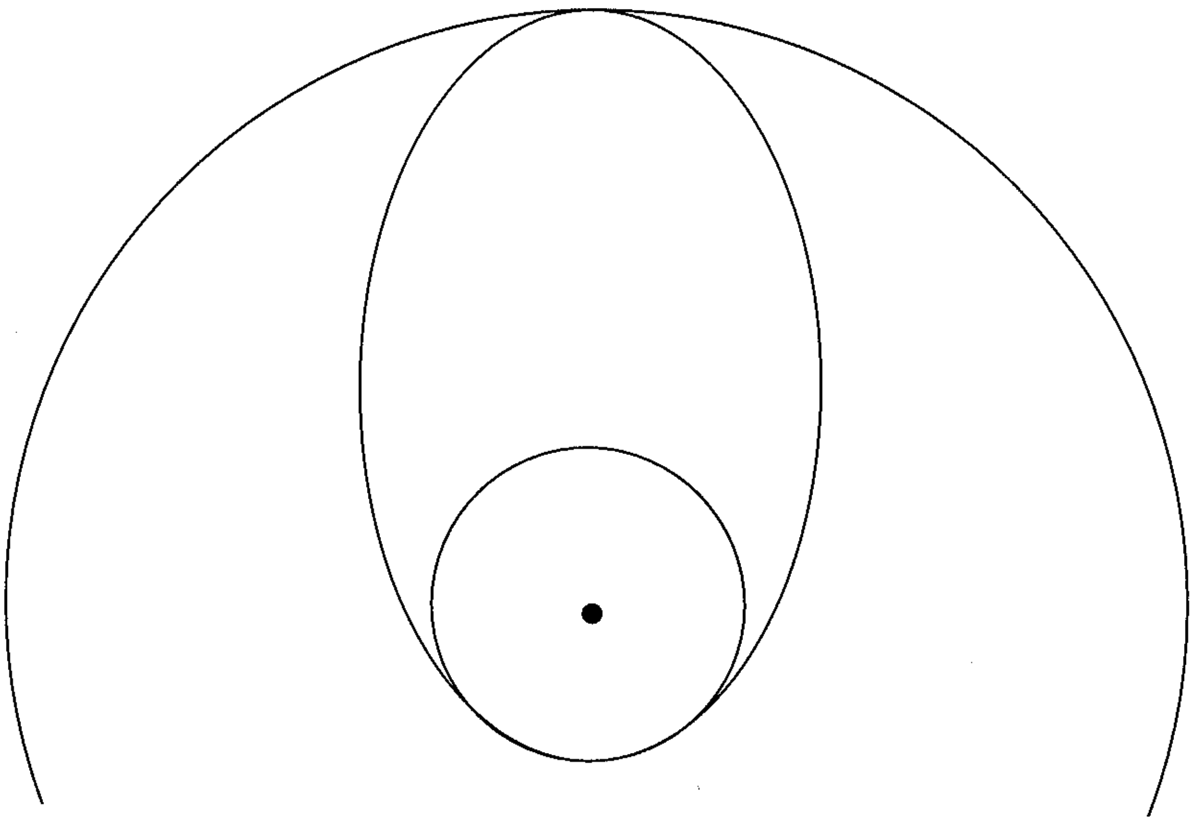
$\delta =$

$\Delta V_{e_\delta} =$

Free Return Trajectories

Look at circular lunar orbit.

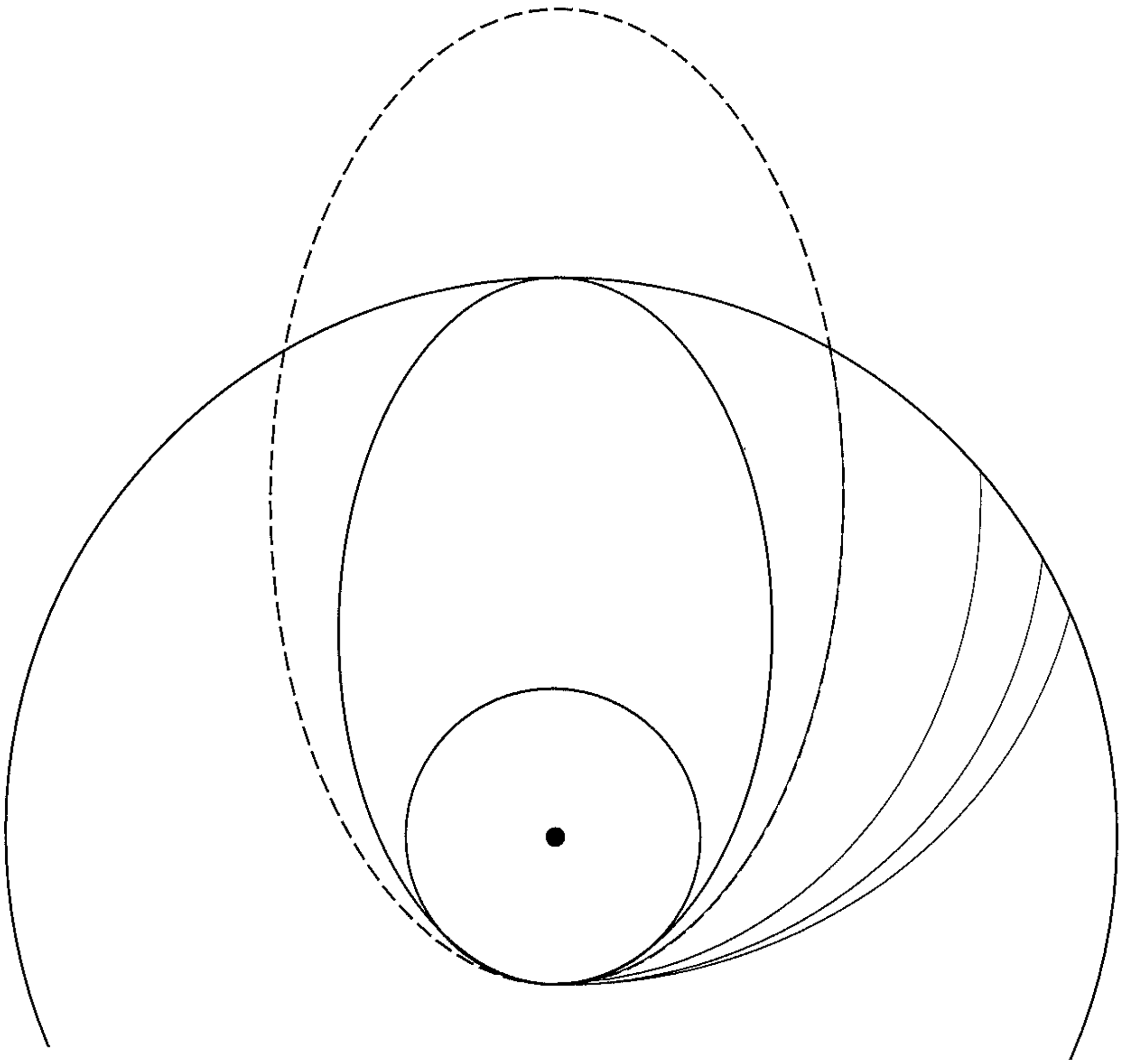
Assume:



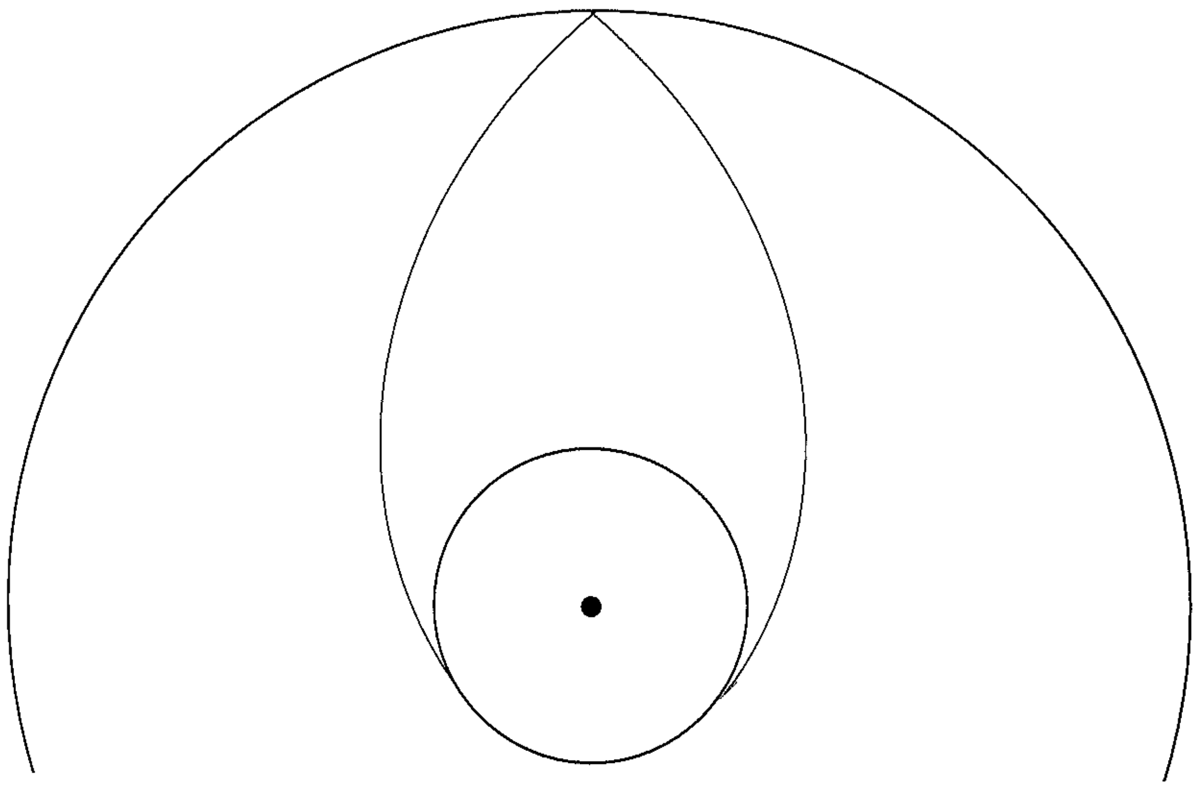
Patched conics less accurate here
than interplanetary.

If Moon has no gravity

- jump to Hohmann ellipse from parking orbit
- At moon w/ no $\Delta\bar{v}$.
-

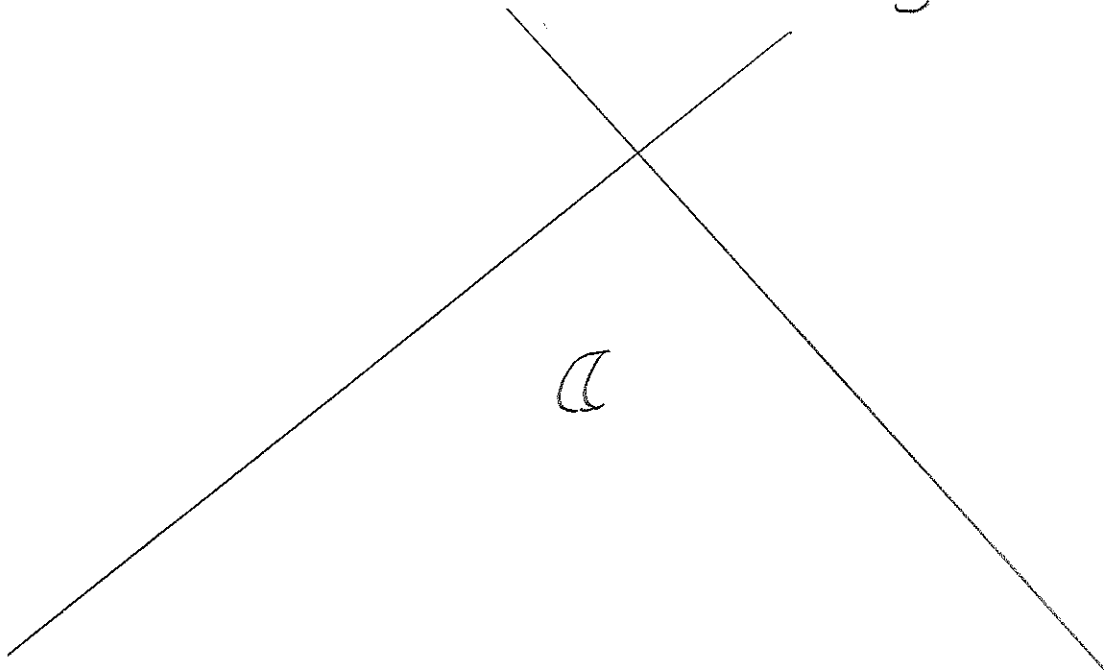


Moon possess gravity, so



Vector diagram

Local trajectory



If you choose "behind" pass,

Flyby angle determined by value of

Know V_∞ and required δ