Orbital Maneuvers

To date, we've only looked at orbit characteristics.

But satellites change orbits so we need to consider maneuvers and estimate velocity changes required for a particular mission.

consider:

Orbit Establishment

Find V_r and V_θ.

Given r, v, γ, how do we characterize the orbit?

$\bar{h} = \bar{r} \times \bar{v} \Rightarrow h = r V_\theta$ (since $\bar{r} = r\hat{r}$) skip

$$\boxed{V_\theta = \frac{h}{r} = }$$

$$V_r = \dot{r} = \frac{dr}{d\theta}\dot{\theta} = \frac{dr}{d\theta}\frac{h}{r^2} = \frac{d}{d\theta}\left(\frac{h^2/\mu}{1+e\cos(\theta-\omega)}\right)\frac{h}{r^2}$$

skip

Rearrange v_θ and v_r

$$e\cos\theta^* = \frac{h v_\theta}{\mu} - 1$$

$$e\sin\theta^* = \frac{h v_r}{\mu} = \frac{r v_\theta v_r}{\mu}$$

$$e^2 = e^2\cos^2\theta^* + e^2\sin^2\theta^*$$

$$\tan\theta^* = \frac{e\sin\theta^*}{e\cos\theta^*}$$

Now from $r, v, \gamma,$ we can find e & θ^*

Single Impulse Adjustments

use single impulse to adjust / change an orbit

Transfer to a with a
impulse is not possible unless the new orbit
the original orbit.

Assume :

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•

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Example: satellite in an established Earth orbit: $a = 3 R_\oplus$ $e = 0.5$

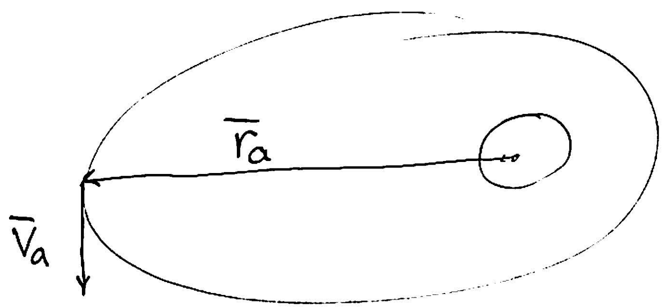
$(r_p = 1.5 R_\oplus)$

Change to: e constant

$a^+ = 4 R_\oplus$

ΔV (thrust) applied at apogee

Determine magnitude and direction of ΔV.



Solution:

1. Current orbit already established

2. Find conditions at thrust point <u>before</u> maneuver

(r, v, γ)

$R_{\oplus} = 6371$ km $\qquad \mu = 398600$ km^3/s^2

Find r, v, γ.

Decrease or increase v for increase in a?

If we increase v and maintain $e+r$, does θ change?

3. Determine (if possible) conditions at thrust point after maneuver.

$r^+ =$

$a^+ = 4 R_\oplus$ $e = 0.5$ given

4. Sketch a vector diagram to scale

If not at apoapsis

Use cosine law to find ΔV.

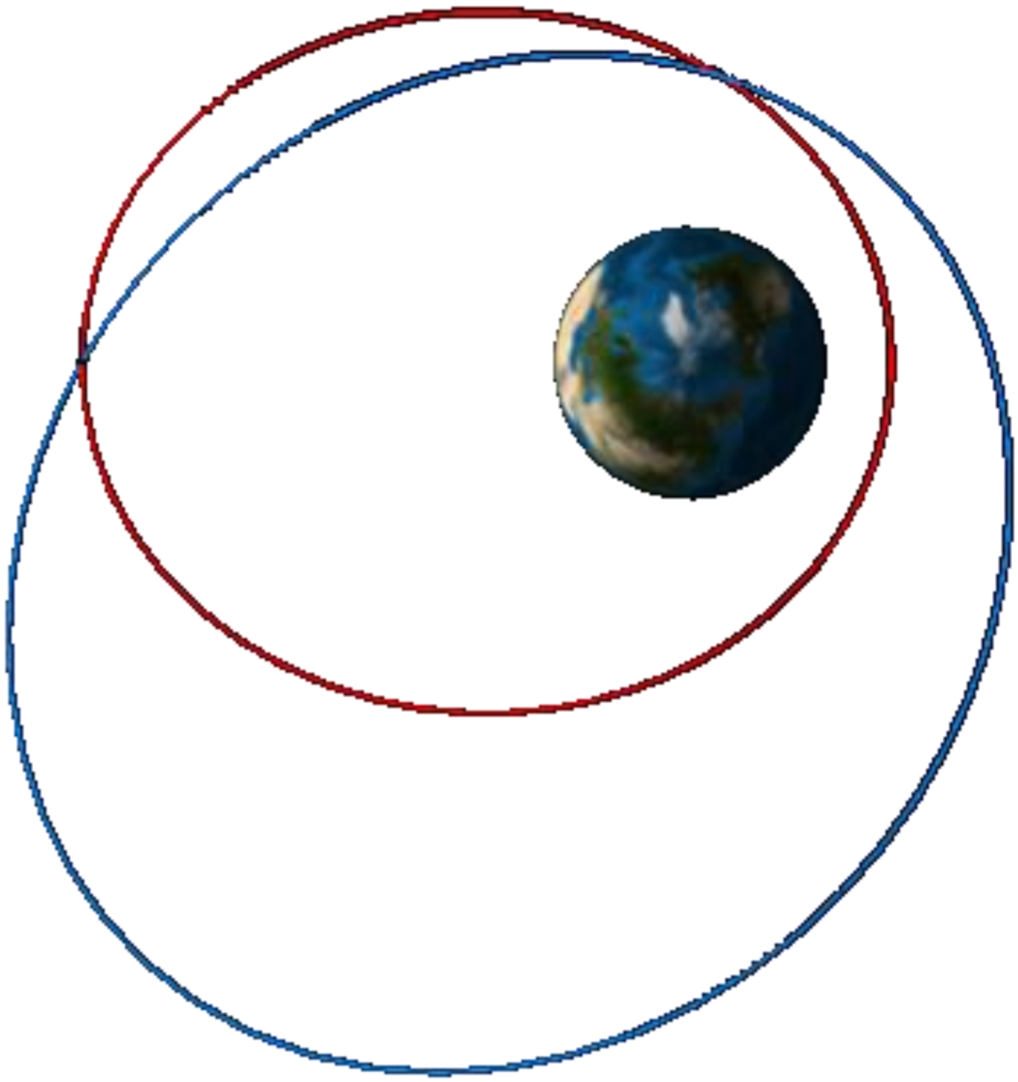
$\Delta V =$

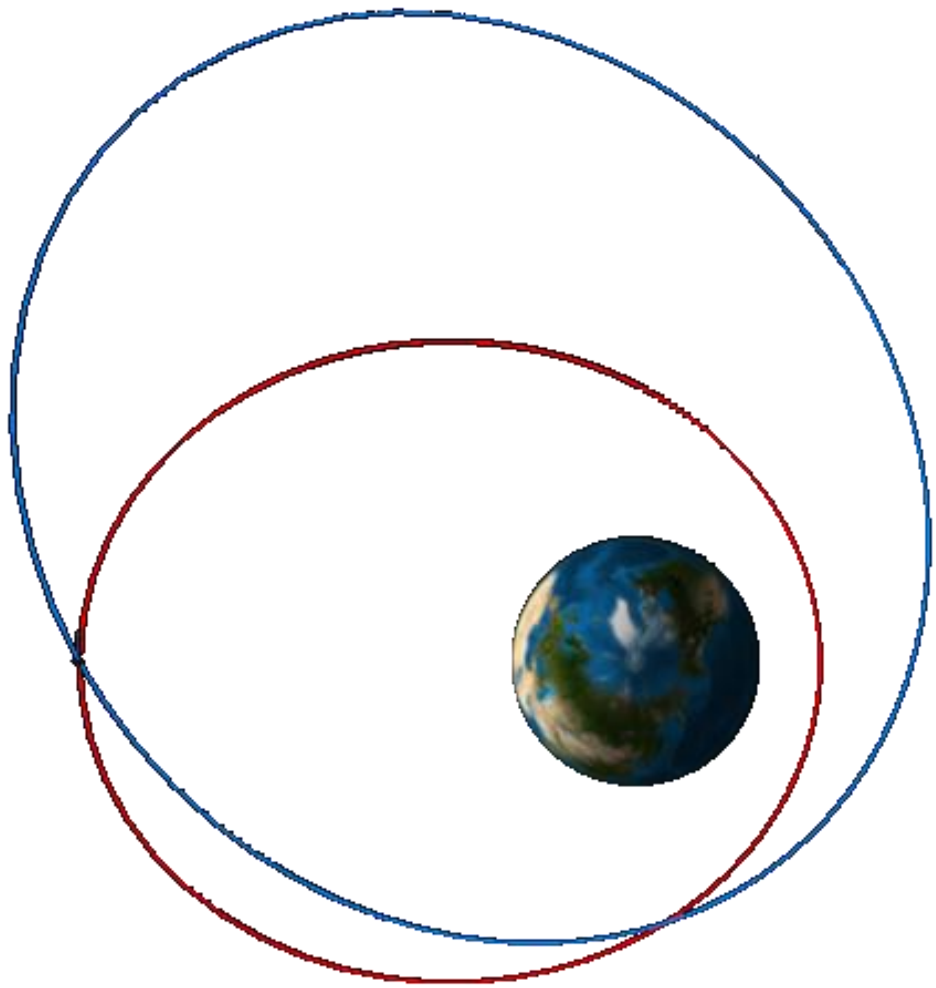
Use law of sines to find α

$\theta^{*+} =$

Originally $\theta^* = 180^\circ \rightarrow$

New orbit:

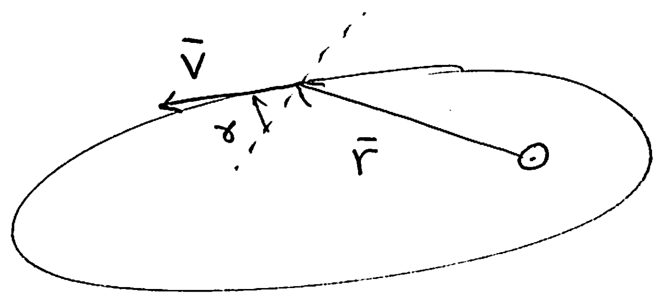




Let's do the same maneuver

$(a = 3R_{\oplus} \Rightarrow a = 4R_{\oplus}\ ,\ e = 0.5)$

but at $\theta^* = 120°$



1. Know current orbit

2. Conditions before

$r =$

$v =$

$\gamma =$

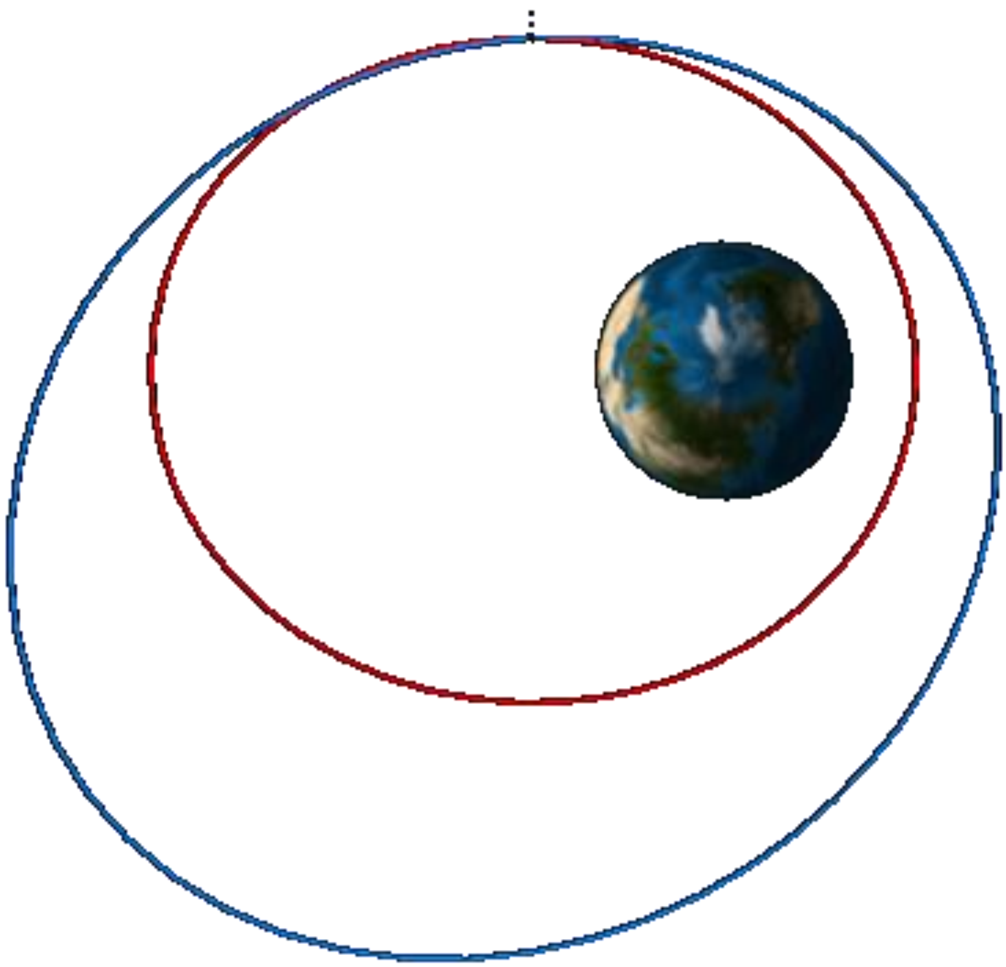
Increase or decrease velocity?

Tangential ΔV possible? (no γ change)

3. Desired conditions

$r^+ = r_a = 3R_\oplus$ $a^+ = 4\ R_\oplus$ $e = 0.5$

4. Vector diagram

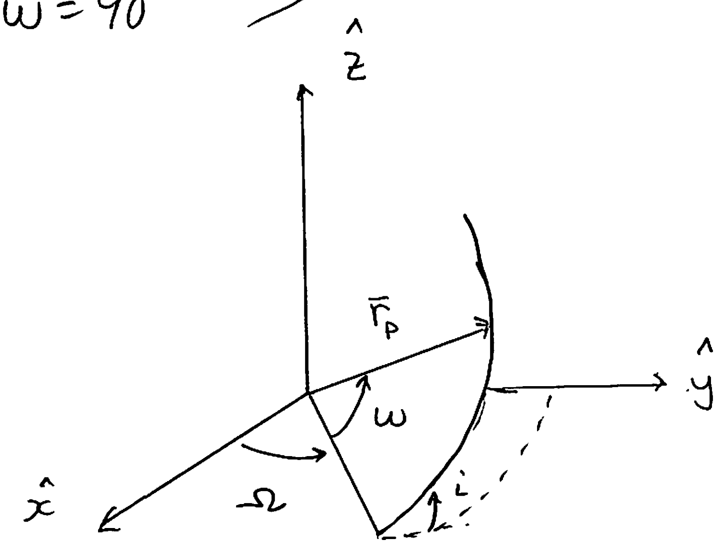


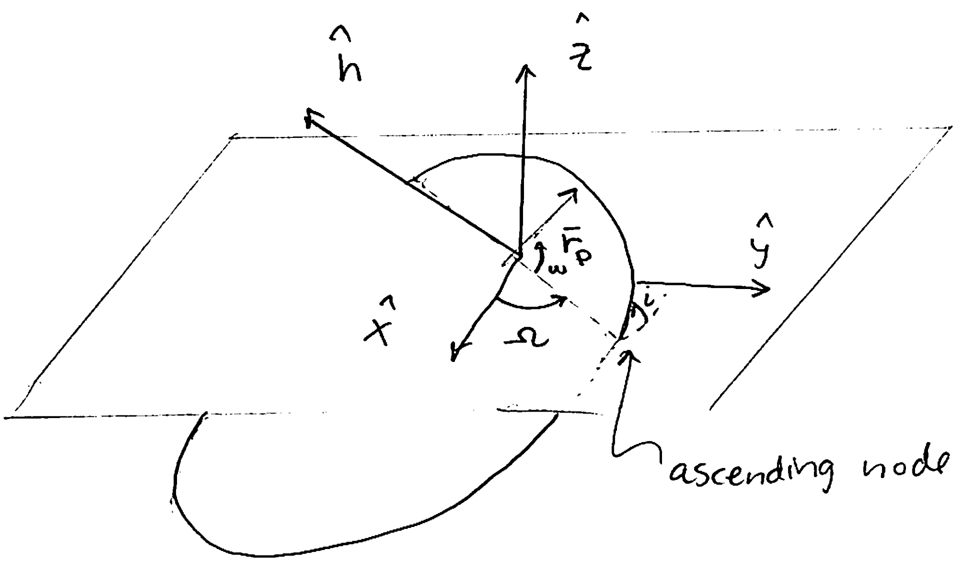
3D Example

Assume s/c is moving in orbit about the Earth

$a = 8 R_\oplus$
$e = 0.7$
$i = 30°$
$\Omega = 60°$
$\omega = 90°$

wrt Earth centered Mean J2000 coordinates



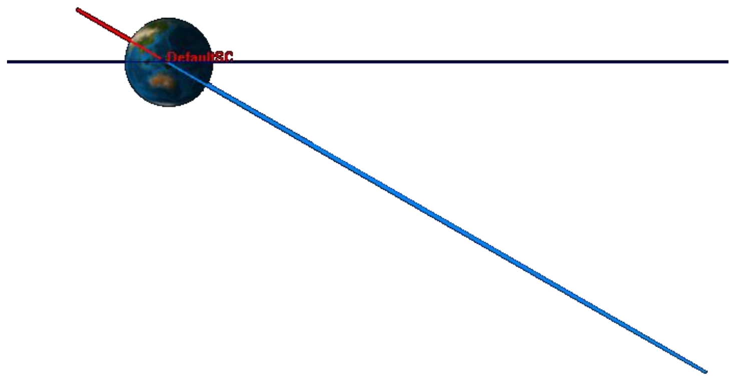


Maneuver at $\theta^* = 90°$ descending node

$r =$

$v =$

$\gamma =$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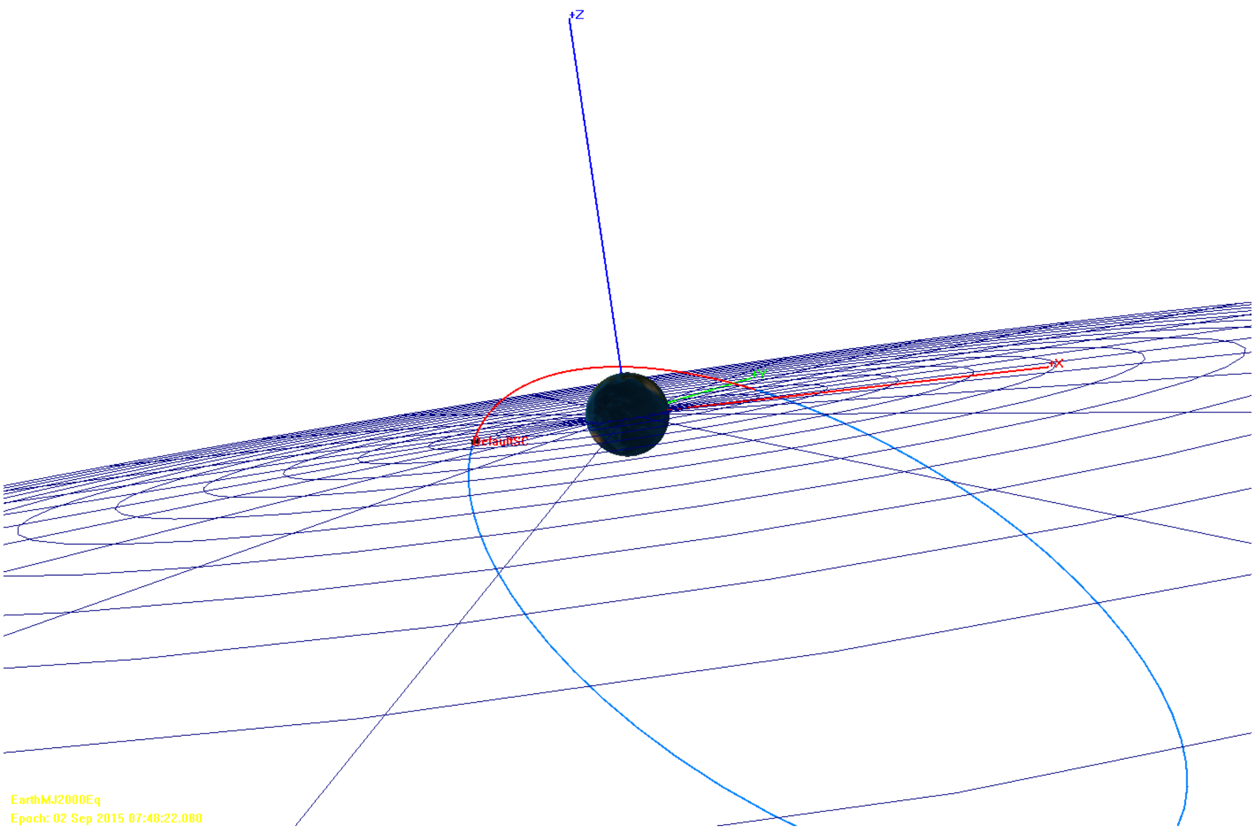
+Z
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EarthMJ2000Eq
Epoch: 02 Sep 2015 07:48:22.080

VNC coordinate Frame

GMAT calls this VNB.

$\hat{V}$:

$\hat{C}$:

$\hat{N}$:

$\Delta \bar{V} =$

$\overline{\Delta V} =$

Assume a maneuver such that

$\Delta v = 2\,km/s \quad \alpha = 0° \quad \beta = 150°$

Find $\Delta \bar{V}$ in VNC + $\hat{r}\hat{\theta}\hat{h}$

Rotate into $\hat{x}-\hat{y}-\hat{z}$ frame.

Add vectors in ECI b/c $\hat{r}-\hat{\theta}$ changes w/ orbit.

$$
{}^{ECI}R^{R\theta} \quad
\begin{array}{c|ccc}
 & \hat{r} & \hat{\theta} & \hat{h} \\
\hline
\hat{x} & c\Omega c\theta - s\Omega\, ci\, s\theta & -c\Omega s\theta - s\Omega\, ci\, c\theta & s\Omega si \\
\hat{y} & s\Omega c\theta + c\Omega\, ci\, s\theta & -s\Omega s\theta + c\Omega\, ci\, c\theta & -c\Omega si \\
\hat{z} & si\, s\theta & si\, c\theta & ci
\end{array}
$$

$$
{}^{ECI}R^{R\theta} = \begin{bmatrix} -.5 & .75 & .433 \\ -.866 & -.433 & -.25 \\ 0 & -.5 & .866 \end{bmatrix}
$$

$\bar{r} =$

$\bar{V} =$

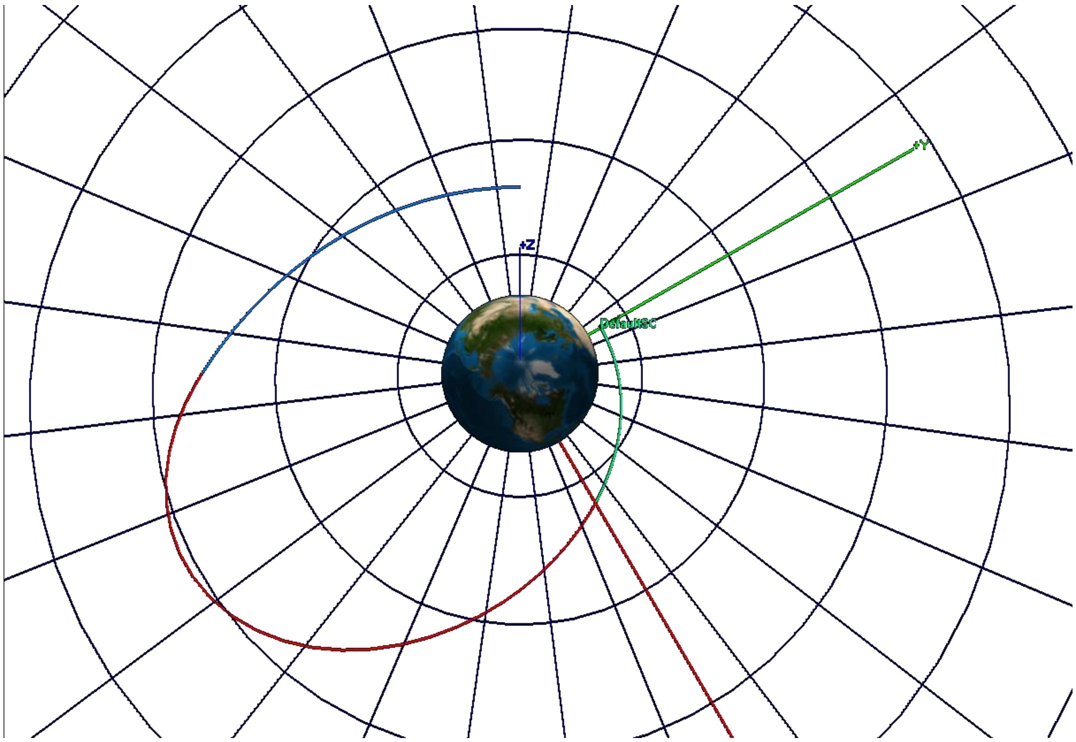
$\bar{V}^{+} =$

Find i and Ω

$\hat{h} = S\Omega si\ \hat{x} - C\Omega si\ \hat{y} + Ci\ \hat{z}$

$|\bar{v}| =$

Find θ^{+}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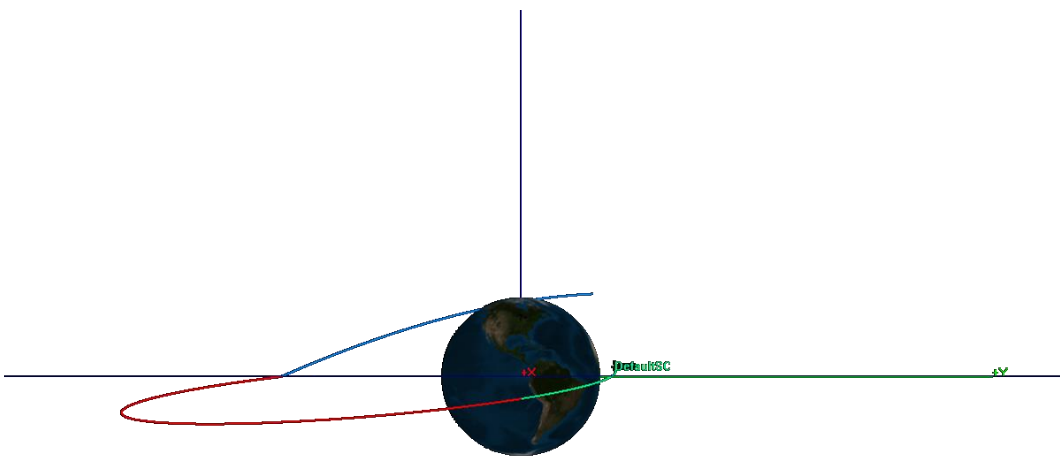
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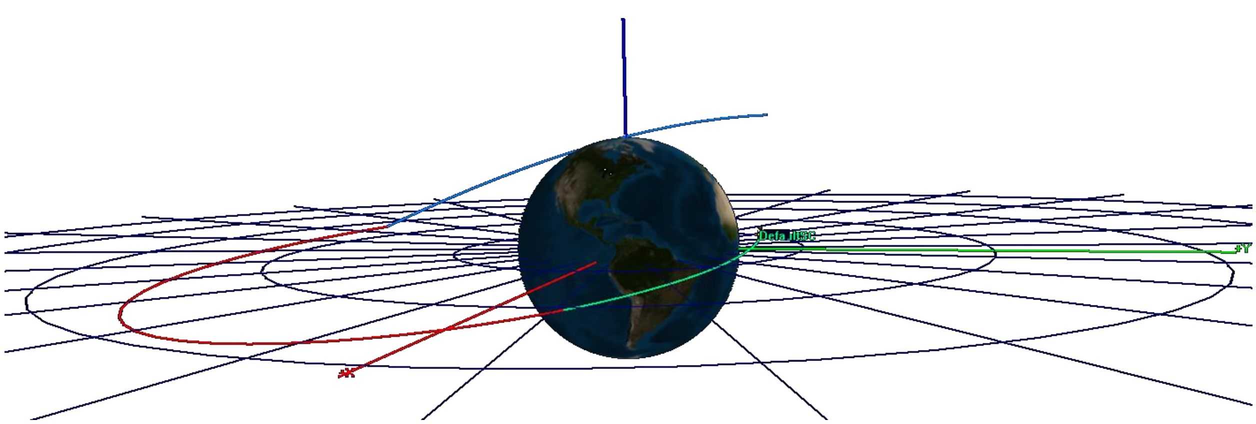
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