Lambert Arcs

Transfer orbits have two approaches to mission planning

1. Given the transfer orbit, find the initial and final positions which are related to time of flight.

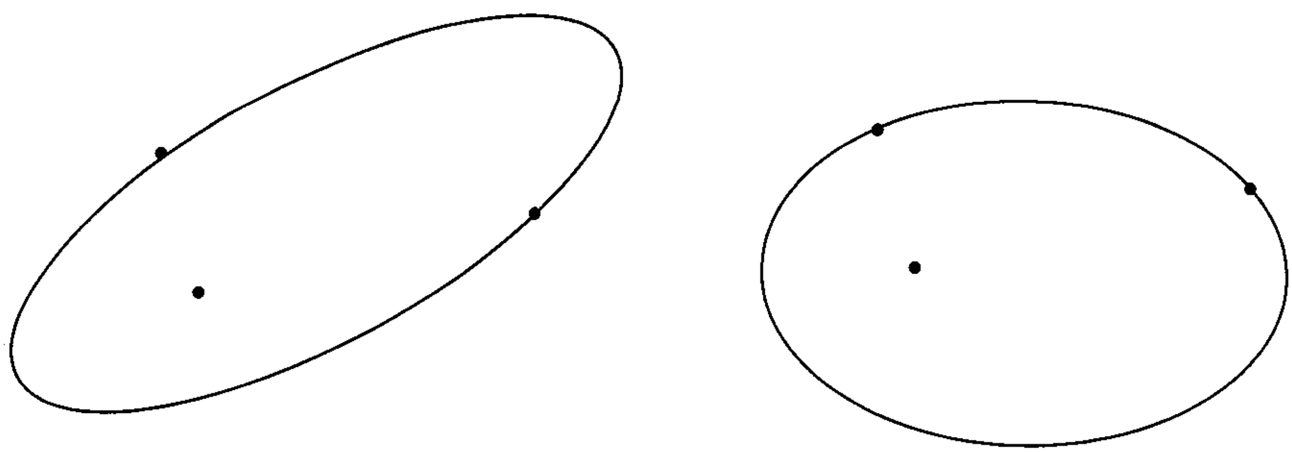
2. Given the initial (departure) and final (target) points, determine the orbit that passes through the points.

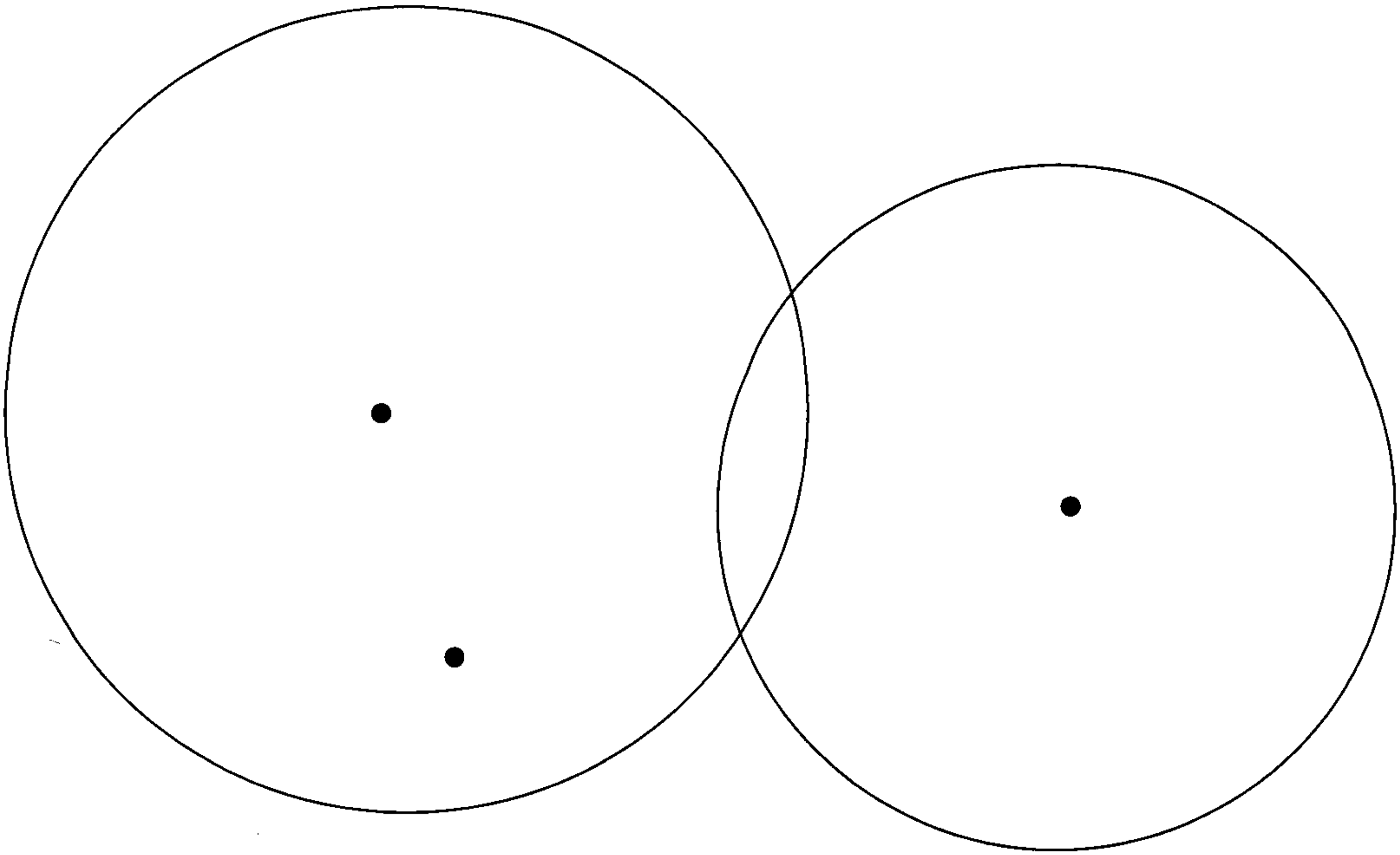
start with geometric relationships.

Assume:

Space Triangle

Ellipses

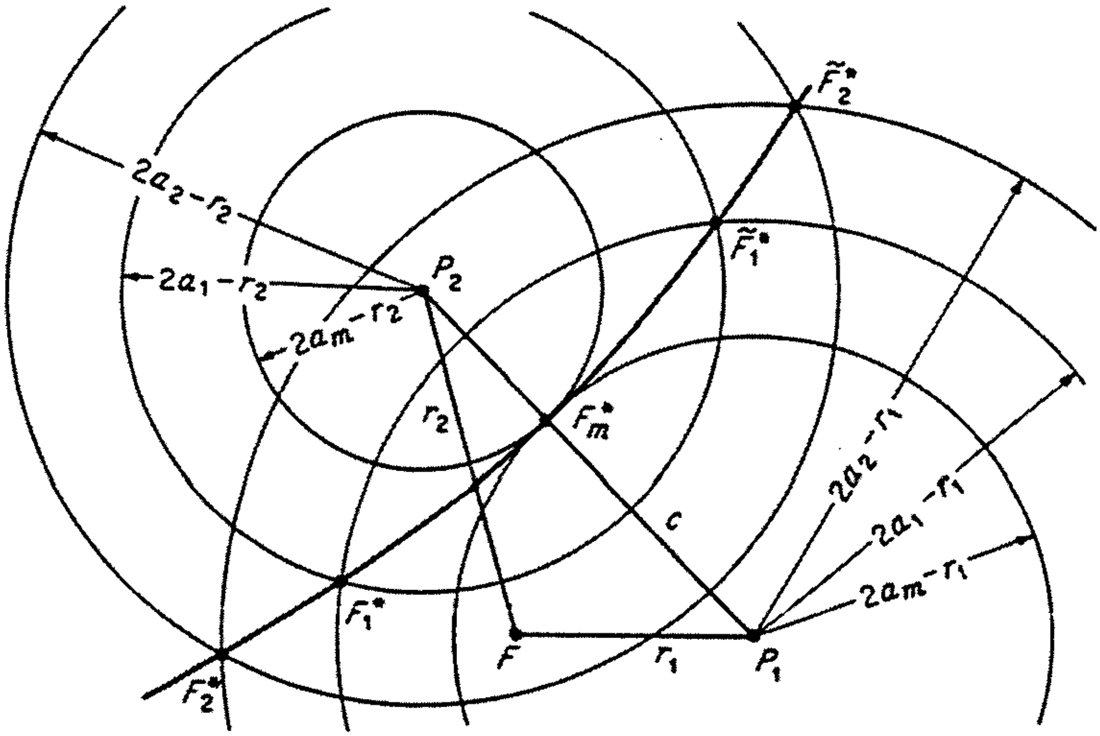




F must

F must

$\tilde{F}$

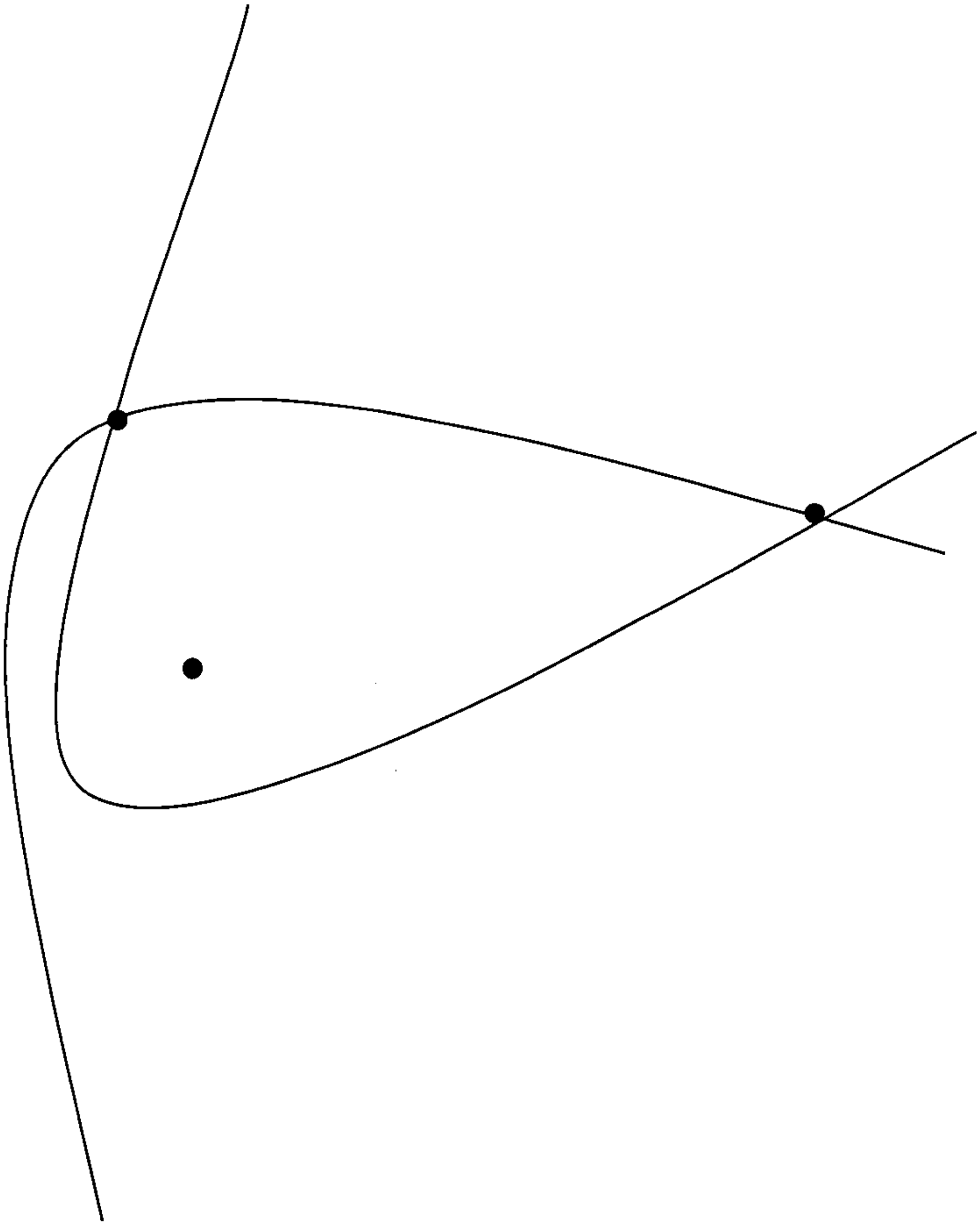


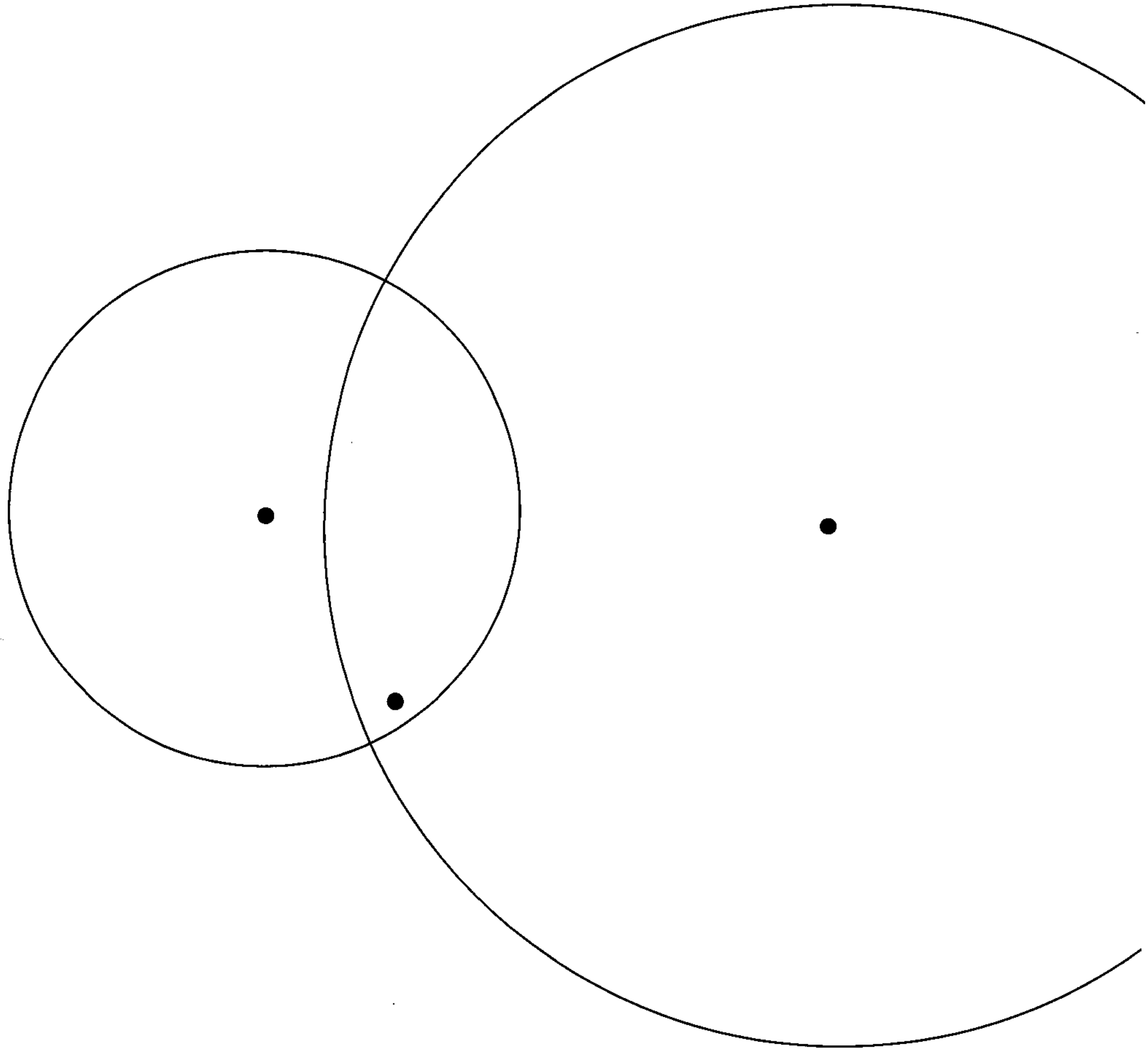
$F_m =$

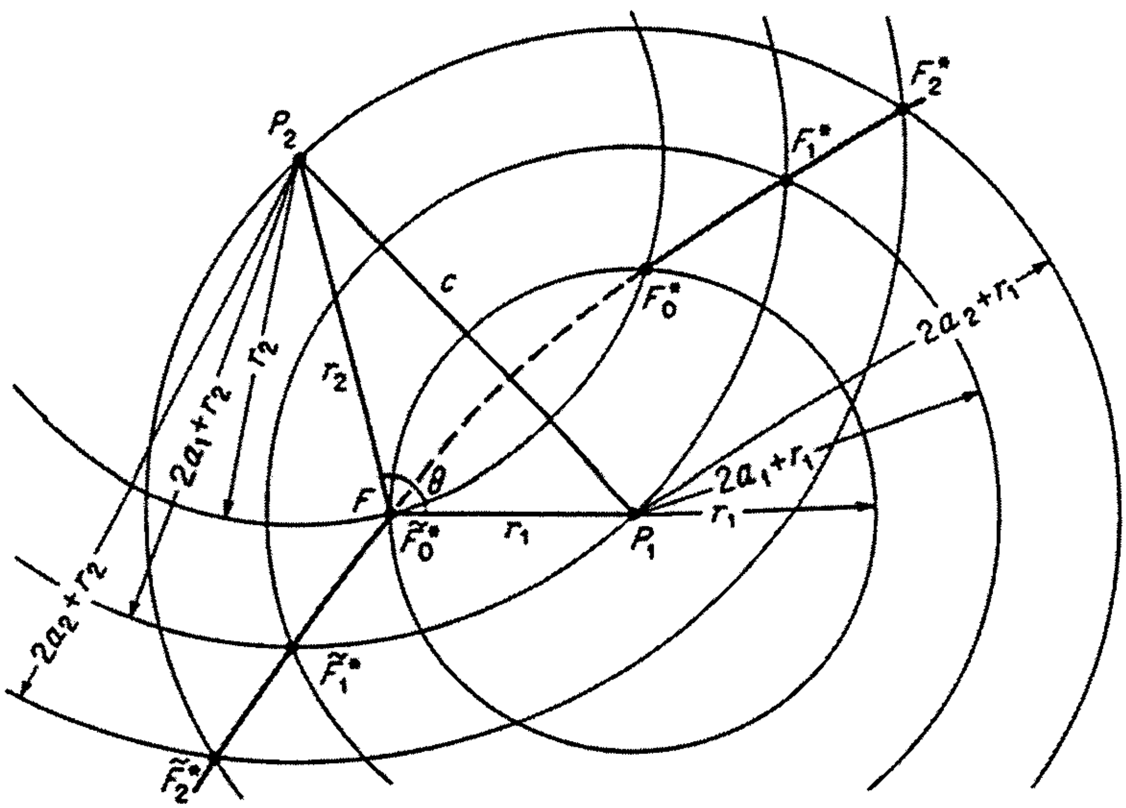
Semi-parameter of space triangle:

Minimum energy transfer (or "a_{min}" transfer)

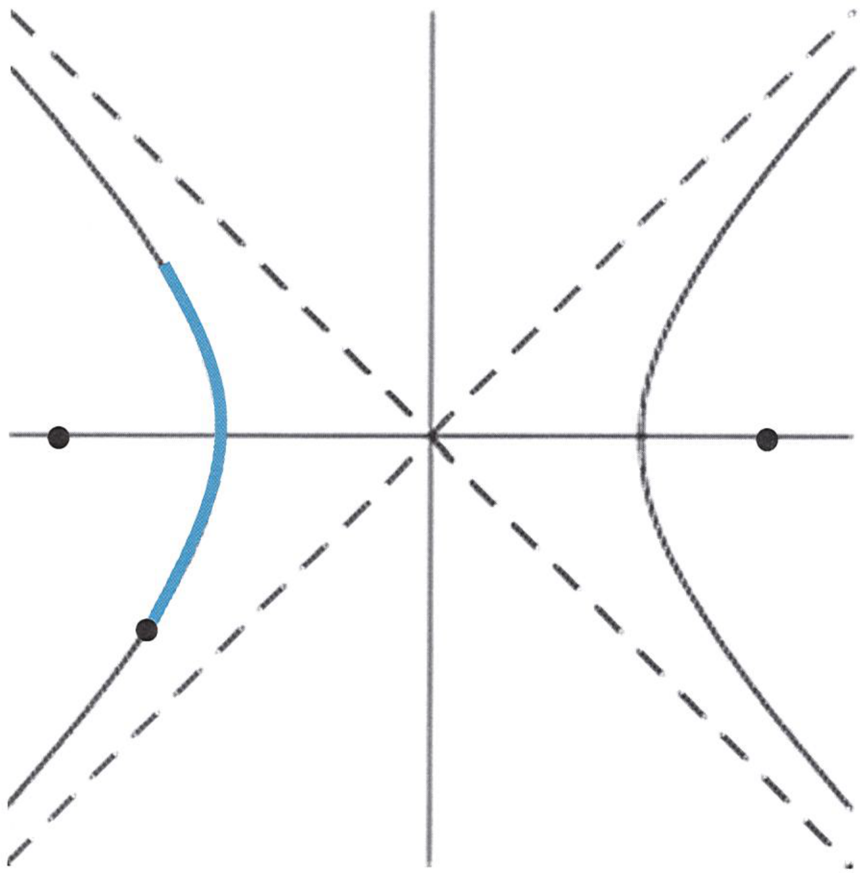
Hyberbola





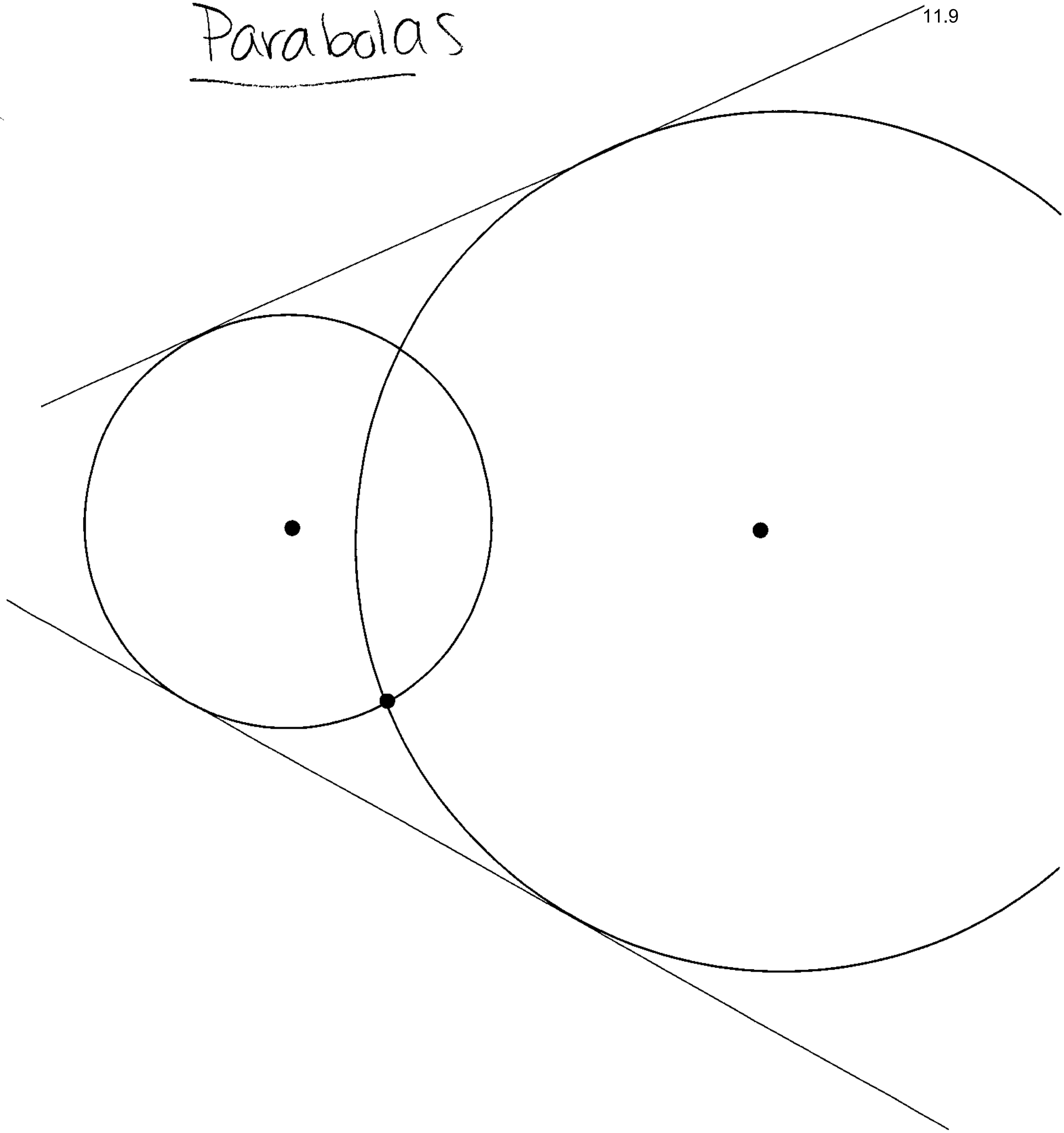


Smallest possible $|a|$ value is

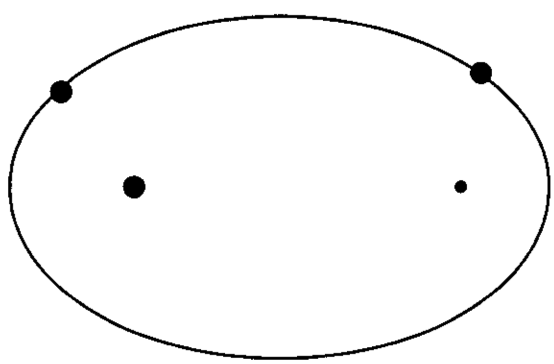


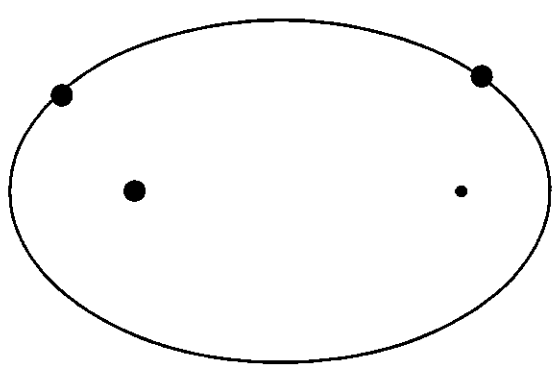
Locus of all possible foci for ellipses and hyperbolas form

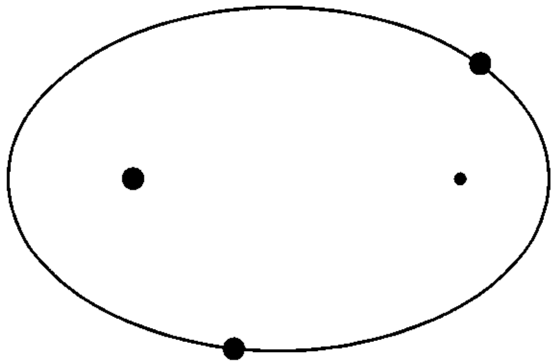
Parabolas

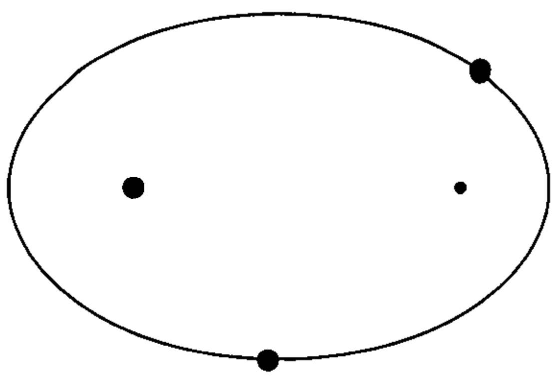


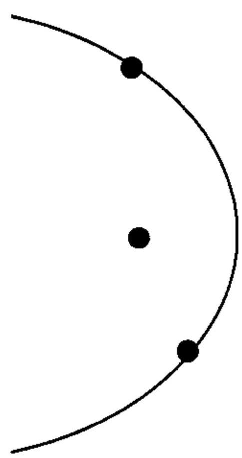
Legend	A – Ellipse (F NOT between chord and arc)
	B – Ellipse (F between chord and arc)
	H – Hyperbola
	1 – Transfer Angle < 180°
	2 – Transfer Angle > 180°

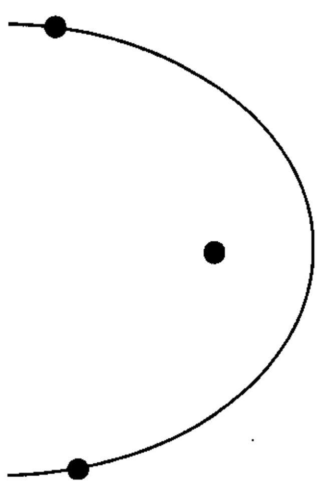


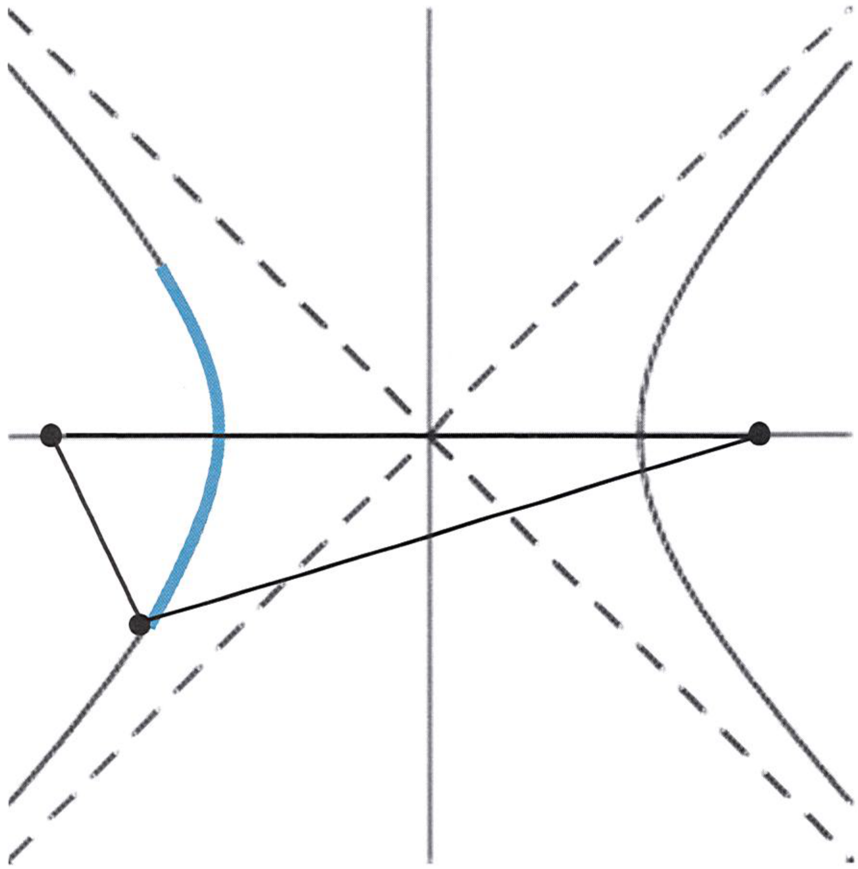












Minimum Energy Transfer

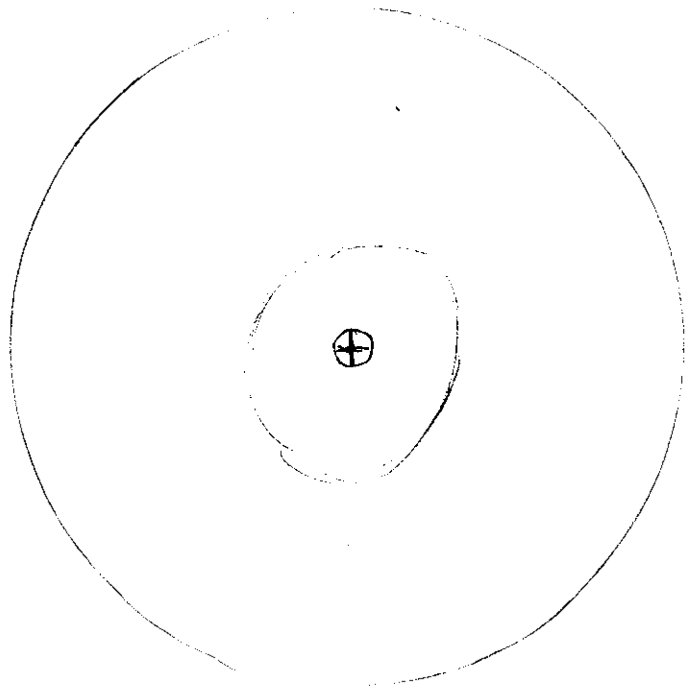
a_{min}

$P_{a_{min}}$

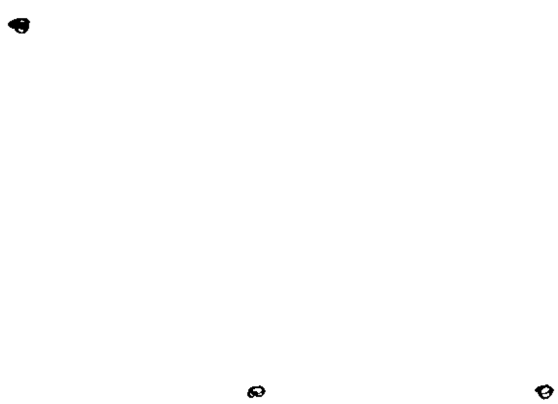
where

Example: The USS Enterprise is currently in a circular Earth orbit at $4R_\oplus$. A shuttlecraft is currently in a $1.25 R_\oplus$ circular, coplanar orbit.

What is the required Δv for the shuttlecraft to rendezvous with the USS Enterprise assumin a minimum energy transfer with a transfer angle of $120°$?



Space triangle:



C =

S =

For a minimum energy transfer,

$a_{min} =$

$p =$

$e =$

Now we have the orbit shape, find the conditions immediately before and after the first maneuver.

Before (circular orbit)

r_1^-

v_1^-

γ_1^-

After (transfer orbit)

r_1^+

V_1^+

γ_1^+

How do we choose correct γ^+?

Vector Diagram

Second Maneuver:

Before (transfer orbit)

$r_2^- =$

$v_2^- =$

$\gamma_2^- =$

After (circular orbit)

$r_2^+ =$

$V_2^+ =$

$\gamma_2^+ =$

Vector Diagram

ΔV_{TOTAL} =

Not optimal Δv for the transfer angle.

Hohmann transfer has a ΔV_{TOTAL} =

with TOF =

TOF for transfer angle of 120° =