f and g Functions

Determine $\bar{r}_2$ and $\bar{v}_2$ from current $\bar{r}_1$ and $\bar{v}_1$.

If given two positions, you can find $\bar{v}_1$.

f and g relationships

For any conic:

$$\bar{r}_2 = \left\{1 - \frac{r_2}{p}\left[1 - \cos\left(\theta_2^* - \theta_1^*\right)\right]\right\}\bar{r}_1 + \frac{r_2 r_1}{\sqrt{\mu p}}\sin\left(\theta_2^* - \theta_1^*\right)\bar{v}_1$$

$$\bar{v}_2 = \left\{\frac{\bar{r}_1 \bullet \bar{v}_1}{p r_1}\left[1 - \cos\left(\theta_2^* - \theta_1^*\right)\right] - \frac{1}{r_1}\sqrt{\frac{\mu}{p}}\sin\left(\theta_2^* - \theta_1^*\right)\right\}\bar{r}_1 + \left\{1 - \frac{r_1}{p}\left[1 - \cos\left(\theta_2^* - \theta_1^*\right)\right]\right\}\bar{v}_1$$

For elliptic orbits

$$\bar{r}_2 = \left\{1 - \frac{a}{r_1}\left[1 - \cos\left(E_2 - E_1\right)\right]\right\}\bar{r}_1 + \left\{\left(t_2 - t_1\right) - \sqrt{\frac{a^3}{\mu}}\left[\left(E_2 - E_1\right) - \sin\left(E_2 - E_1\right)\right]\right\}\bar{v}_1$$

$$\bar{v}_2 = -\frac{\sqrt{\mu a}}{r_2 r_1}\sin\left(E_2 - E_1\right)\bar{r}_1 + \left\{1 - \frac{a}{r_2}\left[1 - \cos\left(E_2 - E_1\right)\right]\right\}\bar{v}_1$$

For hyperbolic orbits

$$\bar{r}_2 = \left\{1 - \frac{|a|}{r_1}\left[\cosh\left(H_2 - H_1\right) - 1\right]\right\}\bar{r}_1 + \left\{\left(t_2 - t_1\right) - \sqrt{\frac{|a|^3}{\mu}}\left[\sinh\left(H_2 - H_1\right) - \left(H_2 - H_1\right)\right]\right\}\bar{v}_1$$

$$\bar{v}_2 = -\frac{\sqrt{\mu |a|}}{r_2 r_1}\sinh\left(H_2 - H_1\right)\bar{r}_1 + \left\{1 - \frac{|a|}{r_2}\left[\cosh\left(H_2 - H_1\right) - 1\right]\right\}\bar{v}_1$$

f and g example

Initially a spacecraft has a position and velocity of

$$\bar{r}_1 = 1.2\, R_\oplus\, \hat{r}$$

$$\bar{v}_1 = 0.79\,\hat{r} + 7.9\,\hat{\theta} \text{ km/s}$$

A. Find a, θ^*, p what type of orbit?

How do you determine the sign?

B. What is the new velocity and position $\Delta\theta^* = 30°$?

Need to find f and g

$f =$

Calculate g.

Now find $\dot{f}$ and $\dot{g}$

$\dot{f} =$

$\dot{g}$

However, we need $\bar{r}_1$ and $\bar{v}_1$ in

Then

To double check, magnitudes should be the same.
They are!

Then

$\bar{r}_2 =$

$\bar{v}_2$

C. How much time has passed?