AE 313
Orbital Mechanics
Exam 2

Instructions: This exam is worth a total of ~~10~~70 points. Work as quickly and accurately as you can. Write your name at the bottom of this page. A table of constants is included.

Read the problems carefully. Write clearly and use diagrams when necessary.

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO.

Partial credit can only be given for:

1. Correct partial steps toward the complete solution which are
2. Clearly labeled in a logical and systematic manner.

Name Exam Solutions

Problem 1 (40 pts)

Jason-2 (which measures sea surface height) is currently in orbit about Earth. At time t_1, tracking data provides the following position and velocity vectors relative to an inertial Earth equatorial coordinate system:

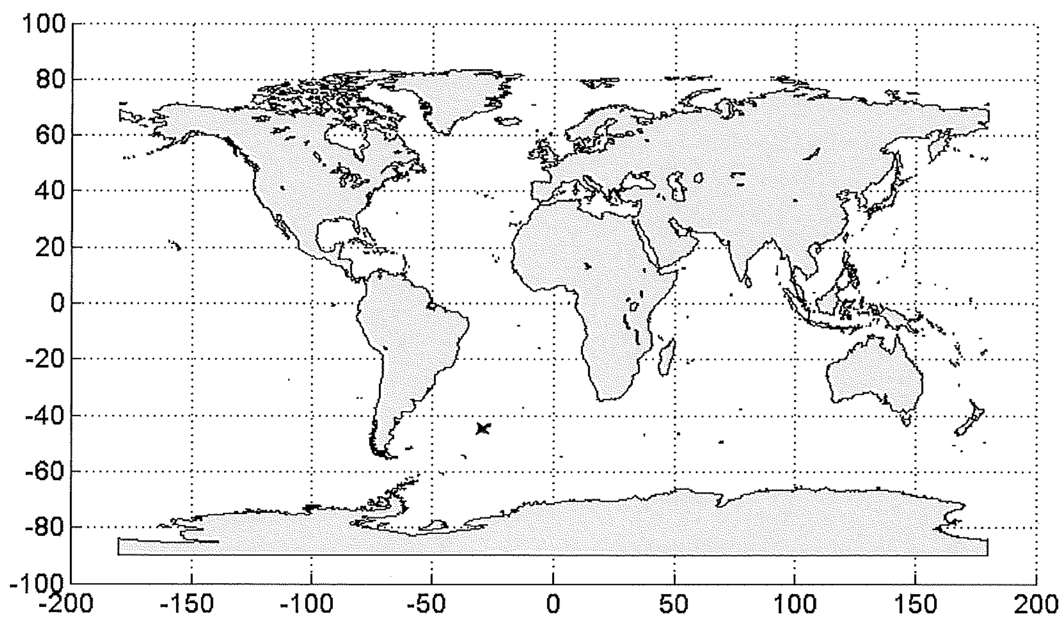
$$\overline{r}_1 = 0\hat{x} + 45{,}000\hat{y} - 45{,}000\hat{z} \ \text{ km}$$

$$\overline{v}_1 = 4\hat{x} + 0\hat{y} - 4\hat{z} \ \text{ km/s}$$

A. Assuming that $\theta_{ERA} = 120°$, determine the latitude and longitude at time t_1.

B. On the map below, approximately mark where the spacecraft is located.

C. Determine the following quantities associated with the vehicle orbit: θ_1, i



A. Find λ and ϕ

$$\bar{r}_{ECEF} = \begin{bmatrix} \cos\theta_{ERA} & \sin\theta_{ERA} & 0 \\ -\sin\theta_{ERA} & \cos\theta_{ERA} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 45{,}000 \\ -45{,}000 \end{Bmatrix}$$

$$= 38971.1\hat{x}' - 22{,}500\hat{y}' - 45{,}000\hat{z}' \text{ Km}$$

Latitude: $\phi = \sin^{-1}\left(\dfrac{\bar{r}_{ECEF} \cdot \hat{z}'}{r}\right)$

$\phi = -45°$ No quad check

$$\boxed{\phi = 45° \text{ S}}$$

Longitude: $\lambda = \tan^{-1}\left(\dfrac{\bar{r}_{ECEF} \cdot \hat{y}'}{\bar{r}_{ECEF} \cdot \hat{x}'}\right)$

$= -30°$ or $150°$

Since $\hat{y}' < 0$ and $\hat{x}' > 0$,

$$\boxed{\lambda = -30° \text{ or } 30° \text{ W}}$$

c. Find θ_1, i

$$\hat{h} = \frac{\bar{r}_1 \times \bar{v}_1}{|\bar{r}_1 \times \bar{v}_1|} = -\frac{1}{\sqrt{3}}\hat{x}' - \frac{1}{\sqrt{3}}\hat{y}' - \frac{1}{\sqrt{3}}\hat{z}'$$

$$\cos i = \hat{h}\cdot\hat{z} = -\frac{1}{\sqrt{3}} \Rightarrow \boxed{i = 125.26^\circ}$$

No quad check

$$\hat{r}_1 = \frac{\bar{r}_1}{r_1} = \frac{1}{\sqrt{2}}\hat{y} - \frac{1}{\sqrt{2}}\hat{z}$$

$$\hat{\theta}_1 = \hat{h} \times \hat{r}_1 = \frac{2}{\sqrt{6}}\hat{x} - \frac{1}{\sqrt{6}}\hat{y} - \frac{1}{\sqrt{6}}\hat{z}$$

$$\cos\theta_1 \sin i = \hat{\theta}_1 \cdot \hat{z} = -\frac{1}{\sqrt{6}} \Rightarrow \theta_1 = \pm 120^\circ$$

$$\sin\theta_1 \sin i = \hat{r}_1 \cdot \hat{z} = -\frac{1}{\sqrt{2}} \Rightarrow \theta_1 = 120^\circ \text{ or } 60^\circ$$

$\theta_1 = 120^\circ$

$$\boxed{\theta_1 = 120^\circ}$$

Problem 2 (30 pts)

Sometime in the future, a Martian base is *finally* in operation. The Earth-Mars-Transport (EMT) vehicle is ready to make another run to Earth. The EMT is currently in a Mars orbit with $r^- = 2R_{Mars}$, $v^- = 2.5$ km/s, and $\gamma^- = -30°$.

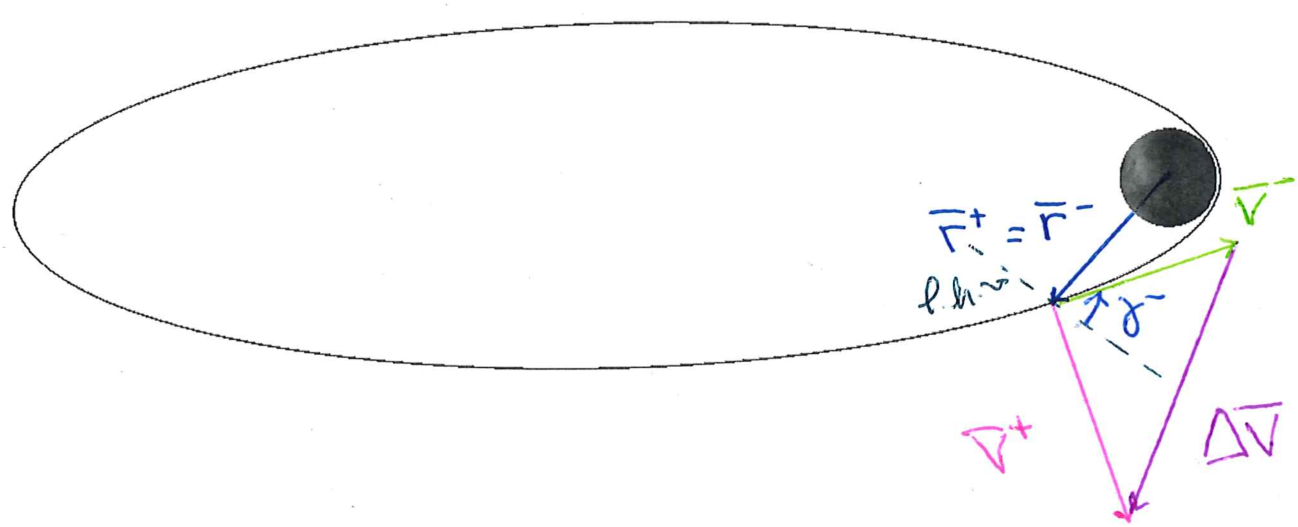
At this time, a maneuver will shift the EMT to a hyperbolic orbit relative to the Moon.

A. Determine a^-, e^-, θ^{*-} at the maneuver point in the original orbit.

B. Immediately after some instantaneous maneuver, $v^+ = 3$ km/s and $\gamma^+ = 30°$, determine the new orbital energy.

C. Is EMT now on a hyperbolic path? Why or why not?

D. On the diagram below of the original orbit, draw $\bar{r}^-, \bar{r}^+, \bar{v}^-, \bar{v}^+, \gamma^-, \Delta\bar{v}$



A. Find a^-, e^-, θ^{*-}

$\mu = 42828\ km^3/s^2$

$R_{Mars} = 3397.0\ km$

$r^- = 2R_{Mars}$
$= 6794\ km$

$$\mathcal{E}^- = \frac{(v^-)^2}{2} - \frac{\mu}{r^-} = -3.1788\ km^2/s^2$$

$$\boxed{a^- = -\frac{\mu}{2\mathcal{E}^-} = 6736.5\ km}$$

$h = r^- v^- \cos\gamma^- = 14709\ km$

$p = h^2/\mu = 5052.0\ km$

$$\boxed{e^- = \sqrt{1 - \frac{p^-}{a^-}} = 0.5}$$

$$\theta^{*-} = \cos^{-1}\left(\frac{p^-}{r^- e^-} - \frac{1}{e^-}\right) = \pm 120.85^\circ$$

Since $\gamma^- < 0$, $\theta^{*-} < 0$

$$\boxed{\theta^{*-} = -120.85^\circ}$$

B. New $\mathcal{E}^+$:

$$\mathcal{E}^+ = \frac{(v^+)^2}{2} - \frac{\mu}{r^+} = -1.8038 \text{ km}^2/\text{s}^2$$

$r^+ = r^-$

C. Since $\mathcal{E}^+ < 0$, still in an elliptical orbit.