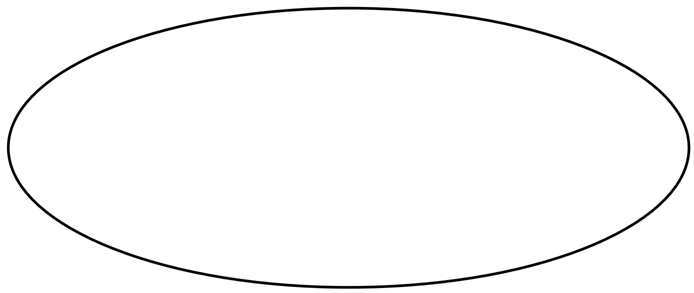
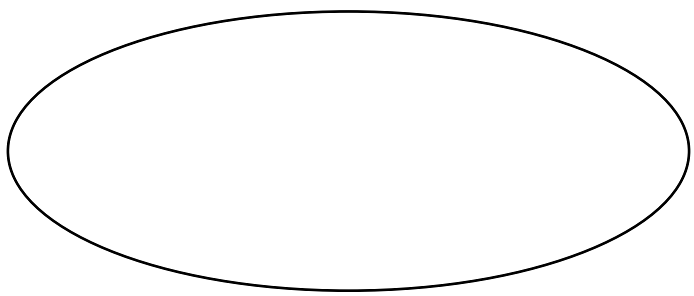
Name:

Class Engagement Worksheet for Wk 14 Fri 4/12 Option 1 (Vector Diagrams/Maneuvers)

1. For an elliptical transfer, say we have a transfer angle of 210 degrees and leave with a true anomaly of 90 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.



2. Repeat with a transfer angle of 120 degrees and leaving with a true anomaly of 15 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.



3. A spacecraft is currently in orbit around Earth with an initial velocity of 8 km/s and a flight path angle of -45 degrees. The spacecraft is going to perform a maneuver with an alpha of 30 degrees and a Δv of 2 km/s. Draw the vector diagram? What is the new velocity magnitude and flight path angle?

4. Now assume a spacecraft has an initial velocity of 10 km/s and an initial flight path angle of 30 degrees. With a Δv of 5 km/s with α of -20 degrees. Draw the vector diagram.

5. Given $v^{-}=5$ km/s, $\gamma^{-}=-30°, v^{+}=8$ km/s, $\gamma^{+}=35°$. Draw the vector diagram. Is the resulting α positive or negative?

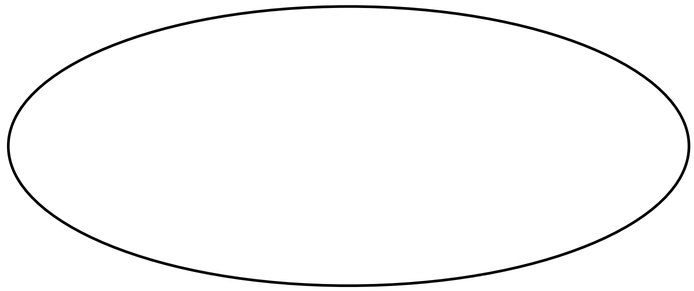
6. Let's do a fly-by at Venus. Initially Venus' heliocentric properties are $v_{Venus}=35$ km/s, $\gamma_{Venus}=20°$. The spacecraft has the following heliocentric properties $v^{-}=20$ km/s, $\gamma^{-}=-45°$. Draw the vector diagram for an ahead pass with a fly-by angle of 15 degrees.

7. For the same problem, what is $v^{-}_{\infty/Venus}$ and the altitude the spacecraft travels above Venus?

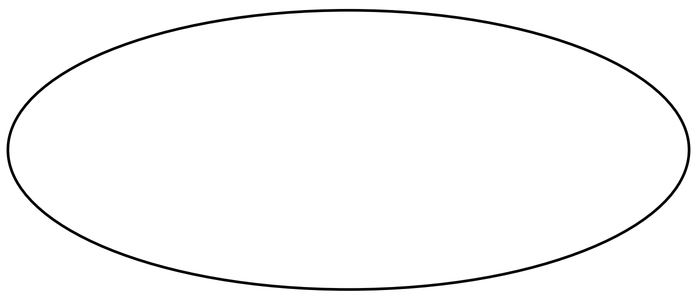
8. Let's instead do a Jupiter fly-by. Initially Jupiter's heliocentric properties are $v_{Jupiter}=5$ km/s, $\gamma_{Jupiter}=-10°$. The spacecraft has the following heliocentric properties $v^{-}=7$ km/s, $\gamma^{-}=-60°$. Draw the vector diagram for a behind pass with a fly-by angle of 55 degrees.

Class Engagement Worksheet for Wk 14 Fri 4/12 Option 2 (Transfers)

1. For an elliptical transfer, say we have a transfer angle of 210 degrees and leave with a true anomaly of 90 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.



2. Repeat with a transfer angle of 120 degrees and leaving with a true anomaly of 15 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.



3. A spacecraft is currently in orbit around Earth with $a_1 = 5R_{Earth}, e_1 = 0.4, \theta_1^* = -130°$. You are planning on transferring into a new orbit circular orbit with $a_3 = 10R_{Earth}$. The transfer arc leaves at periapsis and has a transfer angle of 210 degrees with $a_2 = 51783 \text{ km}, e_2 = 0.3036$. What are the conditions in the original orbit immediately before the maneuver?

4. What are the conditions immediately after the maneuver?

5. Draw the vector diagram. What are Δv_1 and α_1?

6. What are the conditions immediately before and after the second maneuver?

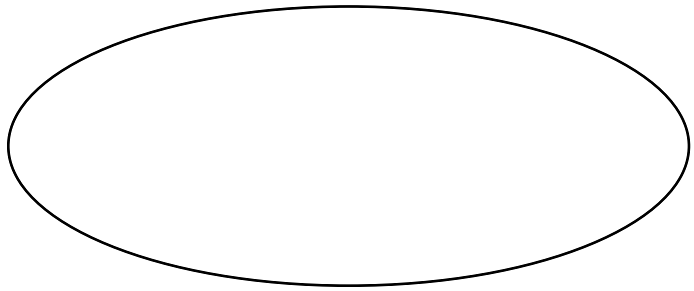
7. Draw the vector diagram for the second maneuver. What are Δv_2 and α_2?

8. Consider a Jupiter fly-by. Initially Jupiter's heliocentric properties are $v_{Jupiter} = 5 \text{ km/s}, \gamma_{Jupiter} = -10°$. The spacecraft has the following heliocentric properties $v^- = 7 \text{ km/s}, \gamma^- = -60°$. Draw the vector diagram for a behind pass with a fly-by angle of 55 degrees.

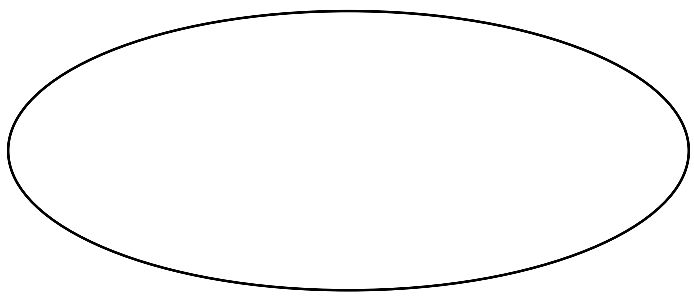
Name:

Class Engagement Worksheet for Wk 14 Fri 4/12 Option 3 (Fly-by)

1. For an elliptical transfer, say we have a transfer angle of 210 degrees and leave with a true anomaly of 90 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.



2. Repeat with a transfer angle of 120 degrees and leaving with a true anomaly of 15 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.



We are going to do a Jupiter fly-by! Assume that the spacecraft and Jupiter are currently 778,570,000 km away from the Sun. Jupiter has $v_J = 13.056 \text{ km/s}$ and $\gamma^- = 2.76°$. The spacecraft's heliocentric properties when it arrives at Jupiter are $v^- = 9.3262 \text{ km/s},\ \gamma^- = -36.551°$.

3. On your whiteboard, draw the vector diagram with a behind pass. Include $\bar{v}^-, \gamma^-, \bar{v}_J, \bar{v}^-_{\infty/J}, \bar{v}^+_{\infty/J}, \delta, \bar{v}^+, \Delta\bar{v}_{eq}$

4. What is $v_{\infty/J}^{-}$?

5. What is $v_{\infty/J}^{+}$?

6. We are approaching Jupiter with a periapsis of 142,984 km. What is the fly-by angle?

7. What is the new velocity (v^{+}) in the heliocentric view?

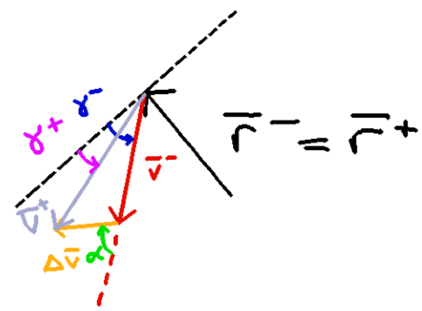
8. What is the change in energy in the heliocentric view?

Lambert 2 Class Engagement Worksheet for Option 1 Answers

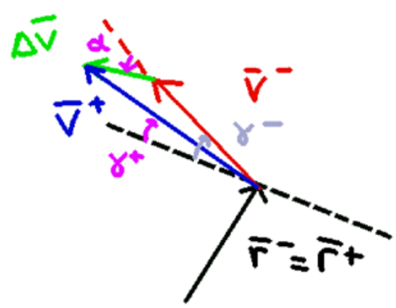
1. For an elliptical transfer, say we have a transfer angle of 210 degrees and leave with a true anomaly of 90 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.
   2B
2. Repeat with a transfer angle of 120 degrees and leaving with a true anomaly of 15 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.
   1A
3. A spacecraft is currently in orbit around Earth with an initial velocity of 8 km/s and a flight path angle of -45 degrees. The spacecraft is going to perform a maneuver with an alpha of 30 degrees and Δv of 2 km/s. Draw the vector diagram? What is the new velocity magnitude and flight path angle?

$$v^{+} = \left[\Delta v^{2} + v^{-\,2} - 2\Delta v v^{-}\cos\left(180° - \alpha\right)\right]^{\frac{1}{2}} = 9.7833 \text{ km}$$

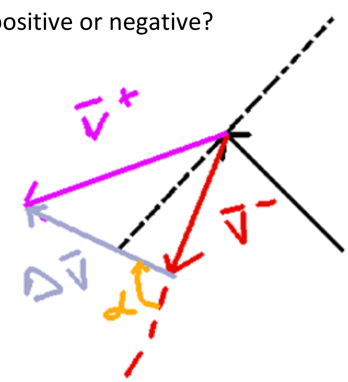
$$\gamma^{+} = \gamma^{-} + \Delta\gamma = \gamma^{-} + \cos^{-1}\left(\frac{\Delta v^{2} - v^{-\,2} - v^{+2}}{-2v^{-}v^{+}}\right) = -39.8°$$



4. Now assume a spacecraft has an initial velocity of 10 km/s and an initial flight path angle of 30 degrees. With a Δv of 5 km/s with α of -20 degrees. Draw the vector diagram.

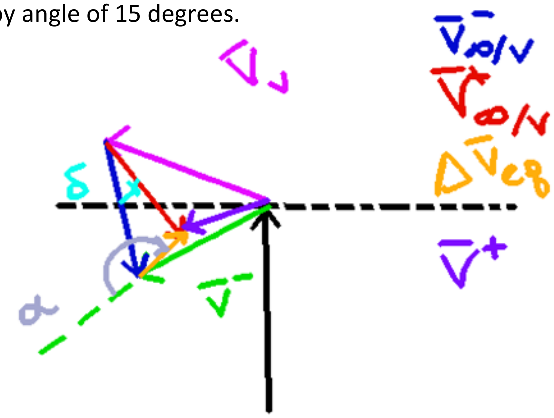


5. Given $v^{-} = 5$ km/s, $\gamma^{-} = -30°, v^{+} = 8$ km/s, $\gamma^{+} = 35°$. Draw the vector diagram. Is the resulting α positive or negative?



Alpha is positive

6. Let's do a fly-by at Venus. Initially Venus' heliocentric properties are $v_{Venus} = 35\text{ km/s},\ \gamma_{Venus} = 20°$. The spacecraft has the following heliocentric properties $v^- = 20\text{ km/s}, \gamma^- = -45°$. Draw the vector diagram for an ahead pass with a fly-by angle of 15 degrees.



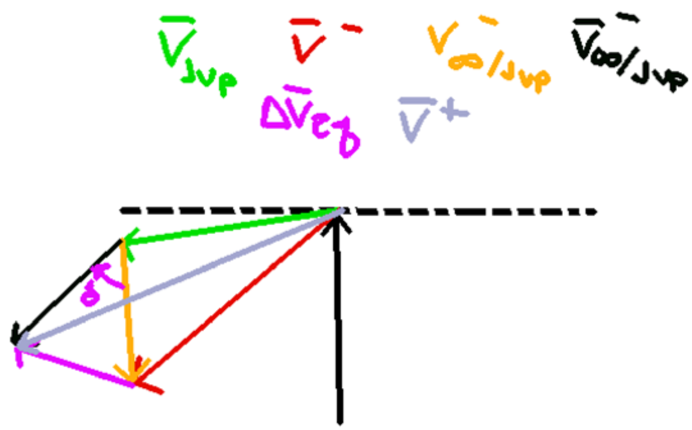
7. For the same problem, what is $v^-_{\infty/Venus}$ and the altitude the spacecraft travels above Venus?

$$v^-_{\infty/Venus} = \sqrt{v^{-2} + v^2_{Veuns} - 2v^- v_{Venus}\cos(\Delta\gamma)} = 32.146\text{ km/s}$$

$$e_H = \frac{1}{\sin\frac{\delta}{2}} = 7.6613, a_H = -\frac{\mu_{Venus}}{v^2_\infty} = -314.38\text{ km}$$

$$r_p = a_H(1 - e_H) = 2094.2\text{ km or -3957.7 km altitude}$$

8. Let's instead do a Jupiter fly-by. Initially Jupiter's heliocentric properties are $v_{Jupiter} = 5\text{ km/s},$ $\gamma_{Jupiter} = -10°$. The spacecraft has the following heliocentric properties $v^- = 7\text{ km/s}, \gamma^- = -60°$. Draw the vector diagram for a behind pass with a fly-by angle of 55 degrees.



Lambert 2 Class Engagement Worksheet Option 2 Answers 4/12

1. For an elliptical transfer, say we have a transfer angle of 210 degrees and leave with a true anomaly of 90 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.
   2B
2. Repeat with a transfer angle of 120 degrees and leaving with a true anomaly of 15 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.
   1A
3. A spacecraft is currently in orbit around Earth with $a_1 = 5R_{Earth}, e_1 = 0.4, \theta_1^* = -130°$. You are planning on transferring into a new orbit circular orbit with $a_3 = 10R_{Earth}$. The transfer arc leaves at periapsis and has a transfer angle of 210 degrees with $a_2 = 51783 \text{ km}, e_2 = 0.3036$. What are the conditions in the original orbit immediately before the maneuver?

$$r_1 = \frac{a_1\left(1-e_1^2\right)}{1+e_1\cos\theta_1^*} = 36,059 \text{ km}$$

$$v_1^- = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_1}} = 3.0998 \text{ km/s}$$

$$\gamma_1^- = \tan^{-1}\left(\frac{r_1 e_1}{p_1}\sin\theta_1^*\right) = -22.415° \text{ no quad check since other value out of range}$$

4. What are the conditions immediately after the maneuver?

$$r_1^+ = r_1$$

$$v_1^+ = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_2}} = 3.7962 \text{ km/s}$$

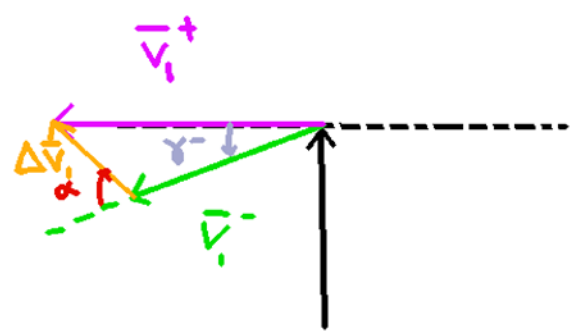
$$\gamma_1^+ = 0° \text{ since periapsis}$$

5. Draw the vector diagram. What are Δv_1 and α_1?

$$\Delta v_1 = \sqrt{v_1^{-2} + v_1^{+2} - 2v^- v^+ \cos\Delta\gamma} = 1.5043 \text{ km/s}$$

$$|\alpha| = 180° - \cos^{-1}\left(\frac{v_1^{+2} - v_1^{-2} - \Delta v_1^2}{-2v_1^- \Delta v_1}\right) = 74.204°$$

Since α away from Earth, $\alpha = 74.204°$



6. What are the conditions immediately before and after the second maneuver?

$$r_2 = 63,780 \text{ km}$$

$$v_2 = \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_2}} = 2.1913 \text{ km/s}$$

$$\gamma_2 = \tan^{-1}\left(\frac{r_2 e_2}{p_2}\sin\theta_2^*\right) = -11.640°$$

$$v_3 = \sqrt{\frac{\mu}{r_2}} = 2.4999 \text{ km/s}$$

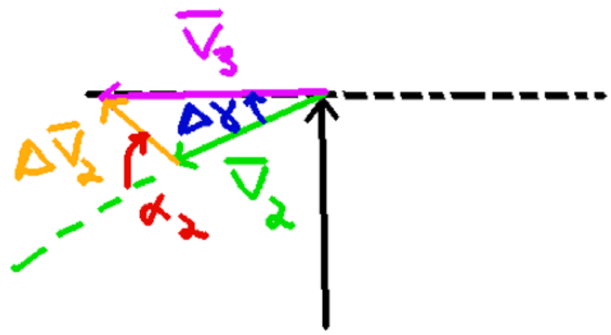
$$\gamma_3 = 0° \text{ since circular}$$

7. Draw the vector diagram for the second maneuver. What are Δv_2 and α_2?

$$\Delta v_2 = \sqrt{v_2^2 + v_3^2 - 2v_2 v_3 \cos\Delta\gamma} = 0.5662 \text{ km/s}$$

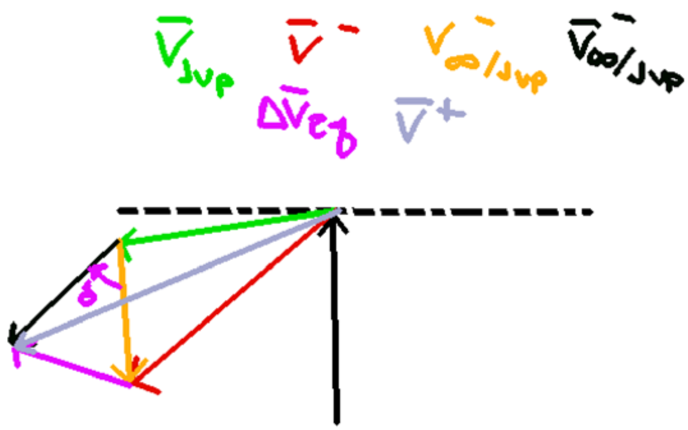
$$|\alpha| = 180° - \cos^{-1}\left(\frac{v_3^2 - v_2^2 - \Delta v_2^2}{-2v_2\Delta v_2}\right) = 62.978°$$

Since α away from Earth, $\alpha = 62.978°$



8. Consider a Jupiter fly-by. Initially Jupiter's heliocentric properties are $v_{Jupiter} = 5$ km/s, $\gamma_{Jupiter} = -10°$.

The spacecraft has the following heliocentric properties $v^- = 7$ km/s, $\gamma^- = -60°$. Draw the vector diagram for a behind pass with a fly-by angle of 55 degrees.



Lambert 2 Class Engagement Worksheet Option 3 Answers 4/12

1. For an elliptical transfer, say we have a transfer angle of 210 degrees and leave with a true anomaly of 90 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.
   2B
2. Repeat with a transfer angle of 120 degrees and leaving with a true anomaly of 15 degrees. Highlight the transfer arc below, draw the space triangle, and determine the type of transfer.
   1A

We are going to do a Jupiter fly-by! Assume that the spacecraft and Jupiter are currently 778,570,000 km away from the Sun. Jupiter has $v_J = 13.056 \text{ km/s}$ and $\gamma^- = 2.76°$. The spacecraft's heliocentric properties when it arrives at Jupiter are
$v^- = 9.3262 \text{ km/s}, \ \gamma^- = -36.551°$.

3. On your whiteboard, draw the vector diagram with a behind pass. Include $\bar{v}^-, \gamma^-, \bar{v}_J, \bar{v}^-_{\infty/J}, \bar{v}^+_{\infty/J}, \delta, \bar{v}^+, \Delta\bar{v}_{eq}$

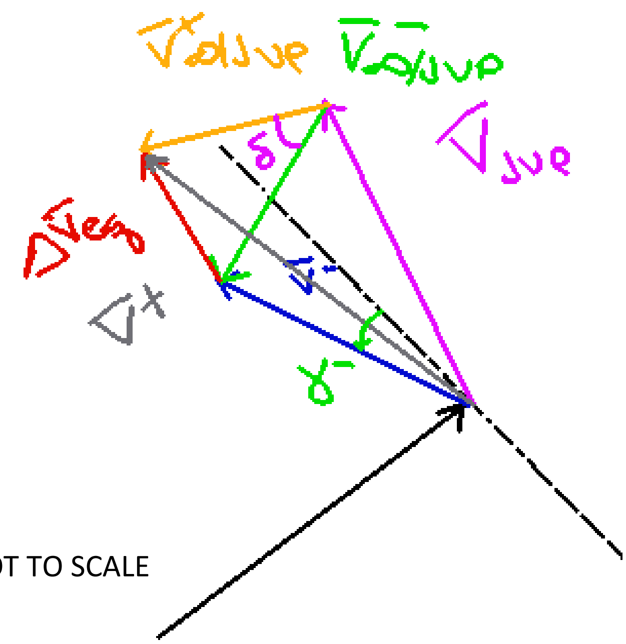


DIAGRAM NOT TO SCALE

4. What is $v^-_{\infty/J}$?
   $$v^-_{\infty/J} = \sqrt{v_J^2 + v^{-2} - 2v_J v^- \cos\left(\gamma_J - \gamma^-\right)} = 8.3076 \text{ km/s}$$
5. What is $v^+_{\infty/J}$?
   $$v^-_{\infty/J} = v^+_{\infty/J} = 8.3076 \text{ km/s}$$

6. We are approaching Jupiter with a periapsis of 142,984 km $\left(\mu_J = 126{,}713{,}000\ \dfrac{\text{km}^3}{\text{s}^2}\right)$.

What is the fly-by angle?

$$a_H = -\frac{\mu_J}{v_{\infty/J}^2} = -1{,}836{,}000 \text{ km}$$

$$e_H = 1 - \frac{r_p}{a_H} = 1.0779$$

$$\delta = 2\sin^{-1}\left(\frac{1}{e_H}\right) = 136.17°$$

7. What is the new velocity (v^+) in the heliocentric view?

$$\eta = \cos^{-1}\left(\frac{v^{-2} - v_J^2 - v_{\infty/J}^2}{-2v_{\infty/J}v_J}\right) = 45.333°$$

$$v^+ = \sqrt{v_J^2 + v_{\infty/J}^2 - 2v_J v_{\infty/J}\cos(\delta + \eta)} = 21.362 \text{ km/s}$$

8. What is the change in energy in the heliocentric view?

$$En^- = \frac{v^{-2}}{2} - \frac{\mu_{Sun}}{r_J} = -126.97\ \frac{\text{km}^2}{\text{s}^2}$$

$$En^+ = \frac{v^{+2}}{2} - \frac{\mu_{Sun}}{r_J} = 57.705\ \frac{\text{km}^2}{\text{s}^2}$$

$$\Delta En = En^+ - En^- = 184.67\ \frac{\text{km}^2}{\text{s}^2}$$