Homework 8

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AE 313
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AE 313 Homework 8

1. Express $\Delta\vec{v}$ in rotating orbit unit vectors $\hat{r}, \hat{\theta}\hat{h}$ as well as inertial unit vectors $\hat{x}, \hat{y}, \hat{z}$.

```
vdv_rth = dv*[cosd(beta)*sind(phi) cosd(beta)*cosd(phi) sind(beta)]';
vdv_eci = rot_rth_eci(RAANm, incm, AOLm) * vdv_rth;
```

$$\begin{aligned} \Delta\vec{v}_{rth} &= \quad < -2.6250,\ 2.2344,\ 2.8925 > km/s \\ \Delta\vec{v}_{eci} &= \quad < 3.9498,\ -0.8593,\ 1.9776 > km/s \end{aligned}$$

2. Determine the position and velocity immediately after the maneuver, $\vec{r}^{+}, \vec{v}^{+}$ in the intertial coordinate system.

```
vrp_eci = vrm_eci;
vvp_eci = vvm_eci + vdv_eci;
```

$$\begin{aligned} \vec{r}^{+}_{eci} &= \quad < -5.9784,\ -4.6680,\ -0.1583 > \cdot 10^{3} km \\ \vec{v}^{+}_{eci} &= \quad < 11.6963,\ -5.4791,\ -1.1101 > km/s \end{aligned}$$

3. Compute the orbital elements $e^{+},\ i^{+},\ \Omega^{+},\ \theta^{+},\ \theta^{*+}$ in the new orbit.

```
FPAp = asind(dot(vrp_eci,vvp_eci)/(norm(vrp_eci)*norm(vvp_eci)));
...
true_ap = 360 - true_ap;
```

$$\begin{aligned} e^{+} &= \quad 2.0151 \\ i^{+} &= \quad 6.2208^\circ \\ \Omega^{+} &= \quad 26.9440^\circ \\ \theta^{+} &= \quad 191.1029^\circ \\ \theta^{*+} &= \quad 320.4327^\circ \end{aligned}$$

4. Find the changes in the elements (including the sign) that occured due to the maneuver, that is, $\Delta e,\ \Delta i,\ \Delta\Omega,\ \Delta\theta$.

```
de = ep-em;
dinc = incp-incm;
dRAAN = RAANp-RAANm;
dAOL = AOLp-AOLm;
```

$$\begin{aligned} \Delta e &= \quad 1.2501 \\ \Delta i &= \quad -14.3792^\circ \\ \Delta\Omega &= \quad -7.8560^\circ \\ \Delta\theta &= \quad 7.7029^\circ \end{aligned}$$

5. Confirm the position, m and new orbital elements (problem 3) in GMAT. Plot the original and new orbit (include XY plane and inertial unit vectors). Mark the maneuver location on the plot.

$$\begin{aligned} \vec{v}^{-}_{eci} &= \quad < 7.7465,\ -4.6198,\ -3.0877 > km/s \\ \vec{v}^{+}_{eci} &= \quad < 12.026,\ -5.1957,\ -1.0988 > km/s \end{aligned}$$

The data in the GMAT orbit is very close to the MATLAB calculations and therefore matches the data from MATLAB calculations.

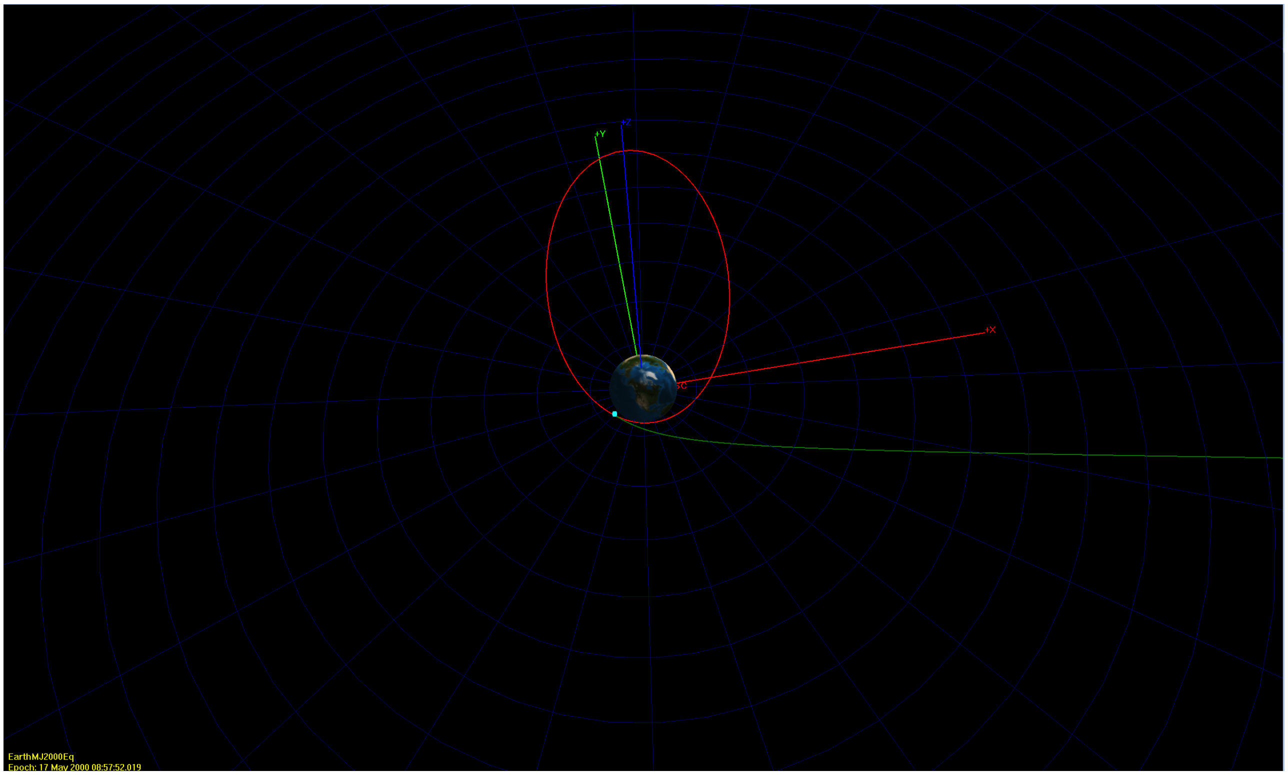

Figure 1: GMAT Plot of Orbits

Turquoise indicates the maneuver point. Clearer view of the maneuver after the code.

6. Would you want to perform this maneuver? Why or why not?
I would not want to perform this maneuver because it results in a massively hyperbolic orbit. The goal of the maneuver is to do an orbital correction, not enter an escape trajectory. With an eccentricity of 2.02, the orbit is very far from being elliptical.

7. Given that the semi-major axes of bi-elliptical transfer are $a_{T1} = 6659\ km$ and $a_{T2} = 6798\ km$, what is the departure phase angle? Include a figure/sketch of the phase angle.

```
at1 = 6658;
at2 = 6798;
a_sc = 6378+430;
TOF = pi*(sqrt(at1^3/MU('Earth'))+sqrt(at2^3/MU('Earth')));
n_sc = sqrt(MU('Earth')/a_sc^3);
phase = 2*pi-n_sc*TOF;
phase = phase*180/pi;
```

$\Delta\Phi = \quad 6.3124°$

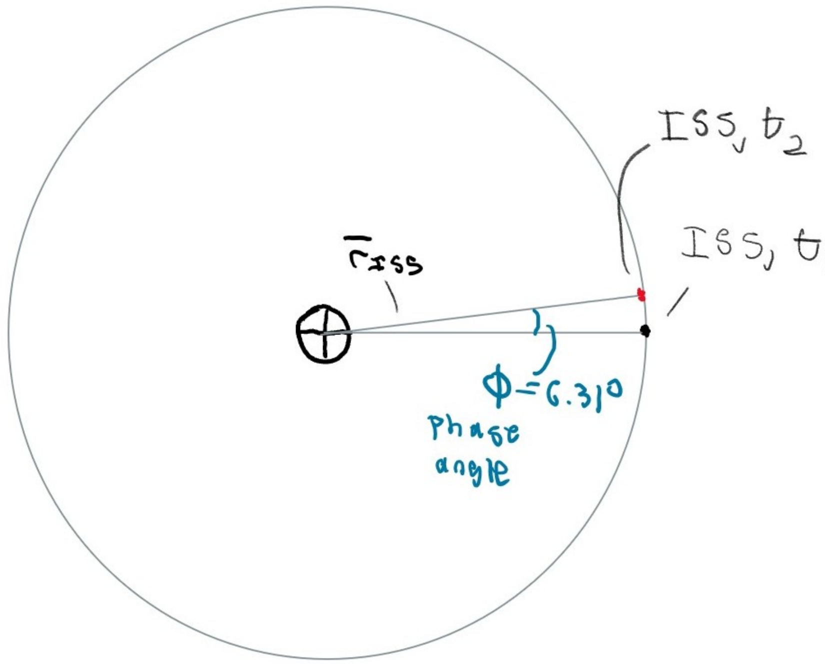

Figure 2: Sketch of the Phase Angle

8. Survey.

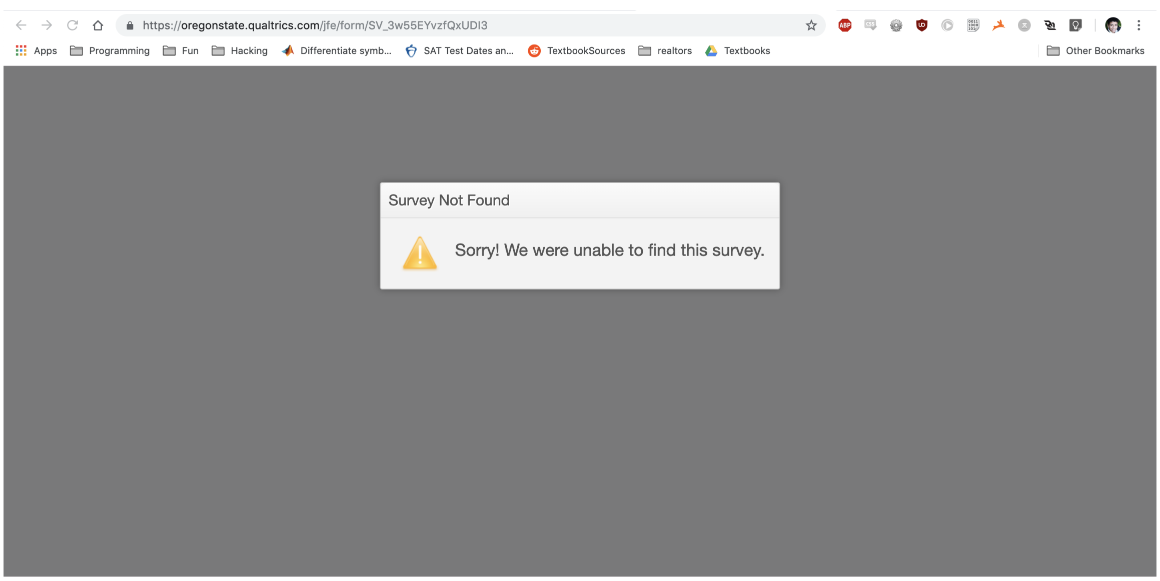

Figure 3: Survey Screenshot

HW8.m

```matlab
clc; clear;
% Constants
planets = {'Sun', 'Moon', 'Mercury', 'Venus', 'Earth', 'Mars', 'Jupiter',
    'Saturn', 'Uranus', 'Neptine', 'Pluto'};
rad_list = [695990.0, 1739.2, 2439.7, 6051.9, 6378.0, 3397.0, 71492.0,
    60268.0, 25559.0, 25269.0, 1162.0];
mu_list = [132712440000.0, 4902.8, 22032.0, 324860.0, 398600.0, 42828.0,
    126713000.0, 37941000.0, 5794500.0, 6836500.0, 981.6];
sma_list = [0.0, 384400.0, 57910000.0, 108210000.0, 149600000.0,
    227920000.0, 778570000.0, 1433530000.0, 2872460000.0, 4495060000.0,
    5906380000.0];
R = containers.Map(planets,rad_list);
MU = containers.Map(planets,mu_list);
r = containers.Map(planets,sma_list);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
am = 28081;
AOPm = 229;
RAANm = 34.8;
em = 0.765;
incm = 20.6;
true_am = -45.6;

AOLm = AOPm + true_am;

vrm_eci = [-5978.4 -4668 -158.31]';
vvm_eci = [7.7465 -4.6198 -3.0877]';

dv = 4.5;
alpha = -30;
beta = 40;
pm = am*(1-em^2);
% FPA = acosd(norm(cross(vrm_eci, vvm_eci))/(norm(vrm_eci)*norm(vvm_eci)))
FPA = atan2d(norm(vrm_eci)*em*sind(true_am),pm);
phi = alpha + FPA;

% 1.

vdv_rth = dv*[cosd(beta)*sind(phi) cosd(beta)*cosd(phi) sind(beta)]';
vdv_eci = rot_rth_eci(RAANm, incm, AOLm) * vdv_rth;

% 2.
vrp_eci = vrm_eci;
vvp_eci = vvm_eci + vdv_eci;
```

```matlab

% 3.
FPAp = asind(dot(vrp_eci,vvp_eci)/(norm(vrp_eci)*norm(vvp_eci)));
ep = sqrt((((norm(vrp_eci)*norm(vvp_eci)^2/MU('Earth'))-1)^2*cosd(FPAp)^2)+sind(FPAp)^2);

syms ap
eq = norm(vvp_eci)^2/2 - MU('Earth')/norm(vrp_eci) == MU('Earth')/(2*ap);
ap = double(solve(eq, ap));
pp = norm(cross(vrp_eci,vvp_eci))^2/MU('Earth');
h_hat = cross(vrp_eci, vvp_eci)/norm(cross(vrp_eci,vvp_eci));

incp = acosd(h_hat(3));

syms RAANp
eq(1) = h_hat(2) == -cosd(RAANp)*sind(incp);
eq(2) = h_hat(1) == sind(RAANp)*sind(incp);
RAANp1 = acosd(-h_hat(2)/sind(incp));
RAANp1 = [-RAANp1 RAANp1];
RAANp2 = asind(h_hat(1)/sind(incp));
RAANp2 = [180-RAANp2 RAANp2];

RAANp = min(RAANp2); %intersecting the two doesnt work due to small delta in solution. It's the negative value.

rp_eci_hat = vrp_eci/norm(vrp_eci);
theta_hat = cross(rp_eci_hat,h_hat);

AOLp1 = asind(rp_eci_hat(3)/sind(incp));
AOLp1 = [180-AOLp1 AOLp1];
AOLp2 = acosd(theta_hat(3)/sind(incp));
AOLp2 = [-AOLp2 AOLp2];
AOLp = min(AOLp2); %same as above. Small delta does not allow me to do intersection
AOLp = 191.1029;

true_ap = acosd((pp/norm(vrp_eci)-1)/ep); %Checked Gmat its neg
true_ap = 360 - true_ap;

% 4.
de = ep-em;
dinc = incp-incm;
dRAAN = RAANp-RAANm;
dAOL = AOLp-AOLm;
```

```

% 5.
vrp_eci; %-7192.67008574393            -3847.023676019902            355.568234258645            303.8898424306826

% 6.
% I would not want to perform this maneuver because it results in a
% massively hyperbolic orbit. The goal of the maneuver is to do an orbital
% correction, not enter an escape trajectory. With an eccentricity of 2.02,
% the orbit is very far from bring elliptical.

% 7.
at1 = 6658;
at2 = 6798;
a_sc = 6378+430;
TOF = pi*(sqrt(at1^3/MU('Earth'))+sqrt(at2^3/MU('Earth')));
n_sc = sqrt(MU('Earth')/a_sc^3);
phase = 2*pi-n_sc*TOF;
phase = phase*180/pi;

function A = rot_rth_eci(o,i,t)

% o : Omega, Longitude of the Ascending Node (RAAN)
% i : i, inclination
% t : theta, Argument of Latitude

A = [cosd(o)*cosd(t)-sind(o)*cosd(i)*sind(t), -cosd(o)*sind(t)-sind(o)*cosd(i)*cosd(t), sind(o)*sind(i); ...
        sind(o)*cosd(t)+cosd(o)*cosd(i)*sind(t), -sind(o)*sind(t)+cosd(o)*cosd(i)*cosd(t), -cosd(o)*sind(i); ...
        sind(i)*sind(t), sind(i)*cosd(t), cosd(i)];

end

function A = rot_eci_ecef(t)

% t : theta_era, Earth rotation angle

A = [cosd(t), sind(t), 0; -sind(t), cosd(t), 0; 0, 0, 1];
end

function A = rot_ecef_sez(l,p)

```

```
% l : lamda_gs, longitude of the ground station
% p : phi_gs, latitude of the ground station

A = [sind(p)*cosd(l), sind(p)*sind(l), -cosd(p);
    -sind(l), cosd(l), 0;
    cosd(p)*cosd(l), cosd(p)*sind(l), sind(p)];

end
```

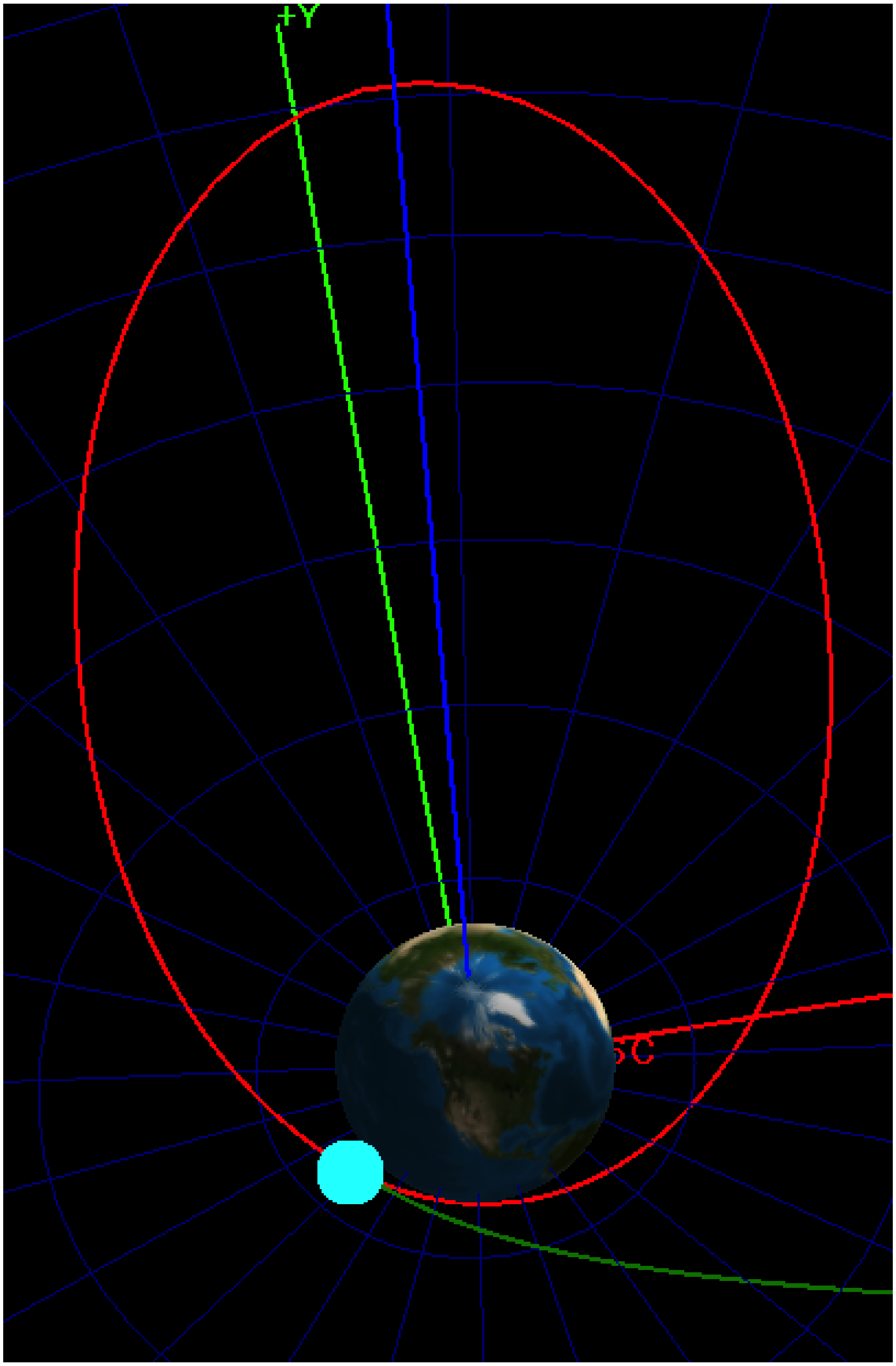

Figure 4: GMAT Modified

Word has changed their background removal functionality such that I cannot get rid of the black. This is a zoomed photo to help if the first one is difficult to read. Turquoise indicates the maneuver point.