Group Exam 2

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Grace Day

JiYoung Hwang

Aaron Scott

Thorne Wolfenbarger

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Professor Kaela Martin
College of Engineering

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Knowns

$$\text{a} = 42{,}170 \text{ km} \quad \text{e} = 0.01053 \quad \Omega = 14.89°$$
$$\omega = 318.4° \quad i = 14.37° \quad \theta_1^* = 184.2°$$

1 Current Position And Velocity

Find the current position and velocity $(\vec{r}_1, \vec{v}_1)$ in ECI and perifocal coordinates.

$$\vec{r}_1 = (-3.9158\hat{x} + 1.5532\hat{y} + 0.6423\hat{z}) \cdot 10^4 \ km = (-4.2498\hat{e} - 0.3121\hat{p}) \cdot 10^4 \ km$$
$$\vec{v}_1 = -1.1820\hat{x} - 2.7383\hat{y} - 0.6002\hat{z} \ {}^{km}/_{s} = 0.2252\hat{e} - 3.0340\hat{p} \ {}^{km}/_{s}$$

```
% 1. Find the current position and velocity in ECI and perifocal coordinates
p = a*(1-e^2);
h = sqrt(MU*p);
r1 = p/(1+e*cosd(true_a1));

vr1_rth = [r1 0 0]';
vv1_rth = [h*e/p*sind(true_a1) h/r1 0]';

AOL1 = AOP + true_a1;
rth_eci = rot_rth_eci(RAAN,inc,AOL1);

vr1_eci = rth_eci*vr1_rth;
vv1_eci = rth_eci*vv1_rth;

vr1_peri = r1*[cosd(true_a1) sind(true_a1) 0]';
vv1_peri = [-h/p*sind(true_a1) h/p*(e+cosd(true_a1)) 0]';
```

2 Time To Send The Message

You need to send the message by the time E = 15°. How long do you have until t_2?
Time until E = 15°: t = $4.5618 \cdot 10^4 \ sec = 12.6716 \ hours = 0.528 \ days$

```
% 2. You need to send the message by the time E=15deg. How long do you have
% until t2?
period = 2*pi*sqrt(a^3/MU);
E2 = 15; %deg
E2_rad = E2*pi/180;
E1 = 2*atand(tand(true_a1/2)/sqrt((1+e)/(1-e)));
E1_rad = E1*pi/180;

```

```
dt = sqrt(a^3/MU)*(E2_rad - E1_rad - (e*sin(E2_rad) - e*sin(E1_rad)));
```

3 New Position And Velocity

What is the new position and velocity $(\vec{r}_2, \vec{v}_2)$ in ECI and perifocal coordinates?

$$\vec{r}_2 = (4.0746\hat{x} - 0.7798\hat{y} - 0.4613\hat{z}) \cdot 10^4 \ km = (4.0289\hat{e} + 1.0914\hat{p}) \cdot 10^4 \ km$$
$$\vec{v}_2 = 0.6526\hat{x} + 2.9573\hat{y} + 0.6892\hat{z} \ {}^{km}/_{s} = -0.8039\hat{e} + 3.0000\hat{p} \ {}^{km}/_{s}$$

```
% 3. What is the new position and velocity in ECI and perifocal
    coordinates?
r2 = a*(1-e*cos(E2_rad));
true_a2 = 2*atan2d(sqrt((1+e)/(1-e))*tan(E2_rad/2),1);
vr2_rth = [r2 0 0]';
vv2_rth = [h*e/p*sind(true_a2) h/r2 0]';

AOL2 = AOP + true_a2;
rth_eci = rot_rth_eci(RAAN,inc,AOL2);

vr2_eci = rth_eci*vr2_rth;
vv2_eci = rth_eci*vv2_rth;

vr2_peri = r2*[cosd(true_a2) sind(true_a2) 0]';
vv2_peri = [-h/p*sind(true_a2) h/p*(e+cosd(true_a2)) 0]';
```

4 Latitude And Longitude

Find the latitiude and longitude at the two positions.

$$\phi_1 = 8.6698^\circ \quad \lambda_1 = -49.9545^\circ$$
$$\phi_2 = -6.3452^\circ \quad \lambda_2 = -49.7477^\circ$$

```
% 4. Find the latitiude and longitide at the two positions
dt_days = dt/60/60/24;
rot_earth = 7.2921151467*10^-5; %rad
tu1 = juliandate(datetime('2019-03-06 03:00:00')) - 2451545;
tu2 = juliandate(datetime('2019-03-06 03:00:00')) - 2451545;

theta_era1 = 2*pi*(0.7790572732640 + 1.00273781191135448*tu1);
theta_era1 = mod(theta_era1,2*pi)*180/pi;
theta_era2 = 2*pi*(0.7790572732640 + 1.00273781191135448*tu2);
theta_era2 = mod(theta_era2,2*pi)*180/pi;
```

```
eci_ecef1 = rot_eci_ecef(theta_era1);
eci_ecef2 = rot_eci_ecef(theta_era2);
vr1_ecef = eci_ecef1*vr1_eci;
vr2_ecef = eci_ecef2*vr2_eci;


alpha1 = atan2d(cosd(inc)*sind(AOL1),cosd(AOL1));
alpha2 = atan2d(cosd(inc)*sind(AOL2),cosd(AOL2));
theta_gr1 = theta_era1+180/pi*rot_earth*0;
theta_gr2 = theta_era2+180/pi*rot_earth*dt;

lat1 = asind(vr1_ecef(3)/r1);
long1 = alpha1 + RAAN - theta_gr1;
lat2 = asind(vr2_ecef(3)/r2);
long2 = alpha2 + RAAN - theta_gr2;
long2 = mod(long2,-360);
```

5 Ground Track

Create a ground track between the two position using MATLAB. Mark the starting location.

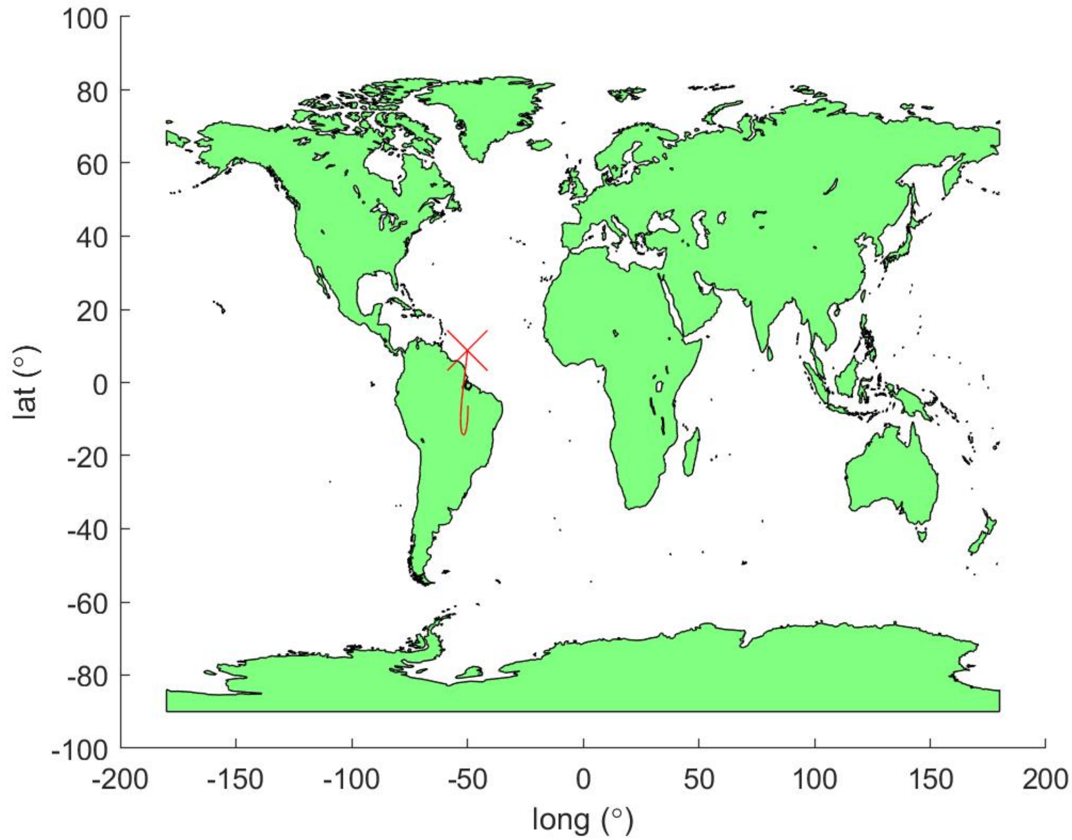

Figure 1: Latitude vs. Longitude

```
% 5. Create a ground track between the two positions using MATLAB. Mark
    the
% starting location.

true_a_array = linspace(true_a1, true_a2+360, 1000);
r_array = p./(1+e*cosd(true_a_array));
% E_array = acosd(-(r_array/a-1)/e).*sign(true_a_array);
E_array = 2*atand(tand(true_a_array/2)/sqrt((1+e)/(1-e)));
E_array_rad = E_array*pi/180;

dt_array = sqrt(a^3/MU)*(E_array_rad - E1_rad - (e*sin(E_array_rad) - e*
    sin(E1_rad)));

```

```
theta_era = theta_era1;
theta_gr_array = theta_era+180/pi*rot_earth*dt_array;


theta_array = AOP + true_a_array;
lat_array = asind(sind(inc)*sind(theta_array));

alpha_array = atan2d(cosd(inc)*sind(theta_array),cosd(theta_array));
long_array = alpha_array + RAAN - theta_gr_array;

figure; hold on;
geoshow("landareas.shp","FaceColor",[0.5 1.0 0.5]);
geoshow(lat_array, long_array,'Color','red');
geoshow(lat_array(1),long_array(1),'DisplayType','Point','Marker','x','
    Markersize',20);
```

6 Can You Send The Message?

Assume that you have a ground station in Prescott (34.54 N, 112.4685 W, 1.64 km above mean equatorial). Plot the elevation angle versus time passed. If the ground station needs a minimum elevation angle of 10 degrees, how much time is the TDRS visible? Can you send your urgent message to the TDRS?

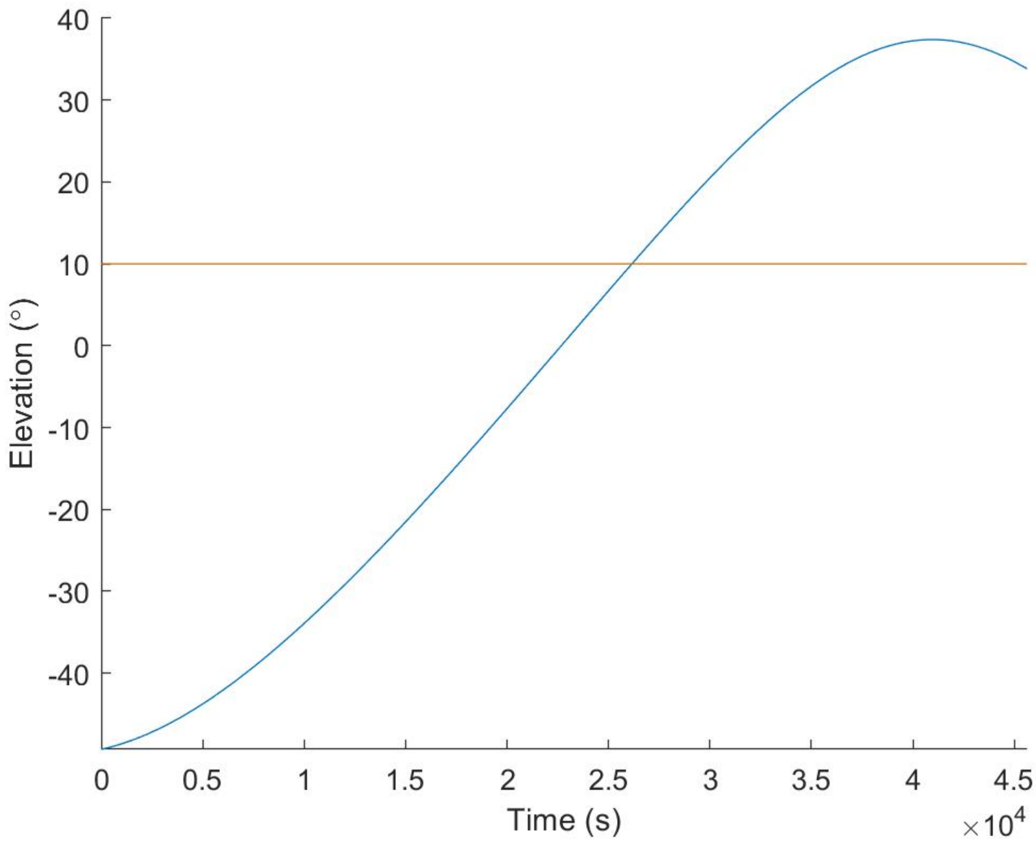

Figure 2: Elevation vs. Time

In Part 2, "Time To Send The Message", we were informed that we have until E = 15 degrees to send our message. We found that it will take 12.6716 hours to reach our E = 15 degrees limit, beginning on March 6, 2019 at 03:00:00.000. As shown in the above plot, it will take only 7.2751 hours for TDRS to become visible to our ground station. Once visible, the TDRS will remain visible for 5.3965 hours. Therefore, because TDRS becomes and remains visible for well within our allowable range, we will be able to send our urgent message as early as 10:16:30.36.

```
% 6. Assume you have the ground station in Prescott. Plot the elevation
    angle
% versus time passed. If the ground station needs a minimum elevation
    angle
```

```
% of 10 degrees, how much time is the TDRS visible? Can you send your
% urgent message to the TDRS?
min_elevation = 10; %deg
radius_e = 6378; %km
lat_gs = 34.54; %deg
long_gs = 112.4685; %deg
h_gs = 1.64; %km
vr_e_gs = [0 0 radius_e+h_gs]'; %km

elevation_array = zeros(1,length(r_array));
for i = 1:length(r_array)
    vr_rth = [r_array(i) 0 0]';
    rth_eci = rot_rth_eci(RAAN, inc, theta_array(i));
    eci_ecef = rot_eci_ecef(theta_era);
    vr_ecef = eci_ecef*rth_eci*vr_rth;

    ecef_sez = rot_ecef_sez(long_gs, lat_gs);
    vr_sez = ecef_sez*vr_ecef;

    vr_e_sc = vr_sez;

    vr_gs_sc = vr_e_sc - vr_e_gs;

    elevation_array(i) = asind(vr_gs_sc(3)/norm(vr_gs_sc));
end

figure(2)
hold on;
plot(dt_array, elevation_array)
line([0 dt_array(end)], [10 10],'Color',[0.78 0.43 0.02])
xlabel('Time (s)')
ylabel('Elevation (\circ)')
q = 1;
while elevation_array(q)<10
    q = q + 1;
end
time_start = dt_array(q);
time_end = dt_array(end);
time_above_minimum = time_end - time_start;
time_above_minimum_hours = time_above_minimum/60/60;
```

7 Why?

Why would the TDRS be in such an orbit?

The TDRS is a relay satellite intended to be in a geosynchronous orbit. The purpose of such an orbit is to maximize the amount of time that signals can be transmitted through TDRS from a well positioned ground station.

Team3_GroupExam2.m

```matlab
%% AE 313 Group Exam 2
% Team 3 - Jiyoung Hwang, Grace Day, Thorne Wolfenbarger, Aaron Scott

clc; clear all; close all;

% Knowns
MU = 398600;
a = 42170; %km
e = 0.01053;
RAAN = 14.89; %deg
AOP = 318.4; %deg
inc = 14.37; %deg
true_a1 = 184.2; %deg
energy = -MU/(2*a);

% 1. Find the current position and velocity in ECI and perifocal coordinates
p = a*(1-e^2);
h = sqrt(MU*p);
r1 = p/(1+e*cosd(true_a1));

vr1_rth = [r1 0 0]';
vv1_rth = [h*e/p*sind(true_a1) h/r1 0]';

AOL1 = AOP + true_a1;
rth_eci = rot_rth_eci(RAAN,inc,AOL1);

vr1_eci = rth_eci*vr1_rth;
vv1_eci = rth_eci*vv1_rth;

vr1_peri = r1*[cosd(true_a1) sind(true_a1) 0]';
vv1_peri = [-h/p*sind(true_a1) h/p*(e+cosd(true_a1)) 0]';
% 2. You need to send the message by the time E=15deg. How long do you have
% until t2?
period = 2*pi*sqrt(a^3/MU);
E2 = 15; %deg
E2_rad = E2*pi/180;

% syms E1
% eq = r1 == a*(1-e*cosd(E1));
% E1 = double(solve(eq,E1))
E1 = 2*atand(tand(true_a1/2)/sqrt((1+e)/(1-e)));
```

```
% E1 = acosd((a*e+r1*cosd(true_a1))/a);
% E1 = acosd(-(r1/a-1)/e)*sign(true_a1);
E1_rad = E1*pi/180;

dt = sqrt(a^3/MU)*(E2_rad - E1_rad - (e*sin(E2_rad) - e*sin(E1_rad)));

% 3. What is the new position and velocity in ECI and perifocal
    coordinates?
r2 = a*(1-e*cos(E2_rad));
true_a2 = 2*atan2d(sqrt((1+e)/(1-e))*tan(E2_rad/2),1);

% f = 1 - a/r1*cos(E2_rad - E1_rad);
% g = dt - sqrt(a^3/MU)*((E2_rad - E1_rad) - sin(E2_rad - E1_rad));
% fdot = sqrt(MU*a)/(r2*r1)*sin(E2_rad - E1_rad);
% gdot = 1-a/r2*(1-cos(E2_rad - E1_rad));
% vr2_eci = f*vr1_eci + g*vv1_eci;
% vv2_eci = fdot*vr1_eci + gdot*vv1_eci;
vr2_rth = [r2 0 0]';
vv2_rth = [h*e/p*sind(true_a2) h/r2 0]';

AOL2 = AOP + true_a2;
rth_eci = rot_rth_eci(RAAN,inc,AOL2);

vr2_eci = rth_eci*vr2_rth;
vv2_eci = rth_eci*vv2_rth;

vr2_peri = r2*[cosd(true_a2) sind(true_a2) 0]';
vv2_peri = [-h/p*sind(true_a2) h/p*(e+cosd(true_a2)) 0]';


% 4. Find the latitiude and longitide at the two positions
dt_days = dt/60/60/24;
rot_earth = 7.2921151467*10^-5; %rad
tu1 = juliandate(datetime('2019-03-06 03:00:00')) - 2451545;
tu2 = juliandate(datetime('2019-03-06 03:00:00')) - 2451545;

theta_era1 = 2*pi*(0.7790572732640 + 1.00273781191135448*tu1);
theta_era1 = mod(theta_era1,2*pi)*180/pi;
theta_era2 = 2*pi*(0.7790572732640 + 1.00273781191135448*tu2);
theta_era2 = mod(theta_era2,2*pi)*180/pi;

eci_ecef1 = rot_eci_ecef(theta_era1);
eci_ecef2 = rot_eci_ecef(theta_era2);
vr1_ecef = eci_ecef1*vr1_eci;
```

```
vr2_ecef = eci_ecef2*vr2_eci;


alpha1 = atan2d(cosd(inc)*sind(AOL1),cosd(AOL1));
alpha2 = atan2d(cosd(inc)*sind(AOL2),cosd(AOL2));
theta_gr1 = theta_era1+180/pi*rot_earth*0;
theta_gr2 = theta_era2+180/pi*rot_earth*dt;

lat1 = asind(vr1_ecef(3)/r1);
long1 = alpha1 + RAAN - theta_gr1;
lat2 = asind(vr2_ecef(3)/r2);
long2 = alpha2 + RAAN - theta_gr2;
long2 = mod(long2,-360);

% 5. Create a ground track between the two positions using MATLAB. Mark the
% starting location.

true_a_array = linspace(true_a1, true_a2+360, 1000);
r_array = p./(1+e*cosd(true_a_array));
% E_array = acosd(-(r_array/a-1)/e).*sign(true_a_array);
E_array = 2*atand(tand(true_a_array/2)/sqrt((1+e)/(1-e)));
E_array_rad = E_array*pi/180;

dt_array = sqrt(a^3/MU)*(E_array_rad - E1_rad - (e*sin(E_array_rad) - e*sin(E1_rad)));

theta_era = theta_era1;
theta_gr_array = theta_era+180/pi*rot_earth*dt_array;


theta_array = AOP + true_a_array;
lat_array = asind(sind(inc)*sind(theta_array));

alpha_array = atan2d(cosd(inc)*sind(theta_array),cosd(theta_array));
long_array = alpha_array + RAAN - theta_gr_array;

figure; hold on;
geoshow("landareas.shp","FaceColor",[0.5 1.0 0.5]);
geoshow(lat_array, long_array,'Color','red');
geoshow(lat_array(1),long_array(1),'DisplayType','Point','Marker','x','Markersize',20);
```

```
% 6. Assume you have the ground station in Prescott. Plot the elevation angle
% versus time passed. If the ground station needs a minimum elevation angle
% of 10 degrees, how much time is the TDRS visible? Can you send your
% urgent message to the TDRS?
min_elevation = 10; %deg
radius_e = 6378; %km
lat_gs = 34.54; %deg
long_gs = 112.4685; %deg
h_gs = 1.64; %km
vr_e_gs = [0 0 radius_e+h_gs]'; %km

elevation_array = zeros(1,length(r_array));
for i = 1:length(r_array)
    vr_rth = [r_array(i) 0 0]';
    rth_eci = rot_rth_eci(RAAN, inc, theta_array(i));
    eci_ecef = rot_eci_ecef(theta_era);
    vr_ecef = eci_ecef*rth_eci*vr_rth;

    ecef_sez = rot_ecef_sez(long_gs, lat_gs);
    vr_sez = ecef_sez*vr_ecef;

    vr_e_sc = vr_sez;

    vr_gs_sc = vr_e_sc - vr_e_gs;

    elevation_array(i) = asind(vr_gs_sc(3)/norm(vr_gs_sc));
end

figure(2)
hold on;
plot(dt_array, elevation_array)
line([0 dt_array(end)], [10 10],'Color',[0.78 0.43 0.02])
xlabel('Time (s)')
ylabel('Elevation (\circ)')
q = 1;
while elevation_array(q)<10
    q = q + 1;
end
time_start = dt_array(q);
time_end = dt_array(end);
time_above_minimum = time_end - time_start;
time_above_minimum_hours = time_above_minimum/60/60;
```

```
% 7. Why would the TDRS be in such an orbit?



function A = rot_rth_eci(o,i,t)

% o : Omega, Longitude of the Ascending Node (RAAN)
% i : i, inclination
% t : theta, Argument of Latitude

A = [cosd(o)*cosd(t)-sind(o)*cosd(i)*sind(t), -cosd(o)*sind(t)-sind(o)*cosd(i)*cosd(t), sind(o)*sind(i); ...
        sind(o)*cosd(t)+cosd(o)*cosd(i)*sind(t), -sind(o)*sind(t)+cosd(o)*cosd(i)*cosd(t), -cosd(o)*sind(i); ...
        sind(i)*sind(t), sind(i)*cosd(t), cosd(i)];

end

function A = rot_eci_ecef(t)

% t : theta_era, Earth rotation angle

A = [cosd(t), sind(t), 0; -sind(t), cosd(t), 0; 0, 0, 1];
end

function A = rot_ecef_sez(l,p)

% l : lamda_gs, longitude of the ground station
% p : phi_gs, latitude of the ground station

A = [sind(p)*cosd(l), sind(p)*sind(l), -cosd(p);
    -sind(l), cosd(l), 0;
     cosd(p)*cosd(l), cosd(p)*sind(l), sind(p)];

end
```