Orbits in 3D

Everything we've done so far has been in 2D. Now we're going to 3D!

We're going to look at two basic types of coordinate systems:

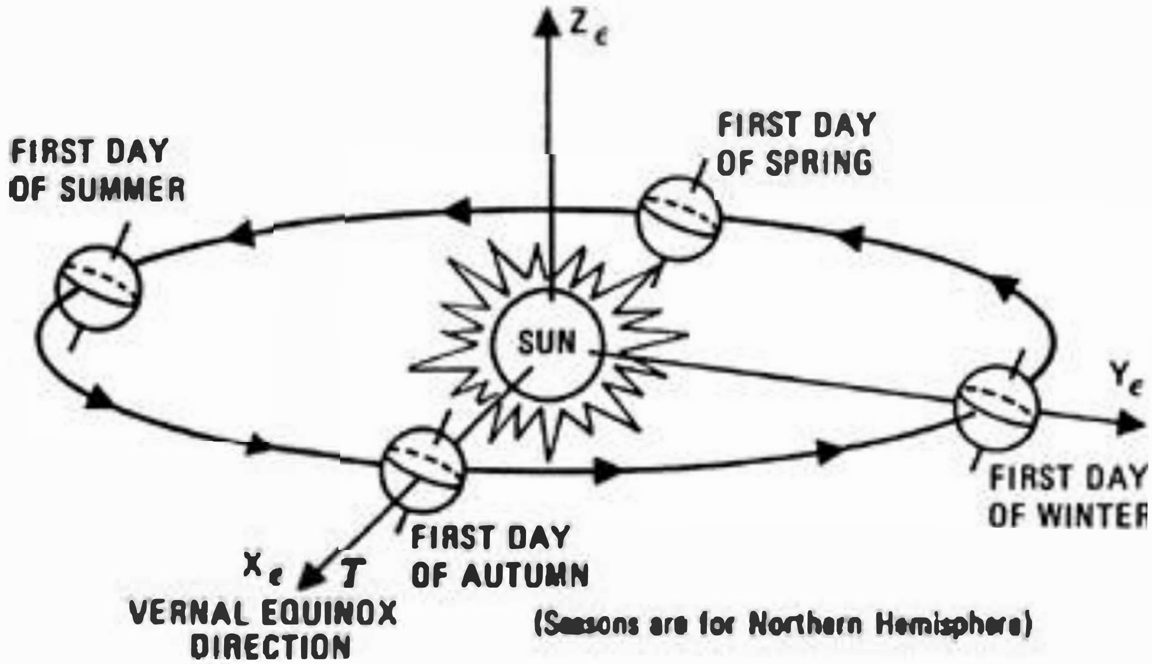
1) Ecliptic system:

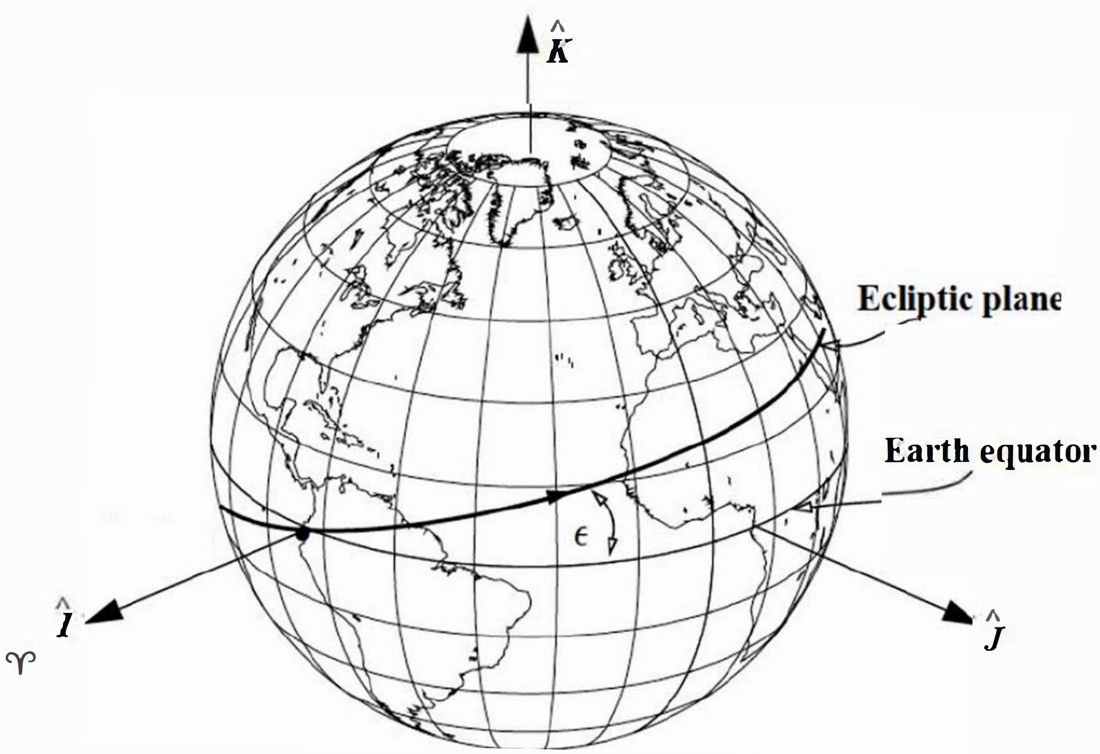
2) Equatorial System:

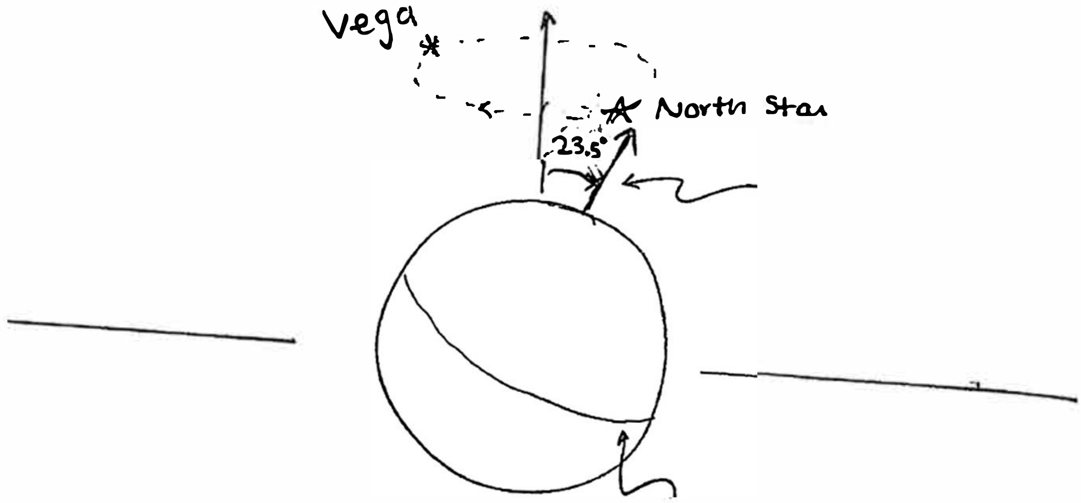
Obliquity of ecliptic (ϵ)

We need a fixed reference direction to use coordinate systems

→ vernal equinox:


Z_e
FIRST DAY
OF SPRING
FIRST DAY
OF SUMMER
SUN
Y_e
FIRST DAY
OF WINTER
FIRST DAY
OF AUTUMN
X_e
Υ
VERNAL EQUINOX
DIRECTION
(Seasons are for Northern Hemisphere)


$\hat{K}$
Ecliptic plane
Earth equator
ϵ
$\hat{I}$
Υ
$\hat{J}$



"precession of the equinoxes"

Caused by ~~pet~~turbing forces on its attitude

Time for complete precession is 26,000 years

Precession means cataloging objects must refer to a specific date or epoch

$\rightarrow$

We will assume γ is fixed over the relatively short intervals of interest.

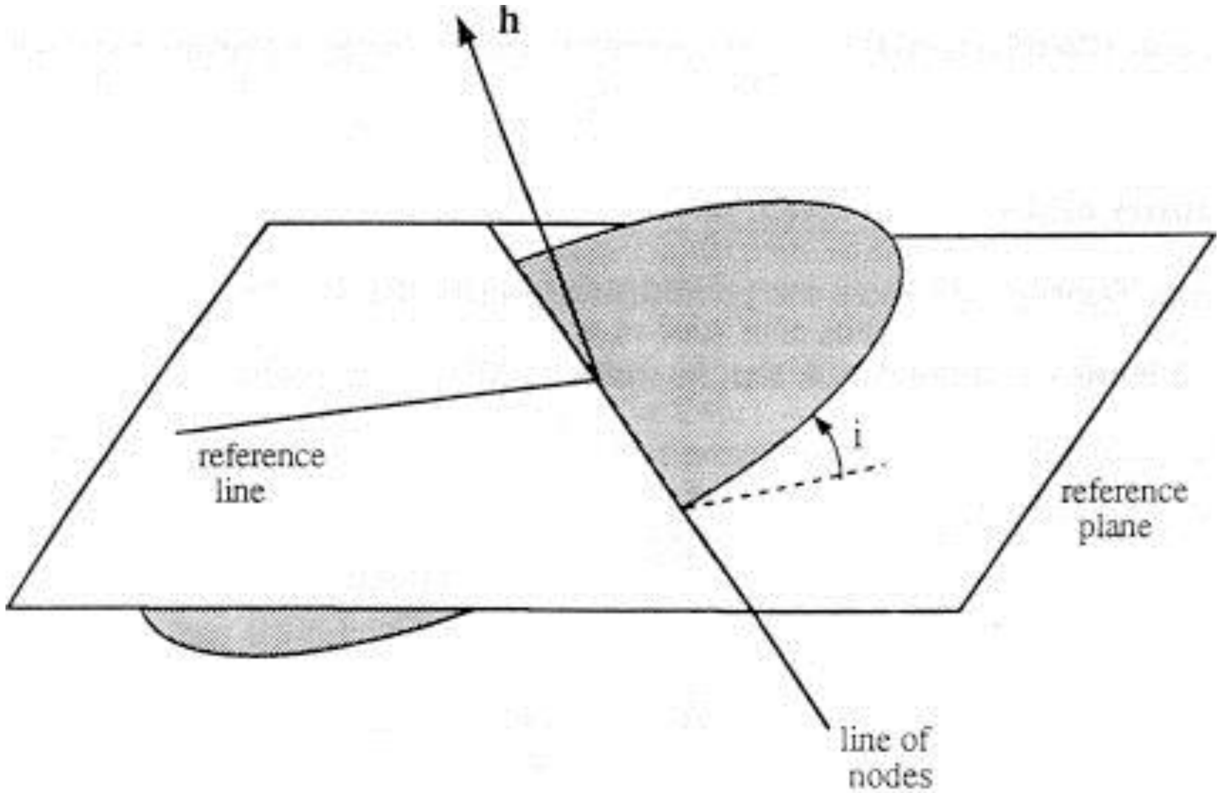
this gives us a

<u>Reference System</u> (ECI)

$\hat{x}$

$\hat{z}$

$\hat{y}$


h
reference line
i
reference plane
line of nodes

1. Locate s/c in orbit: time

2. within orbit plane: orbit size & shape

orbit orientation in orbit plane

3) within space

ω:

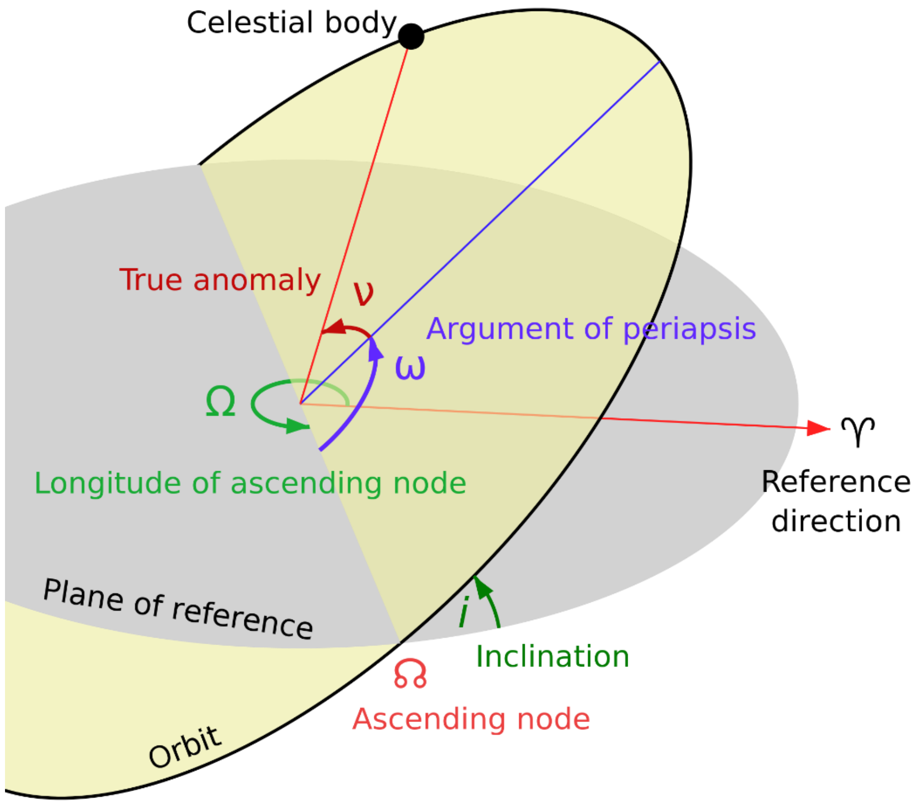
$\Omega + \omega = \bar{\omega}$:

$\bar{\omega} + \theta^* = L$

To transform use 3-1-3 Euler sequence

C: cosine
S: sine

$$
{}_{XYZ}A_{R\theta H} = \begin{array}{c|ccc}
 & \hat{r} & \hat{\theta} & \hat{h} \\
\hat{x} & C\Omega C\theta - S\Omega C i S\theta & -C\Omega S\theta - S\Omega C i C\theta & S\Omega S i \\
\hat{y} & S\Omega C\theta + C\Omega C i S\theta & -S\Omega S\theta + C\Omega C i C\theta & -C\Omega S i \\
\hat{z} & S i S\theta & S i C\theta & C i
\end{array}
$$


Celestial body
True anomaly
ν
Argument of periapsis
ω
Ω
Longitude of ascending node
♈
Reference direction
Plane of reference
i
Inclination
☊
Ascending node
Orbit

Example 1:

Given: $\bar{r}_1 = 1.6772\, R_\oplus\, \hat{x} - 1.6772\, R_\oplus\, \hat{y} + 2.3719\, R_\oplus\, \hat{z}$

$\bar{v}_1 = 3.1574\, \hat{x} + 2.4987\, \hat{y} + 0.4658\, \hat{z}$ km/s

Find: $a, e, i, \Omega, \omega, \theta^*$

Shape? → $r_1 = |\bar{r}_1|$, $v_1 = |\bar{v}_1|$

Find $\mathcal{E}$. What shape is the orbit?

$\mathcal{E} =$

Check for collisions,

Find magnitude of θ_1^*

$\theta_1^* =$

From rotation matrix

Find $\hat{n}$.

Then

We can obtain the remaining elements from

$$\hat{r}_1 = \frac{\bar{r}_1}{|\bar{r}_1|} = 0.5\,\hat{x} - 0.5\,\hat{y} + 0.7071\,\hat{z}$$

$$\hat{\theta}_1 = \hat{h} \times \hat{r}_1 = 0.7071\,\hat{x} + 0.7071\,\hat{y}$$

What is θ_1 ?

Back to θ_1^*. recall

$$\bar{V}_1 = (\bar{v}_1 \cdot \hat{r}_1)\,\hat{r}_1 + (\bar{v}_1 \cdot \hat{\theta}_1)\,\hat{\theta}_1$$

Example 2:

Given: $\bar{r}_1 = 14450.6\,\hat{x} - 1529.9\,\hat{y} - 6524.0\,\hat{z}$ km

$\bar{r}_2 = -6199.5\,\hat{x} + 14699.2\,\hat{y} + 8531.9\,\hat{z}$ km

$p = 2.88\,R_\oplus$

Find: $a, e, i, \Omega, \omega, \theta_1^*, \theta_2^*, \bar{v}_1, \bar{v}_2$

Analysis

know $r_1 = |\bar{r}_1|$ and $r_2 = |r_2|$

Find i, Ω, θ.

To find $\bar{v}_1$ + $\bar{v}_2$

$$\bar{r}_2 = f\bar{r}_1 + g\bar{v}_1 \Rightarrow \bar{v}_1 = \frac{\bar{r}_2 - f\bar{r}_1}{g}$$

use f + g in terms of $\Delta\theta^*$

$$f, g = f(r_1, r_2, \Delta\theta^*, p); \quad g(r_1, r_2, \Delta\theta^*, p)$$

How to find $\Delta\theta^*$?

Now find θ_1^*

Since we have v_1 and r_1, now find a + e