Homework 7

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of the Requirements of

AE 313
Space Mechanics
Spring, 2019

AE 313 Homework 7

1. What is the initial Jason orbit semi-major axis, eccentricity, and true anomaly?

```
syms am
eq = -MU/(2*am) == vm^2/2 - MU/rm;
am = double(solve(eq, am)) % 6.6993e3 km

hm = rm*vm*cosd(FPAm);
pm = hm^2/MU;
em = sqrt(1-pm/am) % 0.5001

true_am = atan2d(rm*vm^2/MU*cosd(FPAm)*sind(FPAm),rm*vm^2/MU*cosd(FPAm)^2-1) % 160.0052
```

$$
\begin{aligned}
a^- &= 6699.3 \text{ km} \\
e^- &= 0.5001^\circ \\
\theta^{*-} &= 160.0052^\circ
\end{aligned}
$$

2. Draw the vector diagram.

On a separate page

3. Determine the velocity and flight path angle immediately following the maneuver. Make sure to justify if $\Delta\gamma$ is positive or negative.

```
syms vp FPAp
vp = sqrt(vm^2 + dv^2 - 2*vm*dv*cosd(180 - abs(alpha))) % 6.1136 km/s
dFPA = acosd(((dv^2)-(vm^2)-(vp^2))/(-2*vm*vp)) %11.5652 deg | negative because the dv is oriented towards earth (indicated by the negative alpha)
FPAp = FPAm - dFPA;
```

$$
\begin{aligned}
v^+ &= 6.1136 \text{ km/s} \\
\gamma^+ &= 6.3148^\circ
\end{aligned}
$$

$\Delta\gamma$ is negative because $|\Delta\vec{v}|$ is oriented towards earth (indicated by the negative alpha).

4. Determine the orbital characteristics following the maneuver: $a^+, e^+, \theta^{*+}, \Delta\omega$

```
FPAp = FPAm - dFPA;
syms ap
rp = rm;
eq = -MU/(2*ap) == vp^2/2 - MU/rp;
ap = double(solve(eq, ap)) % 8.5290e+03 km
ep = sqrt((rp*vp^2/MU - 1)^2*cosd(FPAp)^2 + sind(FPAp)^2) % 0.1560
true_ap = atan2d(rp*vp^2/MU*cosd(FPAp)*sind(FPAp),rp*vp^2/MU*cosd(FPAp)^2 - 1)
true_ap = atan2d(rp*vp^2/MU*cosd(FPAp)*sind(FPAp),rp*vp^2/MU*cosd(FPAp)^2-1); % 141.4710
dAOP = -(true_ap - true_am) % 18.5342
```

$$\begin{aligned} a^+ &= 8529 \text{ km} \\ e^+ &= 0.1260 \\ \theta^{*+} &= 141.71^\circ \\ \Delta\omega &= 18.5342^\circ \end{aligned}$$

θ^{*+}'s value is positive due to the fact that $atan2$ takes into account the signs of the components. $\Delta\omega$ =18.5342° is a positive value due to the fact that it follow's the relationship of $\Delta\omega = \theta^{*-} - \theta^{*+}$.

5. Use GMAT to plot the two orbits. Propagate both orbits at least one period. Draw the vector diagram (vm, vp, alpha, dFPA, dv) and position vector for the maneuver on the printout. Check a^+, e^+, θ^*, v^+.

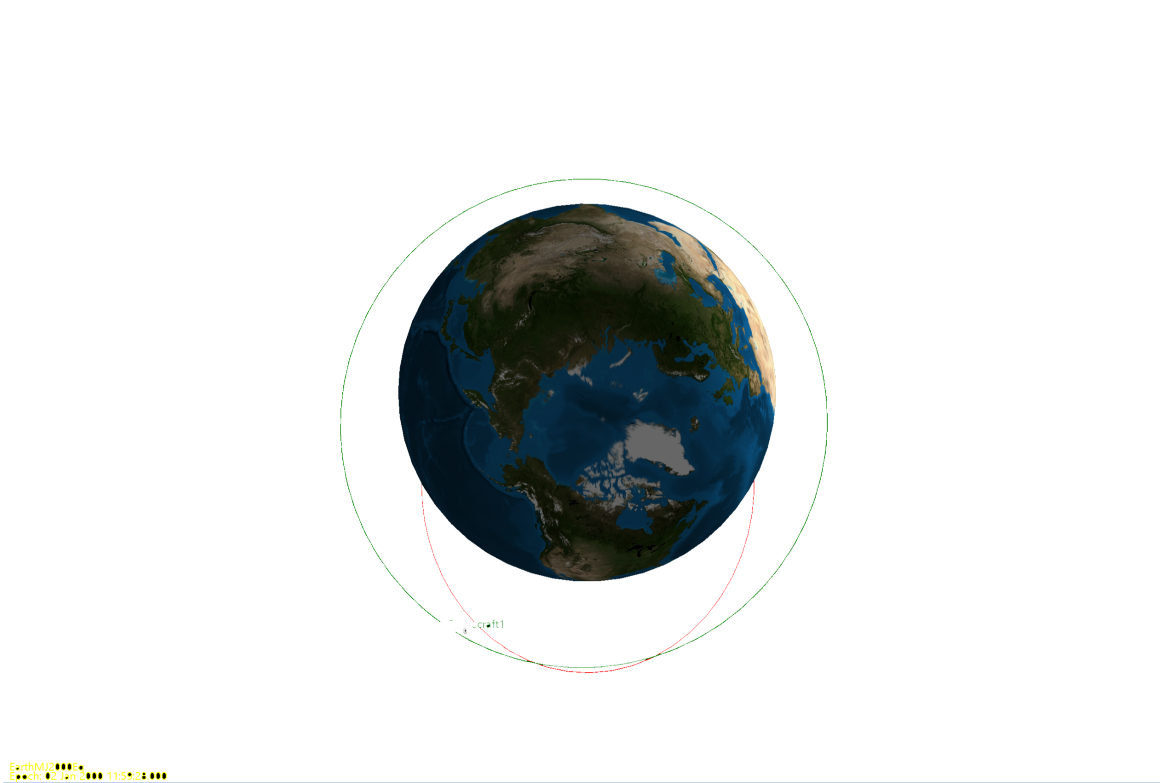

Figure 1: GMAT Plot of Orbits

$$
\begin{aligned}
a^+ &= 8529 \text{ km} \\
e^+ &= 0.1560 \\
\theta^{*+} &= 141.471^\circ \\
v^+ &= 6.1136 \text{ km/s}
\end{aligned}
$$

6. What would happen if you failed to perform this maneuver?
Without the maneuver, ICESAT-2 is on a parabollic collision trajectory with Earth. If the maneuver failed, the spacecraft would crash into Earth. This can be observed in (Fig. 1) where the smaller, red orbit displays the collision trajectory.

HW7.m

```
clc;

MU = 398600; % km^3/sec^2

rm = 9478; % km
vm = 4.961; % km/s
FPAm = 17.88; % deg

dv = 1.6; % km/s
alpha = -50; % deg

% 1. What is the initial Jason orbit semi-major axis, eccentricity, and
% true anomaly?
syms am
eq = -MU/(2*am) == vm^2/2 - MU/rm;
am = double(solve(eq, am)) % 6.6993e3 km

hm = rm*vm*cosd(FPAm);
pm = hm^2/MU;
em = sqrt(1-pm/am) % 0.5001

true_am = atan2d(rm*vm^2/MU*cosd(FPAm)*sind(FPAm),rm*vm^2/MU*cosd(FPAm)^2-1) % 160.0052


% 2. Draw the vector diagram

% 3. Determine the velocity and flight path angle immediately following the
% maneuver. Make sure to justify if dFPA is positive or negative.
syms vp FPAp
vp = sqrt(vm^2 + dv^2 - 2*vm*dv*cosd(180 - abs(alpha))) % 6.1136 km/s
dFPA = acosd(((dv^2)-(vm^2)-(vp^2))/(-2*vm*vp)) %11.5652 deg | positive because the dv is oriented towards earth (indicated by the negative alpha)

% 4. Determine the orbital characteristics following the maneuver: ap, ep,
% true_ap, dAOP
FPAp = FPAm - dFPA;
syms ap
rp = rm;
eq = -MU/(2*ap) == vp^2/2 - MU/rp;
ap = double(solve(eq, ap)) % 8.5290e+03 km
```

```
ep = sqrt((rp*vp^2/MU - 1)^2*cosd(FPAp)^2 + sind(FPAp)^2) % 0.1560

true_ap = atan2d(rp*vp^2/MU*cosd(FPAp)*sind(FPAp),rp*vp^2/MU*cosd(FPAp)^2
    - 1)

true_ap = atan2d(rp*vp^2/MU*cosd(FPAp)*sind(FPAp),rp*vp^2/MU*cosd(FPAp)
    ^2-1); % 141.4710
dAOP = -(true_ap - true_am) % 18.5342

% 5. Use GMAT to plot the two orbits. Propagate both orbits at least one
% period. Draw the vector diagram (vm, vp, alpha, dFPA, dv) and position
% vector for the maneuver on the priintout. check ap, ep, true_ap, vp.

% 6. What would happen if you failed to perform this maneuver?
```