

# TRANSFERS

Goal: Shift to an orbit that does  
NOT intersect the original orbit.  
 Use multiple-impulse transfers.

Limiting factor:

Approach for transfer problems:

1. Define transfer geometry

Given a transfer orbit type

- What are the departure & arrival conditions?
- What are the departure & arrival points?
- Find  $\Delta V_s$

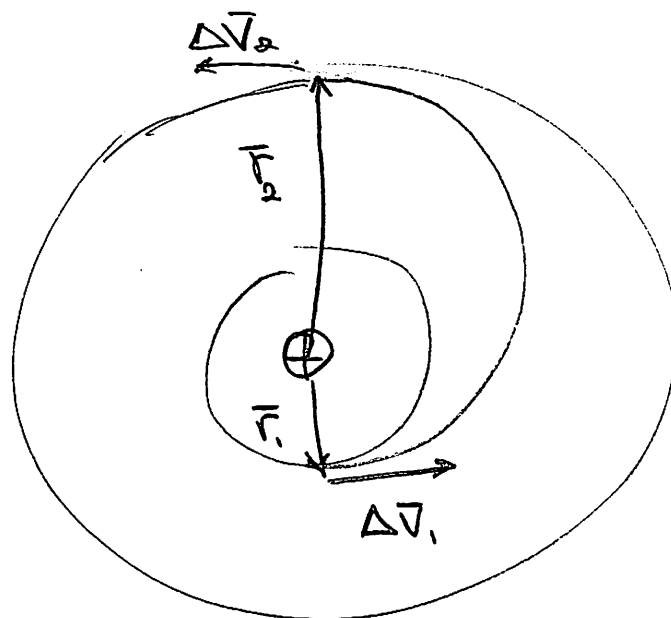
OR 2. Define departure/arrival points

SOLVE for the transfer arc  
itself to meet specifications

Start with simplest 2 - impulse transfer.

## Hohmann Transfer

Simplest version of Hohman is circle-to-circle



## Example.

Given:  $r_1 = 2 R_\oplus$      $r_2 = 4 R_\oplus$

1. Establish current orbit

$$a =$$

$$e =$$

2. Conditions at thrust before  
maneuver

$$r_1 =$$

$$v_1 =$$

$$\gamma_1 =$$

3. Define transfer ellipse

$$a_T$$

$$r_p = a_T(1 - e_T)$$

$$\Rightarrow e_T$$

4. Conditions after maneuver

 $r$  $v_p$  $\gamma$ 

5. Vector Diagram

6. Move to next maneuver point  
in transfer orbit

 $r$  $v$  $\gamma$

7. Conditions required after  
maneuver

$$r_2$$

$$v_2$$

$$\gamma_2$$

8. Vector diagram for  $\Delta \bar{v}_2$

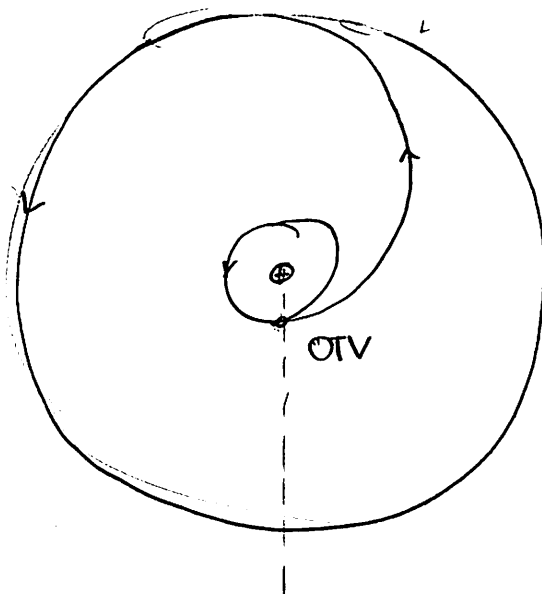
9. Total  $\Delta v$

# Conditions for Rendezvous

Transfers shift vehicles from one planet to another.

Additional complexity if rendezvous.

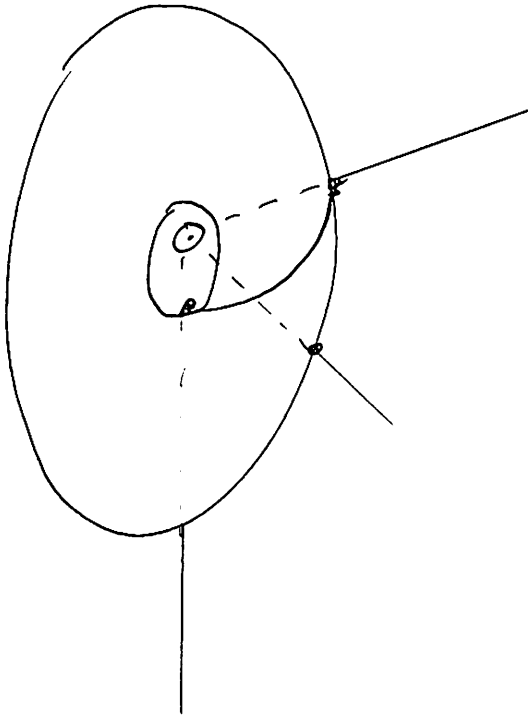
Example:  $\oplus$  orbiting OTV departing LEO to rendezvous w/ space station.



# General Phase Angle

9.7

What happens for noncircular, non-Hohmann orbit?



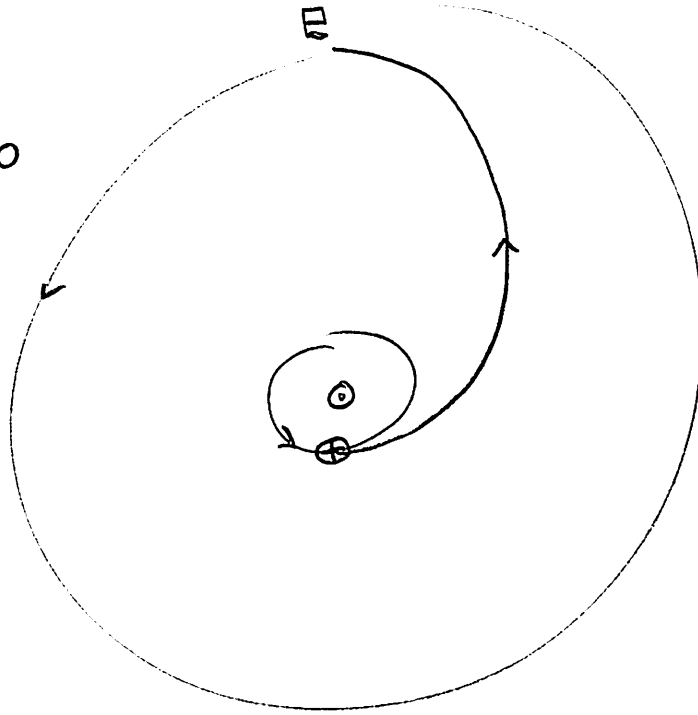
$$\phi =$$

where

# Synodic Period

9.8

Example:  
Earth to Pluto

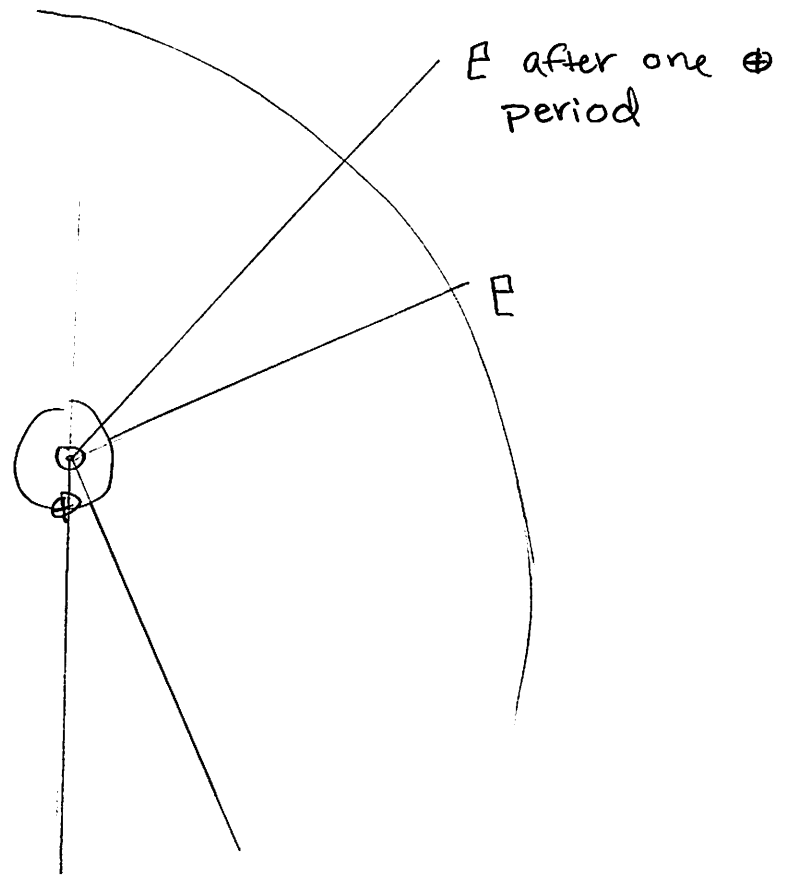


Requirement for rendezvous/interception  
determines initial geometry.

If miss "launch," how long do we wait?

NOTE: Need  $\phi$  to occur again. Don't  
require planets to be located in  
the same places.





$E$  does not move far for one Earth  $P$ ,

After one  $P_\oplus$ , angle now

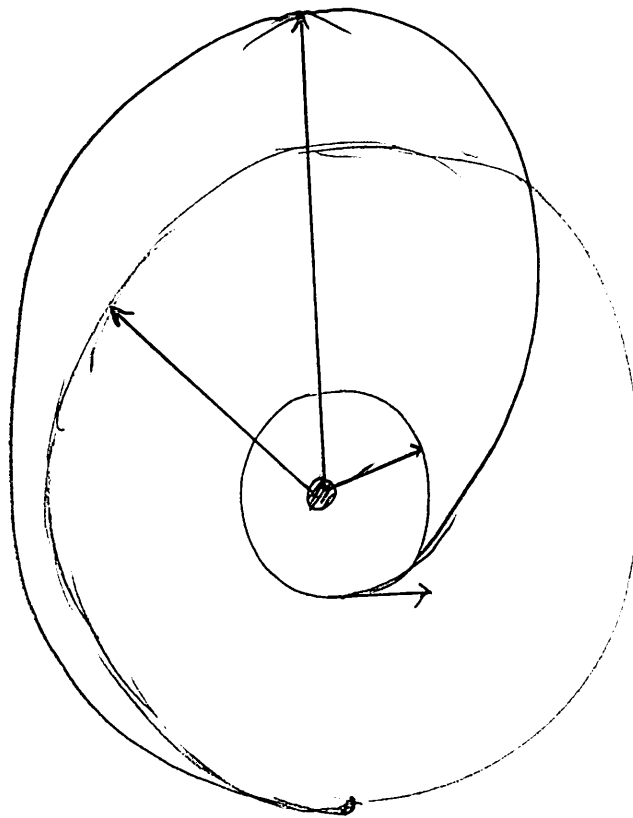
So we let the Earth "catch up"

$$\text{Note: } TP = \frac{2\pi}{n} \Rightarrow nTP = 2\pi$$

# Bi-Elliptical Transfers

9.10

Extend Hohmann transfer to 3 impulses



$T_A =$

$r:$

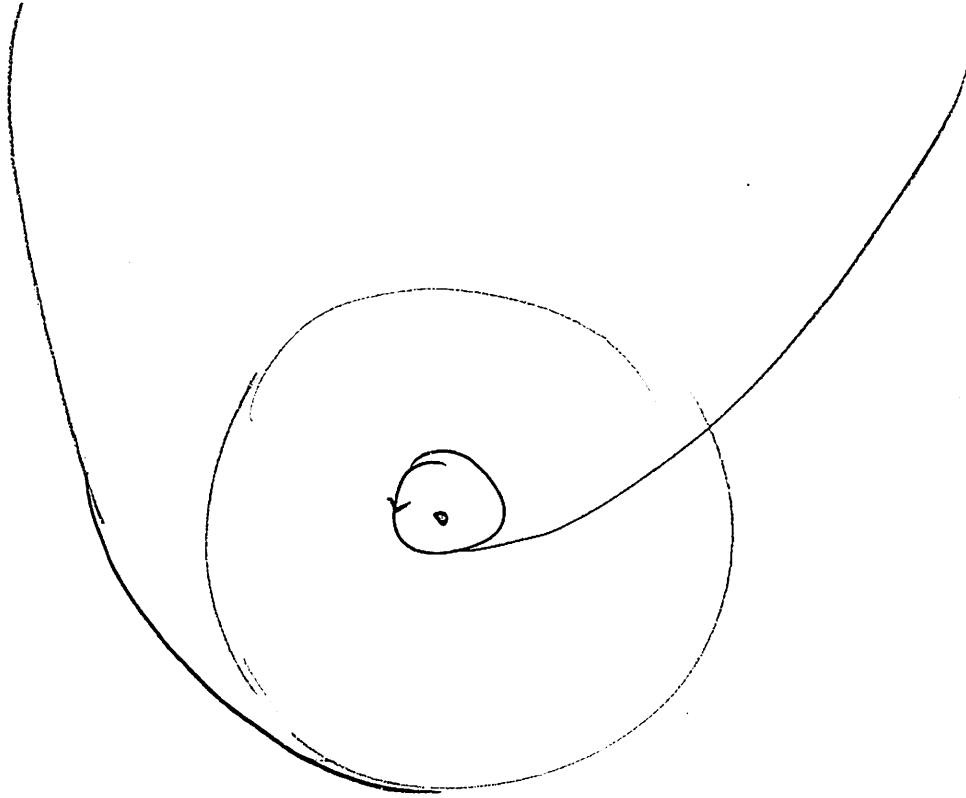
Characteristics:

1. Initial circular orbit
2. 1<sup>st</sup> impulse applied tangentially, shift to periapsis of transfer ellipse #1
3. Apogee on  $E_1 =$   
2<sup>nd</sup> impulse applied tangentially, shift from apoapsis of  $E_1$  to apoapsis of transfer ellipse #2
4. Periapsis on  $E_2 =$   
3<sup>rd</sup> impulse applied tangentially, shift into final circular orbit.
5. Total cost?

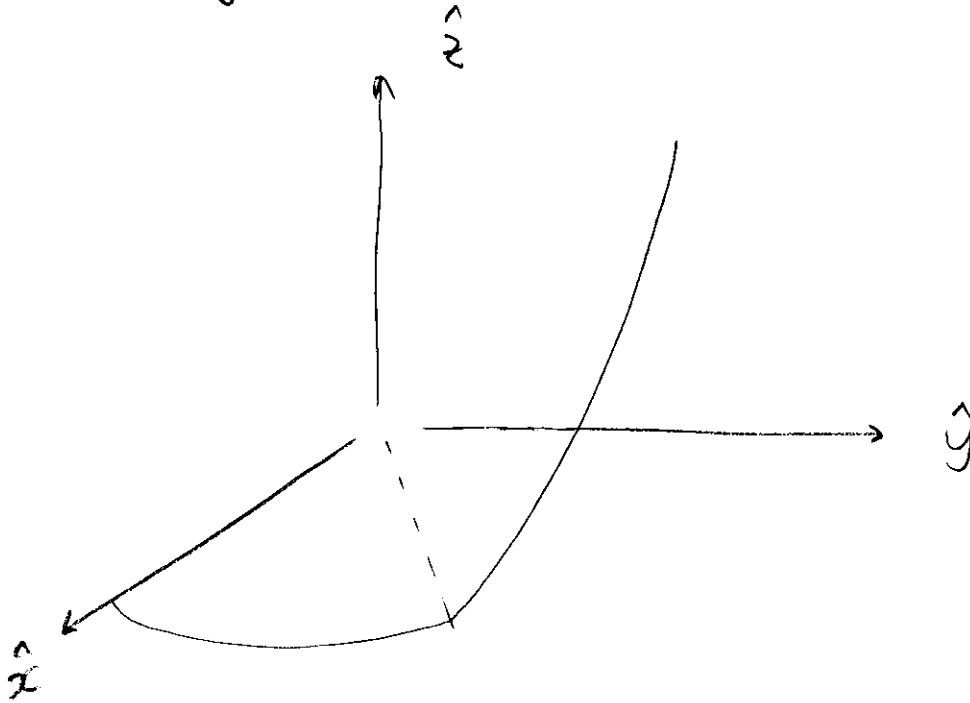
# Bi-Parabolic Transfer

9.11

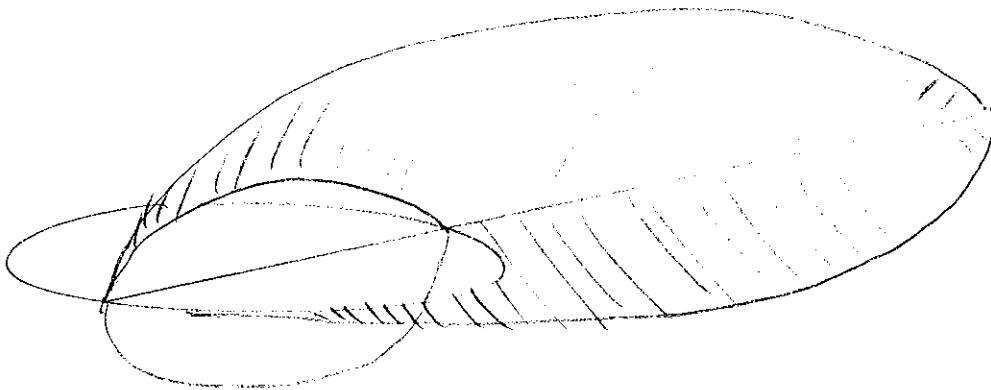
Move intermediate radius to infinity ( $r \rightarrow \infty$ )  
Transfer becomes parabolic



Bi-elliptic useful for inclination changes.



Intermediate orbit to cut costs



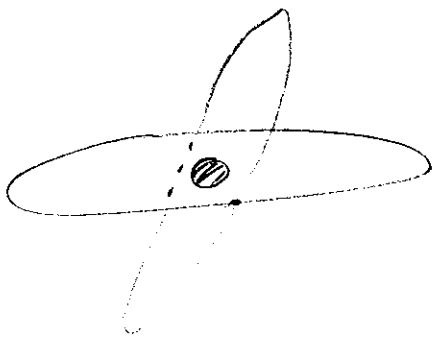
Example: Given a geosynchronous <sup>9.13</sup>

Earth orbit with an inclination of  $30^\circ$ , how much  $\Delta V$  do we need to have an inclination of  $90^\circ$  if we

A. Perform a single impulse maneuver.

B. Perform a bi-elliptic transfer with an intermediate radius of  $40 R_\oplus$

A. Single Impulse



For a geosynchronous orbit,

$$e =$$

$$P =$$

Conditions after maneuver:

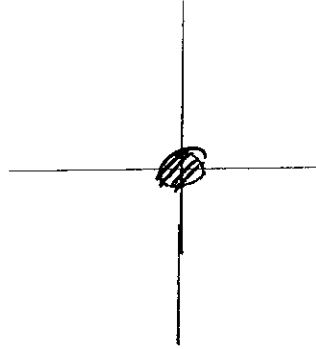
9.14

$$a^+ =$$

$$e =$$

$$v^+ =$$

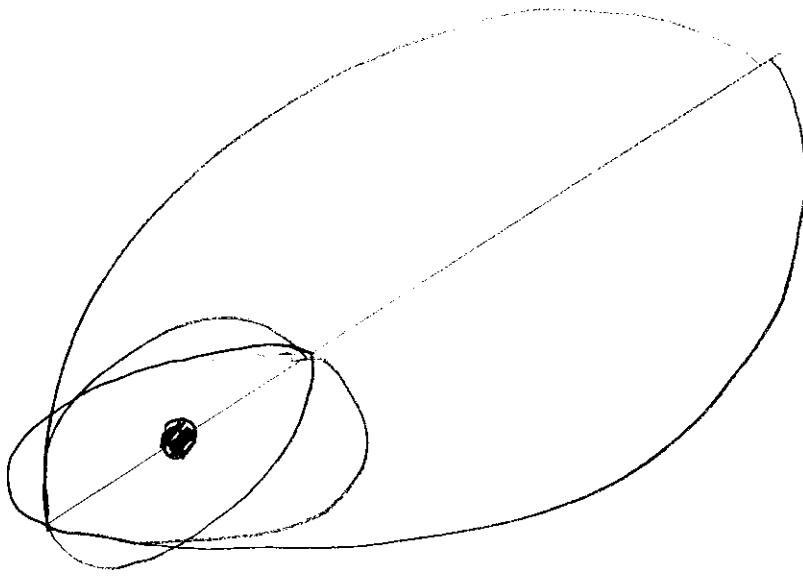
side view:



vector diagram:

## B. Bi-Elliptic

9.15



C1: initial  
circular  
orbit

B1: Burn to E1

E1: 1<sup>st</sup> Elliptical  
orbit

B2: Burn to E2

E2: 2<sup>nd</sup> elliptical  
orbit

B3: Burn from E2  
to C2

C2: Second circular  
orbit

Conditions at C1:

$$a_i =$$

$$e_i =$$

$$v_i =$$

$$\gamma_i =$$

$$L_i =$$

Conditions at E1 perigee

$$a_{T1} =$$

$$r_{p1} =$$

$$V_{P1} =$$

9.16

$$\gamma_{P1} =$$

Vector Diagram:

Conditions at apogee of E1

conditions at apogee of E2



Vector diagram:

Conditions at perigee of  $E_2$

Conditions at  $C_2$

# Vector Diagram

9.18

What is TOF?