The Structure of Convexity

Internal Theory

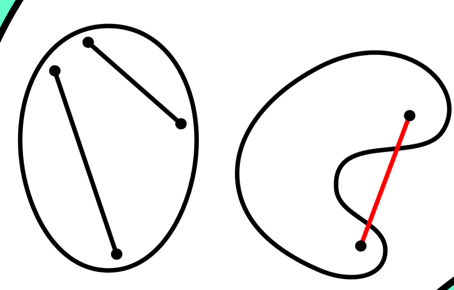
Definition

$(X,\mathcal{C})$ — ***convex space:***

(1) $\emptyset,\ X \in \mathcal{C}$

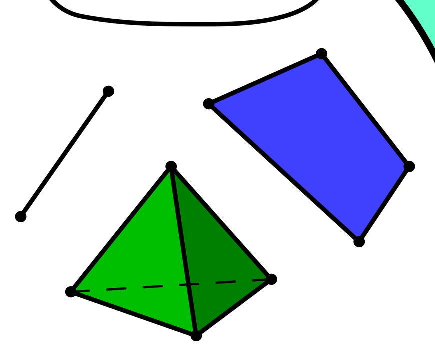
(2) $\mathcal{A} \subset \mathcal{C} \Rightarrow \cap\mathcal{A} \in \mathcal{C}$

(3) $\mathcal{N} \subset \mathcal{C} \Rightarrow \cup\mathcal{N} \in \mathcal{C}$

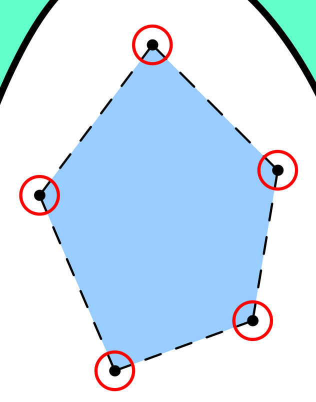


Idea

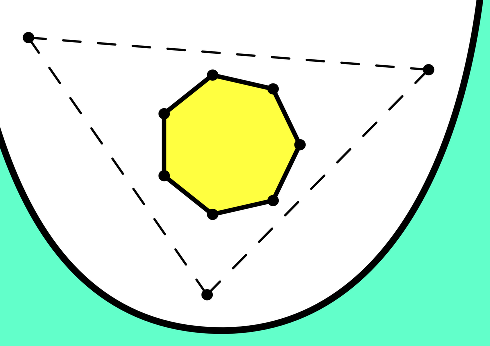
Polytope



Freedom

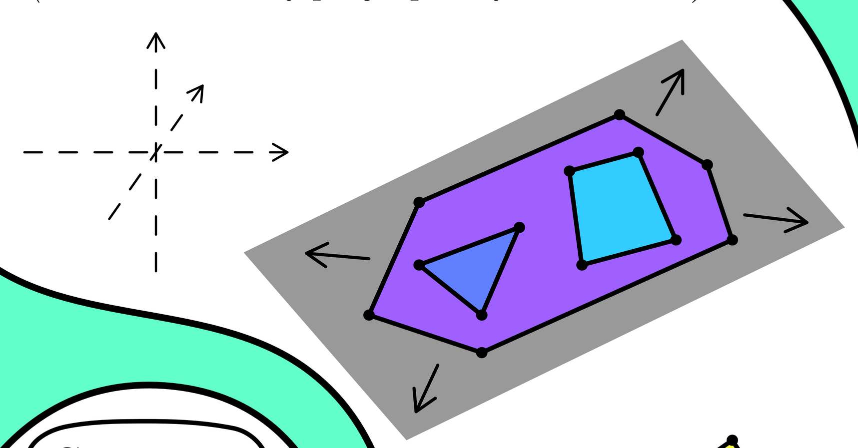


Dimension

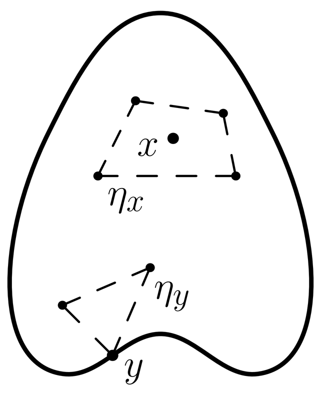


Hyperplane

(Maximal net of polytopes of same dim.)

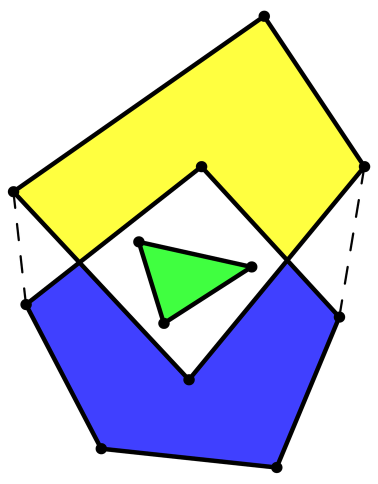


Subsets



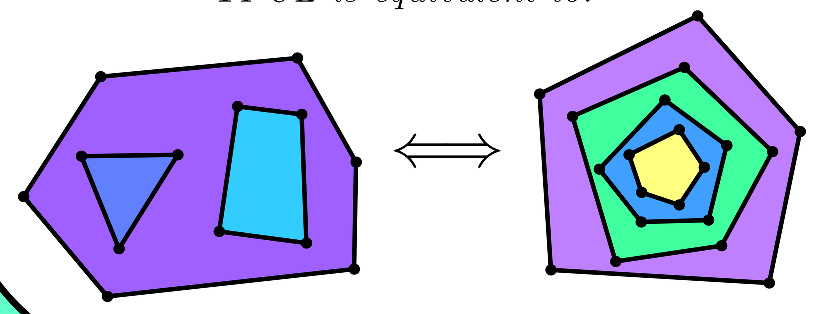
$\eta_x = \eta_y = \dim A$

TPUL



Theorem

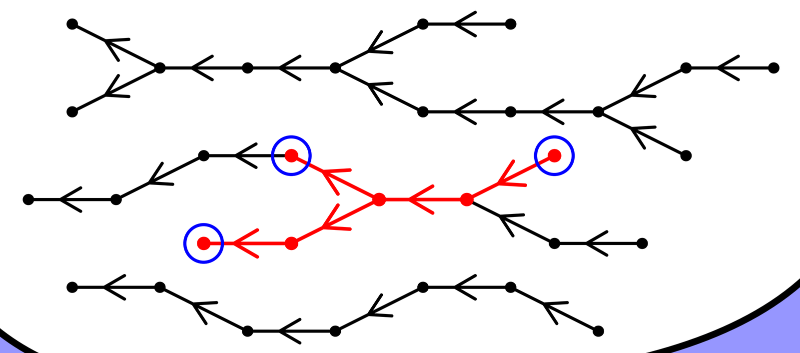
TPUL is equivalent to:



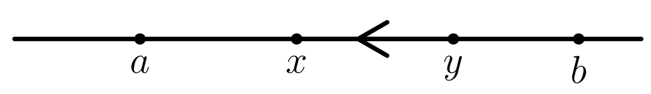
Inducing Structure

Order

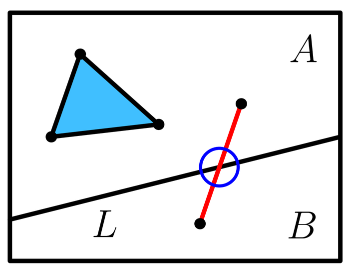
Theorem: *all order convexities are free.*



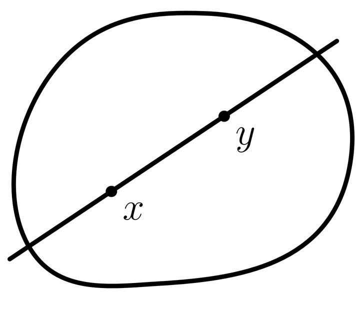
Linearity



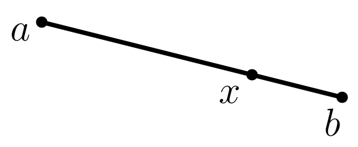
n-Affinity



1-Affinity

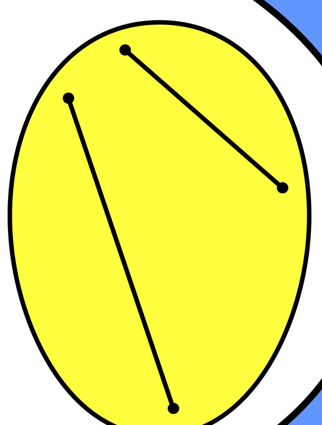


Metric

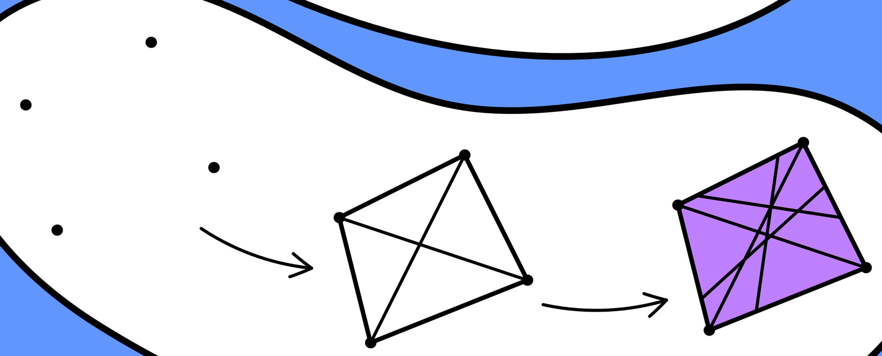


$d(a,b) = d(a,x) + d(x,b)$

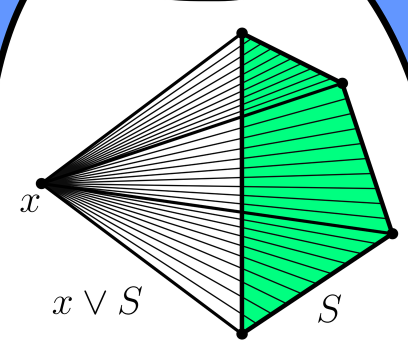
Segmential



Finite



Join



Join-commutative:

$x \vee \langle F\rangle = \langle x \cup F\rangle$

Theorem

Finite-segmential

2-Affine

TPUL

$\Downarrow$

Free

Induced Structure

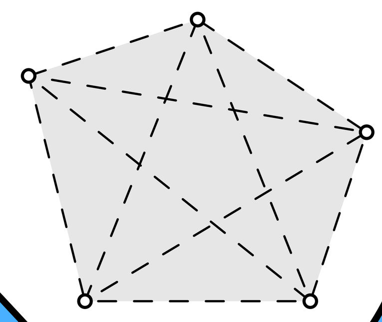
Topology

(1) *The Polytope Intersection Lemma*

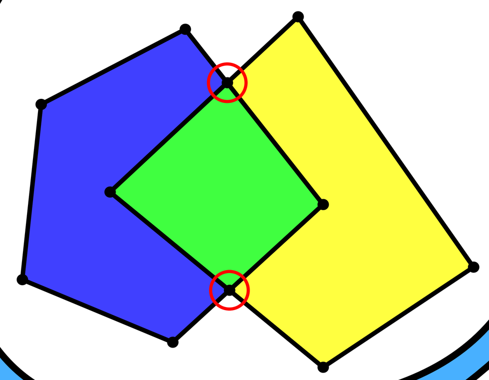
(2) *The Polytope Union Lemma*

(3) *Finite dimension*

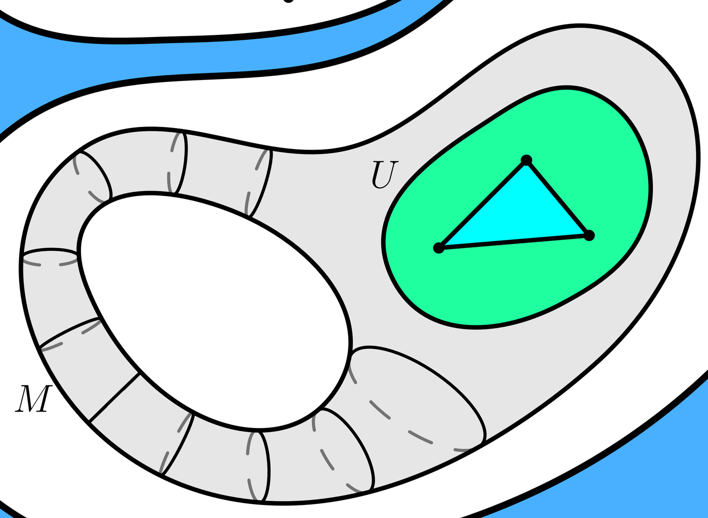
(4) *Freedom*

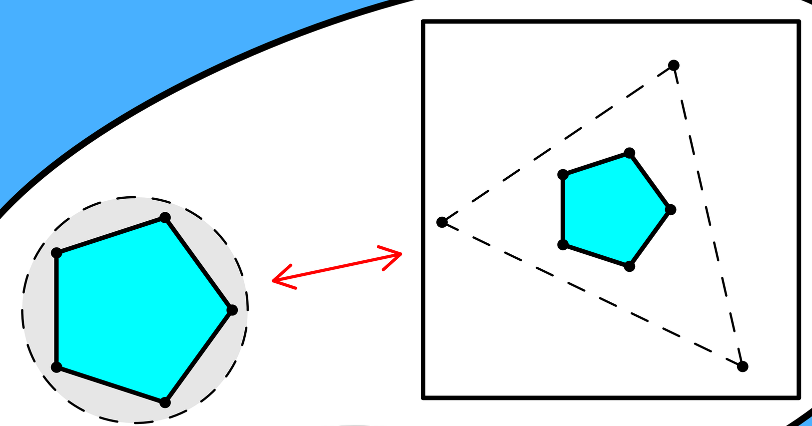


TPIL



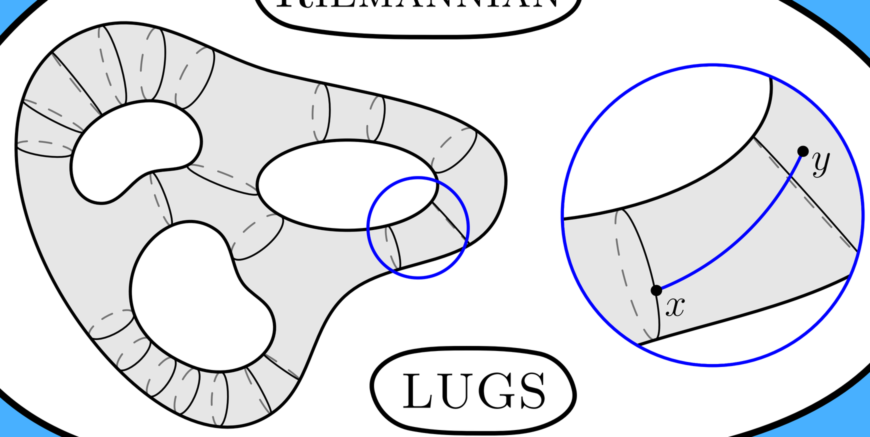
Local Convexity





$E^2 \neq B^2$

Riemannian



LUGS

References

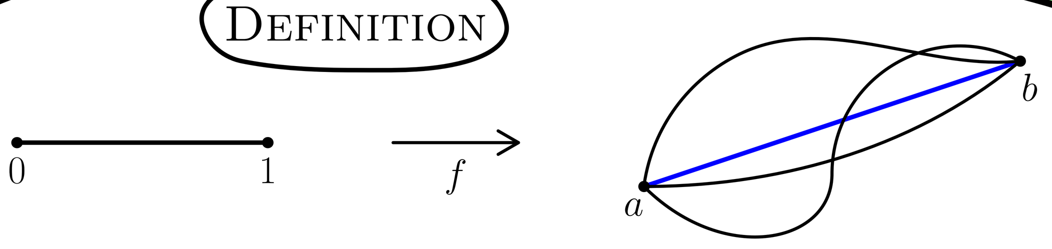
(1) M.L.J. van de Vel, *Theory of convex structures*, North-Holland mathematical library, 1993;

(2) D. C. Kay, E. W. Womble, *Axiomatic convexity theory*, Pacific Journal of Mathematics, 1971;

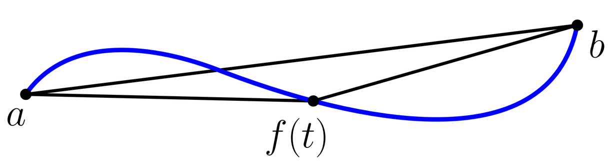
(3) D. Gromoll, W. Klingenberg and W. Meyer, *Riemannsche Geometrie im Grossen*, Springer-Verlag, Berlin, 1968.

Uniquely Geodesic Spaces

Definition



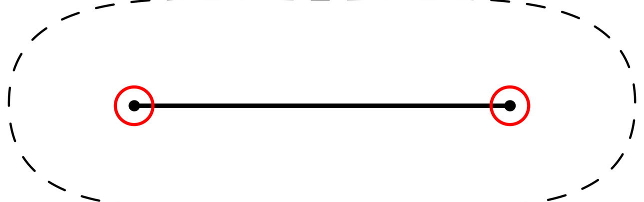
Lemma



$d(a,b) = d(a,f(t)) + d(f(t),b)$

Lemma

In a UGS all metric segments are free polytopes.



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