

# THE STRUCTURE OF CONVEXITY

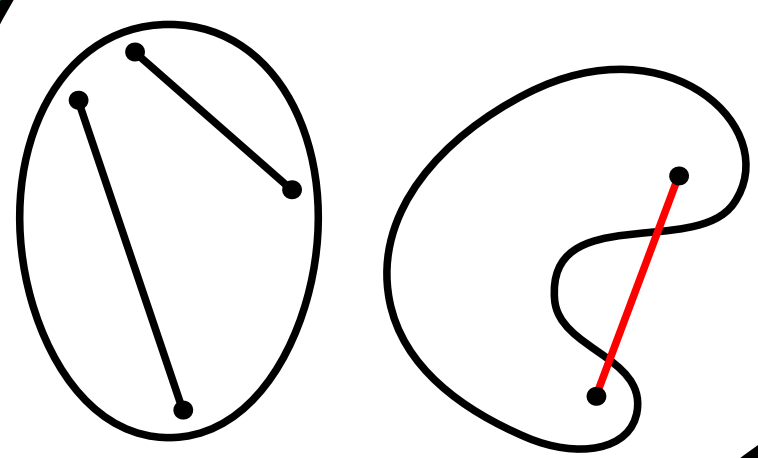
## INTERNAL THEORY

### DEFINITION

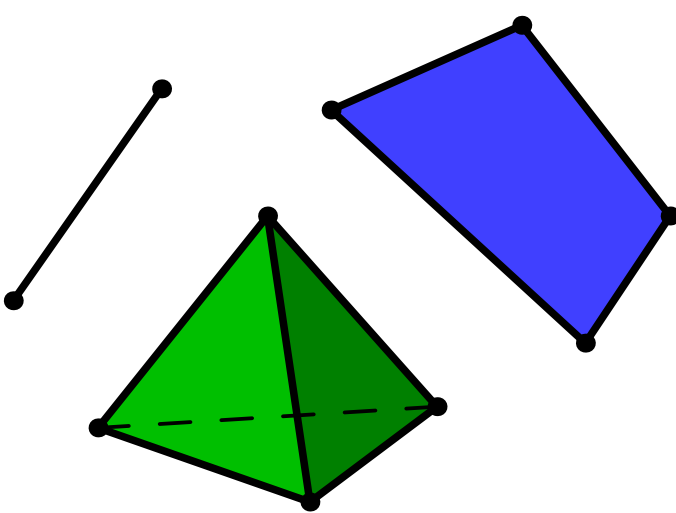
$(X, \mathcal{C})$  — **convex space**:

- (1)  $\emptyset, X \in \mathcal{C}$
- (2)  $A \subset \mathcal{C} \Rightarrow \cap A \in \mathcal{C}$
- (3)  $\mathcal{N} \subset \mathcal{C} \Rightarrow \cup \mathcal{N} \in \mathcal{C}$

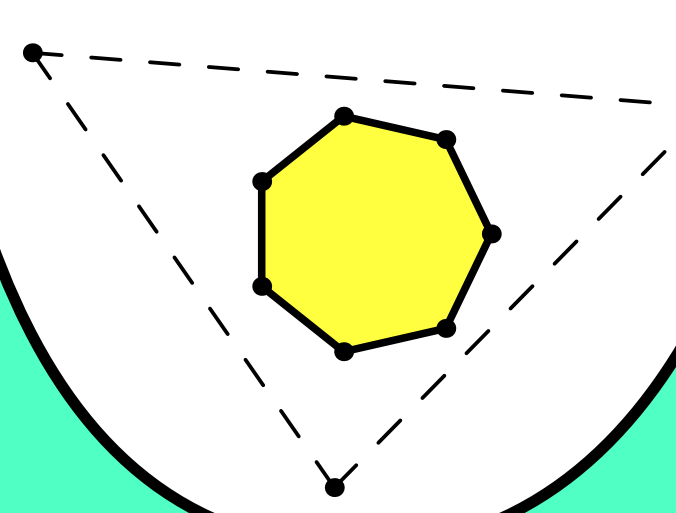
### IDEA



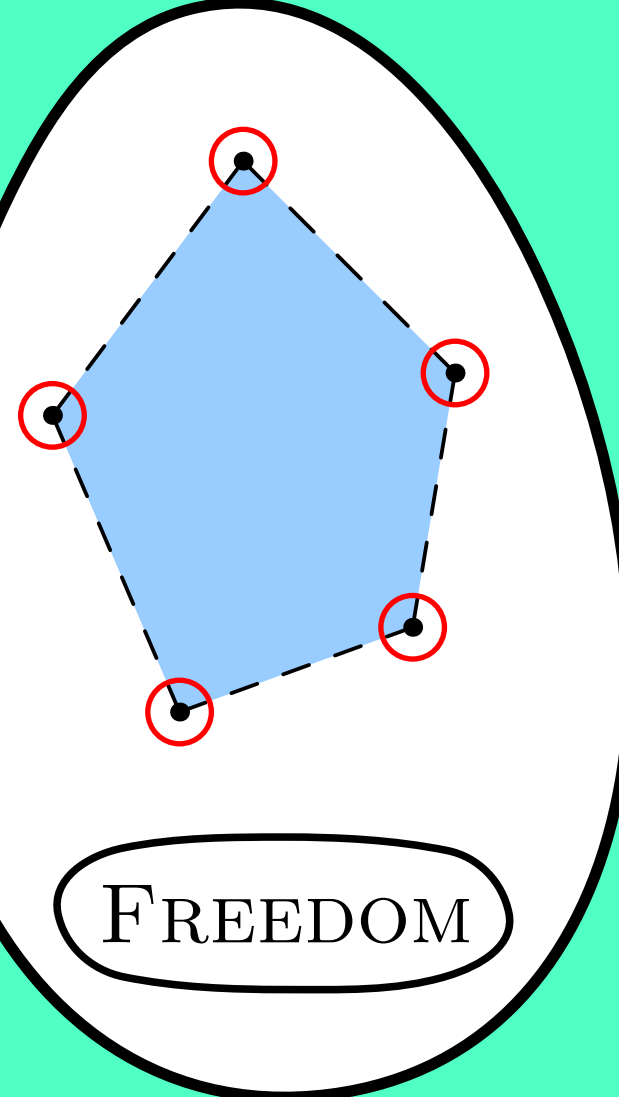
### POLYTOPE



### DIMENSION

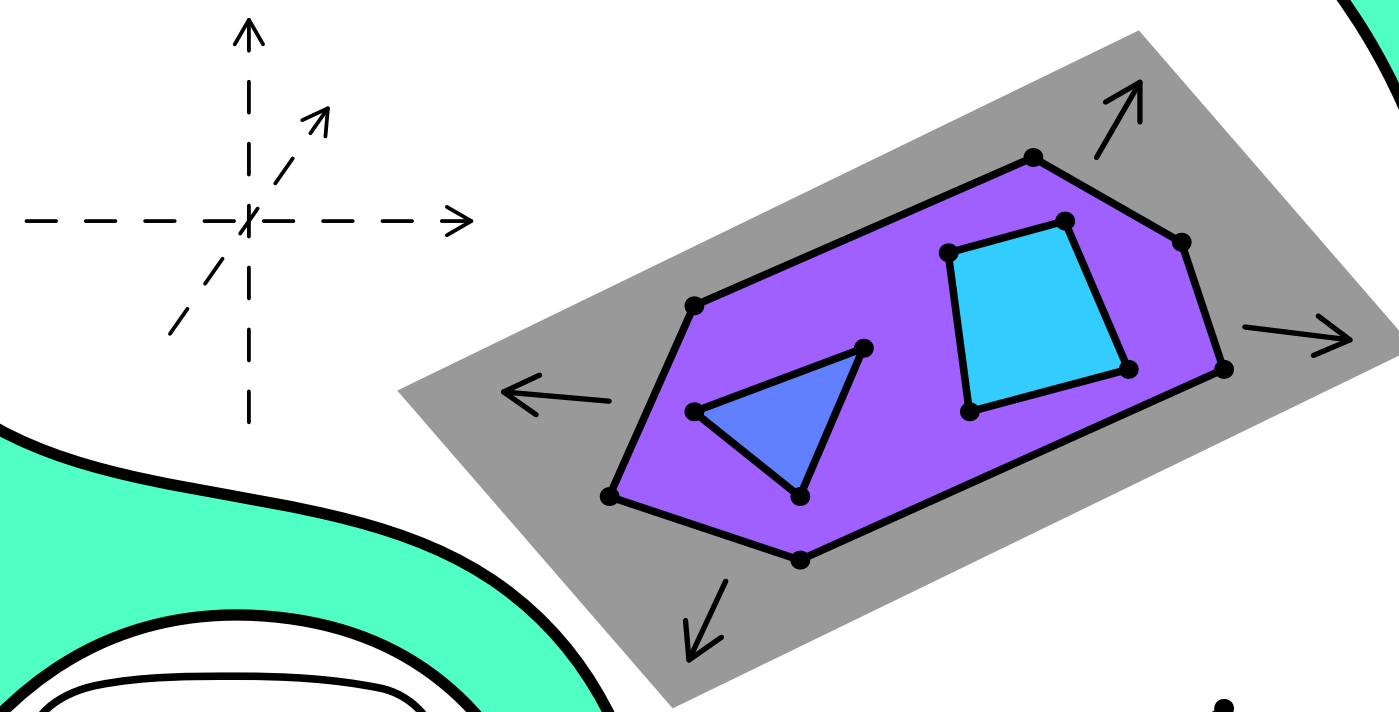


### FREEDOM

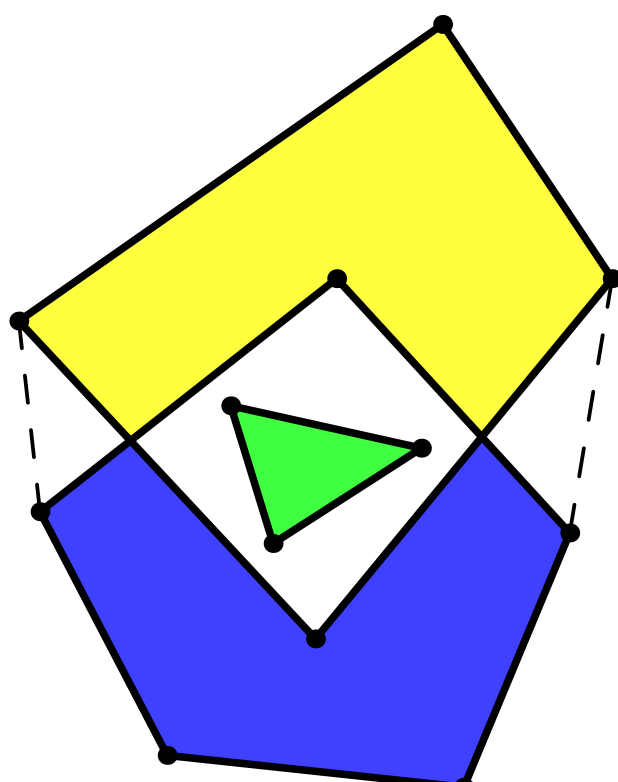
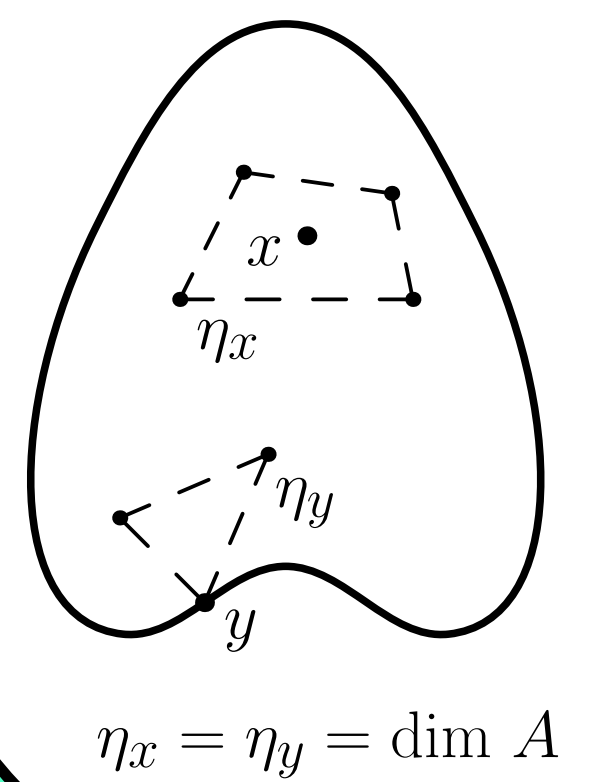


### HYPERPLANE

(Maximal net of polytopes of same dim.)



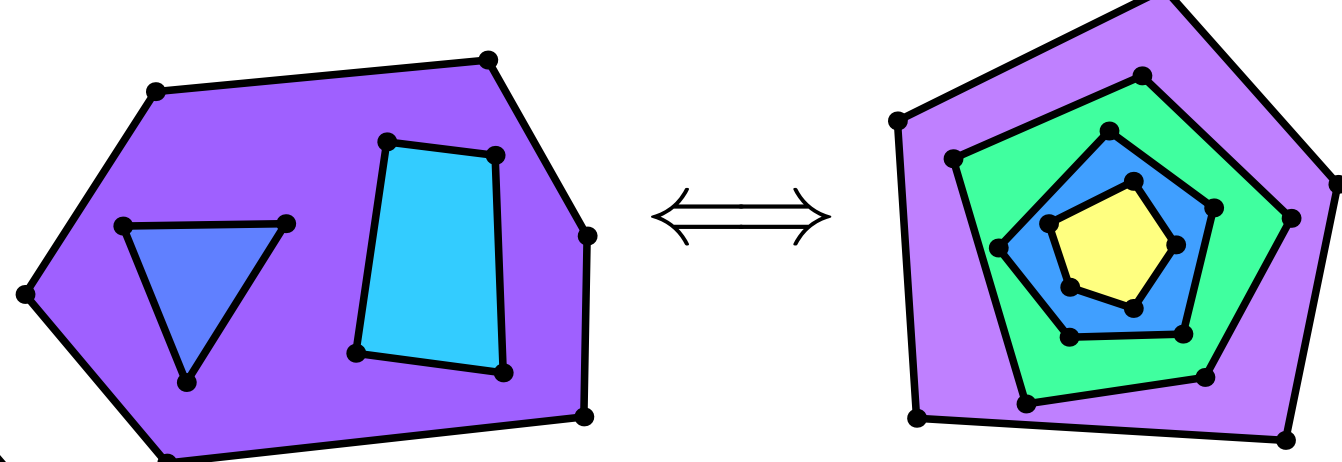
### SUBSETS



### TPUL

### THEOREM

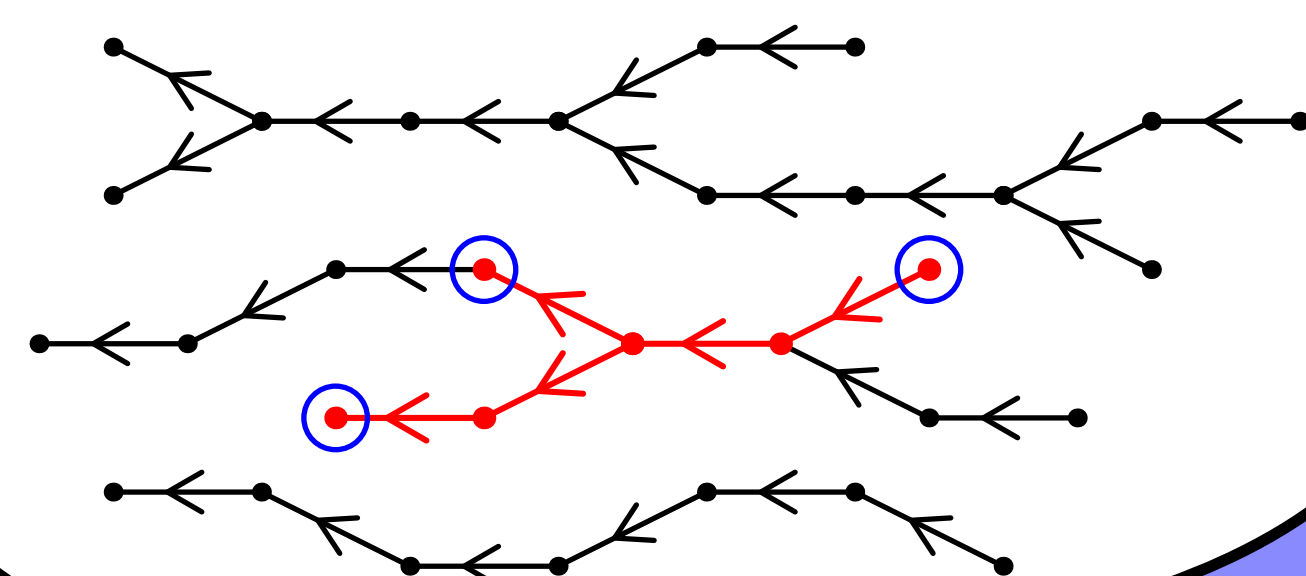
TPUL is equivalent to:



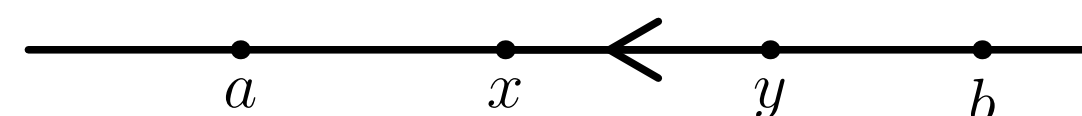
## INDUCING STRUCTURE

### ORDER

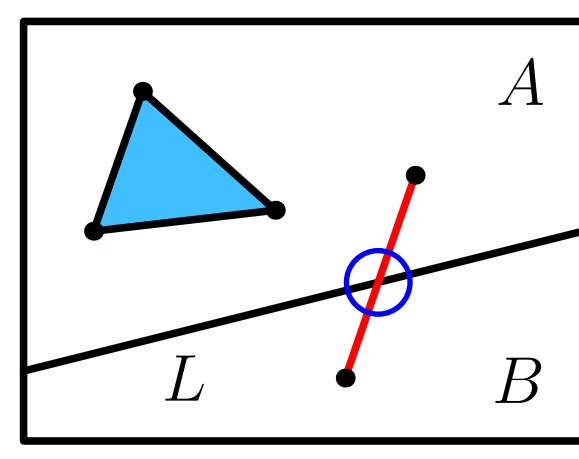
**Theorem:** all order convexities are free.



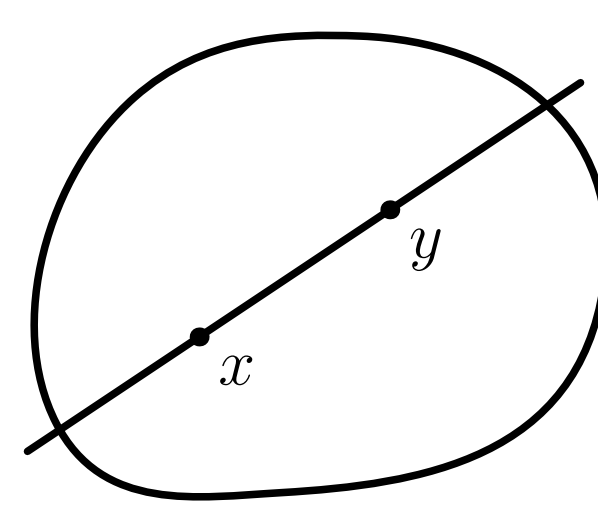
### LINEARITY



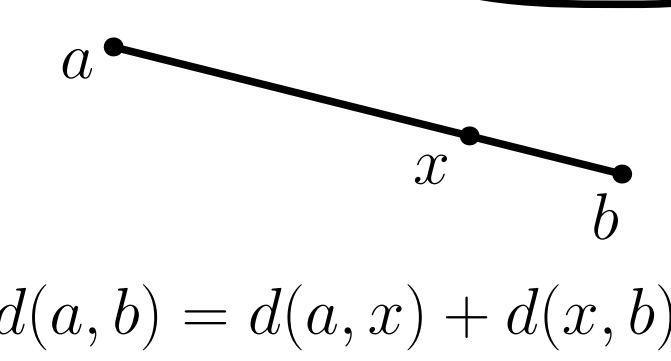
### n-AFFINITY



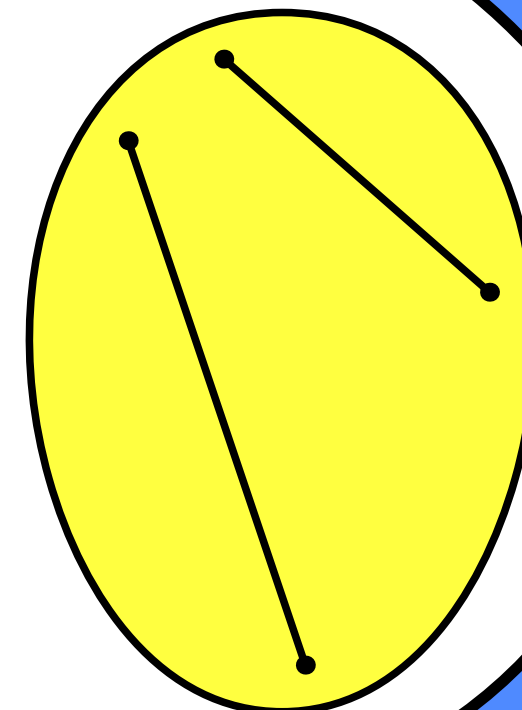
### 1-AFFINITY



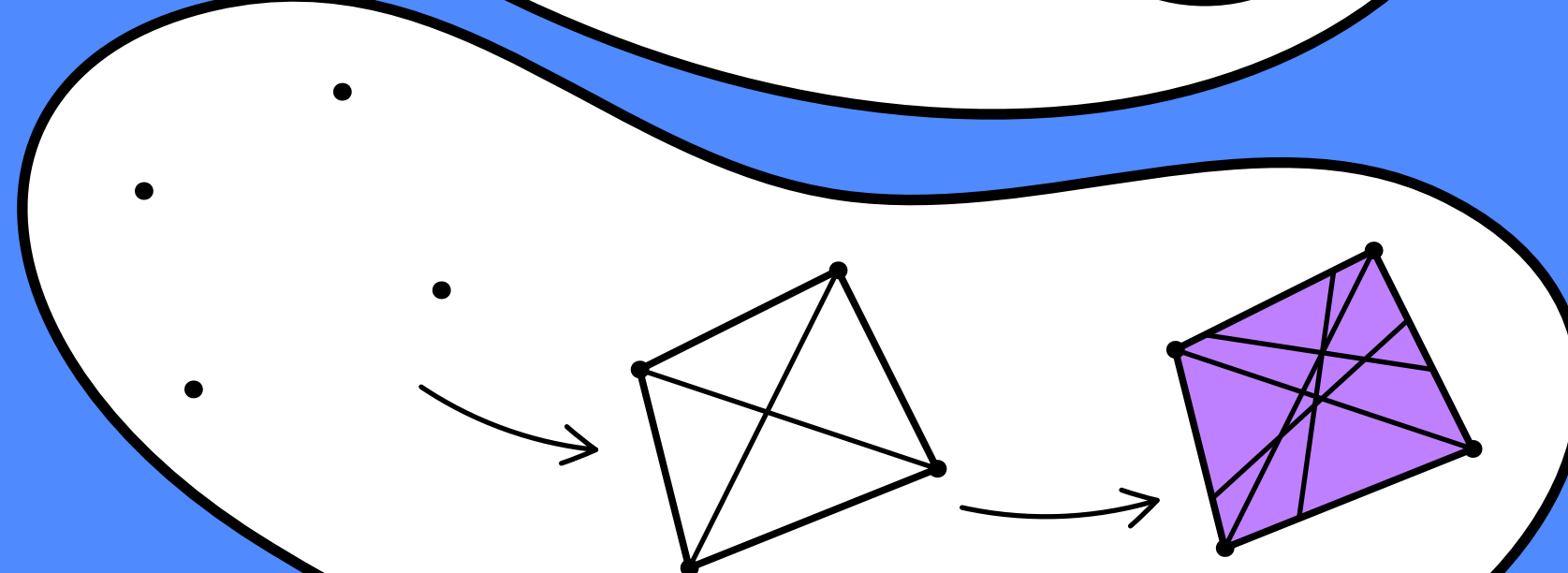
### METRIC



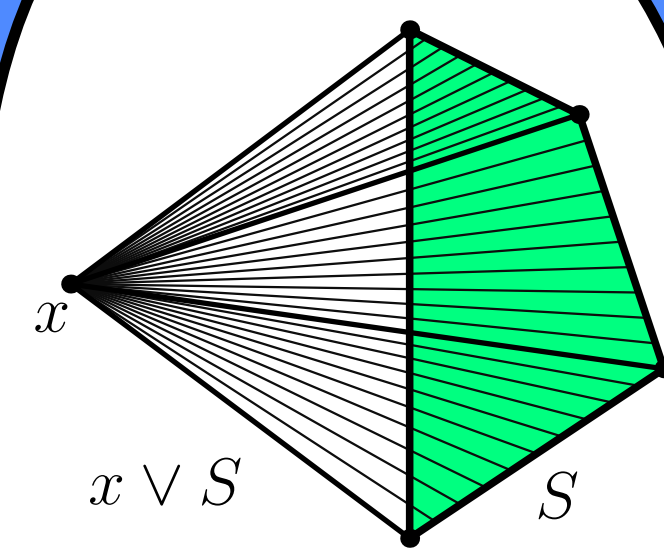
### SEGMENTIAL



### FINITE



### JOIN



Join-commutative:  
 $x \vee \langle F \rangle = \langle x \cup F \rangle$

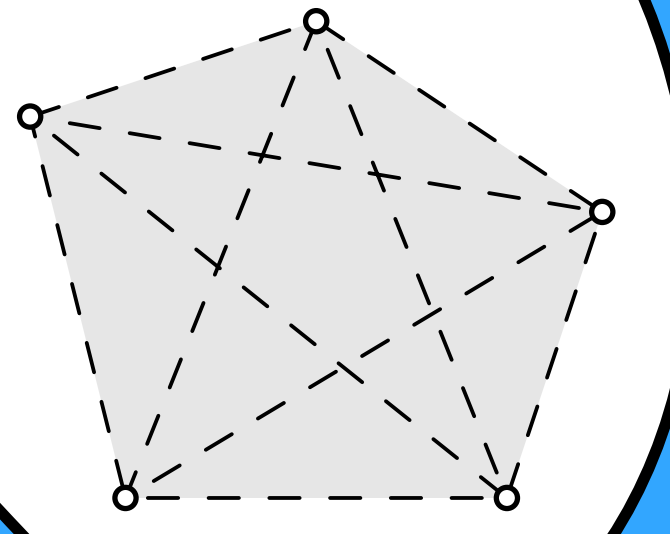
### THEOREM

Finite-segmential  
2-Affine  
TPUL  
 $\Downarrow$   
Free

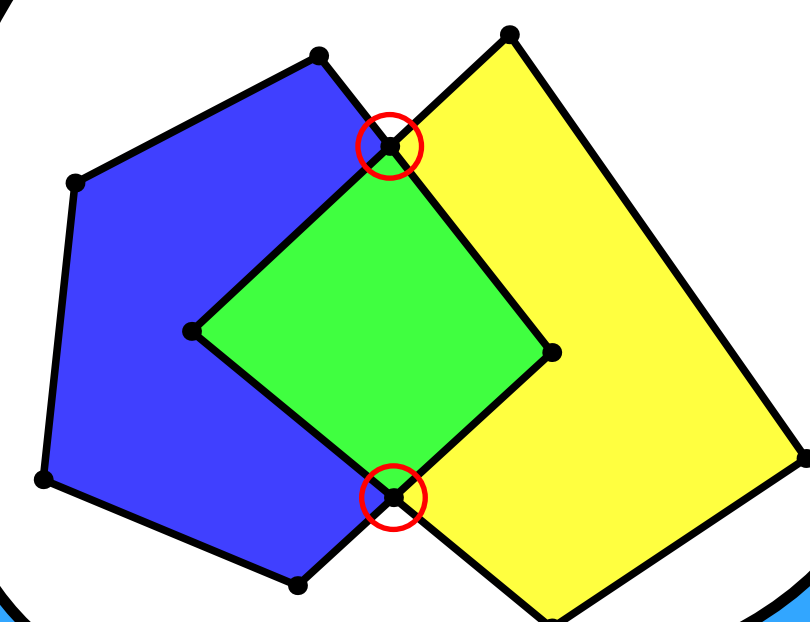
## INDUCED STRUCTURE

### TOPOLOGY

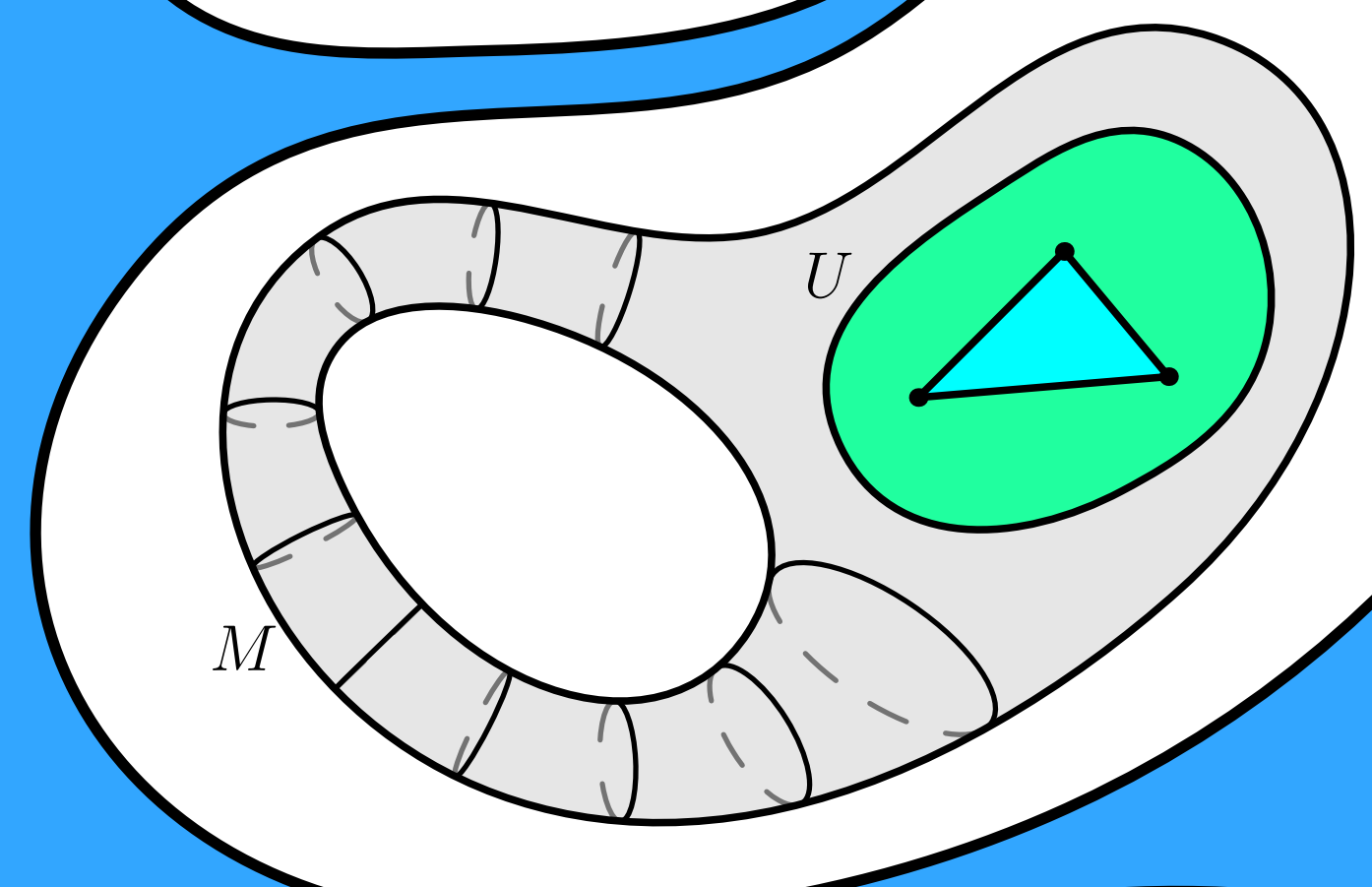
- (1) The Polytope Intersection Lemma
- (2) The Polytope Union Lemma
- (3) Finite dimension
- (4) Freedom



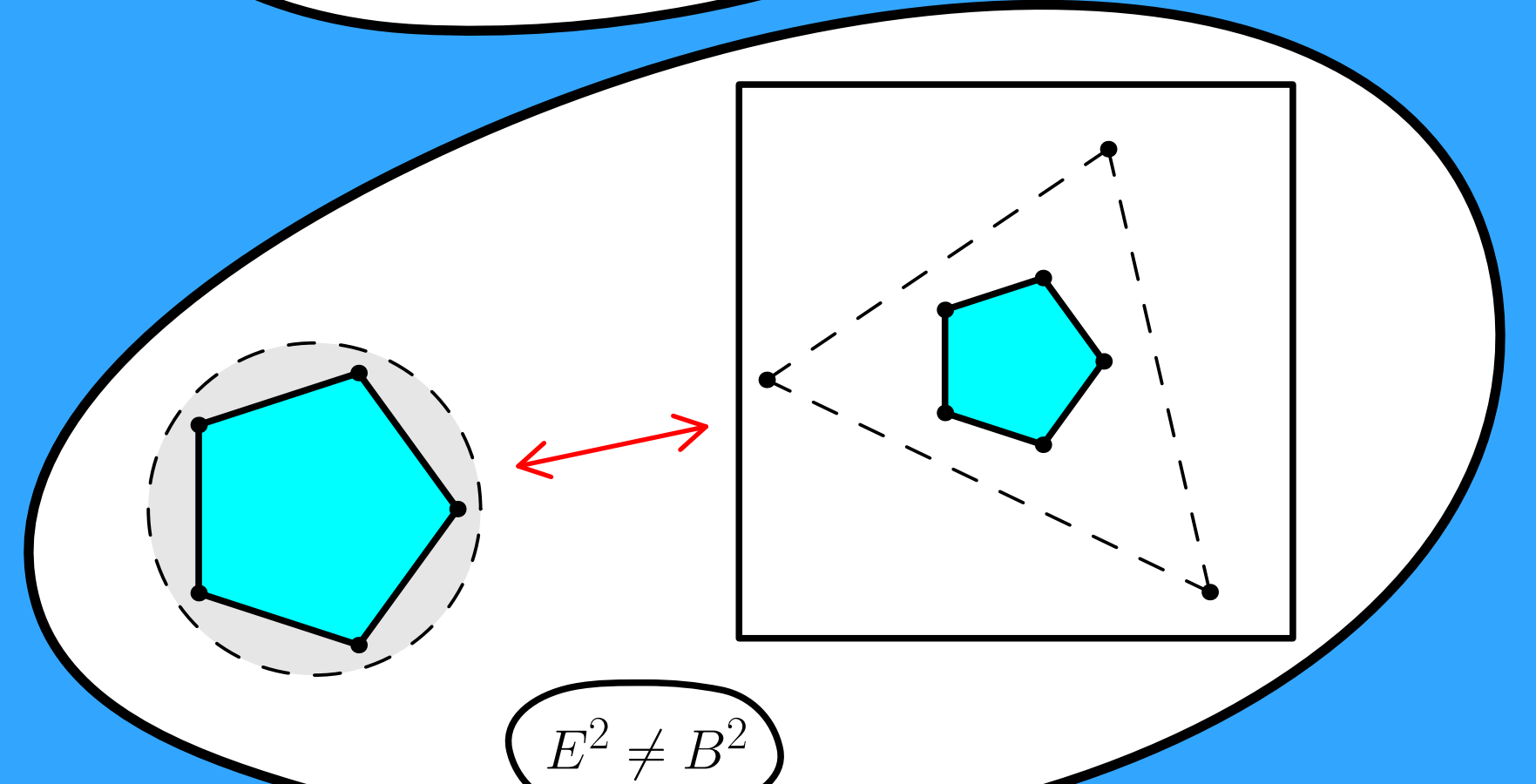
### TPIL



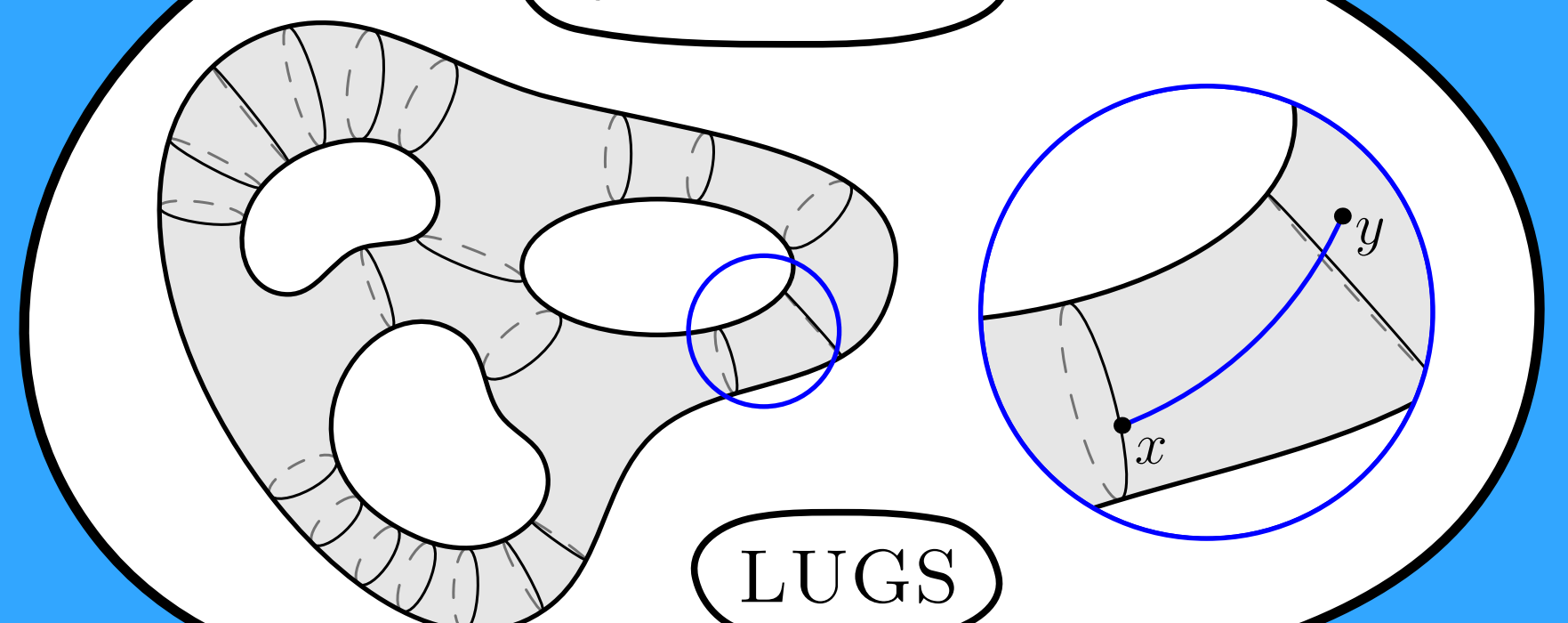
### LOCAL CONVEXITY



### $E^2 \neq B^2$



### RIEMANNIAN



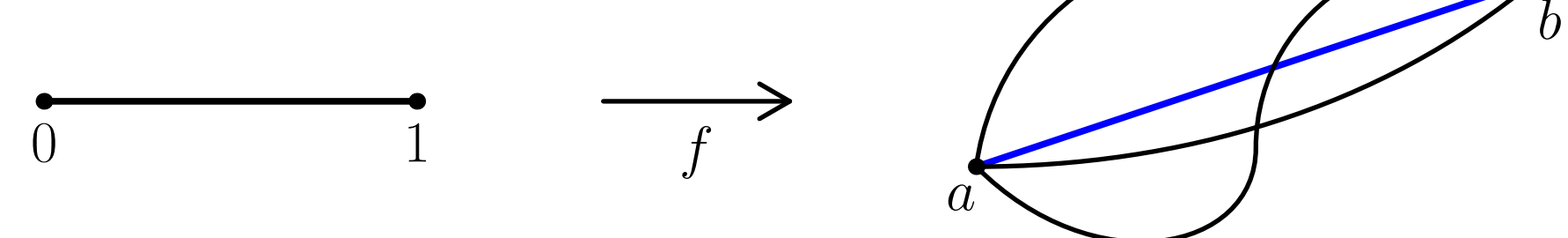
### LUGS

## REFERENCES

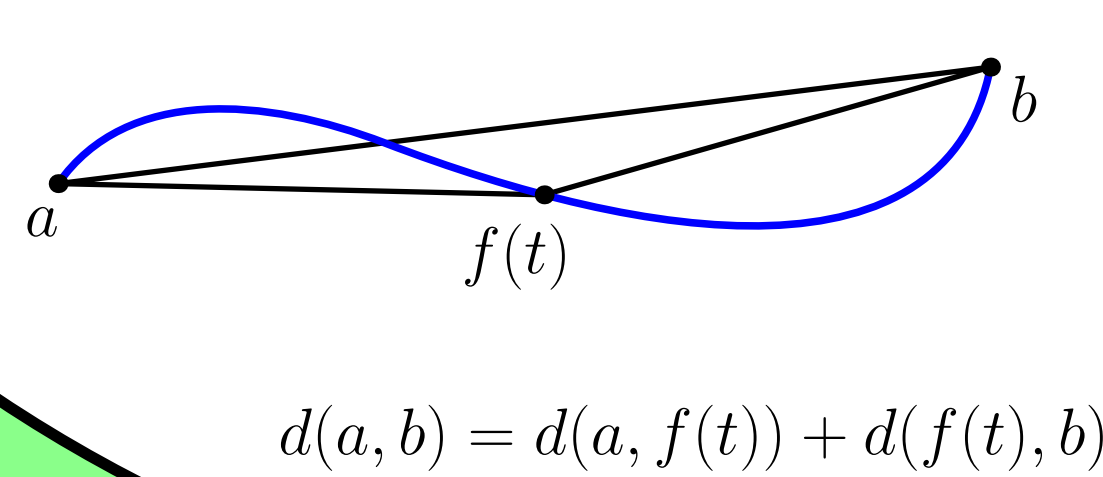
- (1) M.L.J. van de Vel, *Theory of convex structures*, North-Holland mathematical library, 1993;
- (2) D. C. Kay, E. W. Womble, *Axiomatic convexity theory*, Pacific Journal of Mathematics, 1971;
- (3) D. Gromoll, W. Klingenberg and W. Meyer, *Riemannsche Geometrie im Grossen*, Springer-Verlag, Berlin, 1968.

## UNIQUELY GEODESIC SPACES

### DEFINITION



### LEMMA



### LEMMA

In a UGS all metric segments are free polytopes.

