

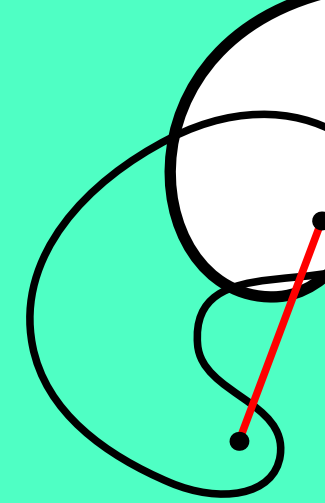
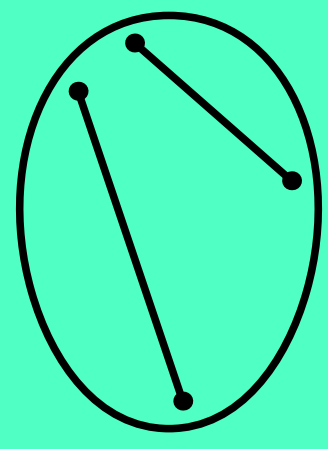
# THE STRUCTURE OF CONVEXITY

## INTERNAL THEORY

### DEFINITION

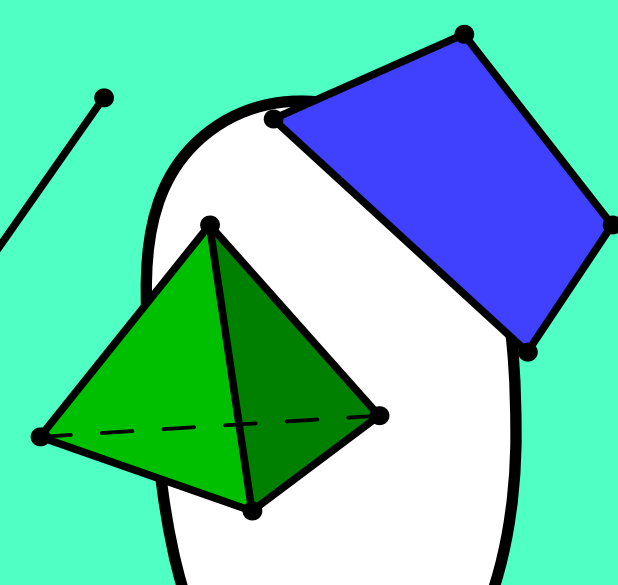
$(X, \mathcal{C})$  — **convex space**:

- (1)  $\emptyset, X \in \mathcal{C}$
- (2)  $A \subset \mathcal{C} \Rightarrow \cap A \in \mathcal{C}$
- (3)  $\mathcal{N} \subset \mathcal{C} \Rightarrow \cup \mathcal{N} \in \mathcal{C}$

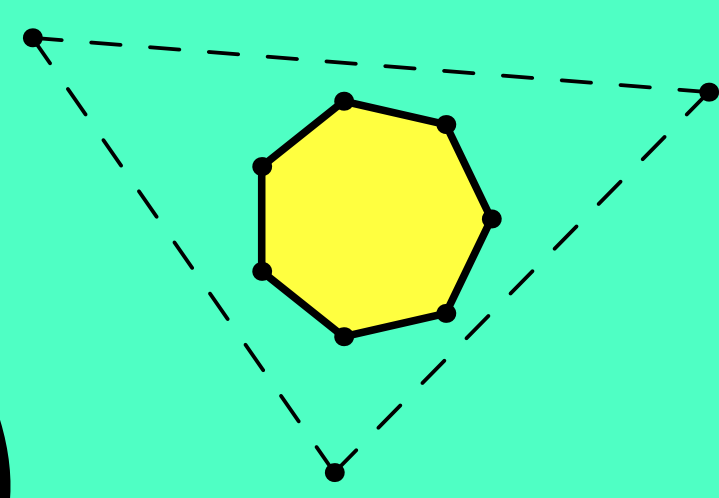


### IDEA

### POLYTOPE



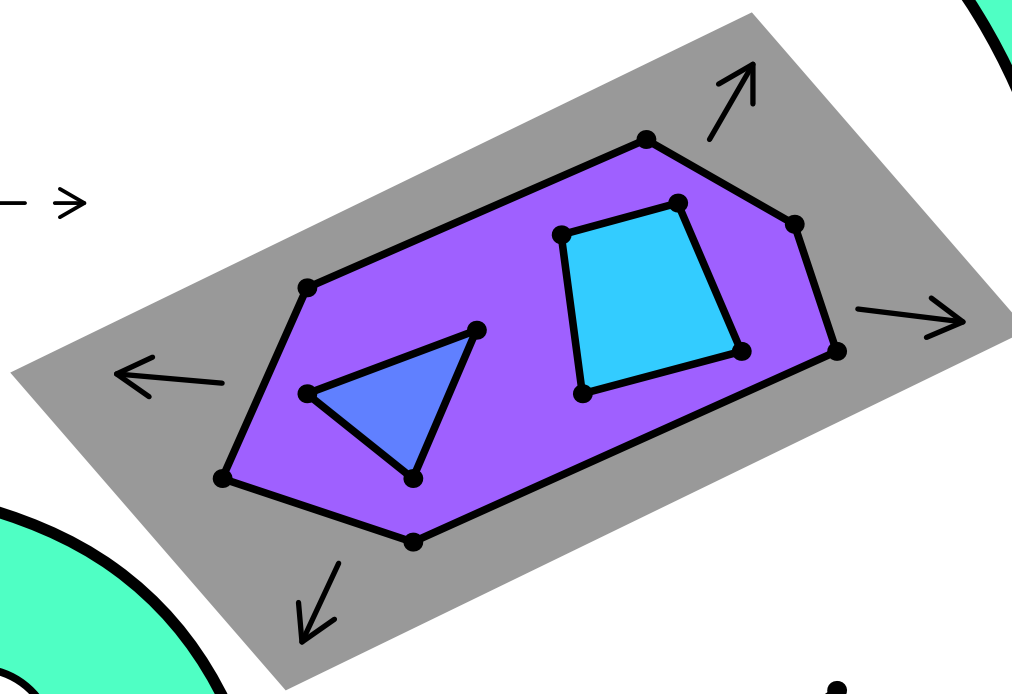
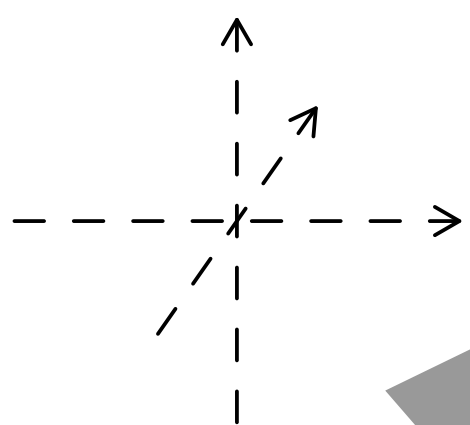
### DIMENSION



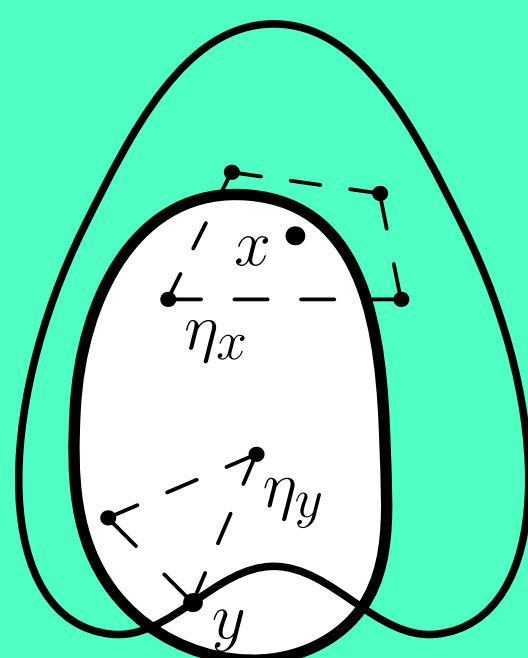
### FREEDOM

### HYPERPLANE

(Maximal net of polytopes of same dim.)

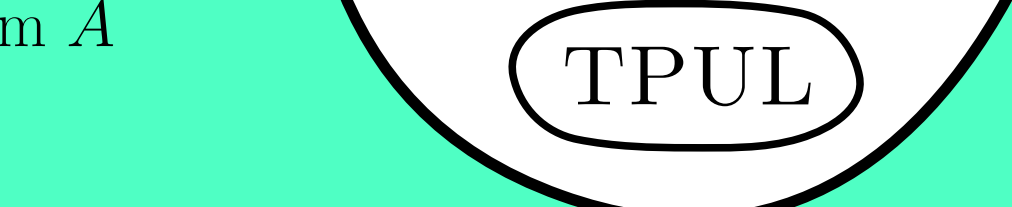


### SUBSETS



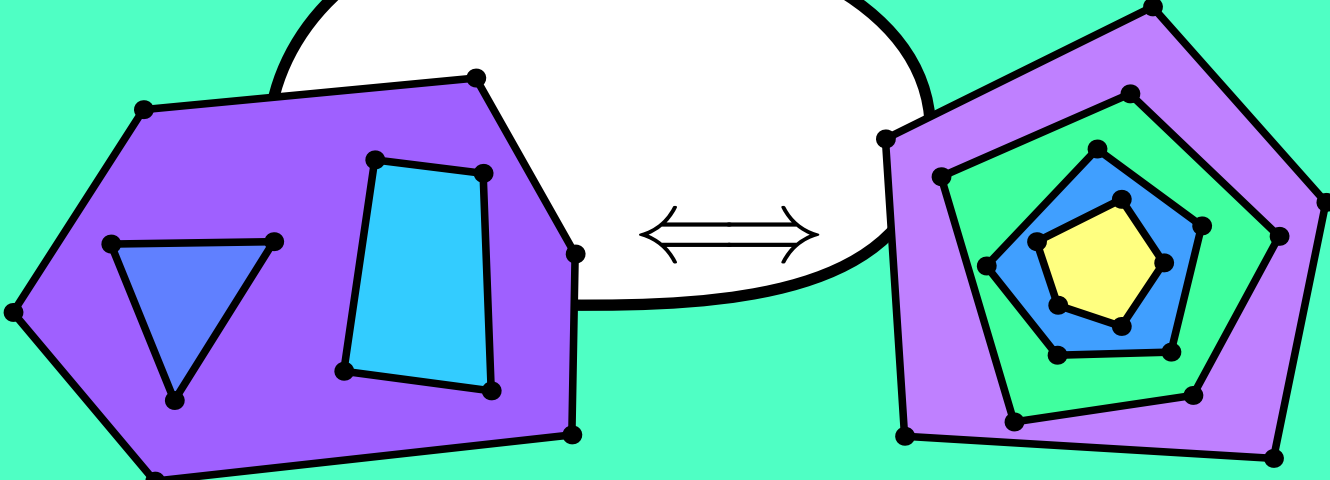
$$\eta_x = \eta_y = \dim A$$

### TPUL



### THEOREM

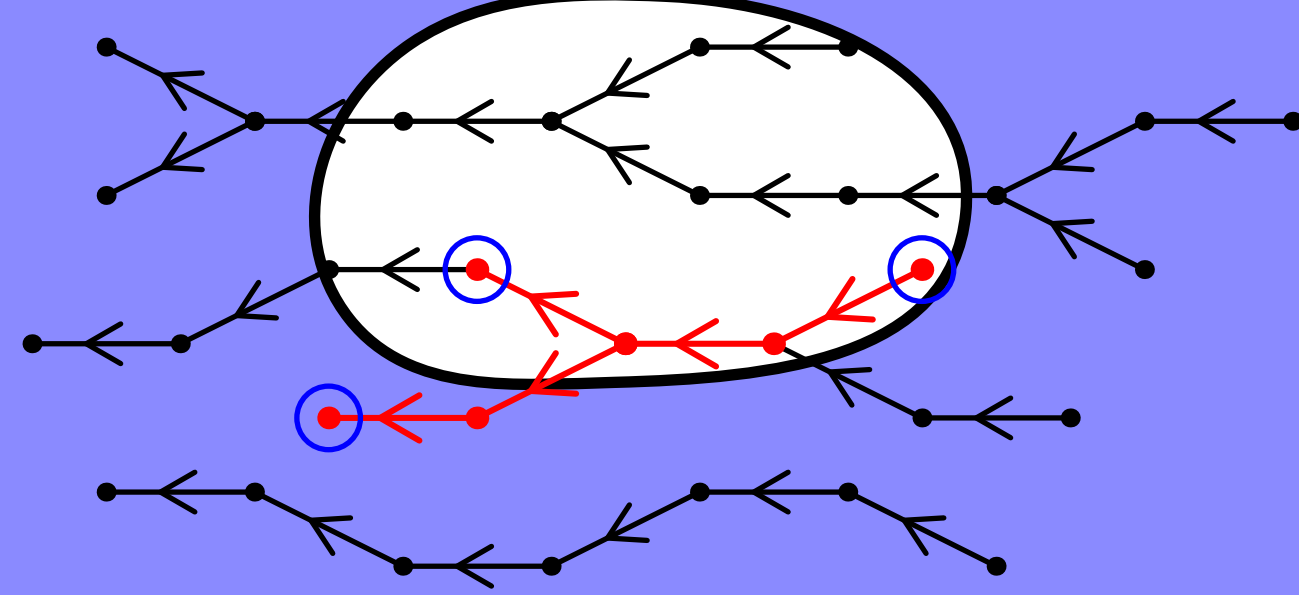
TPUL is equivalent to:



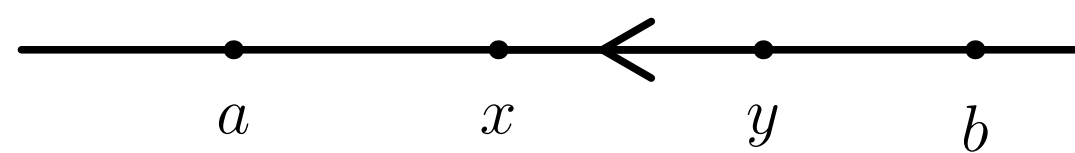
## INDUCING STRUCTURE

### ORDER

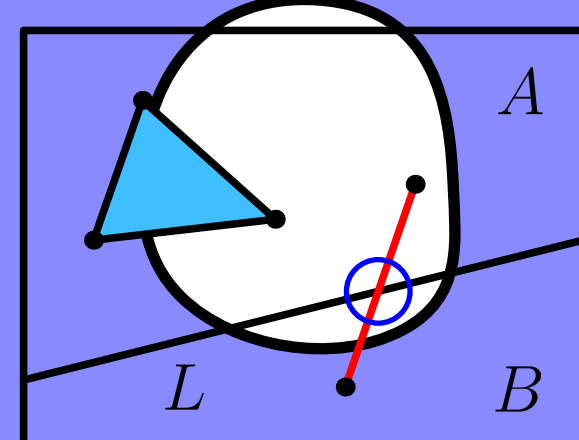
Theorem: all order convexities are free.



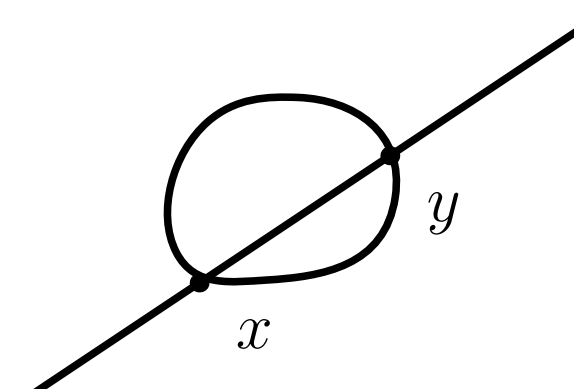
### LINEARITY



### n-AFFINITY



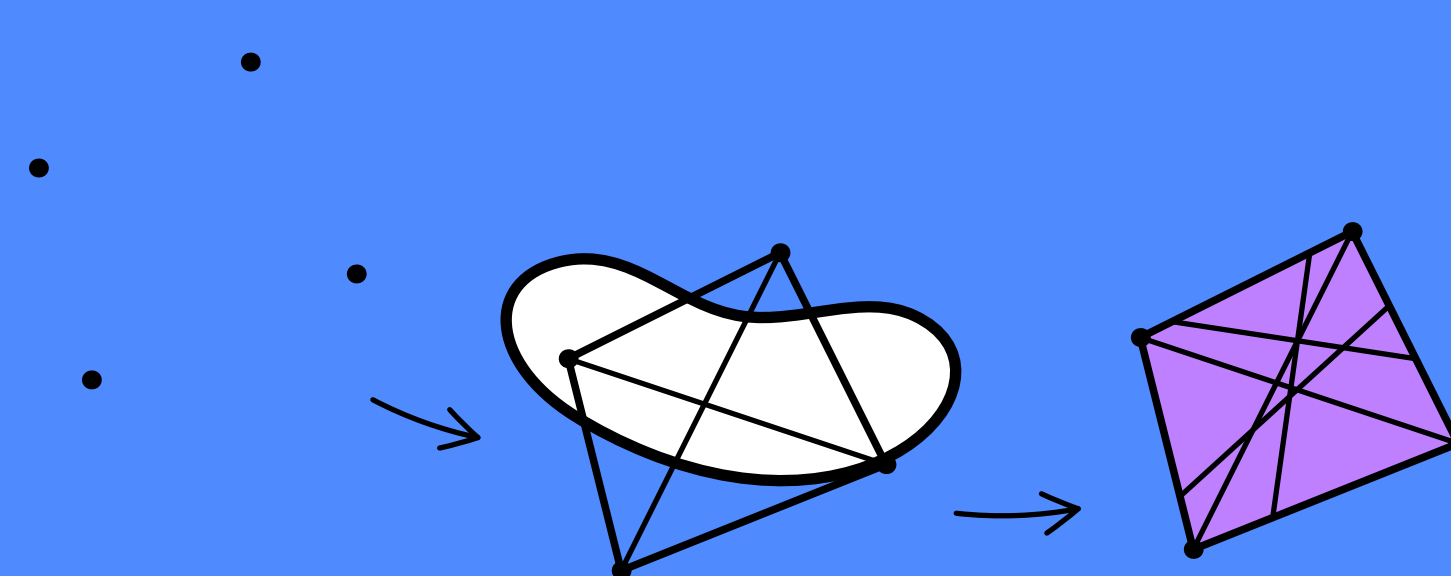
### 1-AFFINITY



### METRIC

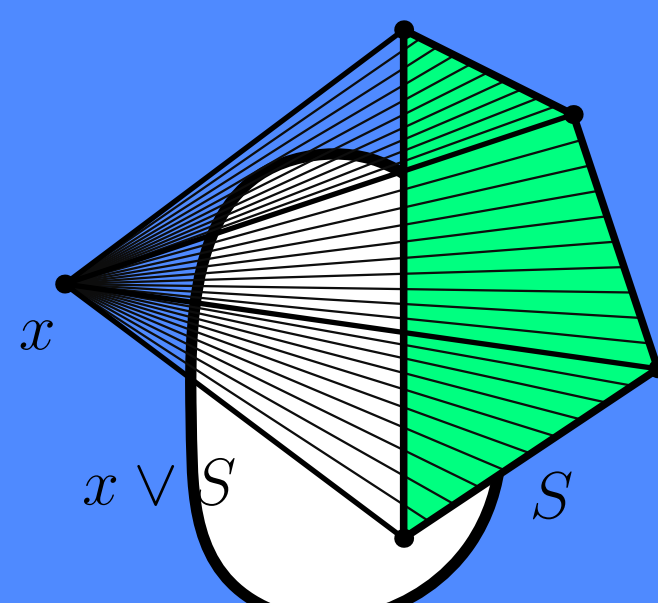
$$d(a, b) = d(a, x) + d(x, b)$$

### SEGMENTIAL



### FINITE

### JOIN



Join-commutative:  
 $x \vee \langle F \rangle = \langle x \cup F \rangle$

### THEOREM

Finite-segmental

2-Affine

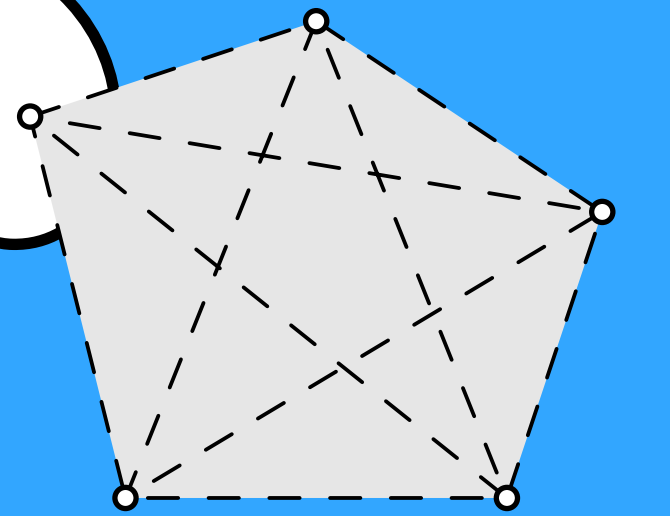
TPUL

Free

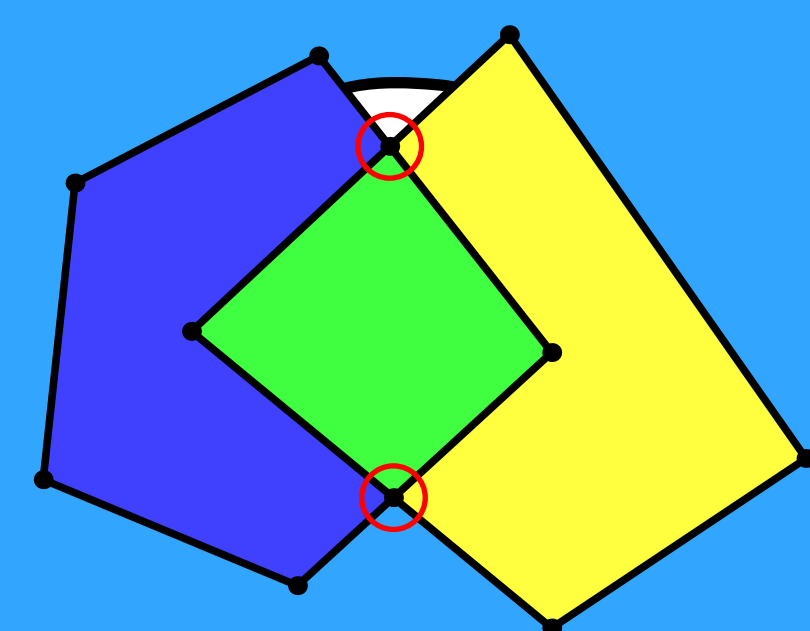
## INDUCED STRUCTURE

### TOPOLOGY

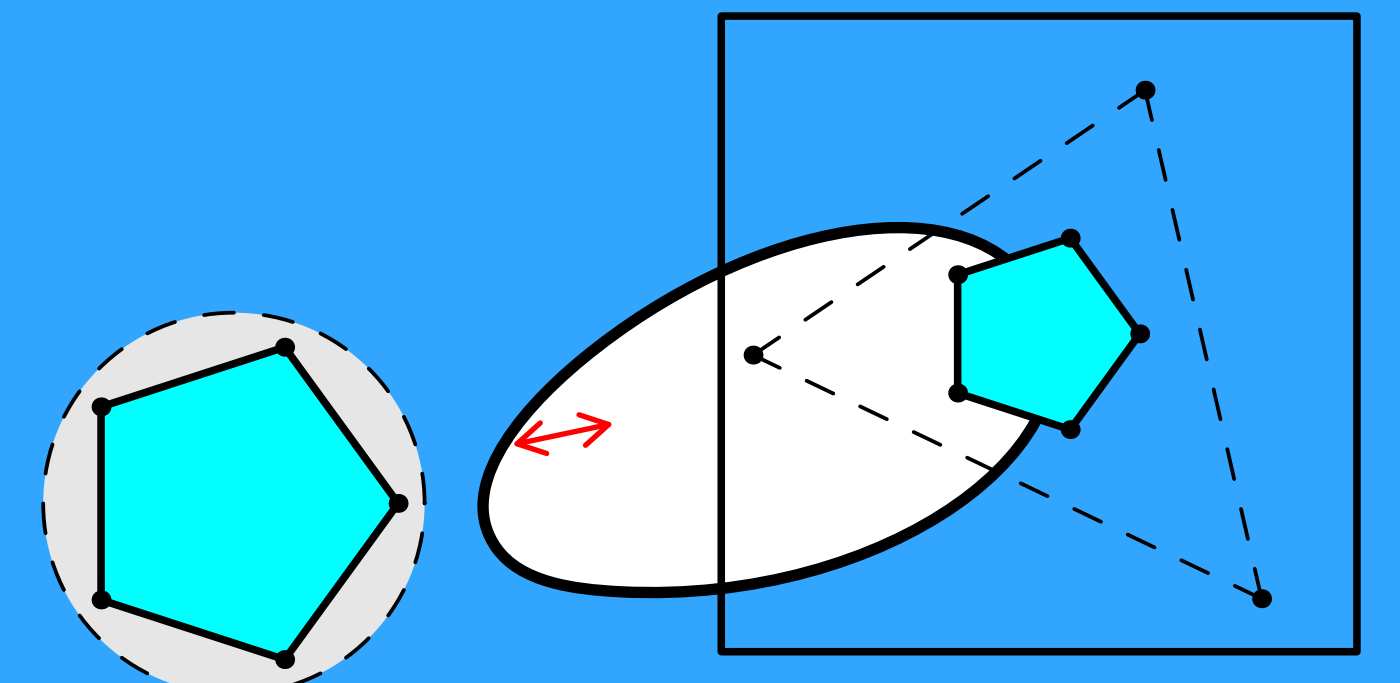
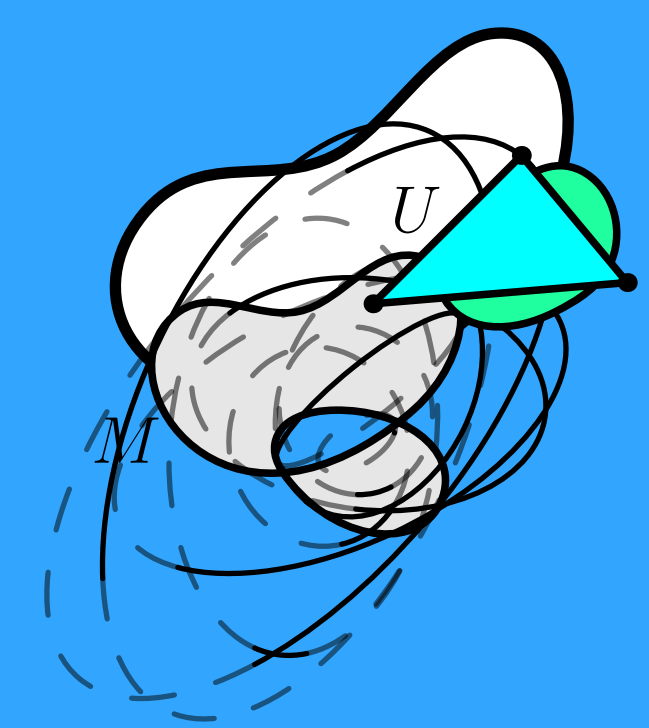
- (1) The Polytope Intersection Lemma
- (2) The Polytope Union Lemma
- (3) Finite dimension
- (4) Freedom



### TPIL

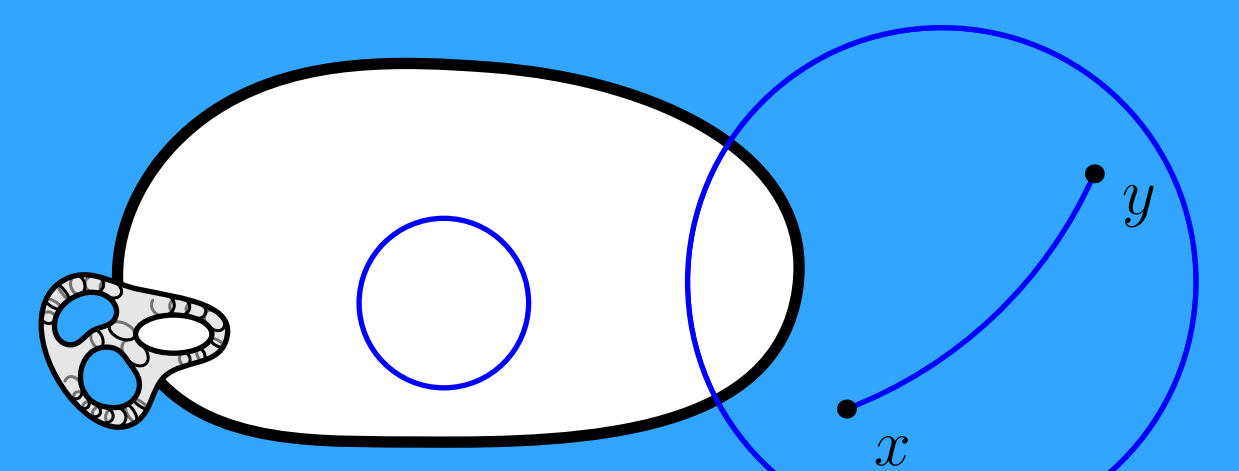


### LOCAL CONVEXITY



$$E^2 \neq B^2$$

### RIEMANNIAN



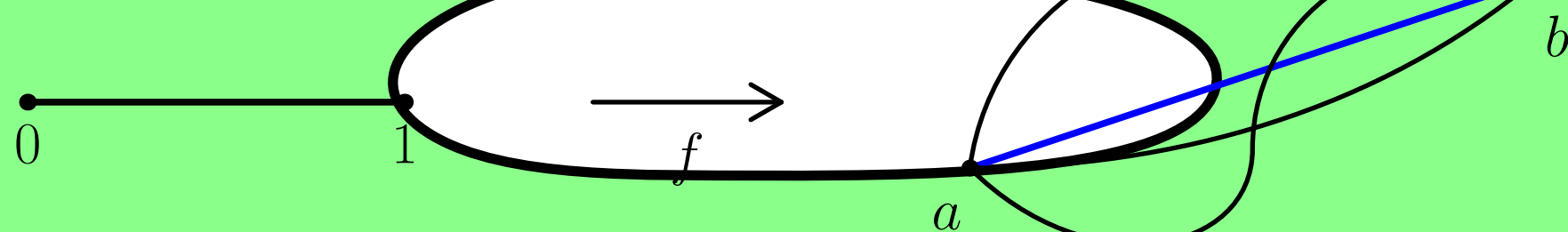
### LUGS

## REFERENCES

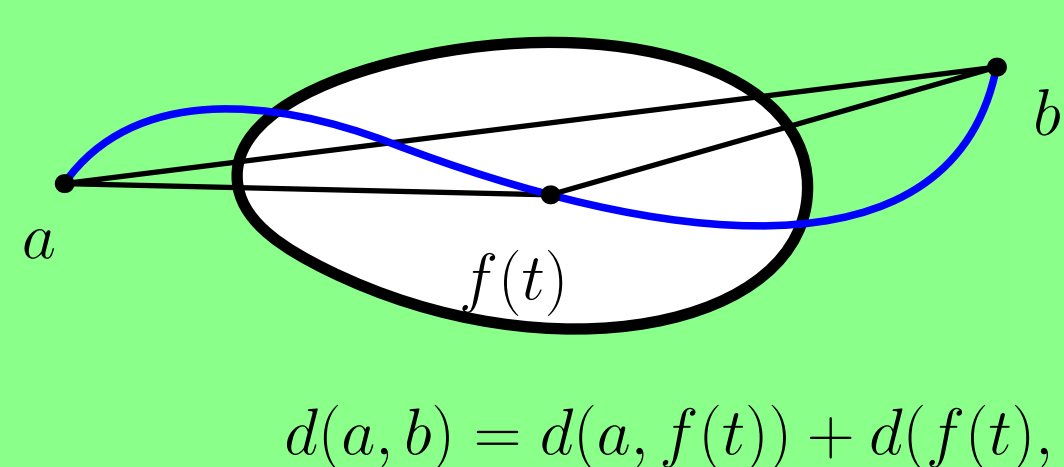
- (1) M.L.J. van de Vel, *Theory of convex structures*, North-Holland mathematical library, 1993;
- (2) D. C. Kay, E. W. Womble, *Axiomatic convexity theory*, Pacific Journal of Mathematics, 1971;
- (3) D. Gromoll, W. Klingenberg and W. Meyer, *Riemannsche Geometrie im Grossen*, Springer-Verlag, Berlin, 1968.

## UNIQUELY GEODESIC SPACES

### DEFINITION



### LEMMA



$$d(a, b) = d(a, f(t)) + d(f(t), b)$$

### LEMMA

In a UGS all metric segments are free polytopes.

