THE STRUCTURE OF CONVEXITY
INTERNAL THEORY
DEFINITION
$(X, \mathcal{C})$ — convex space:
(1) $\emptyset, X \in \mathcal{C}$
(2) $\mathcal{A} \subset \mathcal{C} \Rightarrow \cap\mathcal{A} \in \mathcal{C}$
(3) $\mathcal{N} \subset \mathcal{C} \Rightarrow \cup\mathcal{N} \in \mathcal{C}$
IDEA
POLYTOPE
DIMENSION
FREEDOM
HYPERPLANE
(Maximal net of polytopes of same dim.)
SUBSETS
η_x
η_y
$\eta_x = \eta_y = \dim A$
TPUL
THEOREM
TPUL is equivalent to:
$\Longleftrightarrow$
INDUCING STRUCTURE
ORDER
Theorem: all order convexities are free.
LINEARITY
a x y b
n-AFFINITY
A L B
1-AFFINITY
x y
METRIC
$d(a, b) = d(a, x) + d(x, b)$
SEGMENTIAL
FINITE
JOIN
x $x \vee S$ S
Join-commutative:
$x \vee \langle F \rangle = \langle x \cup F \rangle$
THEOREM
Finite-segmential
2-Affine
TPUL
$\Downarrow$
Free
INDUCED STRUCTURE
TOPOLOGY
(1) The Polytope Intersection Lemma
(2) The Polytope Union Lemma
(3) Finite dimension
(4) Freedom
TPIL
LOCAL CONVEXITY
U
$E^2 \neq B^2$
RIEMANNIAN
x y
LUGS
REFERENCES
(1) M.L.J. van de Vel, Theory of convex structures, North-Holland mathematical library, 1993;
(2) D. C. Kay, E. W. Womble, Axiomatic convexity theory, Pacific Journal of Mathematics, 1971;
(3) D. Gromoll, W. Klingenberg and W. Meyer, Riemannsche Geometrie im Grossen, Springer-Verlag, Berlin, 1968.
UNIQUELY GEODESIC SPACES
DEFINITION
0 1 f a b
LEMMA
a b $f(t)$
$d(a, b) = d(a, f(t)) + d(f(t), b)$
LEMMA
In a UGS all metric segments are free polytopes.
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