The Structure of Convexity

Internal Theory

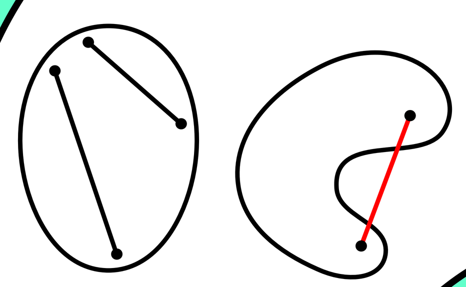
Definition

$(X, \mathcal{C})$ — ***convex space:***

(1) $\emptyset,\ X \in \mathcal{C}$

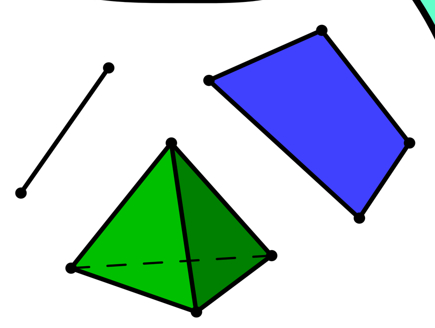
(2) $\mathcal{A} \subset \mathcal{C} \Rightarrow \cap\mathcal{A} \in \mathcal{C}$

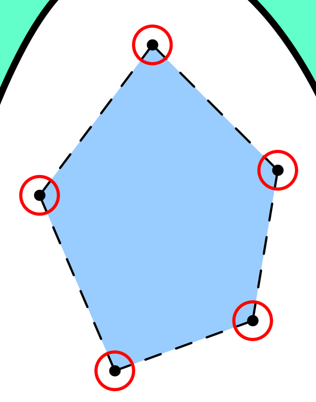
(3) $\mathcal{N} \subset \mathcal{C} \Rightarrow \cup\mathcal{N} \in \mathcal{C}$



Idea

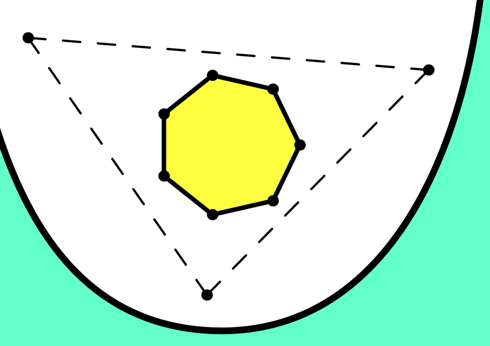
Polytope





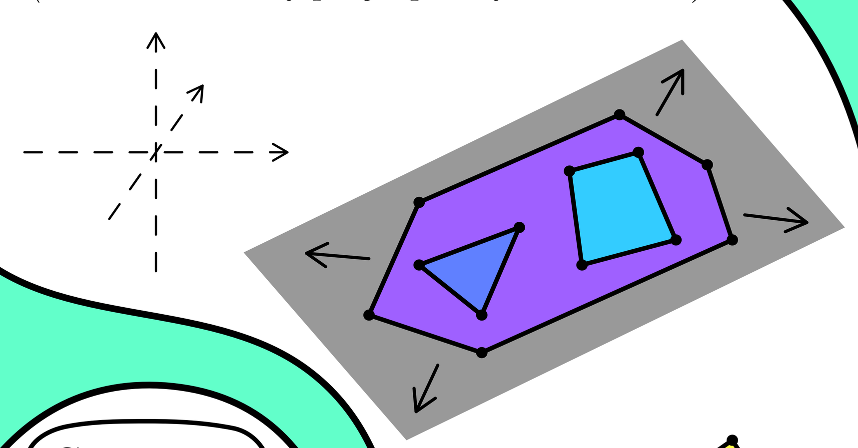
Freedom

Dimension

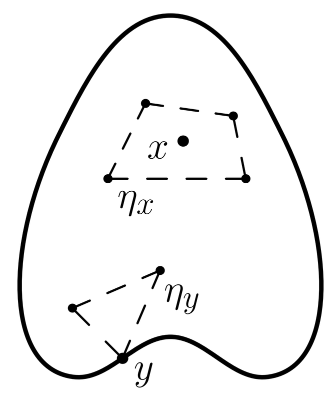


Hyperplane

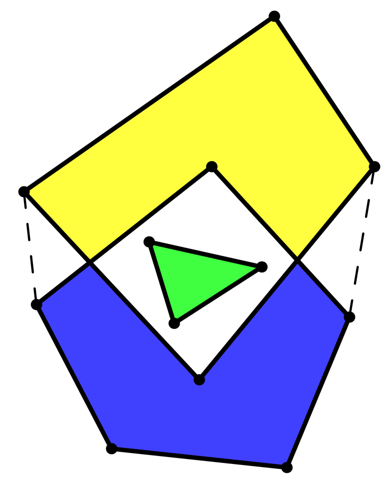
(Maximal net of polytopes of same dim.)



Subsets



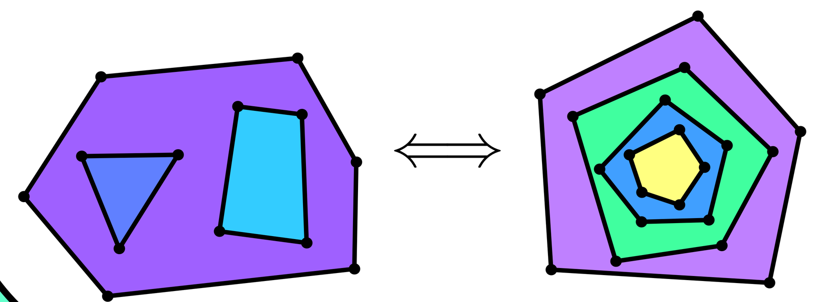
$\eta_x = \eta_y = \dim A$



TPUL

Theorem

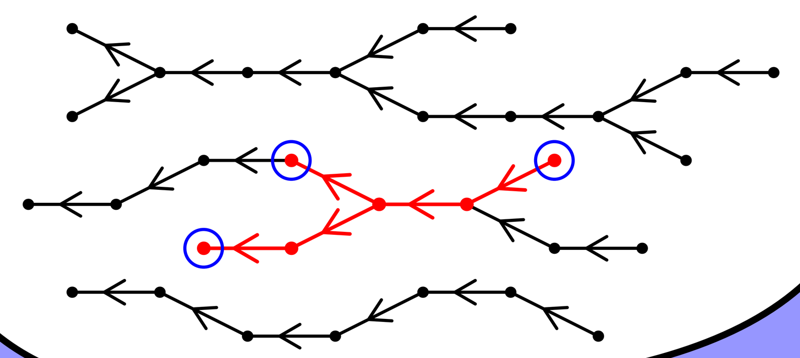
TPUL is equivalent to:



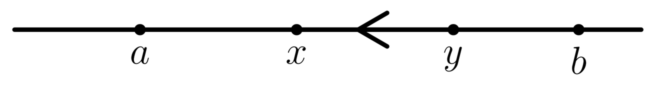
Inducing Structure

Order

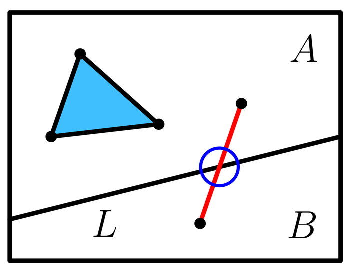
Theorem: *all order convexities are free.*

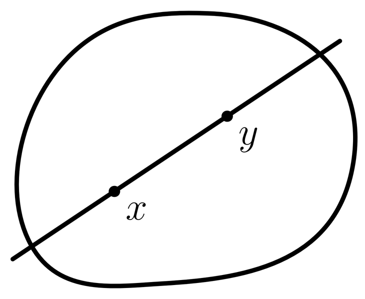


Linearity



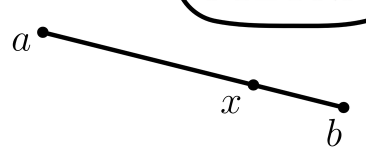
n-Affinity





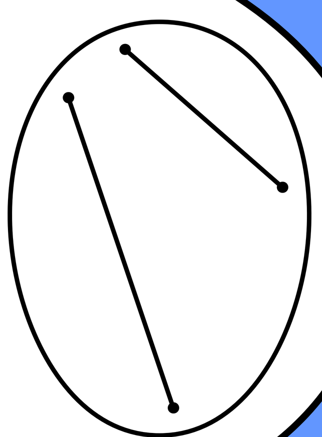
1-Affinity

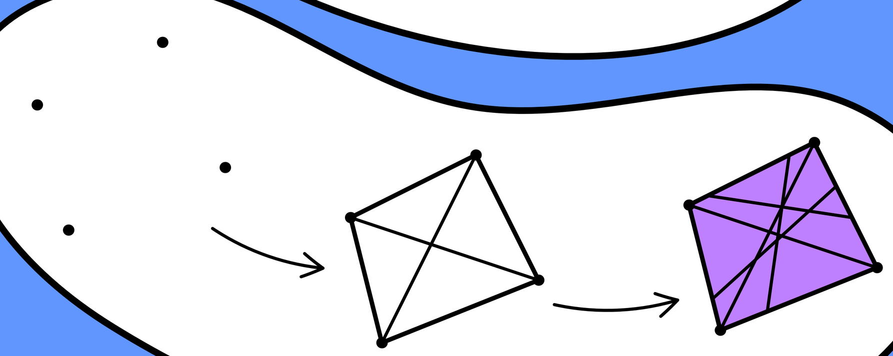
Metric



$d(a, b) = d(a, x) + d(x, b)$

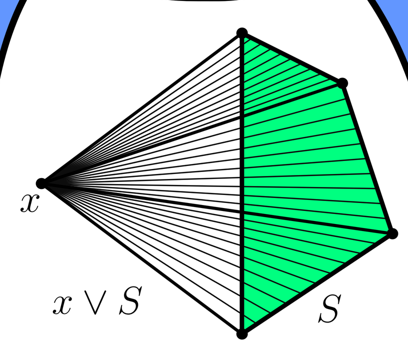
Segmential





Finite

Join



Join-commutative:

$x \vee \langle F \rangle = \langle x \cup F \rangle$

Theorem

Finite-segmential

2-Affine

TPUL

$\Downarrow$

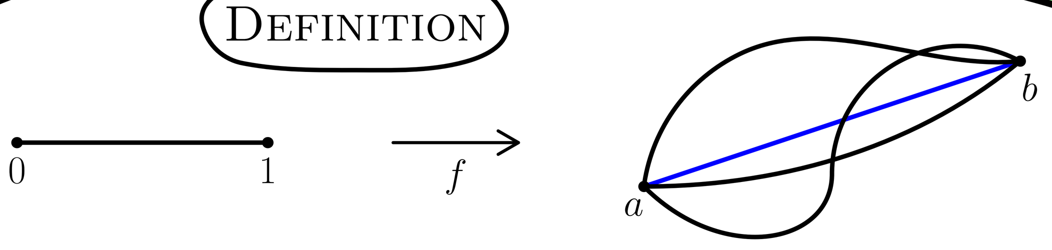
Free

Induced Structure

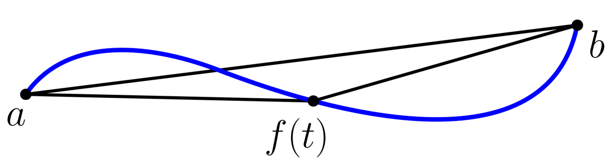
References

Uniquely Geodesic Spaces

Definition



Lemma



$d(a, b) = d(a, f(t)) + d(f(t), b)$

Lemma

In a UGS all metric segments are free polytopes.