

THE STRUCTURE OF CONVEXITY

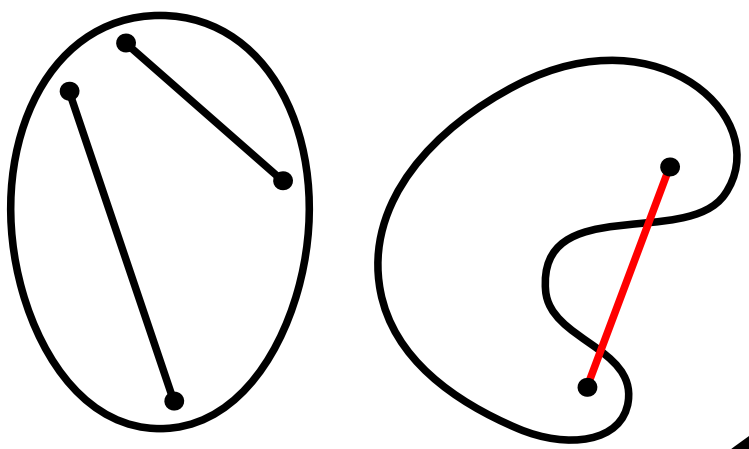
INTERNAL THEORY

DEFINITION

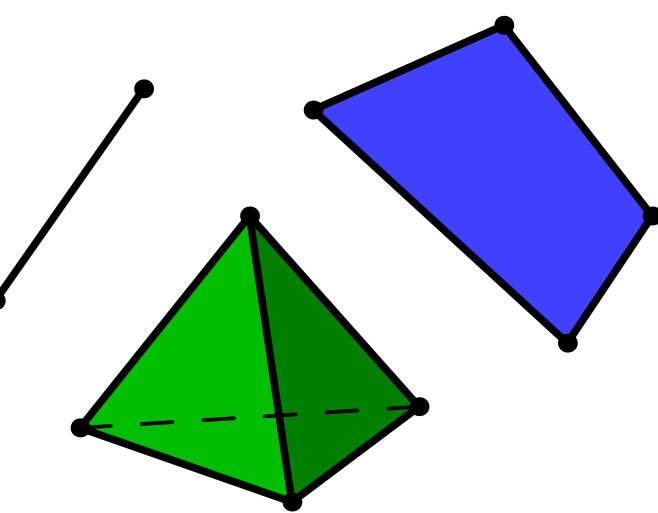
(X, \mathcal{C}) — **convex space**:

- (1) $\emptyset, X \in \mathcal{C}$
- (2) $A \subset \mathcal{C} \Rightarrow \cap A \in \mathcal{C}$
- (3) $\mathcal{N} \subset \mathcal{C} \Rightarrow \cup \mathcal{N} \in \mathcal{C}$

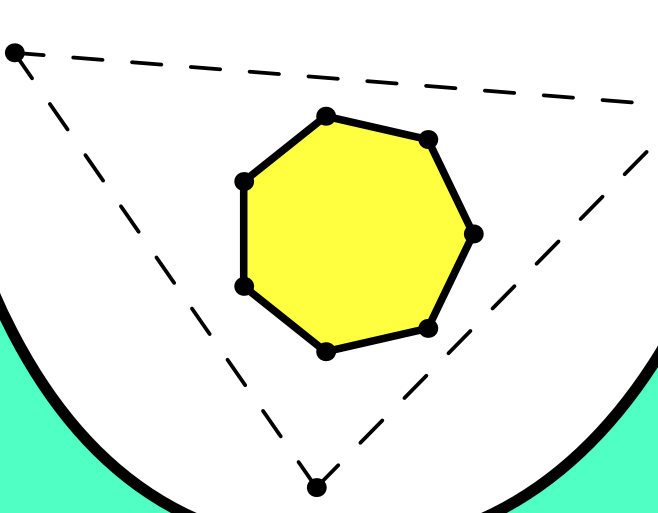
IDEA



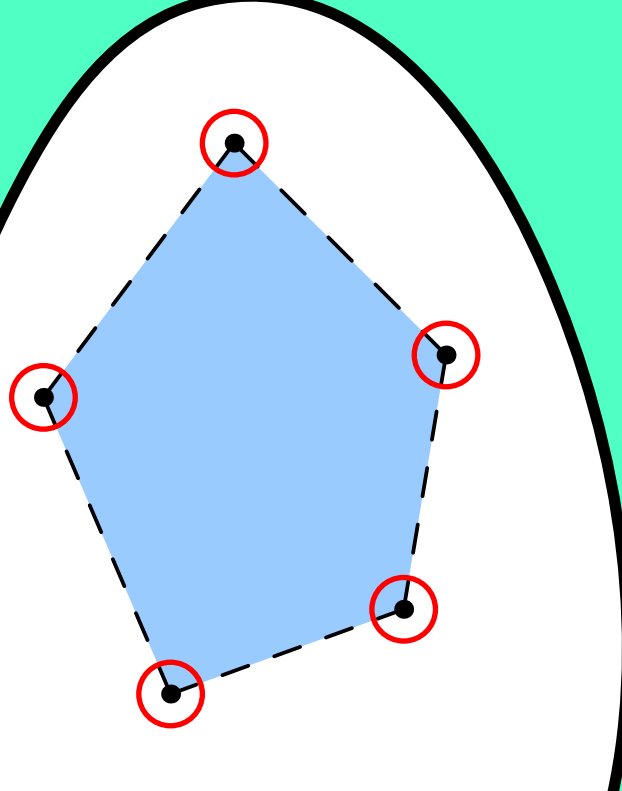
POLYTOPE



DIMENSION

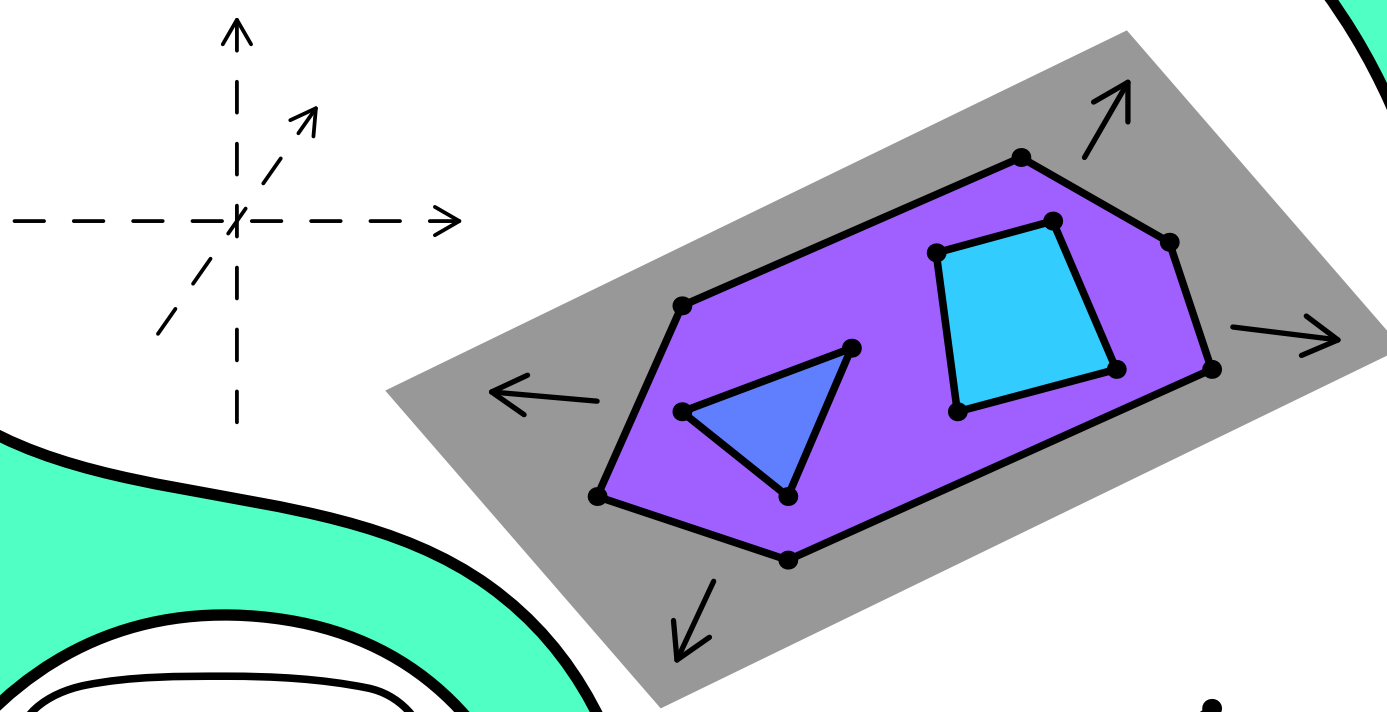


FREEDOM

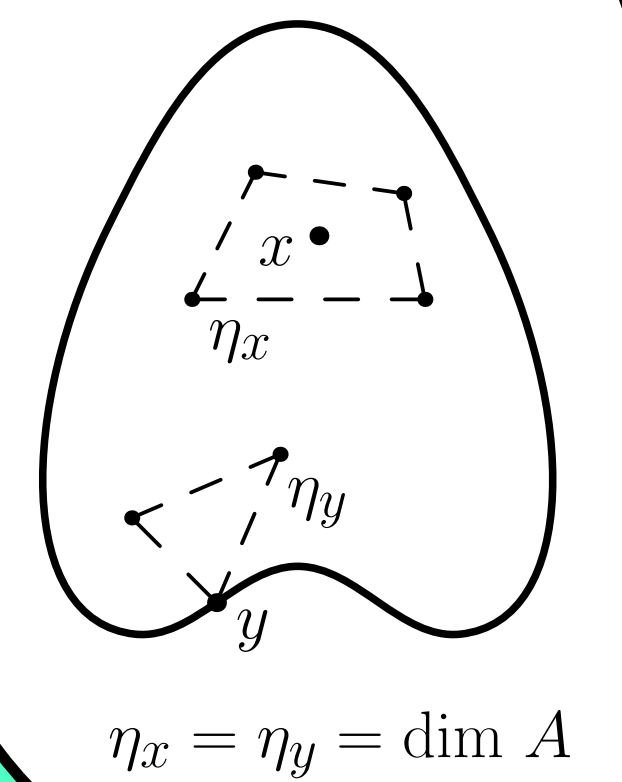


HYPERPLANE

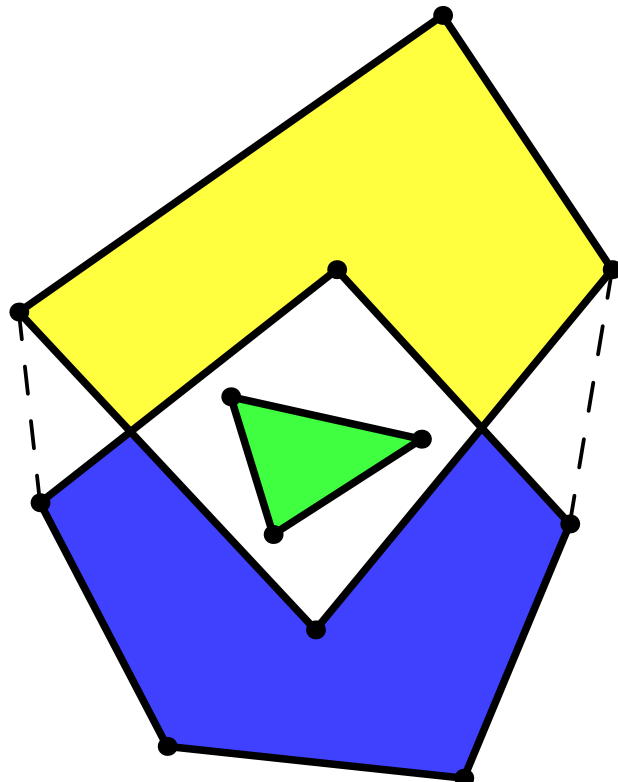
(Maximal net of polytopes of same dim.)



SUBSETS

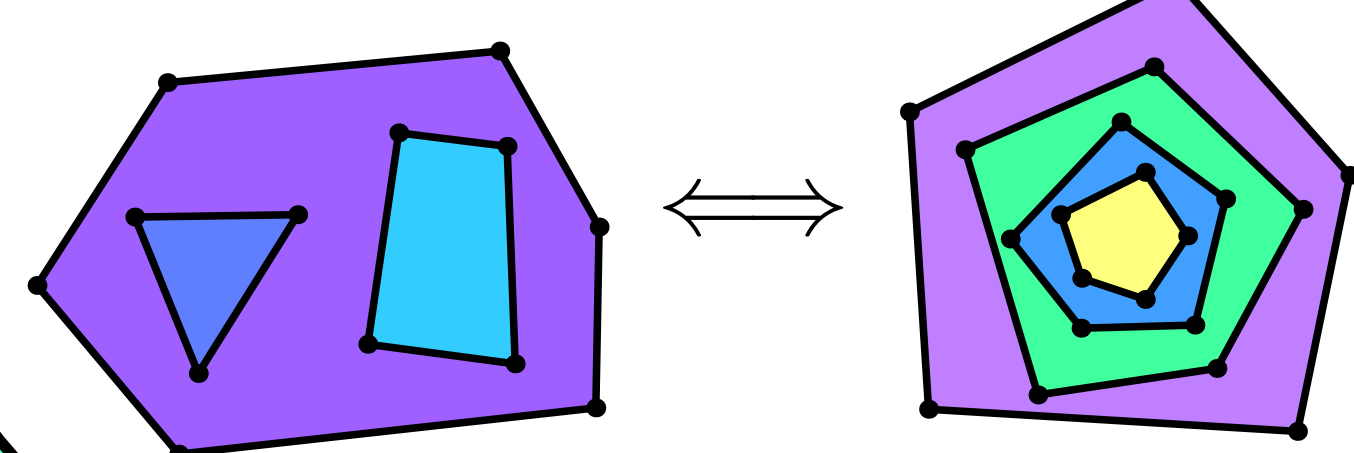


TPUL



THEOREM

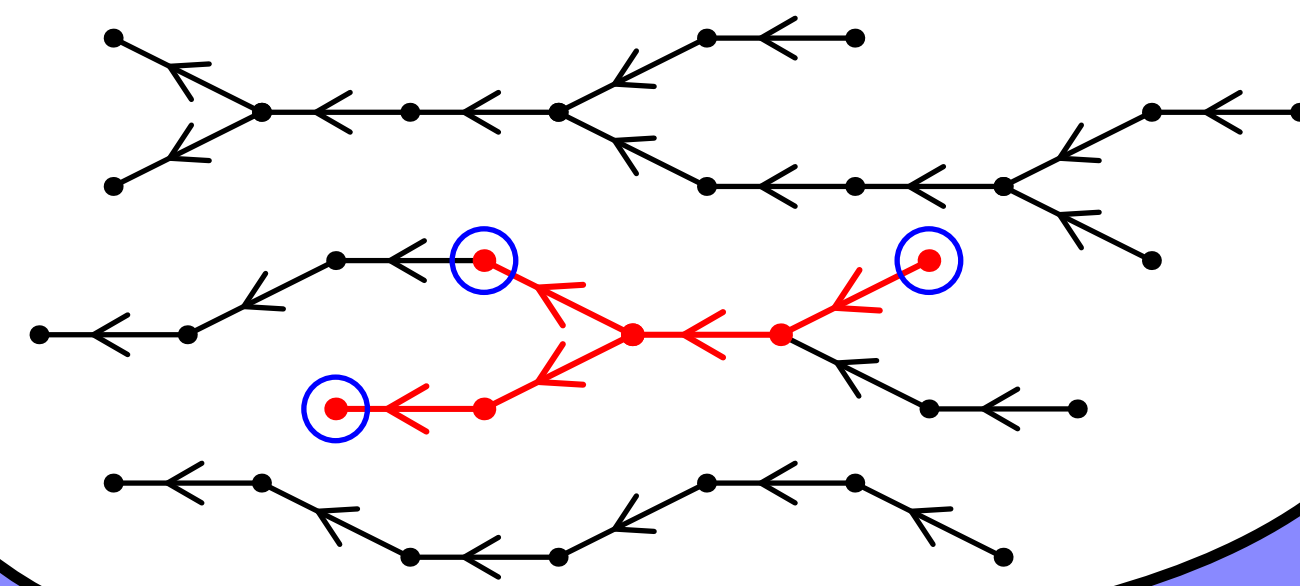
TPUL is equivalent to:



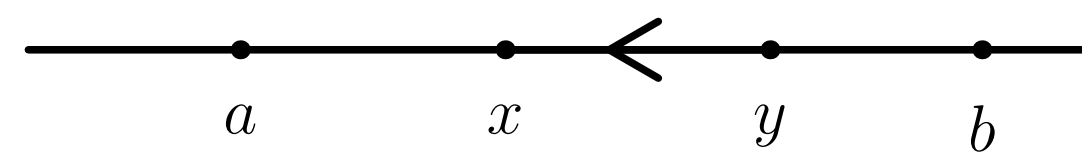
INDUCING STRUCTURE

ORDER

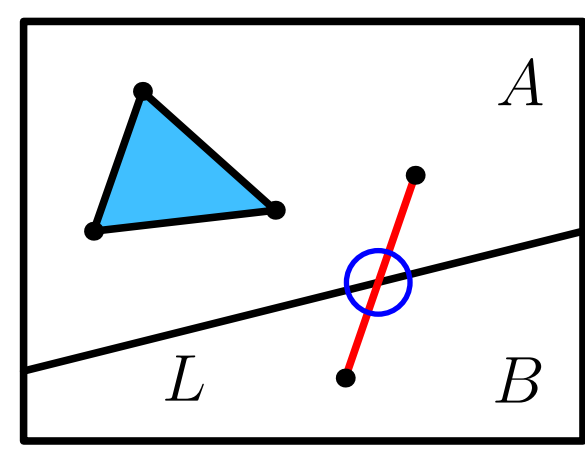
Theorem: all order convexities are free.



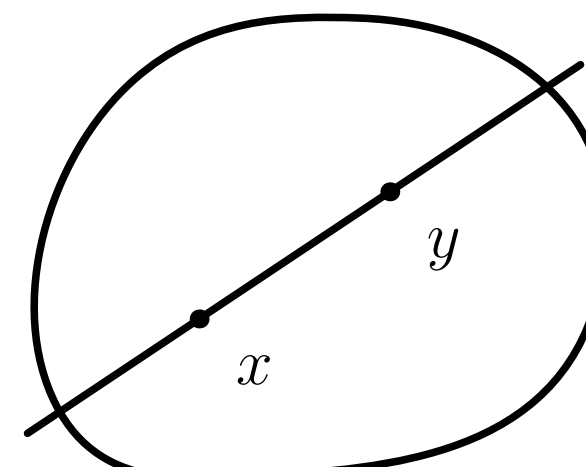
LINEARITY



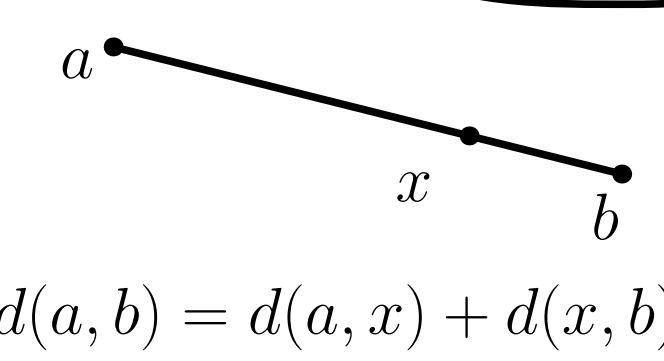
n-AFFINITY



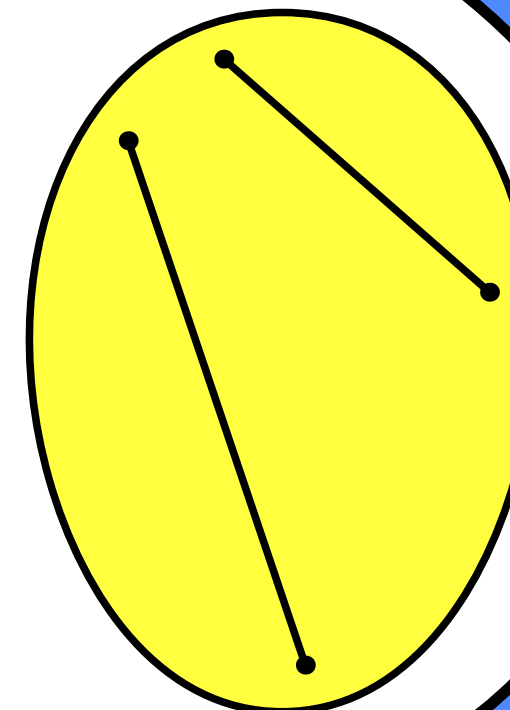
1-AFFINITY



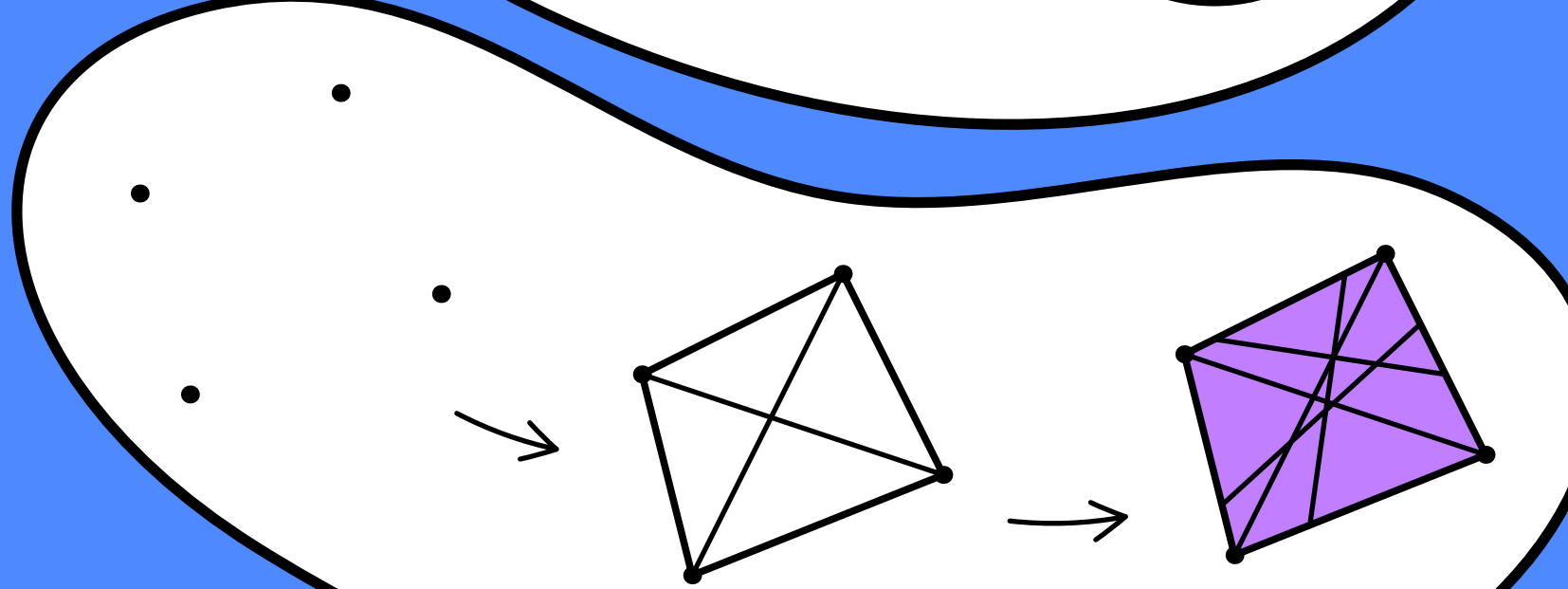
METRIC



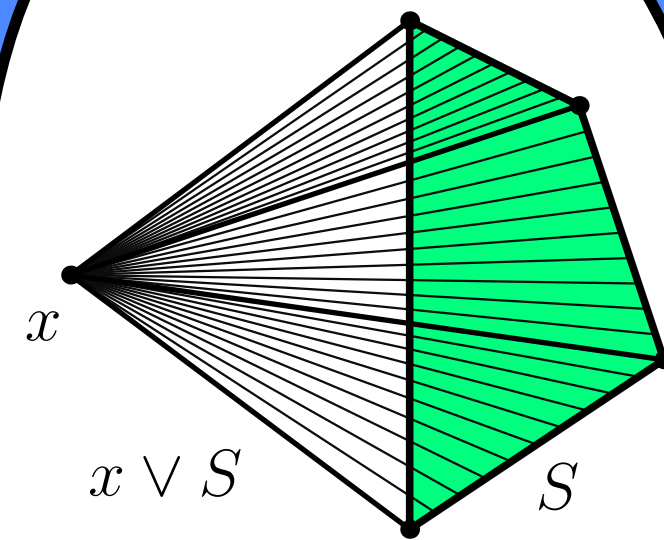
SEGMENTIAL



FINITE



JOIN



Join-commutative:
 $x \vee \langle F \rangle = \langle x \cup F \rangle$

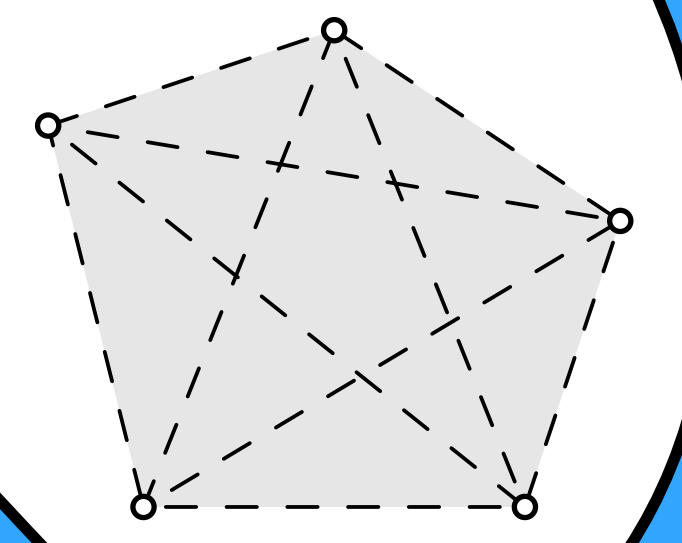
THEOREM

Finite-segmential
2-Affine
TPUL
 \Downarrow
Free

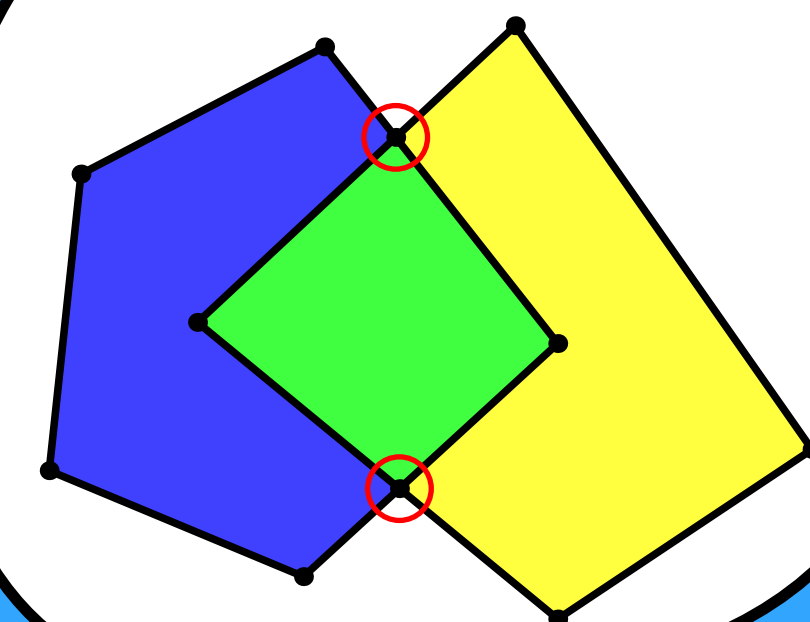
INDUCED STRUCTURE

TOPOLOGY

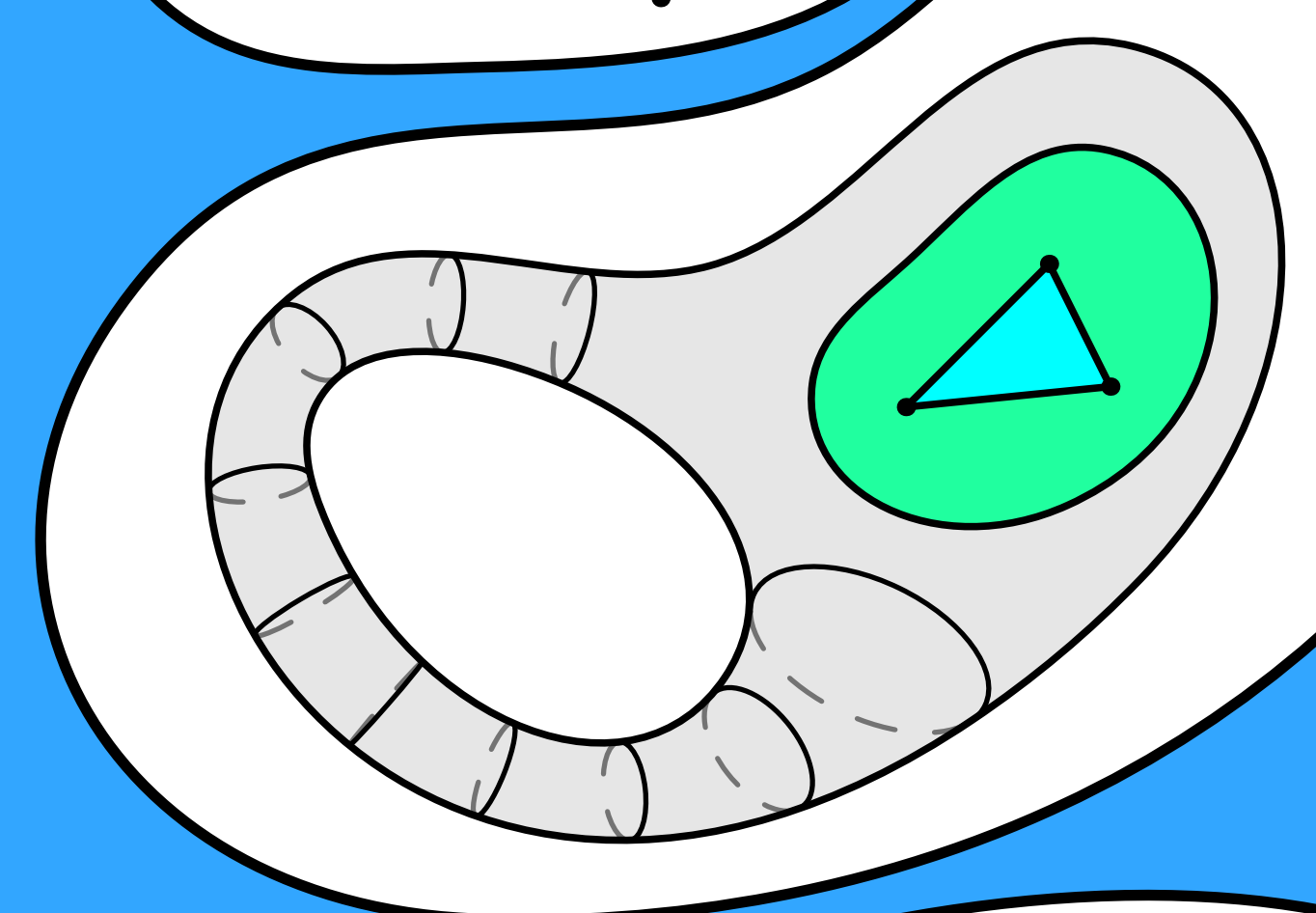
- (1) The Polytope Intersection Lemma
- (2) The Polytope Union Lemma
- (3) Finite dimension
- (4) Freedom



TPIL

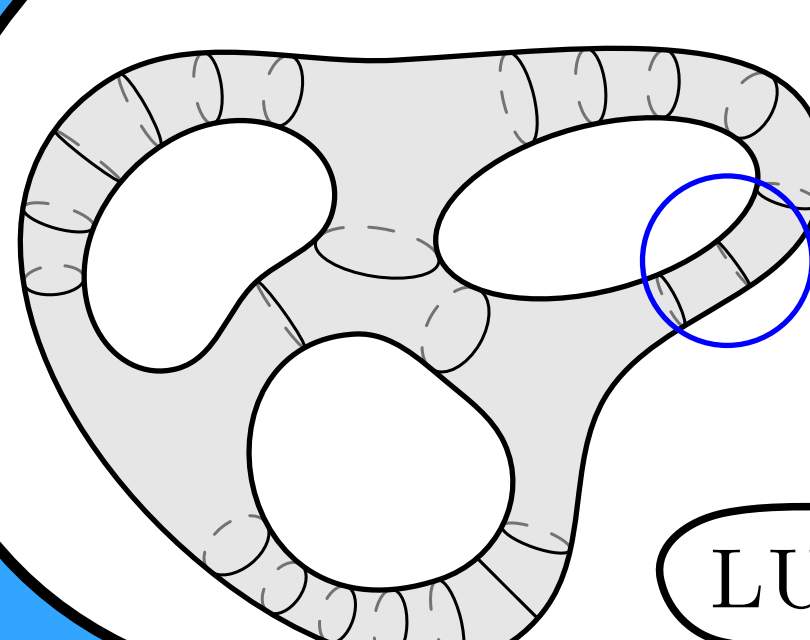


LOCAL CONVEXITY

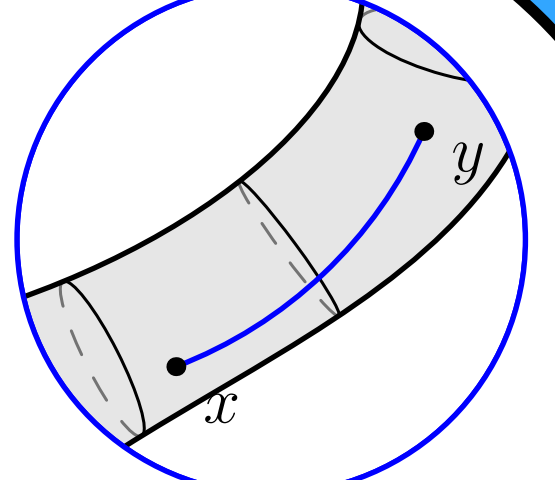


$E^2 \neq B^2$

RIEMANNIAN



LUGS

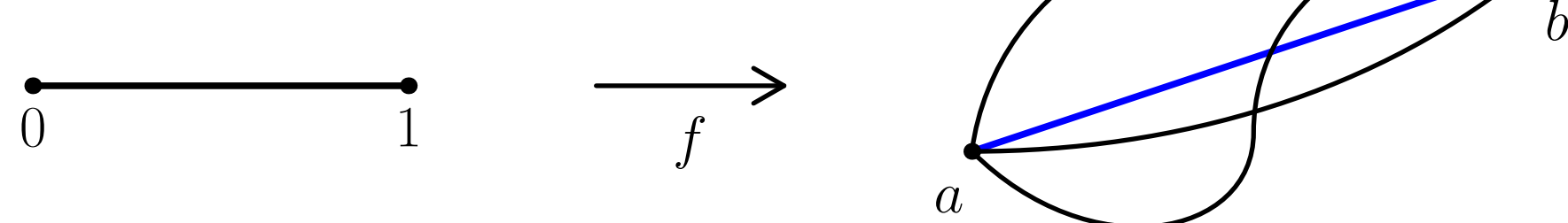


REFERENCES

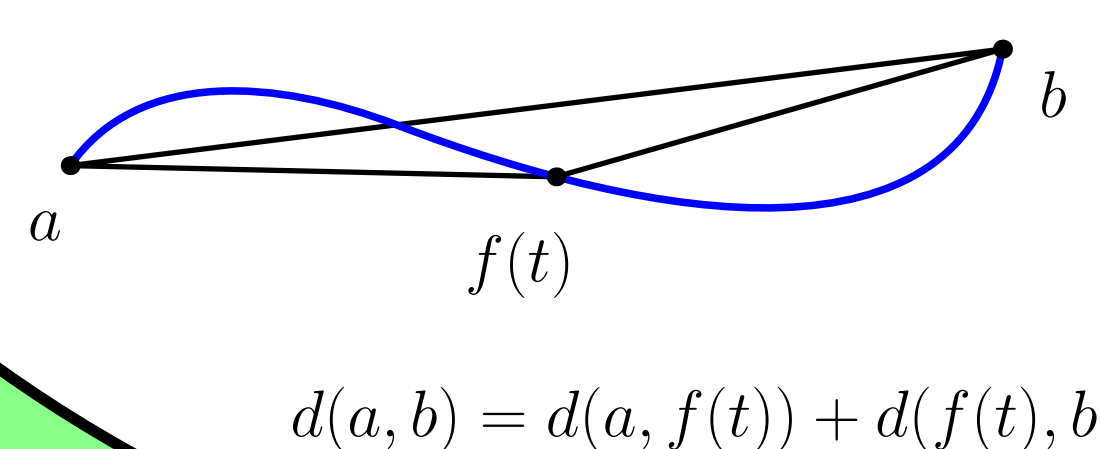
- (1) M.L.J. van de Vel, *Theory of convex structures*, North-Holland mathematical library, 1993;
- (2) D. C. Kay, E. W. Womble, *Axiomatic convexity theory*, Pacific Journal of Mathematics, 1971;
- (3) D. Gromoll, W. Klingenberg and W. Meyer, *Riemannsche Geometrie im Grossen*, Springer-Verlag, Berlin, 1968.

UNIQUELY GEODESIC SPACES

DEFINITION



LEMMA



LEMMA

In a UGS all metric segments are free polytopes.

