MATH1023 Homework, Part 5

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Exercise 1.5.1. Suppose the partial sum $s_n = \frac{n}{2n+1}$. Find the series $\sum x_n$ and its sum.
Solution: First, we find x_n:

$$x_n = s_n - s_{n-1} = \frac{n}{2n+1} - \frac{n-1}{2n-1} = \frac{1}{4n^2-1}.$$

Here, the derivation works for all $n \geq 1$, with s_0 being equal to 0.

Now, we find the sum of the series:

$$\sum_{n=1}^{\infty} x_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}.$$