NEWTON'S LAWS OF MOTION II

Intended Learning Outcomes – after this lecture you will learn:
1. to describe friction in a macroscopic picture and solve problems involving it.
2. to contrast fluid resistance to friction.
3. uniform circular motion and centripetal acceleration
4. to solve problems involving uniform circular motion

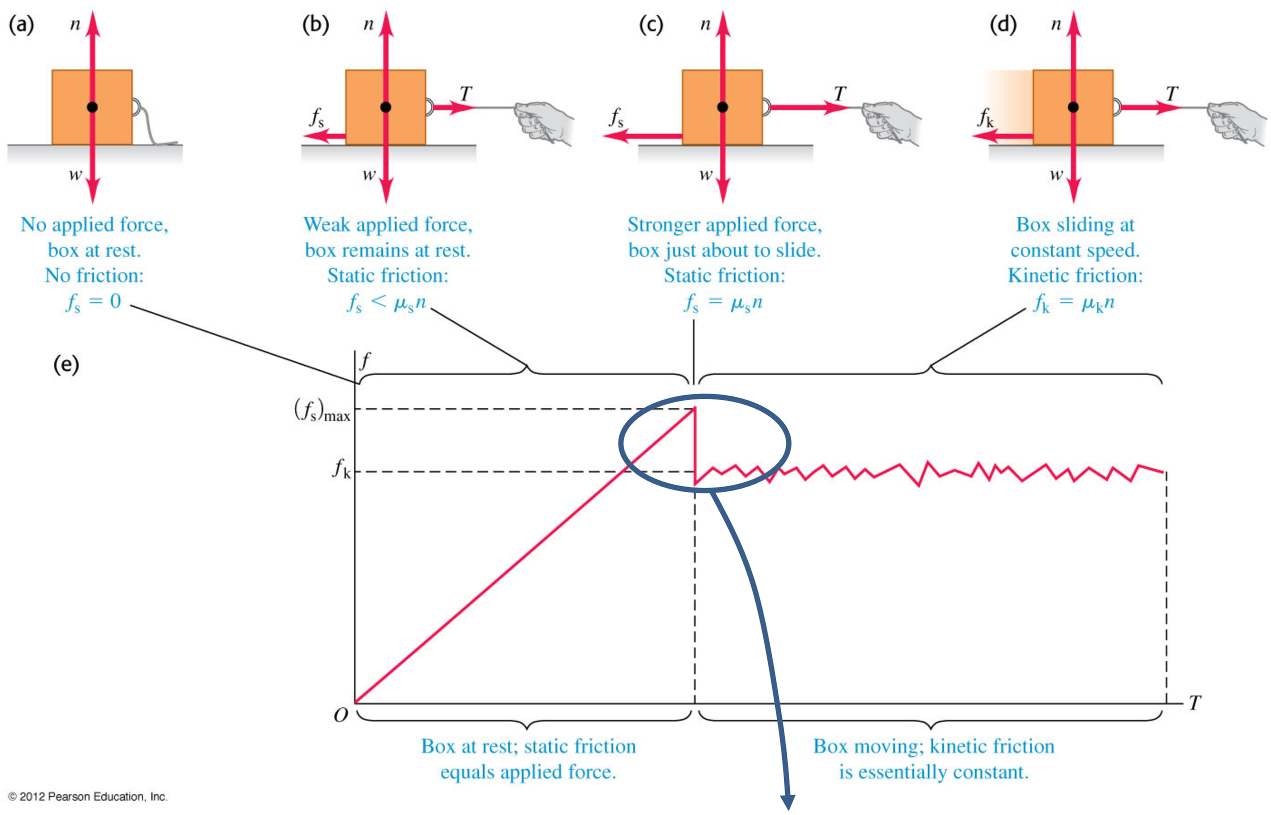
Textbook Reference: Ch 5.3, 3.4, 5.4

Frictional Forces

Microscopic: due to interactions between molecules of surfaces in contact

Macroscopic (phenomenological): ignore microscopic level and look at the outcome only

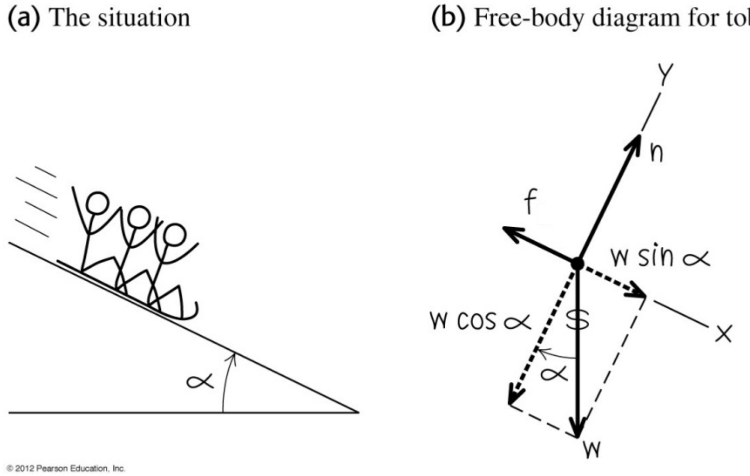
Can be classified into two types: *static* friction, and *dynamic* (or *kinetic*) friction



Interpretation: easier to keep the block moving than to start it moving

⚠ the coefficients of static and kinetic friction μ_s and μ_k depends on the two surfaces in contact
⚠ friction always along contact surface and therefore ⊥ to normal force
⚠ static friction can be less than the maximum value

Example 5.16 and 5.17 P. 173: A block (or toboggan) sliding down an inclined plane



Given: μ_s and μ_k, angle α increases from zero

Before the block starts to slide, friction is (static / kinetic), and equals to ___________

If at a particular α, the block just begin to slide

Right before the block begins to slide, friction is (static / kinetic):

Resolving force ⊥ the plane: $\sum F_y = n - mg\cos\alpha = 0$

along the plane: $\sum F_x = mg\sin\alpha - \mu_s n = 0 \implies \alpha = \tan^{-1}\mu_s$

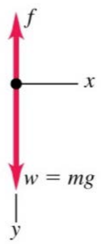
Right after the block begins to slide, friction is (static / kinetic) and the block slides with (constant speed / an acceleration):

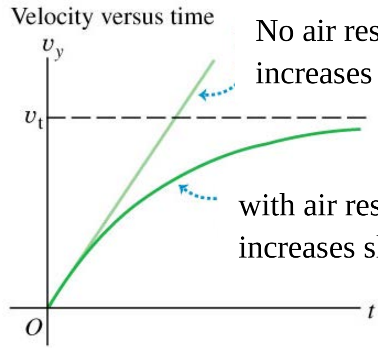
$$\sum F_x = mg\sin\alpha - \mu_k n = ma$$

$$\implies a = g(\sin\alpha - \mu_k\cos\alpha) = g\frac{\mu_s - \mu_k}{\sqrt{1+\mu_s^2}}$$

Fluid Resistance

⚠ fluid resistance depends on speed. At high speed (or non-viscous fluid), $f \propto v^2$, or $f = Dv^2$ e.g. air resistance





No air resistance, v increases linearly

with air resistance, v increases slower and slower

$\sum F_y = mg - Dv^2 = ma$

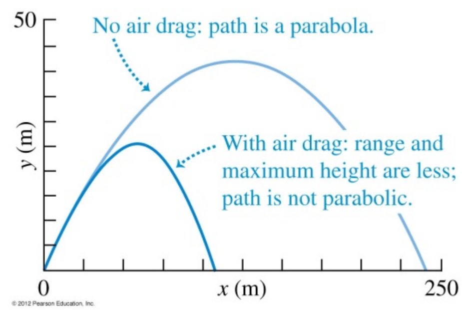
Note:

1) a decreases as v increases
2) there exists a terminal speed $v_t = \sqrt{mg/D}$ when $a = 0$

⚠ heavy bodies fall faster ∵ larger m, i.e. Aristotle was right that heavier objects fall *faster*!

⚠ a sheet of paper falls faster if crumpled into a ball ∵ D smaller

⚠ with air resistance, a projectile is no longer a parabola



Example

Reduce the equation of motion to dimensionless form, $v' = v/v_t$ and $t' = t/t_c$, $t_c = v_t/g$

$$m\frac{dv}{dt} = mg - Dv^2 \quad \Rightarrow \quad \frac{dv'}{dt'} = 1 - v'^2$$

Solve it and verify the v vs t curve. Interpret the physical meaning of t_c.

Ans. $v' = (e^{2t'} - 1)/(e^{2t'} + 1)$ i.e., $v/v_t = (e^{2t/t_c} - 1)/(e^{2t/t_c} + 1)$

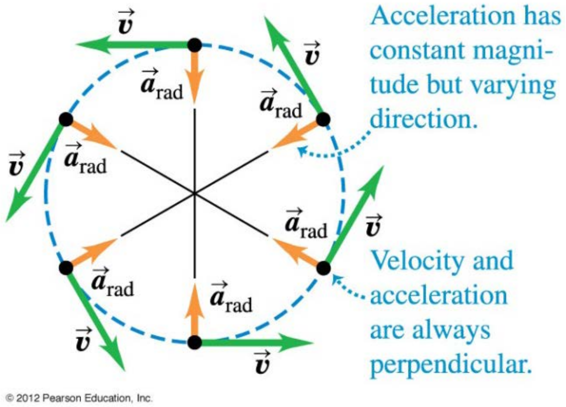
Comment : v_t is the *characteristic* velocity scale and t_c (time taken for the velocity to increase from zero to $(e^2 - 1)/(e^2 + 1) \sim 0.76$ of v_t) is the *characteristic* time scale of the problem. When velocity and time are measured in units of v_t and t_c respectively, the equation of motion is *dimensionless*.

This process is called **nondimensionalization**, see https://en.wikipedia.org/wiki/Nondimensionalization

Exercise

For a particle falling in a viscous fluid, say oil, the fluid resistance is proportional to v, not v^2, i.e., $f = kv$ where k is a positive constant. Repeat the nondimensionalization process above, solve the dimensionless equation assuming the particle falls from rest, and interpret the characteristic time scale.

Dynamics of Uniform Circular Motion Ch 3.4, P. 109

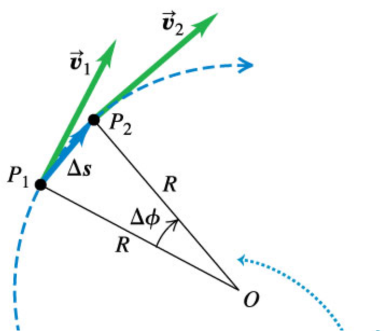


Speed (NOT velocity) constant

$\Rightarrow a_{\parallel} = 0$

$\Rightarrow \vec{\boldsymbol{a}}$ along radial direction (inward / outward)

called centripetal acceleration

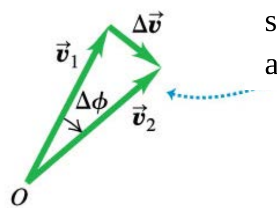


$$\Delta\phi = \frac{\Delta s}{R} = \frac{|\Delta\vec{\mathbf{v}}|}{v}$$

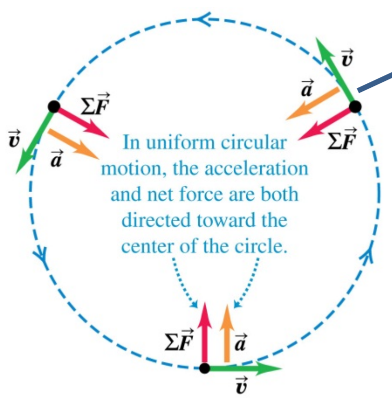
$$a_{\text{rad}} = \frac{|\Delta\vec{\mathbf{v}}|}{\Delta t} = \frac{v}{R}\frac{\Delta s}{\Delta t}$$

$$\frac{ds}{dt} = v$$

$$\therefore \quad \boxed{a_{\text{rad}} = \frac{v^2}{R}}$$



same $\Delta\phi$ $\therefore \vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ are $\perp$ to OP_1 and OP_2



force providing the radially inwards centripetal acceleration, sometimes called the "centripetal force".

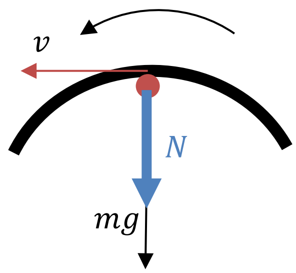
$$F_{net} = ma = m\frac{v^2}{R}$$

In a *rotating frame* where the particle is *at rest*, there is a radially (inward / outward) inertial force mv^2/R, called *centrifugal* force

Demonstration: vertical circular motion

At the top, equation of motion of ball

$$mg + N = m\frac{v^2}{R} \quad \Rightarrow \quad v = \sqrt{R\left(g + \frac{N}{m}\right)}$$

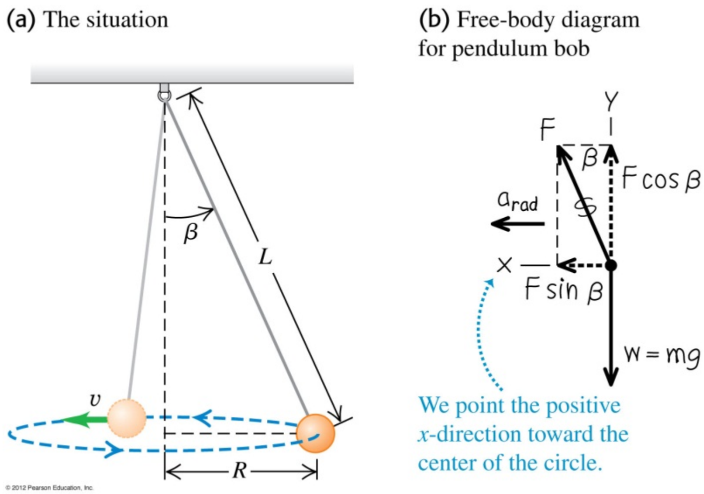


⚠ If $v > \sqrt{Rg}$, $N > 0$, ball in contact with the rotating rim, will not fall

⚠ If $v = \sqrt{Rg}$, $N = 0$, ball just start to fall

⚠ If $v < \sqrt{Rg}$, $N < 0$, equation fail because ball fallen already

Example 5.20 P. 179: A conical pendulum



horizontal uniform circular motion

$$\sum F_x = F\sin\beta = ma$$
$$\sum F_y = F\cos\beta - mg = 0$$
$$\Rightarrow \quad a = g\tan\beta$$

Period of the pendulum:

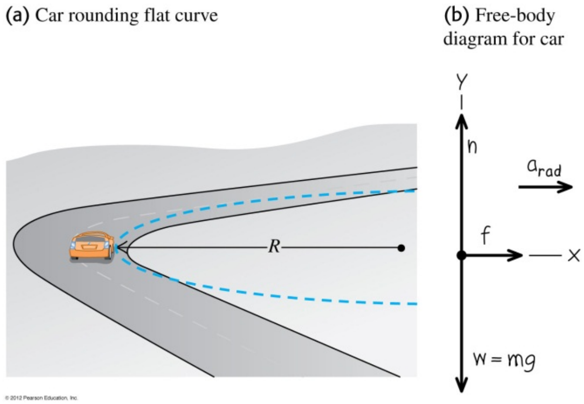
$$T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{L\cos\beta}{g}}$$

c.f. a planar pendulum

Observation: Why banked curves in a racing track help?

Example 5.21 P. 179 Rounding a flat curve

On a flat curve



Assume no skidding, what supplies the centripetal force? (Static / Kinetic) friction!

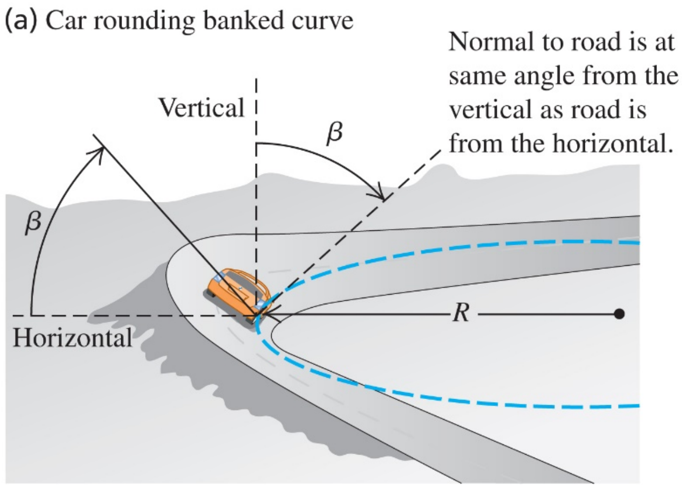
Max. speed:

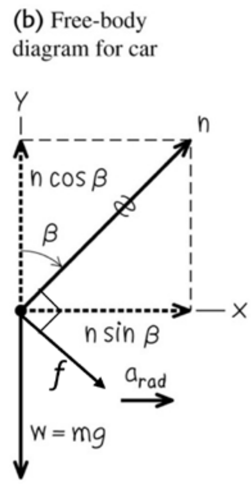
$$f = f_{max} = m\frac{v_{max}^2}{R} \quad \Rightarrow \quad v_{max} = \sqrt{\mu_s g R}$$

$$\mu_s n = \mu_s mg$$

⚠ if no sideway friction f, then the car cannot round a flat curve

If banked at angle β





What supplies the centripetal force? n and f!

$$\sum F_x = n\sin\beta + f\cos\beta = mv^2/R$$

$$\sum F_y = n\cos\beta - f\sin\beta - mg = 0$$

$$\Rightarrow f = m\left(\frac{v^2}{R}\cos\beta - g\sin\beta\right) = \frac{m\cos\beta}{R}(v^2 - gR\tan\beta), n = \frac{m\cos\beta}{R}(v^2\tan\beta + gR)$$

> Digression: solve simultaneous equations in two unknowns using **Cramer's rule**
>
> $$\begin{cases} ax + by = A \\ cx + dy = B \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} A & b \\ B & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, \qquad y = \frac{\begin{vmatrix} a & A \\ c & B \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$
>
> Note: Cramer's rule works for n simultaneous equations in n unknowns (check it out on Wiki). But for $n > 3$ evaluating the determinants becomes too tedious.

$$f \le \mu_s n \Rightarrow v \le v_{max} = \sqrt{\frac{\tan\beta + \mu_s}{1 - \mu_s\tan\beta}gR} \ge \sqrt{\mu_s gR}$$

Interpretation: the car can round a banked curve at a higher speed without skidding

⚠ If no friction, $f = 0$, then this is the same as Example 5.22 P. 180 of the textbook.

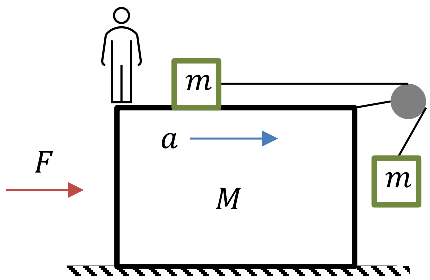
⚠ This is quite similar to the Bridging Problem on P. 186 of the textbook. Work it out.

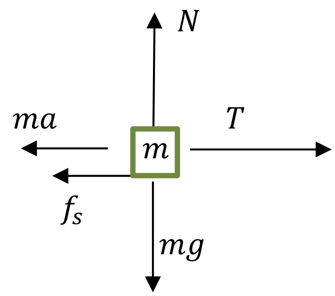
> Challenging Question:
>
> What happen to the friction f if $v < \sqrt{gR\tan\beta}$? How would you interpret this situation?

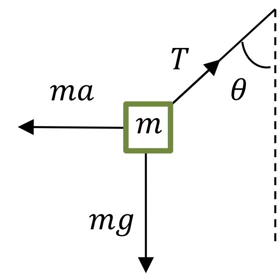
Example

Assuming no friction between the block and the floor, a frictionless pulley, and the static friction coefficient between the block M and mass m is μ_s. What is the minimum force F such that the two masses m are at rest relative to the block M?

Go to the rest frame of the block. Then the masses m are at rest and have inertial forces ma.







$$a = \frac{F}{M + 2m}$$

$$T - f_s - ma = 0$$
$$N - mg = 0$$

$$T\sin\theta - ma = 0$$
$$T\cos\theta - mg = 0$$

$$\Rightarrow \quad f_s = T - ma = ma\left[\sqrt{1 + \left(\frac{g}{a}\right)^2} - 1\right]$$

$$\text{Since } f_s \le \mu_s N = \mu_s mg, \qquad a\left[\sqrt{1 + \left(\frac{g}{a}\right)^2} - 1\right] \le \mu_s g \;\; \Rightarrow a \ge \frac{(1 - \mu_s^2)}{2\mu_s} g$$

The necessary force is $F \ge (M + 2m)g\,\frac{(1-\mu_s^2)}{2\mu_s}$

⚠ interpret the result when $\mu_s > 1$

Clicker Questions:

Q5.10

You are walking on a level floor. You are getting good traction, so the soles of your shoes don't slip on the floor.

Which of the following forces *should* be included in a free-body diagram for your body?

A. the force of kinetic friction that the floor exerts on your shoes

B. the force of static friction that the floor exerts on your shoes

C. the force of kinetic friction that your shoes exert on the floor

D. the force of static friction that your shoes exert on the floor

E. more than one of these

Q3.11

You drive a race car around a circular track of radius 100 m at a constant speed of 100 km/h. If you then drive the same car around a different circular track of radius 200 m at a constant speed of 200 km/h, your acceleration will be

A. 8 times greater.

B. 4 times greater.

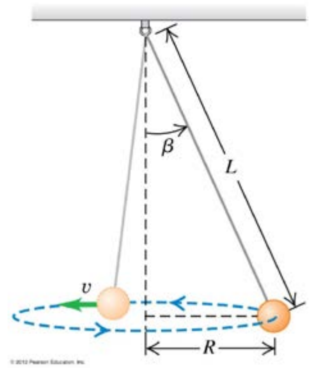
C. twice as great.

D. the same.

E. half as great.

Q5.12

A pendulum bob of mass *m* is attached to the ceiling by a thin wire of length *L*. The bob moves at constant speed in a horizontal circle of radius *R*, with the wire making a constant angle β with the vertical. The tension in the wire



A. is greater than *mg.*

B. is equal to *mg.*

C. is less than *mg.*

D. is any of the above, depending on the bob's speed *v.*

Q5.13

A pendulum of length *L* with a bob of mass *m* swings back and forth. At the low point of its motion (point *Q*), the tension in the string is (3/2)*mg*. What is the speed of the bob at this point?

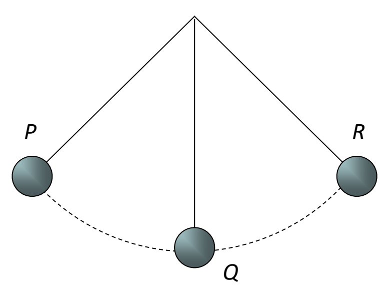
A. $2\sqrt{gL}$

B. $\sqrt{2gL}$

C. $\sqrt{gL}$

D. $\sqrt{\frac{gL}{2}}$

E. $\frac{\sqrt{gL}}{2}$



Ans: Q5.10) B, Q3.11) C, Q5.12) A, Q5.13) D