PHYS1312 Fall 2024

Honors General Physics I

Written Homework I

Upload a pdf version of your solution to Canvas on or before

Fri Sep 20, 2024, 11:59pm

WARNING: You are encouraged to work on the homework in groups. But you should write up your own solution. **Anyone who copy homework or whose homework is copied will get zero point.**

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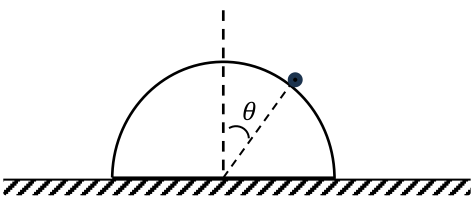
1. A particle is projected from rest at the origin with speed v_0 at angle θ above the ground. Without air resistance it traces out a parabolic trajectory $y(x)$ as shown in Lecture 1. Let's suppose air resistance is proportional to v (not v^2 because will be too difficult), i.e., $-bv$ where b is a constant. By introducing characteristic speed, time, and length for this problem, write the equations of motion in dimensionless form (nondimensionalization) and find the trajectory $y'(x')$, where a prime such as y' means the dimensionless form of the corresponding physical quantity.

   Ans. $y' = x'\left(\tan\theta + \frac{1}{v_0' \cos\theta}\right) + \frac{1}{v_0'} \ln\left(1 - \frac{x'}{\cos\theta}\right)$

   Comment: after solve this problem, you should appreciate nondimensionalization because it greatly simplifies your calculations.

2. A uniform hemisphere of radius R and mass M is placed with its flat side on a smooth surface. A particle of mass m is at rest at the very top of the hemisphere in unstable equilibrium condition. A very small disturbance causes the particle to slide down the spherical surface from rest. At the same time the hemisphere will be pushed by the particle to the left. You can assume no friction at any contact points and surfaces.



   Show that at the moment the particle starts to leave the hemisphere's surface, θ satisfies the following equation,

$$r\cos^3\theta - 3(1+r)\cos\theta + 2(1+r) = 0,$$

   where $r = m/M$.

   Hint: use conservation of energy and momentum in lab frame, and equation of motions in hemisphere's (non-inertial) frame.