PHYSICAL QUANTITIES, VECTORS AND 2D MOTION

Intended Learning Outcomes – after this lecture you will learn:

1. meanings of theories and models in physics.
2. how to define units for fundamental physical quantities.
3. how to use significant figures.
4. vectors and its algebraic operations: addition and subtraction
5. displacement, velocity and acceleration in vector notation
6. to predict the trajectory of projectile motion

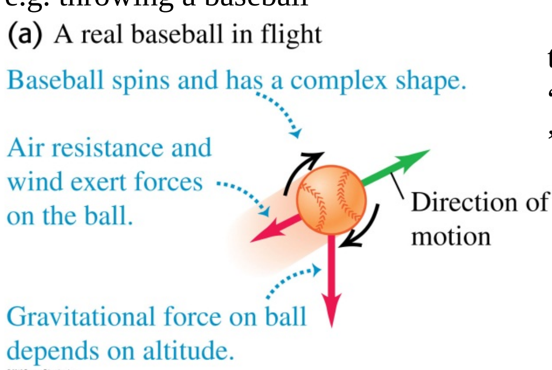
Textbook reference: Ch 1, 3.1 – 3.3

Physics is an *experimental natural* science.

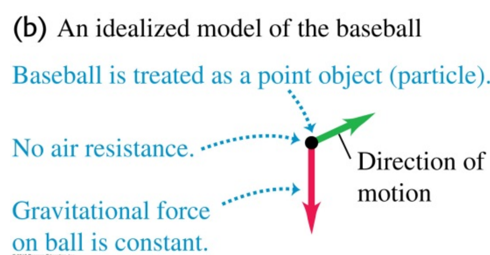
<u>Theory</u>: an explanation of natural phenomena based on observation and accepted fundamental principles, e.g. theory of evolution in biology

<u>Model</u>: a simplified version of a physical system that would be too complicated to analyze in full detail

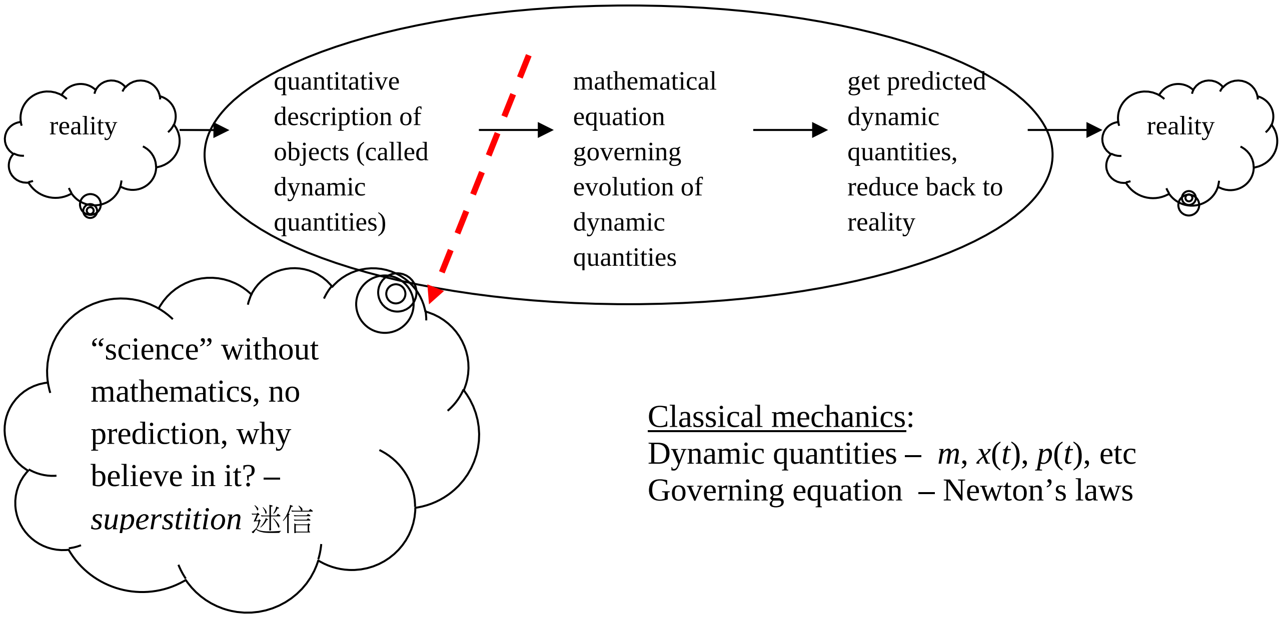
e.g. throwing a baseball



throw away "unimportant" parts



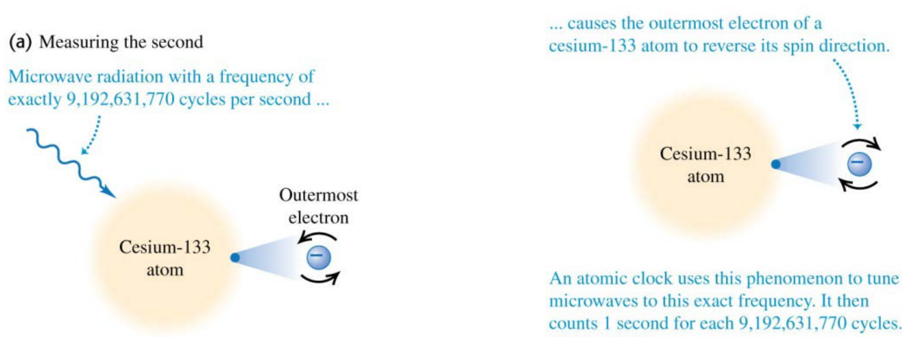
use $\underbrace{\textit{theory} \text{ to make}}_{\text{means calculation}}$ prediction



Classical mechanics:
Dynamic quantities – m, $x(t)$, $p(t)$, etc
Governing equation – Newton's laws

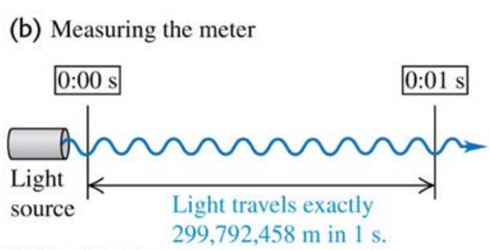
Standards and Units The *International Standard*, or SI (Système International) Units

TIME: in second – starting 1967, defined using an atomic clock



LENGTH: in meter – starting 1983, defined based on the **speed of light** in vacuum, which is *defined* to be (exactly)

$$c = 299{,}792{,}458 \text{ m/s}$$



MASS: in kilogram – starting 2018, kilogram is defined based on a fundamental constant of nature called **Planck's constant**, which is *defined* to be (exactly)

$$h = 6.62607015 \times 10^{-34} \text{ kg}\cdot\text{m}^2/\text{s}$$

Uncertainty and Significant Figures

What is the difference among the following representations of π?

1. 3.14 (means between 3.135 and 3.145, or 3.14 ± 0.005
2. 3.1416 (means 3.1416 ± 0.00005)
3. 22/7 (rational number usually means exact, misleading here, not exact!)

What is the difference between 3 and 3.00?

Be careful about the number of significant figures.

What are the problems with the following representation?

$$2.0176\underbrace{76}_{} \pm 0.0132$$

smaller than error, *meaningless!*

error estimation cannot be so accurate! Usually take 1, at most 2 sig. fig.

keep the same decimal places

should be ____ ± ____

Note: uncertainty propagates in calculations:

Multiplication or division:
Result may have no more significant figures than the starting number with the fewest significant figures:

$$\frac{0.745 \times 2.2}{3.885} = 0.42$$

$$1.32578 \times 10^{7} \times 4.11 \times 10^{-3} = 5.45 \times 10^{4}$$

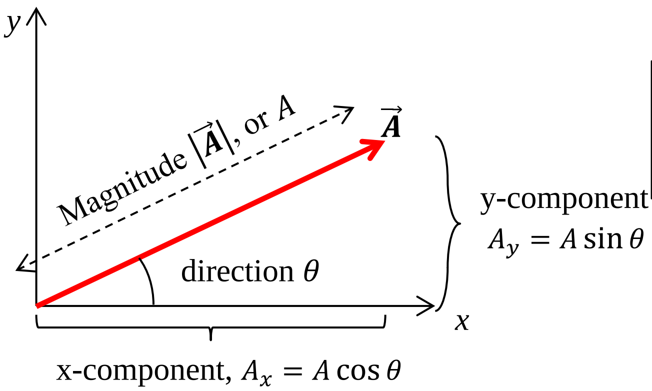
Addition or subtraction:
Number of significant figures is determined by the starting number with the largest uncertainty (i.e., fewest digits to the right of the decimal point):

$$27.153 + 138.2 - 11.74 = 153.6$$

Question: What is the density (in kg/m^3) of a rock of mass 1.80 kg and volume 6.0×10^{-4} m^3?
(a) 3×10^3 kg/m^3, (b) 3.0×10^3 kg/m^3, (c) 3.00×10^3 kg/m^3, (d) 3.000×10^3 kg/m^3
Answer: see inverted text on P. 37 of textbook

Vector

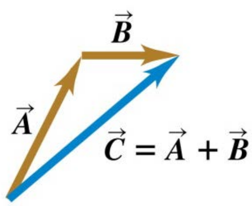
An “arrow” in space, has magnitude (length) and direction
e.g. in 2D Cartesian coordinates (due to René Descartes)



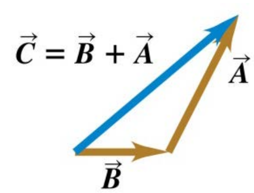
Note: $A = \sqrt{A_x^2 + A_y^2}$ (Pythagoras thm)

$\tan\theta = \frac{A_y}{A_x}$

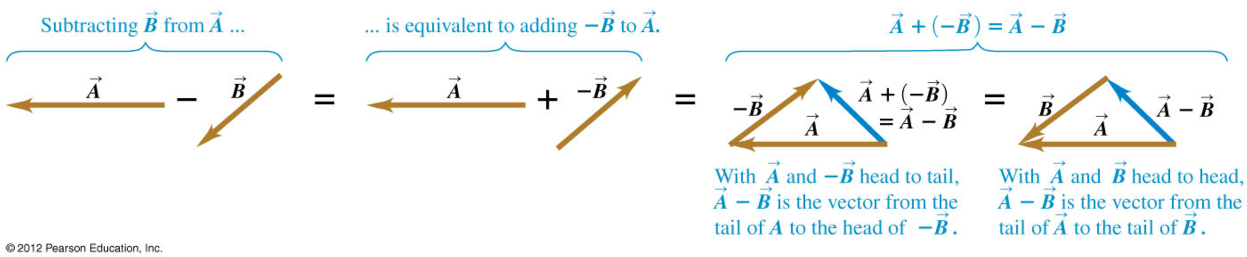
Addition:



or

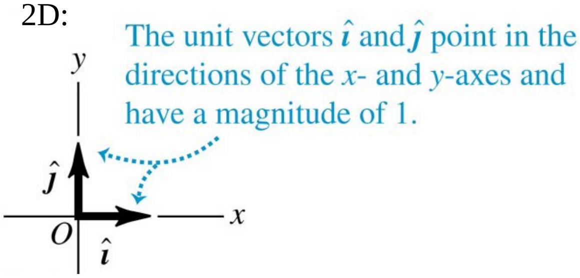


Subtraction:

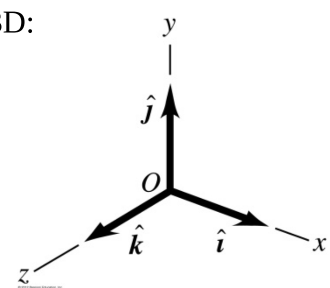


Vectors of unit magnitude are called <u>unit vectors</u>. Most commonly used unit vectors are $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, along x, y, and z directions in Cartesian coordinates

2D:



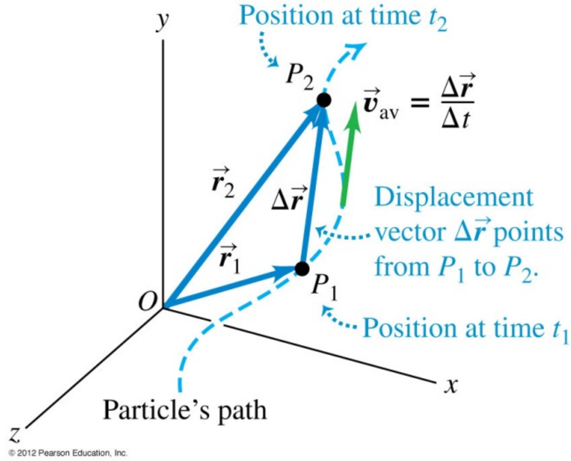
3D:



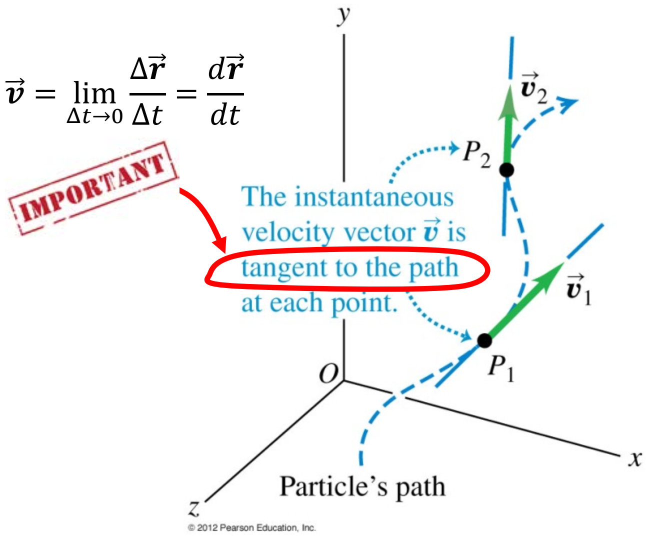
Analytical representation of a vector: $\boxed{\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}}$

Displacement and velocity vectors

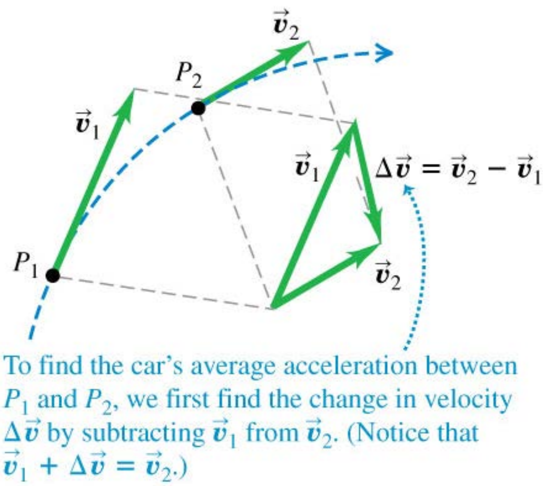
Distance and speed – scalars Displacement and velocity – vectors

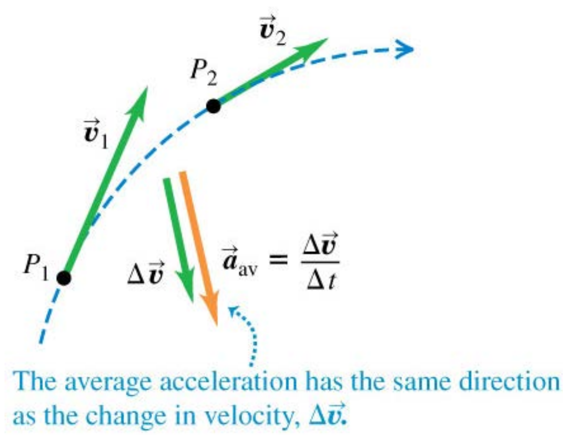


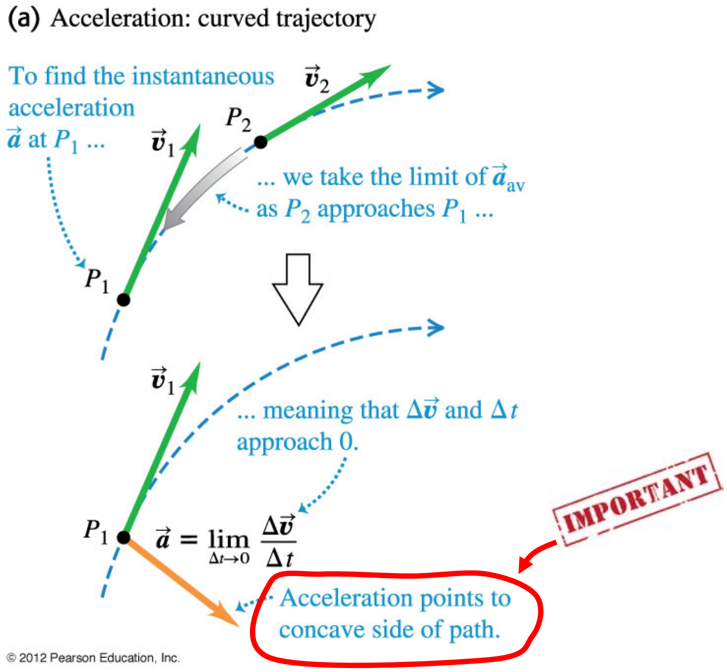
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



Acceleration vector

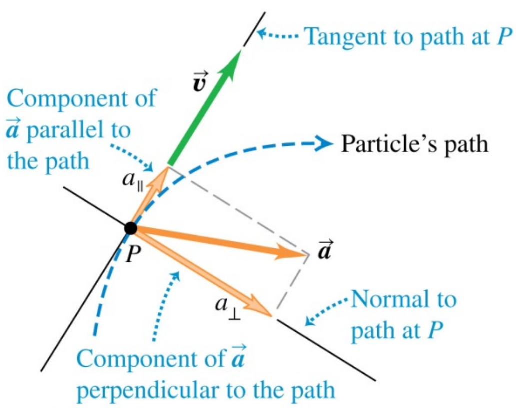


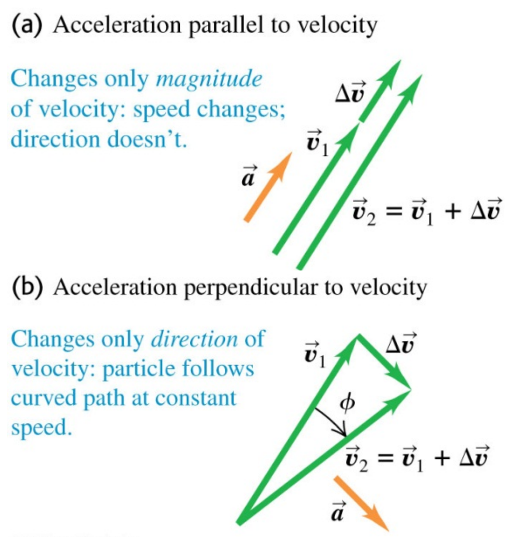




$$\vec{\boldsymbol{a}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{v}}}{\Delta t} = \frac{d\vec{\boldsymbol{v}}}{dt}$$

Resolve into parallel (or <u>tangential</u>) $a_{\parallel}$, and perpendicular (or <u>radial</u>) $a_{\perp}$ components



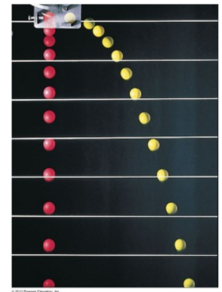


Projectile

Principle: *x* and *y* motions are independent

Vertical motion of red and yellow balls are identical – at the same height at any time

$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = -g\hat{\jmath}$, i.e., $a_x = 0, a_y = -g$



Recall from high school: rectilinear motion with uniform acceleration *a*

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

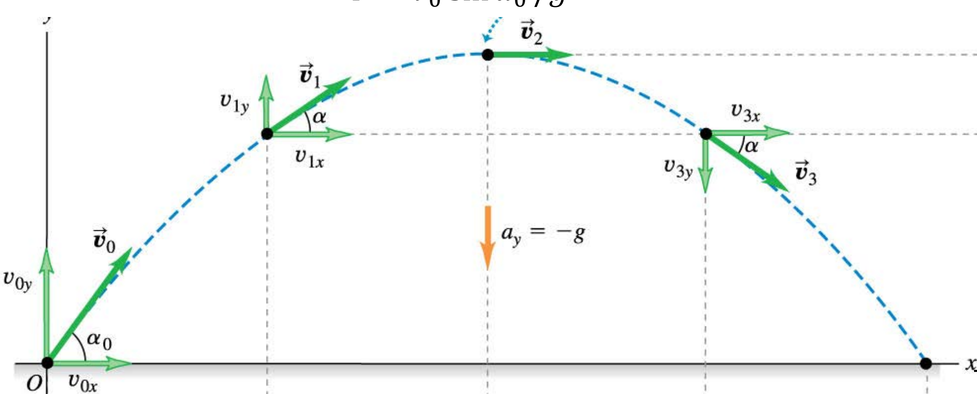
Trajectory: $x(t) = v_0 \cos\alpha_0\, t,\ y(t) = v_0 \sin\alpha_0\, t - \frac{1}{2}gt^2$

Eliminate $t \Longrightarrow \quad y = (\tan\alpha_0)x - \frac{g}{2v_0^2\cos^2\alpha_0}x^2$

i.e. $y = bx - cx^2$ a parabola 抛物線

A typical projectile: at the top, $0 = v_y = v_0 \sin\alpha_0 - gT$

$\Rightarrow T = v_0 \sin\alpha_0 / g$



y motion with uniform downward acceleration *g*, **max. height** $= v_0 \sin\alpha_0\, T - \frac{1}{2}gT^2 = \frac{v_0^2\sin^2\alpha_0}{2g}$

x motion, no acceleration, **range** $= v_0\cos\alpha_0\,(2T) = 2v_0^2 \sin\alpha_0 \cos\alpha_0 / g$

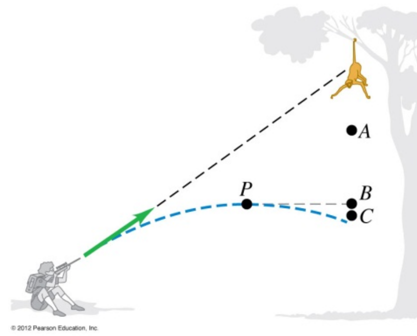
⚠ As derived in the general case, i) $\vec{v}$ is tangential to path, ii) $\vec{a}$ points to concave side

Demonstration: Monkey and Hunter, see Example 3.10 on P. 108 A ball fired at the same instant when the monkey is dropped. Ball *always* hit the monkey, *AMAZING!!* See the textbook for a proof.	

Question:

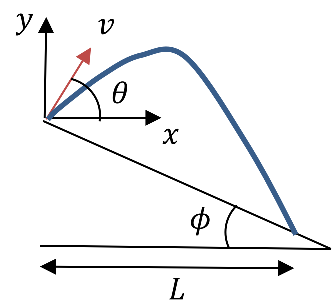
When the ball is at its highest point P, the monkey will be at

(i) point A (higher than P),
(ii) point B (at the same height as P)
(iii) C (lower than P)



Answer: see inverted text on P. 109 of textbook

Example Maximum range on an inclined plane



$$x = vt\cos\theta\,,\quad y = vt\sin\theta - \frac{1}{2}gt^2$$

When it lands, $x = L$, $y = -L\tan\phi$, the time T is

$$\underbrace{-L}_{vT\cos\theta}\tan\phi = vT\sin\theta - \frac{1}{2}gT^2$$

$$\Rightarrow\ T = \frac{2v}{g}(\sin\theta + \cos\theta\tan\phi)$$

$$\Rightarrow\ L = vT\cos\theta = \frac{2v^2}{g}(\sin\theta\cos\theta + \cos^2\theta\tan\phi)$$

$$= \frac{v^2}{g}(\sin 2\theta + \cos 2\theta\tan\phi + \tan\phi)$$

To maximize L, $\frac{dL}{d\theta} = 0 \ \Rightarrow\ \underbrace{\cot 2\theta}_{=\tan\left(\frac{\pi}{2}-2\theta\right)} = \tan\phi \ \Rightarrow\ \boxed{\theta = \frac{\pi}{4} - \frac{\phi}{2}}$

⚠ can this be a local minimum?

Some Problem Solving Ideas:

1. Dimensional Analysis

In mechanics, three fundamental dimensions: [time] = T, [mass] = M, [length] = L

Other quantities have dimensions derived from them, e.g.,

$[v] = \left[\frac{ds}{dt}\right] = \mathrm{LT}^{-1}$, $[a] = \left[\frac{d^2s}{dt^2}\right] = \mathrm{LT}^{-2}$,

Dimensional homogeneity: any physical quantities related by addition, subtraction, equality/inequality sign must have the same dimension, e.g., $v = v_0 + \frac{1}{2}at^2$

Note: dimensional analysis is independent of the underlying units

Example

In the projectile problem, if we assume that the range is proportional to $v_0^a m^b g^c$, then dimensional analysis shows that

$$\mathrm{L} = (\mathrm{LT}^{-1})^a \mathrm{M}^b (\mathrm{LT}^{-2})^c$$

We get $a = 2$, $b = 0$, $c = -1$. Note that dimensional analysis *cannot* be used to find dimensionless constants, which is $2 \sin \alpha_0 \cos \alpha_0$ in this case.

Example

What is the dimension of ω in the solution of the wave equation, $y = A\cos(kx - \omega t)$, where $[x] = [y] = \mathrm{L}$, and $[t] = \mathrm{T}$?

For reference, see libretexts

2. Scaling Analysis

Idea: if x is doubled, how would y change?

I.e., ignore details such as proportionality constants and consider scaling exponents only, $y \sim x^a$.

For example, in projectile:

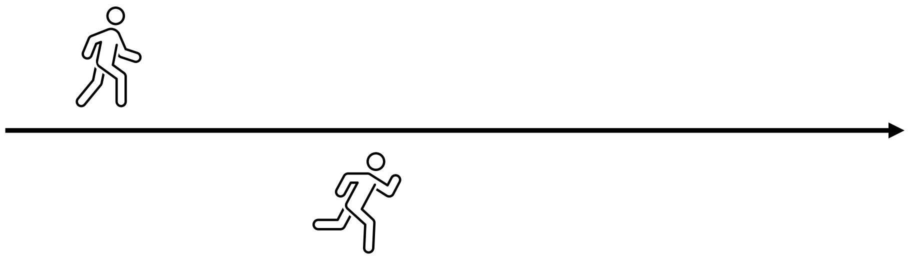
1. If double v_0, range $L \sim (v_0 \cos \alpha_0)T$ and $T \sim (v_0 \cos \alpha_0)$, $L \sim v_0^2$.
2. If double g, reduces T by ½, therefore $L \sim 1/g$
3. If double m, use **Galileo's argument**: imagine two identical projectiles flying side-by-side. It makes no difference to the trajectory if you tie them together and double its mass.

Therefore L is independent of m. Note that we get the same result as dimensional analysis.

3. Invariants

The Newton's second law is a differential equation that describes how a physical system evolves from one instant to another. Quantities that change with time are called *dynamical quantities*. In some cases it is more convenient to make use of *invariants* (those that are time independent), if exist, to relate the system at different instants. Very often invariants exist in the form of conservation laws, such as the conservation of energy and momentum in the elastic collision of Lecture 7. But sometimes it may be other things, such as the center of mass location as in the Example on P. 6 of Lecture 7.

A wrong application of invariant Why running use more energy than walking through *the same distance* (invariant)?



Additional rapid contraction and relaxation of muscle during running cost energy.

A correction application To run or not to run in the rain? From Sanjoy Mahajan, The Art of Insight in Science and Engineering, §3.1.1

Suppose no wind and rains steadily and uniformly

Two ways to get wet:

A. Rain falling vertically on your head, even when you stand still
B. Rain hitting you from the front as you move (due to relative velocity)

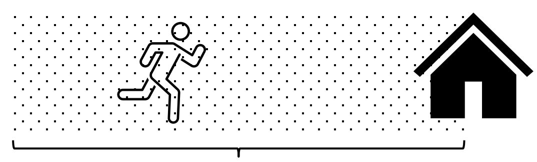
First ignore (A) (either you are really thin, or you put a folder/book over your head)

Question: will running faster make you more wet or less wet?

Two opposite effects:

1. Shorter time to stay in the rain – less wet
2. More raindrops hit you from the front – more wet

Which one dominates?



Volume of rain you cut through

Is there an *invariant*? Yes, the volume of rain you cut through as you move. Independent of your speed. Makes no difference whether to run or not, i.e., the above two effects exactly cancel out each other.

Use of invariant

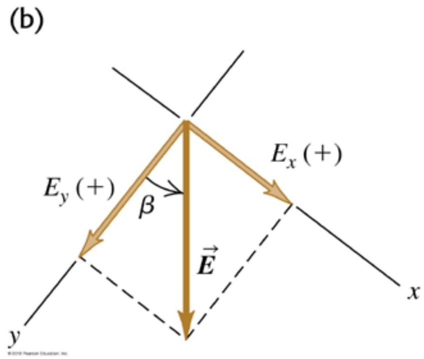
This conclusion is valid only if you ignore (A). The effect of (A) is that the slower you run, the wetter you get.

Dependence of conclusion on assumption

Clicker Questions

Q1.1

What are the x- and y-components of the vector $\vec{E}$?



A. $E_x = E \cos \beta$, $E_y = E \sin \beta$

B. $E_x = E \sin \beta$, $E_y = E \cos \beta$

C. $E_x = -E \cos \beta$, $E_y = -E \sin \beta$

D. $E_x = -E \sin \beta$, $E_y = -E \cos \beta$

E. $E_x = -E \cos \beta$, $E_y = E \sin \beta$

Q1.2

Which of the following statements is correct for *any* two vectors $\vec{A}$ and $\vec{B}$?

A. The magnitude of $\vec{A}+\vec{B}$ is $A + B$

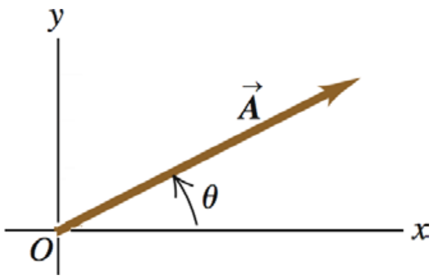
B. The magnitude of $\vec{A}+\vec{B}$ is $A - B$

C. The magnitude of $\vec{A}+\vec{B}$ is greater than or equal to $|A - B|$

D. The magnitude of $\vec{A}+\vec{B}$ is greater than the magnitude of $\vec{A}-\vec{B}$

E. The magnitude of $\vec{A}+\vec{B}$ is $\sqrt{A^2+B^2}$

Q1.3



The angle θ is measured counterclockwise from the positive *x*-axis as shown. For which of these vectors is θ greatest?

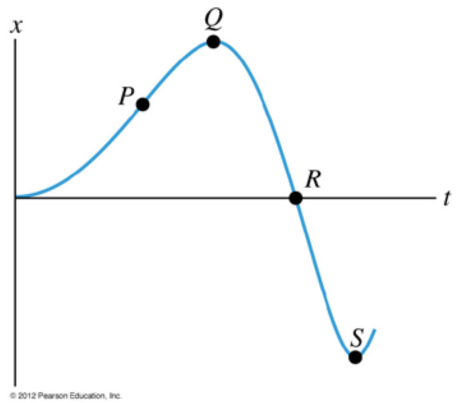
A. $24\hat{\boldsymbol{i}} + 18\hat{\boldsymbol{j}}$

B. $-24\hat{\boldsymbol{i}} - 18\hat{\boldsymbol{j}}$

C. $-18\hat{\boldsymbol{i}} + 24\hat{\boldsymbol{j}}$

D. $-18\hat{\boldsymbol{i}} - 24\hat{\boldsymbol{j}}$

Q2.3



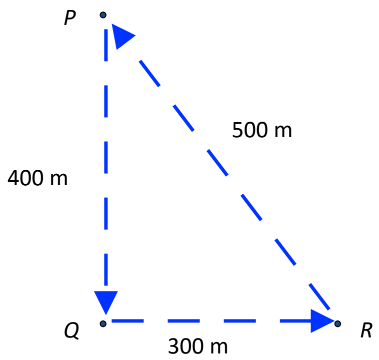
This is the *x*-*t* graph of the motion of a particle. Of the four points *P, Q, R,* and *S*, the acceleration a_x is greatest (most positive) at

A. point *P.* B. point *Q.* C. point *R.* D. point *S.*

E. not enough information in the graph to decide

Q3.1

A bicyclist starts at point *P* and travels around a triangular path that takes her through points *Q* and *R* before returning to point *P*. What is the magnitude of her net displacement for the entire round trip?



A. 100 m

B. 200 m

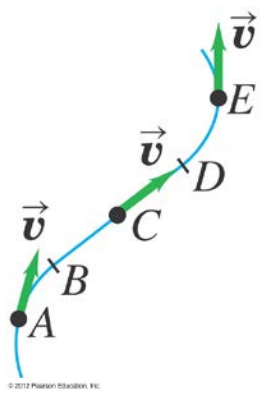
C. 600 m

D. 1200 m

E. zero

Q3.3

The motion diagram shows an object moving along a curved path at constant speed. At which of the points *A, C,* and *E* does the object have *zero* acceleration?



A. point *A* only

B. point *C* only

C. point *E* only

D. points *A* and *C* only

E. points *A, C,* and *E*

Ans. Q1.1) B, Q1.2) C, Q1.3) D, Q2.3) D, Q3.1) E, Q3.3) B

René Descartes

From Wikipedia, the free encyclopedia

"Descartes" redirects here. For other uses, see Descartes (disambiguation).

René Descartes (March 31, 1596 – February 11, 1650), also known as ***Renatus Cartesius*** (latinized form), was a highly influential French philosopher, mathematician, scientist, and writer. Dubbed the "Founder of Modern Philosophy" and the "Father of Modern Mathematics", much of subsequent western philosophy is a reaction to his writings, which have been closely studied from his time down to the present day. His influence in mathematics is also apparent, the Cartesian coordinate system used in plane geometry and algebra being named after him, and he was one of the key figures in the Scientific Revolution.

Descartes frequently contrasted his views with those of his predecessors. In the opening section of the *Passions of the Soul*, he goes so far as to assert that he will write on his topic "as if no one had written on these matters before". Nevertheless many elements of his philosophy have precedents in late Aristotelianism, the revived Stoicism of the 16th century, or in earlier philosophers like Augustine. In his natural philosophy, he differs from the Schools on two major points: first, he rejects the analysis of corporeal substance into matter and form; second, he rejects any appeal to ends—divine or natural—in explaining natural phenomena. In his theology, he insists on the absolute freedom of God's act of creation.

Descartes was a major figure in 17th century continental rationalism, later advocated by Baruch Spinoza and Gottfried Leibniz, and opposed by the empiricist school of thought, consisting of Hobbes, Locke, Berkeley, and Hume. Leibniz, Spinoza and Descartes were all versed in mathematics as well as philosophy, and Descartes and Leibniz contributed greatly to science as well. As the inventor of the Cartesian coordinate system, Descartes founded analytic geometry, that bridge between algebra and geometry crucial to the invention of calculus and analysis. Descartes's reflections on mind and mechanism began the strain of western thought that much later, impelled by the invention of the electronic computer and by the possibility of machine intelligence, blossomed into, e.g., the Turing test. His most famous statement is: *Cogito ergo sum* (French: *Je pense, donc je suis*; English: *I think, therefore I am*), found in §7 of *Principles of Philosophy* (Latin) and part IV of *Discourse on Method* (French).

Western Philosophy
17th-century philosophy

René Descartes

Name:	René Descartes
Birth:	March 31, 1596 (La Haye en Touraine (now Descartes), Indre-et-Loire, France)
Death:	February 11, 1650 (Stockholm, Sweden
School/tradition:	Cartesianism, Rationalism, Foundationalism
Main interests:	Metaphysics, Epistemology, Science, Mathematics
Notable ideas:	Cogito ergo sum, method of doubt, Mathesis Universalis, Cartesian coordinate system, Cartesian dualism, ontological argument for God's existence; regarded as a founder of Modern philosophy
Influences:	Plato, Aristotle, Anselm, Aquinas, Ockham, Suarez, Mersenne, Sextus Empiricus, Michel de Montaigne
Influenced:	Spinoza, Hobbes, Arnauld, Malebranche, Pascal, Locke, Leibniz, More, Kant, Husserl

For more information see http://en.wikipedia.org/wiki/Descartes