MOMENTUM, IMPULSE, AND COLLISIONS I

Intended Learning Outcomes – after this lecture you will learn:

1. impulse as an indication of the effect of a force which is in effect for a short time.
2. the relation between impulse and momentum change – the impulse momentum theorem.
3. conservation of momentum.
4. elastic, inelastic, and completely elastic collisions.
5. rocket propulsion

Textbook Reference: Ch 8.1 – 8.3, 8.6

Define **momentum** $\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$, SI unit: kg·m/s

$$\sum \vec{\boldsymbol{F}} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt}$$ Newton's second law in terms of momentum

Suppose net force $\sum \vec{\boldsymbol{F}}$ is constant:

Define **impulse** $\vec{\boldsymbol{J}} = \sum \vec{\boldsymbol{F}}\,(t_2 - t_1) = \sum \vec{\boldsymbol{F}}\,\Delta t$, SI unit: N·s

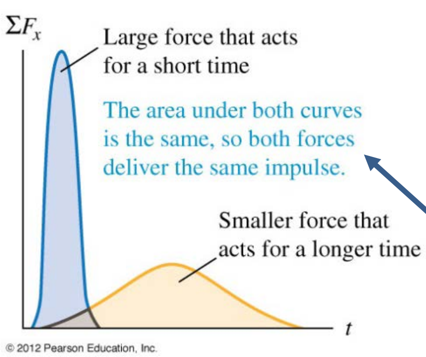
⚠ Most useful if the force is in effect for a short time, i.e., when Δt is small

From Newton's second law

$$\sum \vec{\boldsymbol{F}}\ (t_2 - t_1) = \vec{\boldsymbol{p}}_2 - \vec{\boldsymbol{p}}_1$$

i.e., $\boxed{\vec{\boldsymbol{J}} = \vec{\boldsymbol{p}}_2 - \vec{\boldsymbol{p}}_1}$ **impulse-momentum theorem**:
The change in momentum of a particle during a time interval equals the impulse of the net force acting on the particle during that interval

But in general, $\sum \vec{\boldsymbol{F}}$ is not constant!

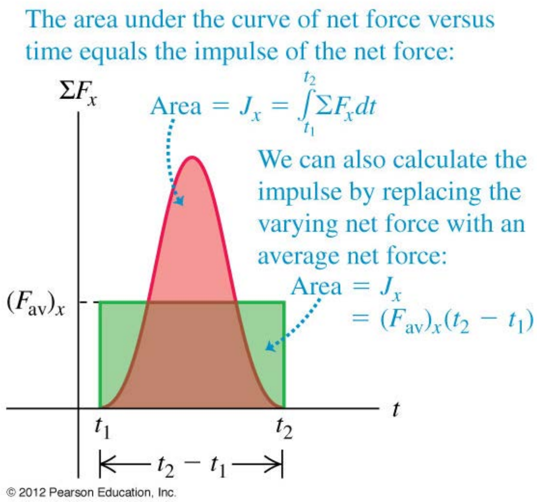


Define impulse as $\vec{\boldsymbol{J}} = \underbrace{\int_{t_1}^{t_2} \sum \vec{\boldsymbol{F}}\, dt}^{\text{area under graph}}$

$$= \int_{t_1}^{t_2} \frac{d\vec{\boldsymbol{p}}}{dt}\, dt = \vec{\boldsymbol{p}}_2 - \vec{\boldsymbol{p}}_1$$

impulse-momentum theorem again !!

⚠ different forces can give the same impulse



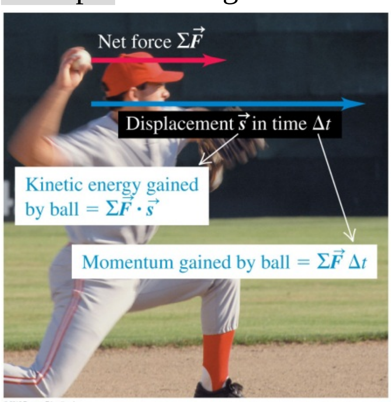
Define <u>average net force</u> $\vec{\boldsymbol{F}}_{\text{av}}$ as the constant force that gives the same impulse

$$\vec{\boldsymbol{J}} = \int_{t_1}^{t_2} \sum \vec{\boldsymbol{F}}\, dt = \vec{\boldsymbol{F}}_{\text{av}}(t_2 - t_1)$$

$$\Rightarrow \qquad \vec{\boldsymbol{F}}_{\text{av}} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \sum \vec{\boldsymbol{F}}\, dt$$

Geometric interpretation: $\vec{\boldsymbol{F}}_{\text{av}}$ is a constant force that has the same area under it as the variable force

Example Catching a ball

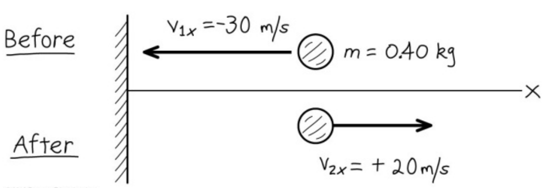


Case 1: 0.50 kg ball moving at 4.0 m/s, p = 2.0 N·s, K = 4.0 J
Case 2: 0.10 kg ball moving at 20 m/s, p = 2.0 N·s, K = 20 J
Which one is easier to catch?

Suppose your hand exerts the same force in both cases:
Both stop within the same time interval (∵ same impulse)
But case 2 stops at 5 times the distance (∵K is 5 times larger)

Example 8.2 P. 267 A ball hits a wall

A ball hits a wall and bounced back. Assume the ball is in contact with the wall for 0.010 s



impulse $J = m(v_{2x} - v_{1x})$
= (0.40 kg)(20 – (– 30) m/s) = 20 N·s

J is a vector, be careful about the direction

average force $F_{av} = \frac{J}{\Delta t} = \frac{20\text{ N·s}}{0.010\text{ s}} = 2000$ N

Demonstration: [velocity amplification]() – impulse can be transmitted from one object to another: impulse due to the normal reaction of the ground on the larger ball is transmitted to the smaller one.

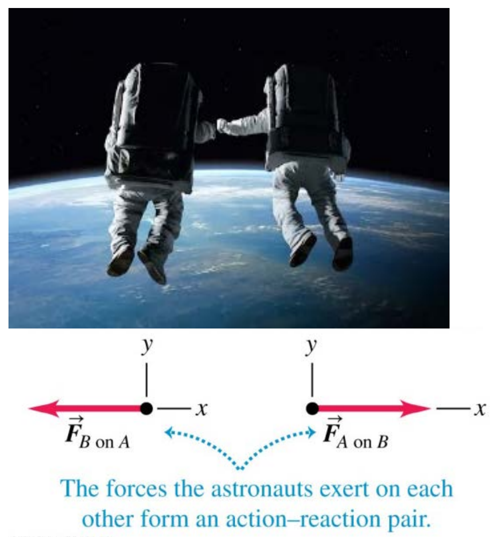
Question: Arrange the following cases in decreasing order of the magnitude of the average net force. In each case a 1000 kg automobile is along a straight east-west road.

a) It is initially moving east at 25 m/s and comes to a stop in 10 s.
b) It is initially moving east at 25 m/s and comes to a stop in 5 s.
c) It is initially at rest, and a 2000 N net force toward the east is applied to it for 10 s.
d) It is initially moving east at 25 m/s, and a 2000 N net force toward the west is applied to it for 10 s.
e) It is initially moving east at 25 m/s. Over a 30 s period, it reverses direction and ends up moving west at 25 m/s.

Answer: see inverted text on P. 269 of textbook

Some terminologies:

A **system** means a collection of bodies, e.g. the 2 astronauts form a system.
Internal forces are forces which individual bodies in the same system exert on others, e.g., the push between the astronauts.
⚠ Internal forces always exist as action and reaction pairs.
External forces are forces exerted on one or more bodies of the system by another object outside it, e.g., gravitational (if any) pull on the astronauts.
A system with no external forces is called an **isolated system**.



Consider a 2 body system,

Net force on A, $\vec{\boldsymbol{F}}_A = \frac{d\vec{\boldsymbol{p}}_A}{dt}$, net force on B, $\vec{\boldsymbol{F}}_B = \frac{d\vec{\boldsymbol{p}}_B}{dt}$

If it is an isolated system, $\vec{\boldsymbol{F}}_A$ and $\vec{\boldsymbol{F}}_B$ are action and reaction pair

$$\vec{\boldsymbol{F}}_A = -\vec{\boldsymbol{F}}_B \quad \Longrightarrow \quad \frac{d\vec{\boldsymbol{p}}_A}{dt} + \frac{d\vec{\boldsymbol{p}}_B}{dt} = 0$$

Define **total momentum** of the system $\vec{\boldsymbol{P}} = \vec{\boldsymbol{p}}_A + \vec{\boldsymbol{p}}_B \quad \Rightarrow \quad \frac{d\vec{\boldsymbol{P}}}{dt} = 0$, $\vec{\boldsymbol{P}}$ is constant or conserved

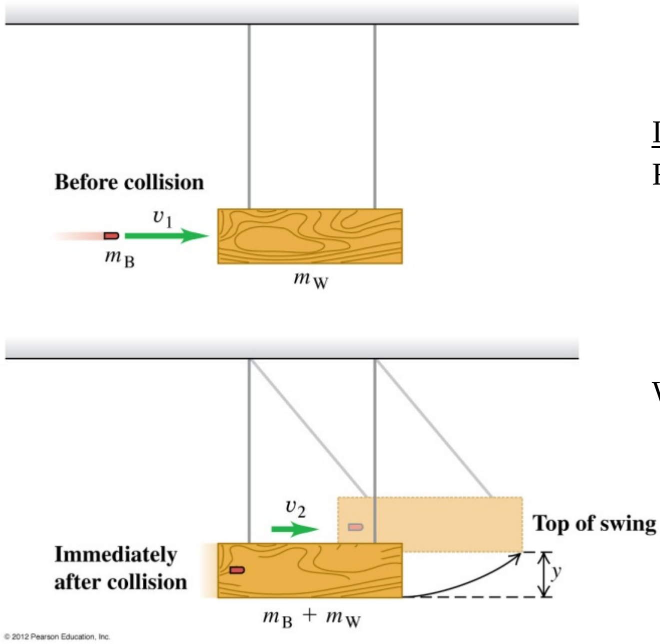
Question: A spring-loaded toy sits at rest on a horizontal, frictionless surface. When the spring releases, the toy breaks into three equal mass pieces, *A*, *B*, and *C*, which slide along the surface. *A* moves off in the negative *x* direction, while *B* moves off in the negative *y* direction.

a) What are the signs of the velocity components of *C* along the *x* and *y* directions?
b) Which of the three pieces is moving the fastest?

Answer: see inverted text on P. 273 of textbook.

⚠ Under no net external force, momentum always conserved, but not mechanical energy.
In an **elastic collision**, the KE is the same before and after the collision. (No change in PE during the impact.)
In an **inelastic collision**, the KE before the collision is larger.
In a **completely inelastic collision**, the bodies stick together after collision.

Example 8.8 P. 275 The ballistic pendulum – one way to measure the speed of a bullet



Incorrect solution:
From conservation of energy

$$\tfrac{1}{2}m_B v_1^2 = (m_B + m_W)gy$$

$$\Rightarrow v_1 = \sqrt{\frac{2(m_B + m_W)gy}{m_B}}$$

What is wrong?

Correct solution:
Conservation of momentum:

$$m_B v_1 = (m_B + m_W)v_2 \Rightarrow \quad v_2 = \frac{m_B v_1}{m_B + m_W}$$

Conservation of energy after collision:

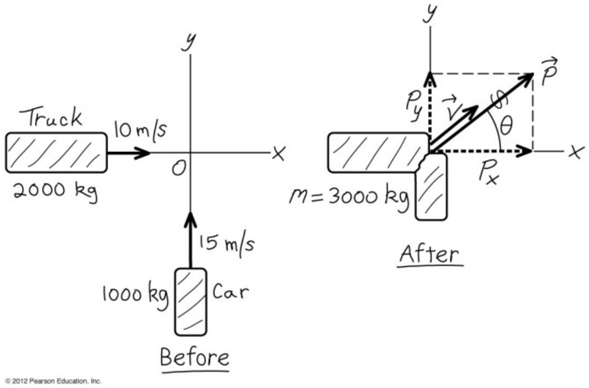
$$\tfrac{1}{2}(m_B + m_W)v_2^2 = (m_B + m_W)gy$$

$$\Rightarrow \frac{1}{2}\left(\frac{m_B v_1}{m_B + m_W}\right)^2 = gy \quad \Rightarrow \quad v_1 = \frac{m_B + m_W}{m_B}\sqrt{2gy}$$

Put in realistic numbers, $m_B = 5.00$ g, $m_W = 2.00$ kg, $y = 3.00$ cm, then $v_1 = 307$ m/s
KE before impact is ½(0.00500 kg)(307 m/s)^2 = 236 J
KE after impact is (0.00500 + 2.00 kg)(9.80 m/s^2)(0.0300 m) = 0.590 J
Most of the original KE is lost! What happens to this amount of energy?

Example 8.9 P 276 An automobile collision

A 1000-kg car traveling north collides with a 2000-kg truck traveling east. Just before the collision, the speed of the car is 15 m/s and that of the truck is 10 m/s. The two vehicles move away from the impact point as one. Find the velocity just after the collision.



By conservation of momentum:

$$(m_C + m_T)V_x = m_T v_{Tx} + m_C v_{Cx}$$

$$\Rightarrow \quad V_x = \frac{m_T\, v_{Tx}}{(m_C + m_T)} = 6.7 \text{ m/s}$$

$$(m_C + m_T)V_y = m_T v_{Ty} + m_C v_{Cy}$$

$$\Rightarrow \quad V_y = \frac{m_C\, v_{Cy}}{(m_C + m_T)} = 5.0 \text{ m/s}$$

$$\therefore V = \sqrt{V_x^2 + V_y^2} = 8.4 \text{ m/s}$$

$$\tan\theta = \frac{V_y}{V_x} = 0.75 \Rightarrow \quad \theta = 37^{\circ}$$

⚠ Are there external forces acting on the vehicles during impact? Yes! Then how to justify using conservation of momentum?

Weight and normal reaction: cancel each other, does not contribute to the net external force.

Friction: contribute to the net external force, but can we neglect it?

The friction f between the vehicles and the road has finite magnitude. Suppose the collision is ideal and takes time $\Delta t \to 0$, then the impulse is $f\Delta t \to 0$. Hence friction can be neglected. **In general, any external forces with bounded magnitude can be neglected in ideal collisions**.

Of course no collision is ideal in the real word. From the given speeds, it is reasonable to assume that the collision takes a time $\Delta t \sim 0.1$s. Suppose $\mu_k = 0.5$. Then the frictions are of the order $\mu_k mg \sim (0.5)(2000 \text{ kg})(10 \text{ m/s}^2) = 10^4$ N. The impulses are of the order $\sim 10^4 \text{ N} \times 0.1 \text{ s} = 10^3 \text{ N} \cdot \text{s}$. The initial momenta of the vehicles are of the order of $2 \times 10^4 \text{ N} \cdot \text{s}$. Therefore momentum is conserved approximately and we can simplify the question by neglecting friction.

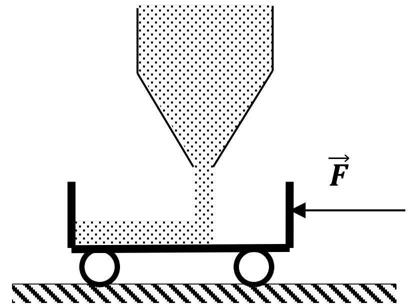
Question: For each situation, state whether the collision is elastic, inelastic, or completely inelastic.

a) You drop a ball from your hand. It collides with the floor and bounces back up so that it just reaches your hand.
b) You drop a different ball from your hand and let it collide with the ground. This ball bounces back up to half the height from which it was dropped.
c) You drop a ball of clay from your hand. When it collides with the ground, it stops.

Answer: refer to the inverted text on P. 277 of textbook

Example

A cart is pulled by a force F while sand is being dumped onto it by a fixed funnel at a rate 500 kg/s. Find the force F needed to keep the cart going at a constant speed 3.00 m/s.



$$dp = (m + dm)v - mv = vdm$$

$$F = \frac{dp}{dt} = v\frac{dm}{dt} = (3.00\ \mathrm{m/s})(500\ \mathrm{kg/s}) = 1.50\ \mathrm{kN}$$

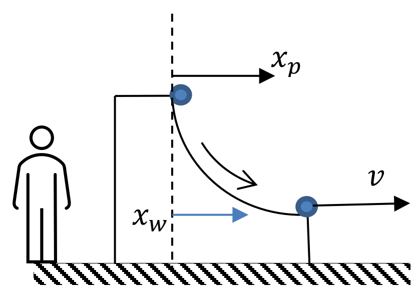
Example

A wedge of mass M with a quarter circular surface of radius R sits on a smooth floor. And mass m slides down from rest from the top of the circular surface. When it reaches the lowest point of the circular surface, how far has the wedge moved relative to the floor?

Let x_w and x_p be the horizontal displacements of the wedge and mass relative to the floor

$$M\frac{dx_w}{dt} + m\frac{dx_p}{dt} = 0 \quad \Rightarrow \quad M\int_0^{x_w} dx_w = -m\int_0^{x_p} dx_p$$

$$\Rightarrow \quad x_p = -\frac{M}{m}x_w$$



Horizontal displacement of mass relative to wedge is $x_p - x_w$, when the mass is at the bottom of the circular surface, $x_p - x_w = R$, therefore $x_w = -\left(\frac{m}{m+M}\right)R$.

⚠ Does it matter whether there is friction along the circular surface?

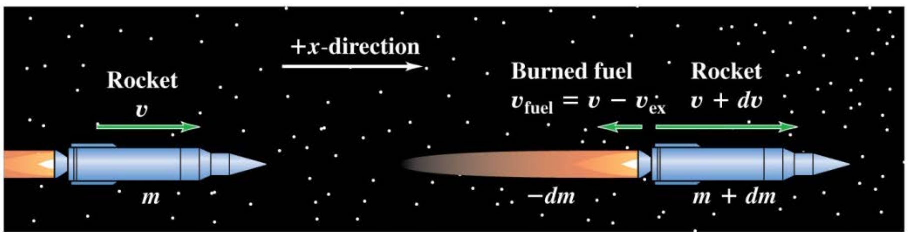
⚠ Does it matter if the surface is straight instead of circular?

Rocket Propulsion – an example of variable mass

Propeller/jet engines in airplanes rely on reaction by air to push the plane. Can they work in space where there is no air to provide reaction?

Yes, first proposed by Robert Goddard, based on *conservation of momentum*

During an interval dt, rocket ejects an amount $-dm$ (⚠ $dm < 0$) of burned fuel with speed v_{ex} relative to the rocket.



$$\underbrace{mv}_{\text{initial momentum}} = \underbrace{(m+dm)(v+dv)}_{\text{final momentum of rocket}} + \underbrace{(-dm)(v-v_{\text{ex}})}_{\text{final momentum of burned fuel}}$$

$$\Rightarrow \quad 0 = mdv + \underbrace{dmdv}_{\sim 0} + v_{\text{ex}}dm$$

$$\Rightarrow \quad m\frac{dv}{dt} = -v_{\text{ex}}\frac{dm}{dt} = \text{thrust (force) on the rocket}$$

initial condition: at $t = 0$, $v = v_0$, $m = m_0$

acceleration $$a = \frac{dv}{dt} = -\frac{v_{\text{ex}}}{m}\frac{dm}{dt} \Longrightarrow dv = -v_{\text{ex}}\frac{dm}{m} \Rightarrow v = -v_{\text{ex}}\ln m + C$$

$$\boxed{v - v_0 = v_{\text{ex}}\ln\frac{m_0}{m}}$$

Example 8.15 and 8.16 P. 286

A rocket ejects burned fuel at a constant rate. In the first second it ejects 1/120 of its initial mass m_0 at a relative speed of 2400 m/s.

The rocket's initial acceleration is

$$a = -\frac{v_{\text{ex}}}{m}\frac{dm}{dt} = -\frac{2400\ \text{m/s}}{m_0}\left(\frac{m_0/120}{1\ \text{s}}\right) = 20\ \text{m/s}^2$$

If the total mass of fuel is $\frac{3}{4}m_0$, then it will be fully consumed at $\frac{3}{4}/\frac{1}{120} = 90$ s, the rocket's speed

$$v_{\text{final}} = v_0 + v_{\text{ex}}\ln\frac{m_0}{m} = 0 + (2400\ \text{m/s})\left(\ln\frac{m_0}{m_0/4}\right) = 3327\ \text{m/s}$$

Clicker Questions

Q8.2

You are testing a new car using crash test dummies. Consider two ways to slow the car from 90 km/h (56 mi/h) to a complete stop:

(i) You let the car slam into a wall, bringing it to a sudden stop.

(ii) You let the car plow into a giant tub of gelatin so that it comes to a gradual halt.

In which case is there a greater *impulse* of the net force on the car?

A. In case (i).

B. In case (ii).

C. The impulse is the same in both cases.

D. The answer depends on how rigid the front of the car is.

E. The answer depends on how rigid the front of the car is and on the mass of the car.

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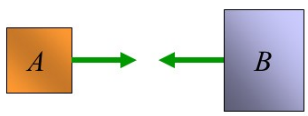
Q8.5

Two objects with different masses collide with and *stick* to each other. Compared to *before* the collision, the system of two objects *after* the collision has



A. the same amount of total momentum and the same total kinetic energy.

B. the same amount of total momentum but less total kinetic energy.

C. less total momentum but the same amount of total kinetic energy.

D. less total momentum and less total kinetic energy.

E. Not enough information is given to decide.

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Q8.7

Block A has mass 1.00 kg and block B has mass 3.00 kg. The blocks collide and stick together on a level, frictionless surface. After the collision, the kinetic energy (KE) of block A is

A. one-ninth the KE of block B.

B. one-third the KE of block B.

C. three times the KE of block B.

D. nine times the KE of block B.

E. the same as the KE of block B.

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Q8.9

Block A on the left has mass 1.00 kg. Block B on the right has mass 3.00 kg. The blocks are forced together, compressing the spring. Then the system is released from rest on a level, frictionless surface. After the blocks are released, how does K_A (the kinetic energy of block A) compare to K_B (the kinetic energy of block B)?

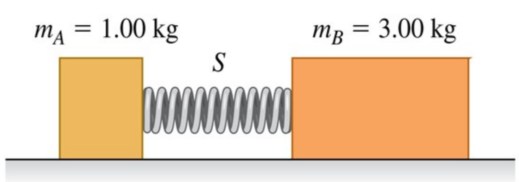
A. $K_A = K_B/9$

B. $K_A = K_B/3$

C. $K_A = K_B$

D. $K_A = 3K_B$

E. $K_A = 9K_B$



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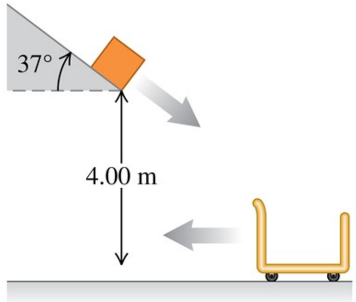
Q8.10

An open cart is rolling to the left on a horizontal surface. A package slides down a chute and lands in the cart. Which quantity or quantities have the same value just *before* and just *after* the package lands in the cart?



A. the horizontal component of total momentum

B. the vertical component of total momentum

C. the total kinetic energy

D. two of A, B, and C

E. all of A, B, and C

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Ans: Q8.2) C, Q8.5) B, Q8.7) B, Q8.9) D, Q8.10) A

Robert H. Goddard

From Wikipedia, the free encyclopedia

For other persons with the name Robert Goddard, see Robert Goddard (disambiguation).

Robert Hutchings Goddard (October 5, 1882 – August 10, 1945)[1] was an American engineer, professor, physicist, and inventor who is credited with creating and building the world's first liquid-fueled rocket.[2] Goddard successfully launched his rocket on March 16, 1926, which ushered in an era of space flight and innovation. He and his team launched 34 rockets[3] between 1926 and 1941, achieving altitudes as high as 2.6 km (1.6 mi) and speeds as fast as 885 km/h (550 mph).[3]

Goddard's work as both theorist and engineer anticipated many of the developments that would make spaceflight possible.[4] He has been called the man who ushered in the Space Age.[5]:xiii Two of Goddard's 214 patented inventions, a multi-stage rocket (1914), and a liquid-fuel rocket (1914), were important milestones toward spaceflight.[6] His 1919 monograph *A Method of Reaching Extreme Altitudes* is considered one of the classic texts of 20th-century rocket science.[7][8] Goddard successfully pioneered modern methods such as two-axis control (gyroscopes and steerable thrust) to allow rockets to control their flight effectively.

Although his work in the field was revolutionary, Goddard received little public support, moral or monetary, for his research and development work.[9]:92, 93 He was a shy person, and rocket research was not considered a suitable pursuit for a physics professor.[10]:12 The press and other scientists ridiculed his theories of spaceflight. As a result, he became protective of his privacy and his work. He preferred to work alone also because of the aftereffects of a bout with tuberculosis.[10]:13

Years after his death, at the dawn of the Space Age, Goddard came to be recognized as one of the founding fathers of modern rocketry, along with Robert Esnault-Pelterie, Konstantin Tsiolkovsky, and Hermann Oberth.[11][12][13] He not only recognized early on the potential of rockets for atmospheric research, ballistic missiles and space travel but also was the first to scientifically study, design, construct and fly the precursory rockets needed to eventually implement those ideas.[14]

NASA's Goddard Space Flight Center was named in Goddard's honor in 1959. He was also inducted into the International Aerospace Hall of Fame in 1966, and the International Space Hall of Fame in 1976.[15]

Robert H. Goddard

Robert Hutchings Goddard (1882–1945)

Born	October 5, 1882[1] Worcester, Massachusetts, U.S.
Died	August 10, 1945 (aged 62)[1] Baltimore, Maryland, U.S
Nationality	American
Education	Worcester Polytechnic Institute Clark University
Occupation	Professor, aerospace engineer, physicist, inventor
Known for	First liquid-fueled rocket
Spouse(s)	Esther Christine Kisk (m. 1924–1945)
Awards	Congressional Gold Medal (1959) Langley Gold Medal (1960) Daniel Guggenheim Medal (1964)

For more information see https://en.wikipedia.org/wiki/Robert_H._Goddard