# NEWTON'S LAWS OF MOTION I

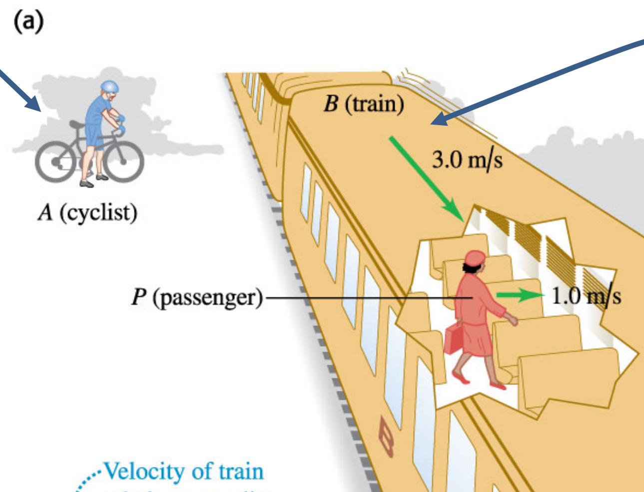
Intended Learning Outcomes – after this lecture you will learn:

1. the concept of frame of reference and relative velocity
2. to use the Newton's three laws of motion to solve problems.
3. the concept of inertial frame of reference.
4. the meanings of apparent weight and weightlessness.

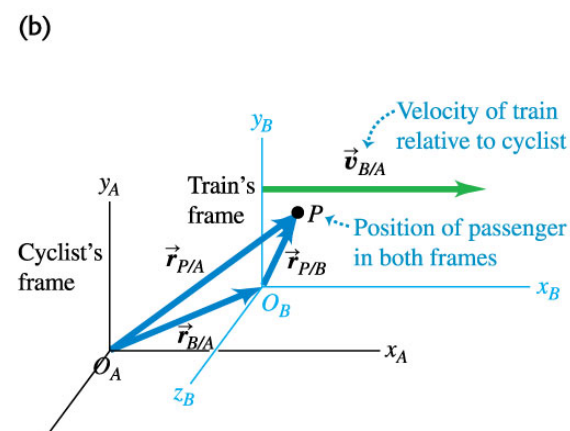
Textbook Reference: Ch 3.5, 4, 5.1, 5.2

**Reference frame –** An observer making observation/measurement forms a reference frame. e.g.

Reference Frame $A$
$1^{st}$ observer $A$, standing still on the platform




Reference Frame $B$
$2^{nd}$ observer $B$, sitting still inside the train

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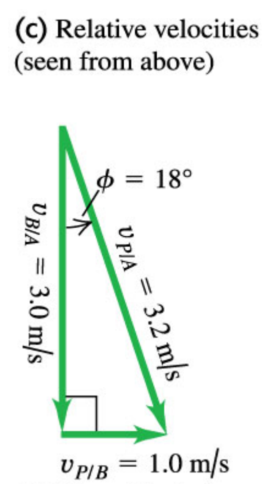
$$\vec{r}_{P/A} = \vec{r}_{B/A} + \vec{r}_{P/B}$$

$$\Rightarrow \vec{v}_{P/A} = \vec{v}_{B/A} + \vec{v}_{P/B}$$

velocity of $P$ measured in frame $A$

velocity of frame $B$ measured in

velocity of $P$ measured in frame $B$

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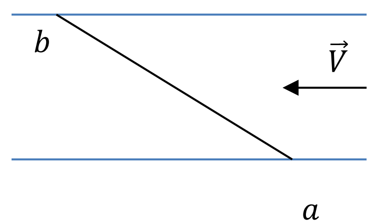
Demonstration: [Relative motion]

**Example**

Two boats A and B can sail in still water with speeds $u_A$ and $u_B$ respectively. They are initial at two points $a$ and $b$ on opposite sides of a river. They start sailing at the same time with velocities $\vec{u}_A$ and $\vec{u}_B$ respectively relative to still water. Assuming water flow with uniform constant velocity $\vec{V}$ relative to the ground as shown, how should A and B choose the directions of $\vec{u}_A$ and $\vec{u}_B$ such that the boats meet somewhere in the river, and what is the shortest time when this occurs?



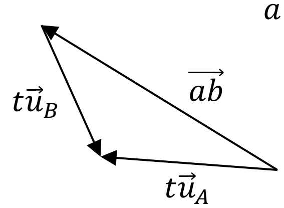

Work in the reference frame of water. When they meet,

$$t\vec{u}_A - t\vec{u}_B = \overrightarrow{ab} \quad \Rightarrow \quad t = \frac{|\overrightarrow{ab}|}{|\vec{u}_A - \vec{u}_B|}$$

i.e., condition to meet is $\vec{u}_A - \vec{u}_B$ along the direction of $\overrightarrow{ab}$.

Time is minimum when $|\vec{u}_A - \vec{u}_B| = |u_A + u_B|$,

$$t_{\min} = \frac{|\overrightarrow{ab}|}{|u_A + u_B|}$$



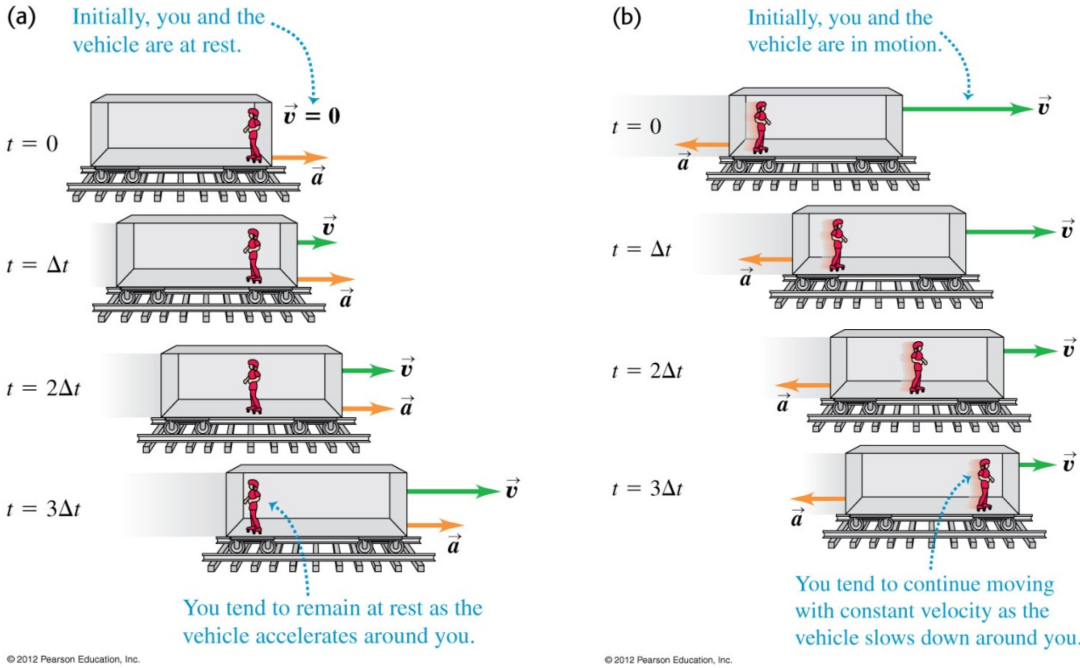

**Newtons' first law of motion**

A body acted on by no net force moves with constant velocity

$\sum \vec{\boldsymbol{F}} = 0$ body in equilibrium

⚠ may be moving

Inertial Frame of Reference

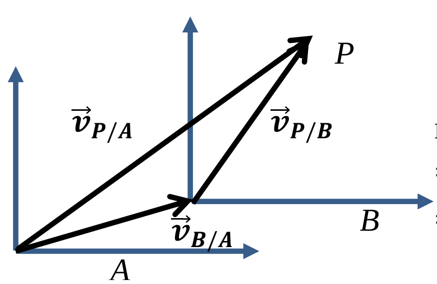



Passenger (in roller skate) accelerates inside the train, but net force is zero. Violate Newton's first law?? The train is not an inertial frame.

Definition: a frame of reference in which Newton's first law is valid is called an <u>inertial frame</u>

Note: 1.Is the earth an inertial frame? Only approximately

2. Given an inertial frame $A$,




$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

no net force

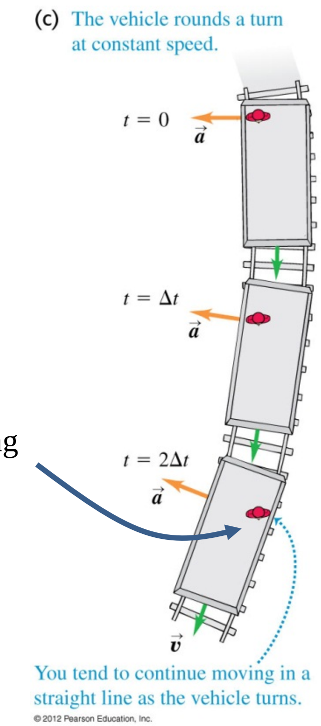
$\Rightarrow$ $\vec{v}_{P/A}$ constant

$\Rightarrow$ $\vec{v}_{P/B}$ constant provided $\vec{v}_{B/A}$ constant

any frame $B$ moving with constant $\vec{v}_{B/A}$ (can be zero) is also an inertial frame

⚠ In a non-inertial frame of reference, may feel like being acted on by a (non-existing) force.

feel like a force pushing you aside




Question: In which of the following situations is there zero net force on the body?

a) an airplane flying due north at a steady speed and at a constant altitude, assuming that the earth is flat and is an inertial frame;
b) a car driving straight up a hill with constant slope at constant speed;
c) a hawk circling at constant speed and constant height above an open field;
d) a box with slick, frictionless surfaces in the back of a truck as the truck accelerates forward on a level road at constant acceleration.

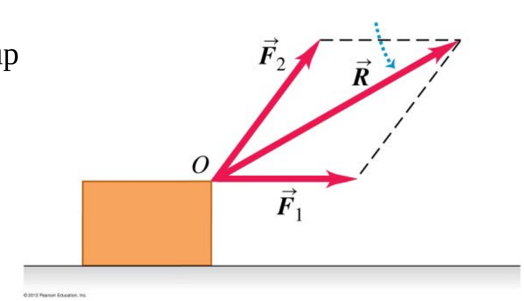
Answer: see inverted text on P. 135 of textbook

**Superposition of forces**

Forces are vectors and can be added up

$\vec{R}$ is called the <u>net</u> or <u>resultant</u> force

The SI unit of force is newton,
1 N = 1 kg·m/s$^2$



## Newton's second law

If a *net* external force acts on a body, the body accelerates according to

$\sum \vec{F} = m\vec{a}$ <u>inertial mass</u> – how reluctant the body is to change its velocity

⚠ Make sure $\sum \vec{F}$ is the net force, see Demonstration: [fan car]

| |
|---|
| Question: Suppose an astronaut landed on a planet where $g$= 19.6 m/s$^2$. Compared to earth, it would be (easier / harder / just as easy) for her to walk around. It would be (easier / harder / just as easy) for her to catch a ball that is moving horizontally at 12 m/s.<br>Answer: see inverted text on P. 143 of textbook |

## Newton's third law of motion

If body $A$ exerts a force on body $B$ (an "action"), then body $B$ exerts a force on body $A$ (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on different bodies.

| |
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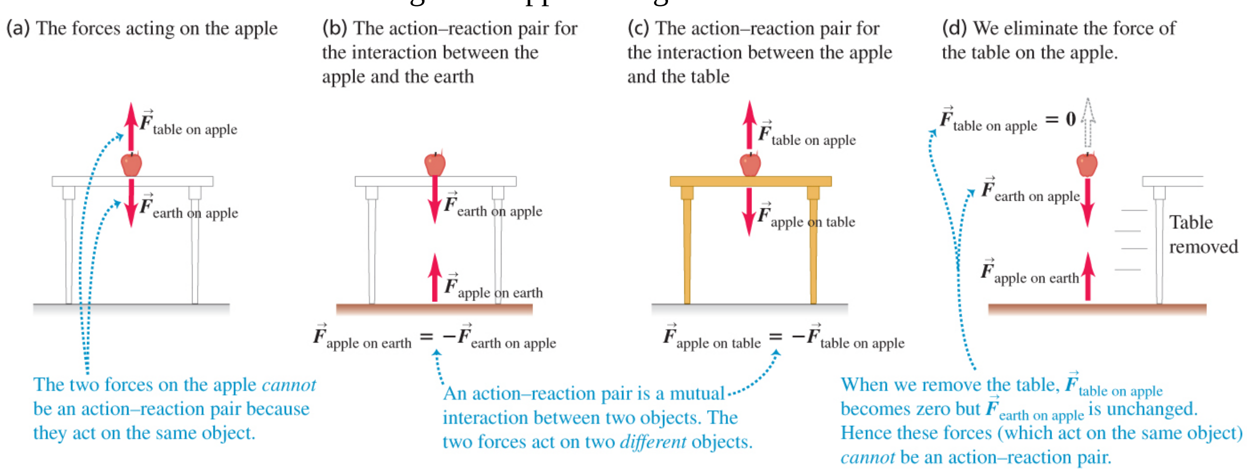
| Question: Since action and reaction are equal and opposite, should they cancel each other? |

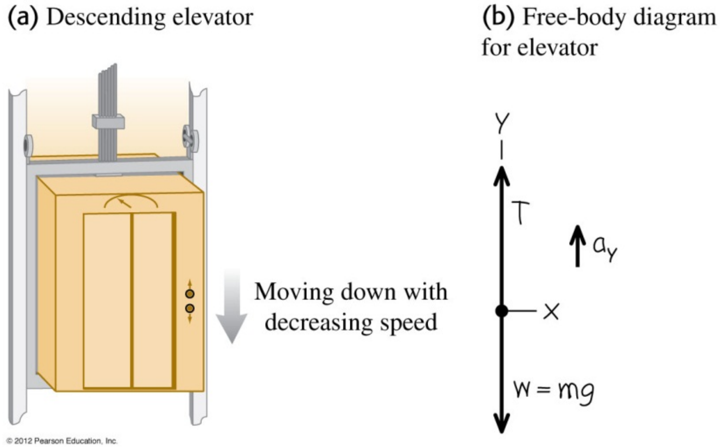
Example 4.9 P. 144

Action and reaction forces acting on an apple sitting on a table




| |
|---|
| Question: The buoyance force experienced by a scuba diver is one half of an action-reaction pair. What force is the other half of this pair?<br>a) the weight of the diver;<br>b) the forward thrust force;<br>c) the backward drag force;<br>d) the downward force that the swimmer exerts on the water. |

Example  5.8 and 5.9 P. 164: Tension in an elevator cable
An elevator, mass 800 kg, moving downwards at 10.0 m/s
If it comes to a stop in a distance of 25.0 m




To find deceleration $a_y$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

0 m/s  -10.0 m/s  -25.0 m

$$\Longrightarrow a_y = 2.00 \text{ m/s}^2$$

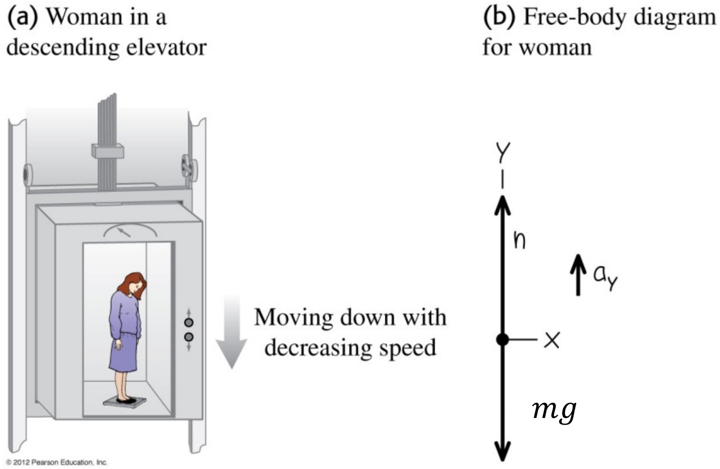
Tension in the cable

$$\sum F_y = T - w = ma_y$$

$$\Rightarrow \quad T = m\left(g + a_y\right) = 9440 \text{ N}$$

Apparent weight and weightlessness

A person standing on a scale in an elevator, reading of the scale is $n$




$$\sum F_y = n - mg = ma_y$$

$\Rightarrow \quad n = m(g + a_y)$ apparent weight of the person

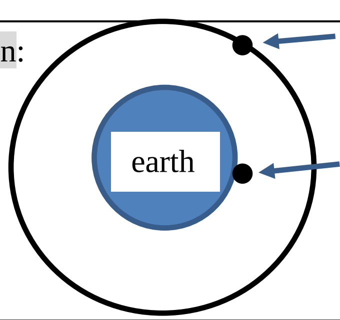
What if free falling, i.e., $a_y = -g$?

$n = 0$  apparent weightlessness

Her feet effectively lose contact with the floor

Question: One of your very clever classmates says, "If your elevator has a broken cable and is falling freely to the ground, you can save yourself by jumping up at the instant the elevator hits the ground." Will this work?

Question:

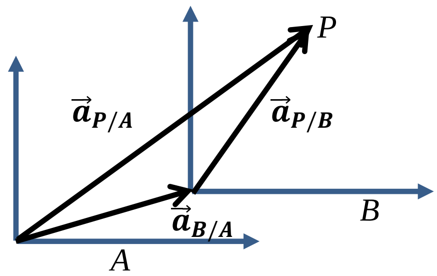



astronaut orbiting around the earth

person standing on the surface of the earth

Both are under the gravitational attraction of the earth. Why does the person have weight but the astronaut is weightless?

## Working in a Non-inertial Frame of Reference




$$\vec{a}_{P/A} = \vec{a}_{P/B} + \vec{a}_{B/A}$$

Equation of motion by $A$ is $\vec{F} = m\vec{a}_{P/A}$

But the equation of motion by $B$ is NOT $\vec{F} = m\vec{a}_{P/B}$ because it is a non-inertial frame. To get the correct one, start from the equation of motion by $A$

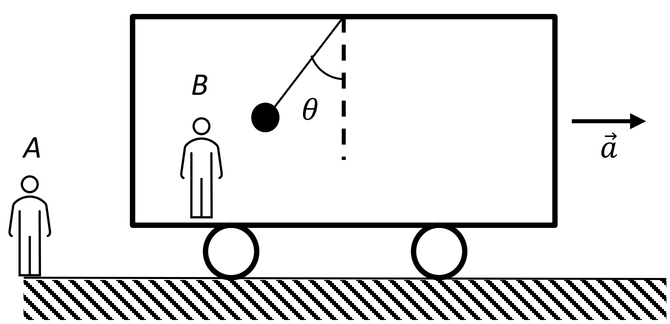
$$\vec{F} = m\vec{a}_{P/A} = m(\vec{a}_{P/B} + \vec{a}_{B/A})$$

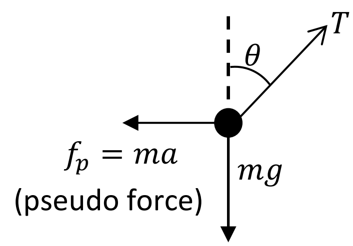
$$\Rightarrow \quad \vec{F} + \vec{f}_p = m\vec{a}_{P/B}$$

$\vec{f}_p = -m\vec{a}_{B/A}$ is a pseudo force

Conclusion: in a non-inertial frame with acceleration $\vec{a}$ (relative an inertial frame such as the ground), a pseudo (or fictitious, or inertial) force $\vec{f}_p = -m\vec{a}$ is needed to make the Newton's second law hold.

E.g., a vertical mass hanging from the ceiling of an accelerating car. If the acceleration is constant, the mass does not move as observed by $B$, but at an inclined angle.




Equations of motion by $A$
(mass is accelerating)

$$T\sin\theta = ma$$
$$T\cos\theta - mg = 0$$

$\longleftrightarrow$ identical

Equations of motion by $B$
(as if mass is in equilibrium)

$$\left.\begin{aligned} T\sin\theta - f_p &= 0 \\ T\cos\theta - mg &= 0 \end{aligned}\right\} \Rightarrow \quad \theta = \tan^{-1}\frac{a}{g}$$

⚠ The concept of inertial force is redundant, but is convenient in more complicated non-inertial frames such as rotating frames.

⚠ Inertial force has no reaction pair, and never satisfy the Newton's third law.

Example

A wedge of mass $M$ sits on the ground. A small mass $m$ is allowed to slide down along its inclined surface. Assuming all contact surfaces are smooth, find the acceleration of the wedge relative to the ground.

Work in the frame of the wedge. For the wedge:

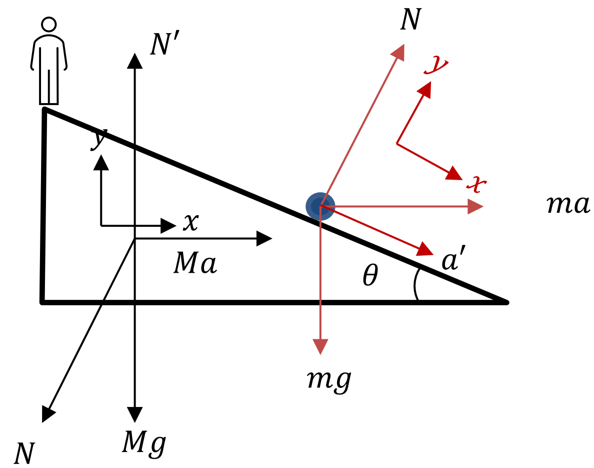
$$Ma - N\sin\theta = 0$$
$$N' - Mg - N\cos\theta = 0$$

For the mass:

$$ma\cos\theta + mg\sin\theta = ma'$$
$$N + ma\sin\theta - mg\cos\theta = 0$$

We get

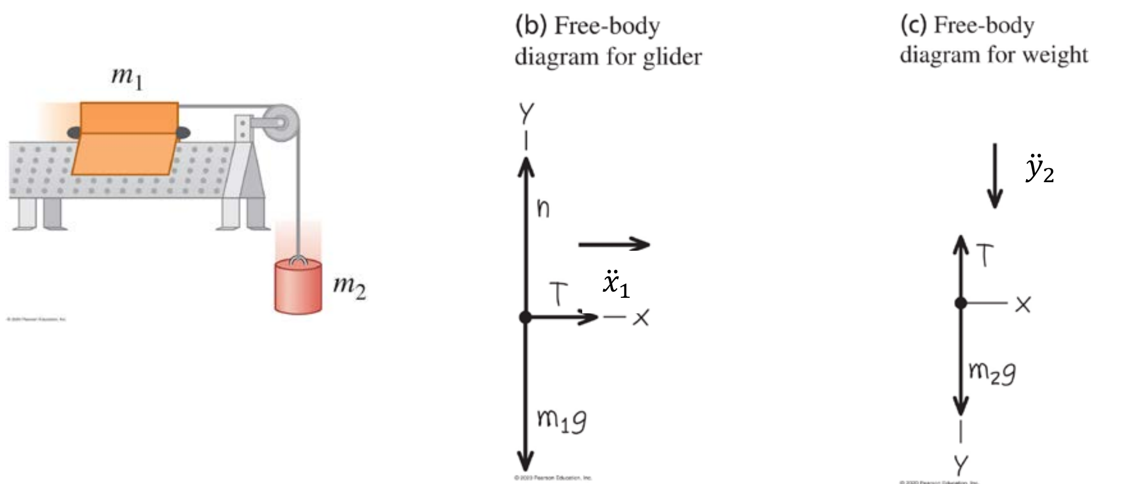
$$a = \frac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}$$




Check: go to the ground (an inertial frame) and write down exactly the same equations for the the accelerating mass.

**Working with geometric constraints**

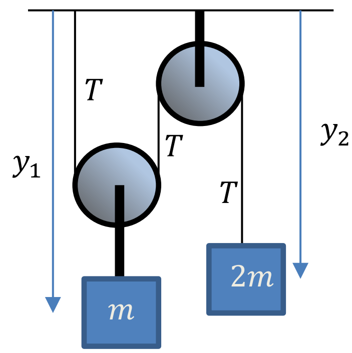
For example, an inextensible massless string (Example 5.12 P. 140)




$T = m_1\ddot{x}_1$ $\qquad$ $m_2 g - T = m_2\ddot{y}_2$

Constraint: $\Delta x_1 = \Delta y_2 \Rightarrow \dot{x}_1 = \dot{y}_2$ and $\ddot{x}_1 = \ddot{y}_2 = a \Rightarrow a = \dfrac{m_2}{m_1 + m_2} g$

Example an Atwood machine with massless pulley and inextensible string




Equations of motion of the masses:

$$mg - 2T = m\ddot{y}_1$$
$$2mg - T = 2m\ddot{y}_2$$

Geometric constraint: $\Delta y_2 = -2\Delta y_1 \quad \Rightarrow \quad \ddot{y}_2 = -2\ddot{y}_1$

We get $\ddot{y}_1 = -g/3$, $\ddot{y}_2 = 2g/3$

## Clicker Questions:

Q3.12

The pilot of a light airplane with an airspeed of 200 km/h wants to fly due west. There is a strong wind of 120 km/h blowing from the north.

If the pilot points the nose of the airplane north of west so that her ground track is due west, what will be her ground speed?

A. 80 km/h

B. 120 km/h

C. 160 km/h

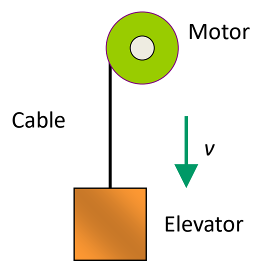
D. 180 km/h

E. It would impossible to fly due west in this situation.

Q4.6

An elevator is being lowered at constant speed by a steel cable attached to an electric motor. There is no air resistance, nor is there any friction between the elevator and the walls of the elevator shaft.

The upward force exerted on the elevator by the cable has the same magnitude as the force of gravity on the elevator, but points in the opposite direction. Why?



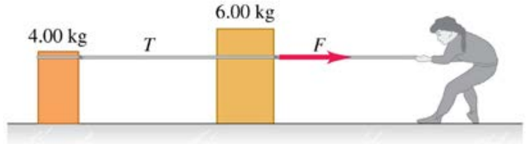

A. Newton's first law

B. Newton's second law

C. Newton's third law

Q4.12

A woman pulls on a 6.00-kg crate, which in turn is connected to a 4.00-kg crate by a light rope. It is given that both crates have non-zero accelerations and the light rope remains taut.




Compared to the 6.00-kg crate, the lighter 4.00-kg crate

A. is subjected to the same net force and has the same acceleration.

B. is subjected to a smaller net force and has the same acceleration.

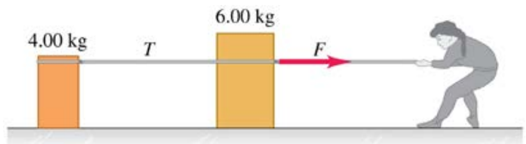
C. is subjected to the same net force and has a smaller acceleration.

D. is subjected to a smaller net force and has a smaller acceleration.

E. none of the above

Q4.14

A woman pulls on a 6.00-kg crate, which in turn is connected to a 4.00-kg crate by a light rope. It is given that both crates have non-zero accelerations and the light rope remains taut.




A. the 6.00-kg crate exerts more force on the 4.00-kg crate than the 4.00-kg crate exerts on the 6.00-kg crate.

B. the 6.00-kg crate exerts less force on the 4.00-kg crate than the 4.00-kg crate exerts on the 6.00-kg crate.

C. the 6.00-kg crate exerts as much force on the 4.00-kg crate as the 4.00-kg crate exerts on the 6.00-kg crate.

Ans: Q3.12) C, Q4.6) A, Q4.12) B, Q4.14) C

# Isaac Newton

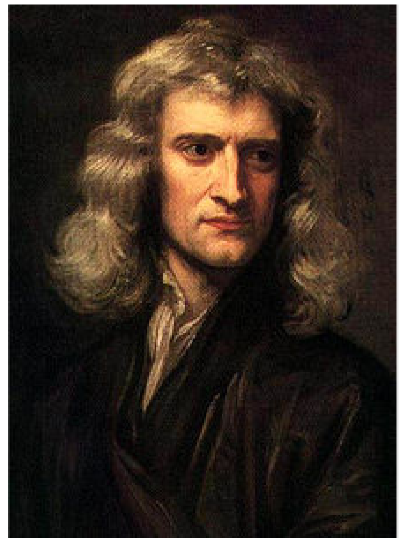
From Wikipedia, the free encyclopedia

**Sir Isaac Newton** PRS MP (25 December 1642 – 20 March 1727 [NS: 4 January 1643 – 31 March 1727])[1] was an English physicist, mathematician, astronomer, natural philosopher, alchemist and theologian, who has been "considered by many to be the greatest and most influential scientist who ever lived."[7] His monograph *Philosophiæ Naturalis Principia Mathematica*, published in 1687, lays the foundations for most of classical mechanics. In this work, Newton described universal gravitation and the three laws of motion, which dominated the scientific view of the physical universe for the next three centuries. Newton showed that the motions of objects on Earth and of celestial bodies are governed by the same set of natural laws, by demonstrating the consistency between Kepler's laws of planetary motion and his theory of gravitation, thus removing the last doubts about heliocentrism and advancing the Scientific Revolution.

The *Principia* is generally considered to be one of the most important scientific books ever written, due, independently, to the specific physical laws the work successfully described, and for the style of the work, which assisted in setting standards for scientific publication down to the present time. Newton built the first practical reflecting telescope[8] and developed a theory of colour based on the observation that a prism decomposes white light into the many colours that form the visible spectrum. He also formulated an empirical law of cooling and studied the speed of sound. In mathematics, Newton shares the credit with Gottfried Leibniz for the development of differential and integral calculus. He also demonstrated the generalised binomial theorem, developed Newton's method for approximating the roots of a function, and contributed to the study of power series.

Newton, although an unorthodox Christian, was deeply religious, and wrote more on Biblical hermeneutics and occult studies than on science and mathematics. Newton secretly rejected Trinitarianism, and feared being accused of refusing holy orders.[9]

**Sir Isaac Newton**



Godfrey Kneller's 1689 portrait of Isaac Newton (age 46)

| | |
|---|---|
| **Born** | 25 December 1642<br>[NS: 4 January 1643][1]<br>Woolsthorpe-by-Colsterworth<br>Lincolnshire, England |
| **Died** | 20 March 1727 (aged 84)<br>[NS: 31 March 1727][1]<br>Kensington, Middlesex, England |
| **Residence** | England |
| **Nationality** | English |
| **Fields** | Physics, mathematics, astronomy, natural philosophy, alchemy, Christian theology |
| **Institutions** | University of Cambridge<br>Royal Society<br>Royal Mint |
| **Alma mater** | Trinity College, Cambridge |
| **Academic advisors** | Isaac Barrow[2]<br>Benjamin Pulleyn[3][4] |
| **Notable students** | Roger Cotes<br>William Whiston |

For more information see http://en.wikipedia.org/wiki/Isaac_Newton