PHYS1312 Homework, Part 2

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Problem 1.

Imagine a mass M hanging from a ceiling suspended on two springs S_1, S_2 with coefficients k_1 and k_2. We will show that the resulting string coefficient (as the system were one large spring) is $k_1 + k_2$. Divide the mass M into two smaller masses m_1, m_2 such that they, suspended from S_1 and S_2, produce the same extension x in both springs. We write Newton's third law together with Hooke's law:

$$m_1 g = k_1 x,$$
$$m_2 g = k_2 x.$$

Now, if we tie the masses together with a rope, there will be no change in their positions or in the tension of the springs. However, the result will be M suspended from both of the springs simultaneously with extension x. We again write Newton's third law and Hooke's law for the system:

$$Kx = Mg = (m_1 + m_2)g = m_1 g + m_2 g = k_1 x + k_2 x = (k_1 + k_2)x,$$
$$K = k_1 + k_2,$$

and we are done.

Problem 2.

The Sierpinski triangle consists of three smaller triangles, each similar to the whole, but scaled down by a factor of $1/2$. Let the mass of the original triangle be M. Then, the mass of each of the three sub-triangles will be $m = M/3$.
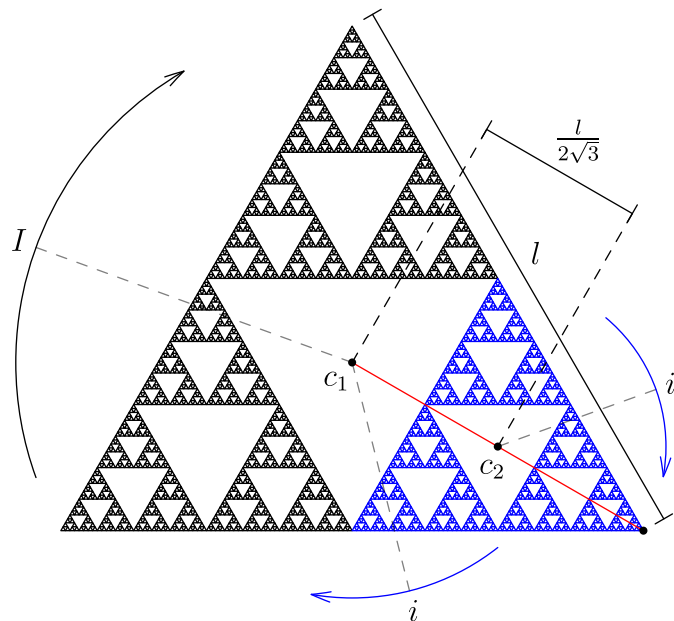

Figure 1: The Sierpinski triangle and moments of inertia at different points

Note that the moment of inertia of the entire triangle about c_1 (denoted I) is three times the moment of inertia of the blue sub-triangle about c_1 (denoted i). In turn, i can be expressed in terms of the moment about the center of mass of the blue triangle, i', as follows:

$$\frac{I}{3} = i = i' + md^2 = i' + \frac{M}{3} \cdot \left(\frac{l}{2\sqrt{3}}\right)^2 = i' + \frac{Ml^2}{36}. \tag{1}$$

On the other hand, the big triangle has double the size of the blue one, and triple the mass. Since the moment of inertia around the center of mass is proportional to mass and to the square of side length (by definition), we have

$$I = 3 \cdot 2^2 \cdot i' = 12i'. \tag{2}$$

Substituting Equation 2 into Equation 1, we have

$$\frac{I}{3} = \frac{I}{12} + \frac{Ml^2}{36},$$
$$\frac{I}{4} = \frac{Ml^2}{36},$$
$$I = \frac{1}{9}Ml^2,$$

and we are done.

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