

## New CA Classifiers (random colours)

## Wolfram Classes of ECAs

```
In[1]:= CAClasses = {1, 2, 2, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 3, 2, 2, 2, 2, 2,
2, 2, 3, 2, 1, 2, 2, 2, 2, 2, 2, 2, 1, 3, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2, 2, 4, 2, 2,
2, 2, 2, 3, 2, 2, 2, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
3, 2, 2, 3, 3, 2, 2, 2, 2, 2, 1, 3, 2, 2, 2, 3, 3, 2, 2, 3, 3, 3, 2, 2, 4, 2, 2, 2, 2, 2,
2, 2, 2, 2, 2, 3, 3, 3, 2, 4, 2, 3, 2, 1, 3, 2, 2, 2, 2, 2, 3, 1, 4, 2, 2, 2, 2, 2, 2, 2,
2, 2, 3, 4, 2, 3, 3, 3, 2, 3, 2, 2, 2, 2, 2, 2, 1, 3, 2, 2, 2, 3, 2, 2, 1, 3, 2, 2,
2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 2, 2, 2, 2, 2, 2, 2, 1, 4, 2, 3, 2, 2, 2, 2,
2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2,
2, 2, 2, 2, 2, 2, 1, 1, 2, 2, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1};
```

## Functions for creating net and random datasets (ECAs, all 4 classes)

```

In[2]:= RandomRuleC[n_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[n, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules -> {0 -> RandomColor[], 1 -> RandomColor[],
      3 -> RandomColor[], 4 -> RandomColor[], 5 -> RandomColor[],
      6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
netC[W_Integer, H_Integer] := NetInitialize@NetChain[
  {ConvolutionLayer[16, {2, 3}], Ramp, PoolingLayer[{H, W} - {1, 2}], FlattenLayer[],
    LinearLayer[256], SoftmaxLayer[]], "Input" -> NetEncoder[{"Image", {W, H}}],
  "Output" -> NetDecoder[{"Class", Range[0, 255]}]]
netTwoCC[W_Integer, H_Integer] := NetInitialize@
  NetChain[<|"conv1" -> ConvolutionLayer[16, {2, 3}], "ramp1" -> Ramp,
    "conv3" -> ConvolutionLayer[16, {2, 3}], "ramp2" -> Ramp,
    "pooling" -> PoolingLayer[{H, W} - {2, 4}], "flatten" -> FlattenLayer[],
    "linear" -> 512, "linear2" -> 4, "softmax" -> SoftmaxLayer[]>,
  "Input" -> NetEncoder[{"Image", {W, H}}], "Output" -> NetDecoder[{"Class", Range[1, 4]}]]
dataC[W_Integer, H_Integer, n_Integer] :=
  Table[RandomRuleC[i, W, H] -> CAClasses[[i + 1]], {i, RandomChoice[Range[0, 255], n]}]

```

```

In[6]:= netThreeCC[W_Integer, H_Integer] :=
  NetInitialize@NetChain[<|"conv1" → ConvolutionLayer[16, {2, 3}],
    "ramp1" → Ramp, "conv2" → ConvolutionLayer[16, {2, 3}],
    "ramp2" → Ramp, "conv3" → ConvolutionLayer[16, {2, 3}], "ramp3" → Ramp,
    "pooling" → PoolingLayer[{H, W} - {4, 8}], "flatten" → FlattenLayer[],
    "linear" → 512, "linear2" → 4, "softmax" → SoftmaxLayer[]|>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]

```

```

In[7]:= netThreeCC1024[W_Integer, H_Integer] :=
  NetInitialize@NetChain[<|"conv1" → ConvolutionLayer[16, {2, 3}],
    "ramp1" → Ramp, "conv2" → ConvolutionLayer[16, {2, 3}],
    "ramp2" → Ramp, "conv3" → ConvolutionLayer[16, {2, 3}], "ramp3" → Ramp,
    "pooling" → PoolingLayer[{H, W} - {4, 8}], "flatten" → FlattenLayer[],
    "linear" → 1024, "linear2" → 4, "softmax" → SoftmaxLayer[]|>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]

```

```

In[8]:= netFourCC512[W_Integer, H_Integer] :=
  NetInitialize@NetChain[<|"conv1" → ConvolutionLayer[32, {2, 3}],
    "ramp1" → Ramp, "conv3" → ConvolutionLayer[32, {2, 3}], "ramp2" → Ramp,
    "pooling" → PoolingLayer[{H, W} - {2, 4}], "flatten" → FlattenLayer[],
    "linear" → 512, "linear2" → 4, "softmax" → SoftmaxLayer[]|>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]

```

```

In[9]:= netFiveCC512[W_Integer, H_Integer] := NetInitialize@
  NetChain[<|"conv1" → ConvolutionLayer[32, {2, 3}], "bat1" → BatchNormalizationLayer[],
    "ramp1" → Ramp, "conv3" → ConvolutionLayer[32, {2, 3}],
    "bat2" → BatchNormalizationLayer[], "ramp2" → Ramp,
    "pooling" → PoolingLayer[{H, W} - {2, 4}], "flatten" → FlattenLayer[],
    "linear" → 512, "linear2" → 4, "softmax" → SoftmaxLayer[]|>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]

```

```

In[10]:= netSixCC512drop[W_Integer, H_Integer] := NetInitialize@
  NetChain[<|"drop1" → DropoutLayer[0.2], "conv1" → ConvolutionLayer[32, {3, 3}],
    "bat1" → BatchNormalizationLayer[], "ramp1" → Ramp,
    "conv3" → ConvolutionLayer[32, {3, 3}], "bat2" → BatchNormalizationLayer[],
    "ramp2" → Ramp, "pooling" → PoolingLayer[{H, W} - {4, 8}],
    "flatten" → FlattenLayer[], "linear" → 512, "drop2" → DropoutLayer[0.2],
    "linear2" → 4, "softmax" → SoftmaxLayer[]|>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]

```

```
In[11]:= netSevenCC512drop[W_Integer, H_Integer] :=
  NetInitialize@NetChain[<|"conv1" → ConvolutionLayer[24, {3, 3}],
    "bat1" → BatchNormalizationLayer[], "ramp1" → Ramp,
    "conv3" → ConvolutionLayer[24, {3, 3}], "bat2" → BatchNormalizationLayer[],
    "ramp2" → Ramp, "pooling" → PoolingLayer[{H, W} - {4, 8}],
    "flatten" → FlattenLayer[], "linear" → 512, "drop2" → DropoutLayer[0.2],
    "linear2" → 4, "softmax" → SoftmaxLayer[]>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]
```

```
In[12]:= netEightCC512drop[W_Integer, H_Integer] :=
  NetInitialize@NetChain[<|"conv1" → ConvolutionLayer[24, {3, 3}],
    "bat1" → BatchNormalizationLayer[], "ramp1" → Ramp,
    "conv2" → ConvolutionLayer[16, {2, 3}], "bat2" → BatchNormalizationLayer[],
    "ramp2" → Ramp, "conv3" → ConvolutionLayer[24, {3, 3}],
    "bat3" → BatchNormalizationLayer[], "ramp3" → Ramp, "pooling" →
    PoolingLayer[{H, W} - {8, 16}], "flatten" → FlattenLayer[], "linear" → 1024,
    "drop2" → DropoutLayer[0.2], "linear2" → 4, "softmax" → SoftmaxLayer[]>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]
```

```
In[13]:= netNineCC512drop[W_Integer, H_Integer] :=
  NetInitialize@NetChain[<|"conv1" → ConvolutionLayer[24, {3, 3}],
    "bat1" → BatchNormalizationLayer[], "ramp1" → Ramp,
    "conv2" → ConvolutionLayer[24, {3, 3}], "bat2" → BatchNormalizationLayer[],
    "ramp2" → Ramp, "conv3" → ConvolutionLayer[24, {3, 3}],
    "bat3" → BatchNormalizationLayer[], "ramp3" → Ramp, "pooling" →
    PoolingLayer[{H, W} - {12, 12}], "flatten" → FlattenLayer[], "linear" → 512,
    "drop2" → DropoutLayer[0.2], "linear2" → 4, "softmax" → SoftmaxLayer[]>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]
```

```
In[180]:= netTenCC1024drop[W_Integer, H_Integer] :=
  NetInitialize@NetChain[<|"conv1" → ConvolutionLayer[24, {3, 3}],
    "bat1" → BatchNormalizationLayer[], "ramp1" → Ramp,
    "conv2" → ConvolutionLayer[24, {3, 3}], "bat2" → BatchNormalizationLayer[],
    "ramp2" → Ramp, "conv3" → ConvolutionLayer[24, {3, 3}],
    "bat3" → BatchNormalizationLayer[], "ramp3" → Ramp,
    "conv4" → ConvolutionLayer[24, {3, 3}], "bat4" → BatchNormalizationLayer[],
    "ramp4" → Ramp, "pooling" → PoolingLayer[{H, W} - {12, 12}],
    "flatten" → FlattenLayer[], "linear" → 1024,
    "drop2" → DropoutLayer[0.3], "linear2" → 4, "softmax" → SoftmaxLayer[]>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]
```

```

In[288]:= netElevenCC1024drop[W_Integer, H_Integer] :=
  NetInitialize@NetChain[<|"conv1" → ConvolutionLayer[48, {3, 3}],
    "bat1" → BatchNormalizationLayer[], "ramp1" → Ramp,
    "pooling1" → PoolingLayer[3, 3], "conv2" → ConvolutionLayer[48, {3, 3}],
    "bat2" → BatchNormalizationLayer[], "ramp2" → Ramp,
    "pooling2," → PoolingLayer[3, 3] "conv3" → ConvolutionLayer[48, {3, 3}],
    "bat3" → BatchNormalizationLayer[], "ramp3" → Ramp,
    "pooling3" → PoolingLayer[3, 3], "conv4" → ConvolutionLayer[48, {3, 3}],
    "bat4" → BatchNormalizationLayer[], "ramp4" → Ramp,
    "pooling4" → PoolingLayer[3, 3], "flatten" → FlattenLayer[], "linear" → 1024,
    "drop2" → DropoutLayer[0.3], "linear2" → 4, "softmax" → SoftmaxLayer[]>,
    "Input" → NetEncoder[{"Image", {W, H}}, "Output" → NetDecoder[{"Class", Range[1, 4]}]]

```

## Functions for creating datasets (1D totalistic CAs)

### k=3, r=1 totalistic (class 4 only)

```

In[14]:= gen3TC[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {3, 1}}, RandomInteger[1, W], H - 1],
    ImageSize → {W, H}, ColorRules →
      {0 → RandomColor[], 1 → RandomColor[], 3 → RandomColor[], 4 → RandomColor[],
        5 → RandomColor[], 6 → RandomColor[], 7 → RandomColor[]}, Frame → False]]
data3T2C[W_Integer, H_Integer, n_Integer] := Table[gen3TC[i, W, H] → 4,
  {i, RandomChoice[{1635, 1815, 2007, 2043, 2049, 1388, 1041}, n]}]

```

### k=4, r=1 totalistic (class 4 only, 1 example)

```

In[16]:= gen4TC[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {4, 1}}, RandomInteger[1, W], H - 1],
    ImageSize → {W, H}, ColorRules →
      {0 → RandomColor[], 1 → RandomColor[], 3 → RandomColor[], 4 → RandomColor[],
        5 → RandomColor[], 6 → RandomColor[], 7 → RandomColor[]}, Frame → False]]
data4TC[W_Integer, H_Integer, n_Integer] := Table[gen4TC[1 004 600, W, H] → 4, n]

```

## k=2, r=2 totalistic (all 4 classes)

In[18]:=

```

gen2r2C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {2, 1}, 2}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
data2r2c4C[W_Integer, H_Integer, n_Integer] :=
  Table[gen2r2C[i, W, H] -> 4, {i, RandomChoice[{20, 52}, n]}]
data2r2c3C[W_Integer, H_Integer, n_Integer] := Table[gen2r2C[i, W, H] -> 3,
  {i, RandomChoice[{2, 6, 10, 12, 14, 18, 22, 26, 28, 30, 34, 38, 42, 44, 46, 50}, n]}]
data2r2c2C[W_Integer, H_Integer, n_Integer] :=
  Table[gen2r2C[i, W, H] -> 2, {i, RandomChoice[{8, 24, 56}, n]}]
data2r2c1C[W_Integer, H_Integer, n_Integer] := Table[gen2r2C[i, W, H] -> 1,
  {i, RandomChoice[{0, 4, 16, 32, 36, 40, 48, 54, 58, 60, 62}, n]}]
genData2r2C[W_Integer, H_Integer, n_Integer] :=
  Join[data2r2c4C[W, H, n], data2r2c3C[W, H, n], data2r2c2C[W, H, n], data2r2c1C[W, H, n]]

```

## k=5, r=1 totalistic (class 4 only)

In[24]:=

```

gen5T4C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {5, 1}}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
data5T4C[n_Integer, W_Integer, H_Integer] := Table[gen5T4C[i, W, H] -> 4,
  {i, RandomChoice[{781 130 654, 772 514 435, 1 151 319 452, 309 095 787, 880 862 046,
    973 835 714, 779 446 817, 345 466 505, 535 500 975, 793 363 571, 1 052 373 865,
    455 984 785, 339 227 109, 1 050 973 846, 513 368 817, 91 315 820, 113 925 357}, n]}]

```

## k=5, r=1 totalistic (classes 2/3/4)

```
In[26]:= gen5TC[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {5, 1}, 1}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
data5T4CC[W_Integer, H_Integer, n_Integer] := Table[gen5TC[i, W, H] -> 4,
  {i, RandomChoice[{644 218 533, 491 739 943, 6 889 640, 986 144 962, 1 099 816 682,
    988 971 204, 300 829 994, 272 622 024, 304 100 638, 626 595 633}, n]}]
data5T3CC[W_Integer, H_Integer, n_Integer] := Table[gen5TC[i, W, H] -> 3,
  {i, RandomChoice[{889 082 395, 541 068 260, 807 907 479, 816 180 062, 650 485 139,
    643 827 745, 753 940 864, 871 525 323, 351 440 311, 83 501 460}, n]}]
data5T2CC[W_Integer, H_Integer, n_Integer] := Table[gen5TC[i, W, H] -> 2,
  {i, RandomChoice[{525 735 659, 1 022 330 944, 1 007 796 739, 495 633 437, 1 036 827 943}, n]}]
genData5TCC[W_Integer, H_Integer, n_Integer] :=
  Join[data5T4CC[W, H, n], data5T3CC[W, H, n], data5T2CC[W, H, n]]
```

## Generate test datasets

### k=2, r=2 non-totalistic

```
In[31]:= genk2r2C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, 2, 2}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
datak2r2C[W_Integer, H_Integer, n_Integer] := Table[genk2r2C[i, W, H] -> i,
  {i, RandomChoice[Range[0, 4 294 967 295], n]}]
```

### k=2, r=3 non-totalistic

```
In[33]:= genk2r3NT[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, 2, 3}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
datak2r3NT[W_Integer, H_Integer, n_Integer] :=
  Table[genk2r3NT[i, W, H] -> i, {i, RandomInteger[2^2^7 - 1, n]}]
```

### k=3, r=1 non-totalistic

```
In[35]:= genk3r1NT[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, 3}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules -> {0 -> RandomColor[], 1 -> RandomColor[],
      3 -> RandomColor[], 4 -> RandomColor[], 5 -> RandomColor[],
      6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
datak3r1NT[W_Integer, H_Integer, n_Integer] :=
  Table[genk3r1NT[i, W, H] -> i, {i, RandomInteger[3^3^3 - 1, n]}]
```

### k=3, r=2 totalistic

```
In[37]:= genk3r2C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {3, 1}, 2}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
    {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
      5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
datak3r2C[W_Integer, H_Integer, n_Integer] := Table[genk3r2C[i, W, H] -> i,
  {i, RandomChoice[Range[0, 177 146], n]}]
```

### k=3, r=3 totalistic

```
In[39]:= genk3r3C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {3, 1}, 3}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
    {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
      5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
datak3r3C[W_Integer, H_Integer, n_Integer] := Table[genk3r3C[i, W, H] -> i,
  {i, RandomChoice[Range[0, 14 348 906], n]}]
```

### k=4, r=1 non-totalistic

```
In[41]:= genk4r1NT[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, 4}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules -> {0 -> RandomColor[], 1 -> RandomColor[],
      3 -> RandomColor[], 4 -> RandomColor[], 5 -> RandomColor[],
      6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
datak4r1NT[W_Integer, H_Integer, n_Integer] :=
  Table[genk4r1NT[i, W, H] -> i, {i, RandomInteger[4^4^3 - 1, n]}]
```

### k=4, r=1 totalistic

```

In[43]:= genk4r1C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {4, 1}}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
datak4r1C[W_Integer, H_Integer, n_Integer] := Table[genk4r1C[i, W, H] -> i,
  {i, RandomChoice[Range[0, 1048575], n]}]

```

### k=4, r=2 totalistic

```

In[45]:= genk4r2C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {4, 1}, 2}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
datak4r2C[W_Integer, H_Integer, n_Integer] := Table[genk4r2C[i, W, H] -> i,
  {i, RandomChoice[Range[0, 4294967295], n]}]

```

### k=5, r=1 totalistic

```

In[47]:= gen5T2C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {5, 1}, 1}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
data5T2C[n_Integer, W_Integer, H_Integer] := Table[gen5T2C[i, W, H] -> i,
  {i, RandomChoice[Range[0, 1220703125], n]}]

```

### k=6, r=1 totalistic

```

In[49]:= gen6TC[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {6, 1}, 1}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
data6TC[n_Integer, W_Integer, H_Integer] := Table[gen6TC[i, W, H] -> i,
  {i, RandomInteger[2821109907455, n]}]

```



## k=6, r=2 totalistic

```
In[51]:= gen6T2C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {6, 1}, 2}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
data6T2C[n_Integer, W_Integer, H_Integer] := Table[gen6T2C[i, W, H] -> i,
  {i, RandomInteger[170 581 728 179 578 208 255, n]}]
```

## k=7, r=1 totalistic

```
In[53]:= gen7TC[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {7, 1}, 1}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
data7TC[n_Integer, W_Integer, H_Integer] := Table[gen7TC[i, W, H] -> i,
  {i, RandomInteger[11 398 895 185 373 142, n]}]
```

## k=8, r=1 totalistic

```
In[55]:= gen8TC[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {8, 1}, 1}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
data8TC[n_Integer, W_Integer, H_Integer] := Table[gen8TC[i, W, H] -> i,
  {i, RandomInteger[73 786 976 294 838 206 463, n]}]
```

## k=8, r=2 totalistic

```
In[338]:= gen8T2C[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {8, 1}, 2}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules ->
      {0 -> RandomColor[], 1 -> RandomColor[], 3 -> RandomColor[], 4 -> RandomColor[],
        5 -> RandomColor[], 6 -> RandomColor[], 7 -> RandomColor[]}, Frame -> False]]
data8T2C[n_Integer, W_Integer, H_Integer] := Table[gen8T2C[i, W, H] -> i,
  {i, RandomInteger[324 518 553 658 426 726 783 156 020 576 255, n]}]
```

## k=9, r=1 totalistic

```

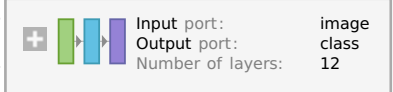
In[335]:= gen9TC[p_Integer, W_Integer, H_Integer] :=
  Image[ArrayPlot[CellularAutomaton[{p, {9, 1}, 1}, RandomInteger[1, W], H - 1],
    ImageSize -> {W, H}, ColorRules -> {0 -> RandomColor[], 1 -> RandomColor[],
      3 -> RandomColor[], 4 -> RandomColor[], 5 -> RandomColor[],
      6 -> RandomColor[], 7 -> RandomColor[], 8 -> RandomColor[]}, Frame -> False]]
data9TC[n_Integer, W_Integer, H_Integer] := Table[gen9TC[i, W, H] -> i,
  {i, RandomInteger[717 897 987 691 852 588 770 248, n]}]

```

## Network XIII - Two convolutions, dropout on linear only, BatchNorm

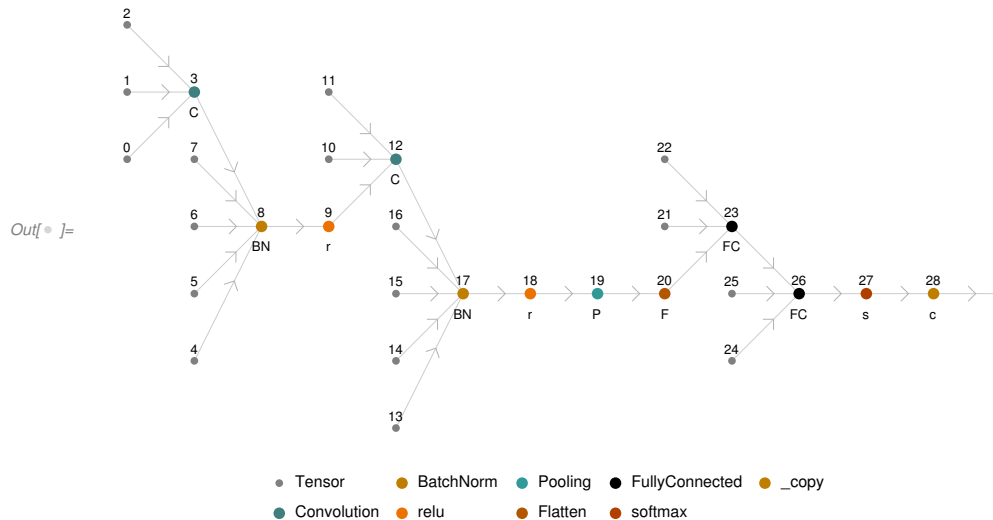
```
In[ ]:= netECA13 = netSevenCC512drop[128, 128]
```

```
Out[ ]:= NetChain[
```

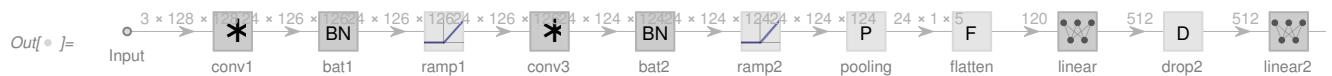


```
]
```

```
In[ ]:= NetInformation[netECA13, "MXNetNodeGraphPlot"]
```



```
In[ ]:= NetInformation[netECA13, "SummaryGraphic"]
```



```
In[ ]:= dataECA13 = dataC[128, 128, 8192];
```

```
In[ ]:= dataTotalistic2BigC13 = genData2r2C[128, 128, 1024];
```

```
In[ ]:= dataTotalistic3BigC13 = data3T2C[128, 128, 1024];
```

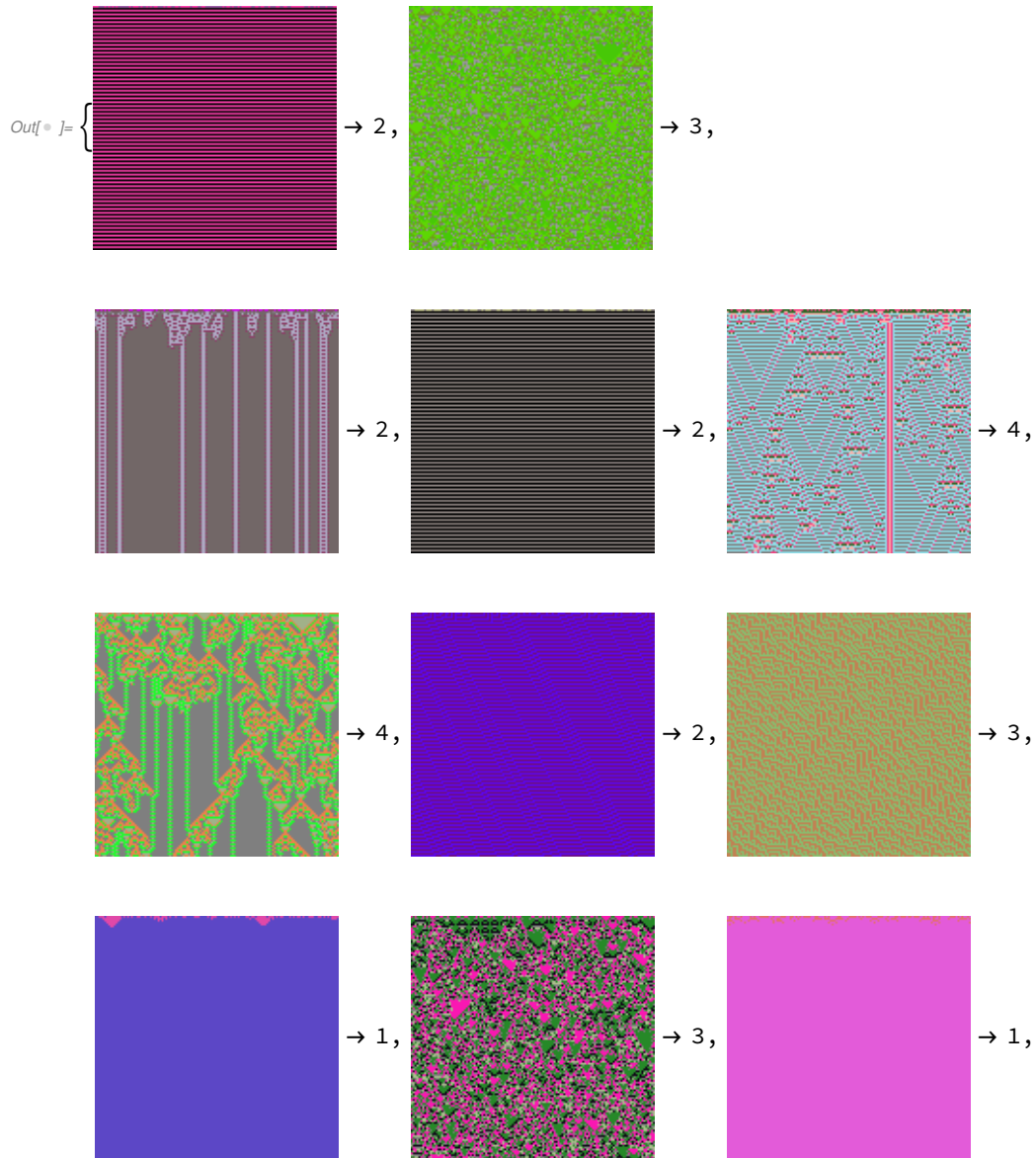
```
In[ ]:= dataTotalistic4BigC13 = data4TC[128, 128, 1024];
```

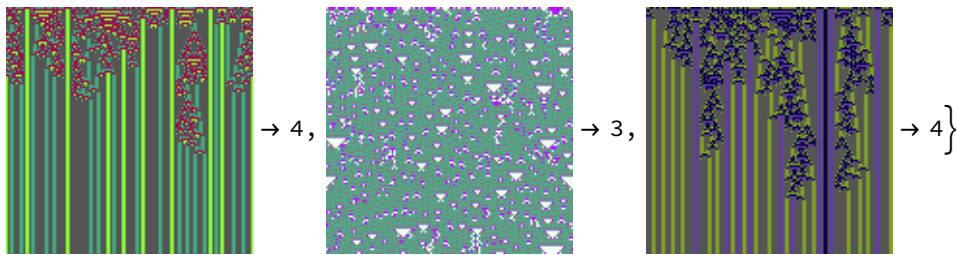
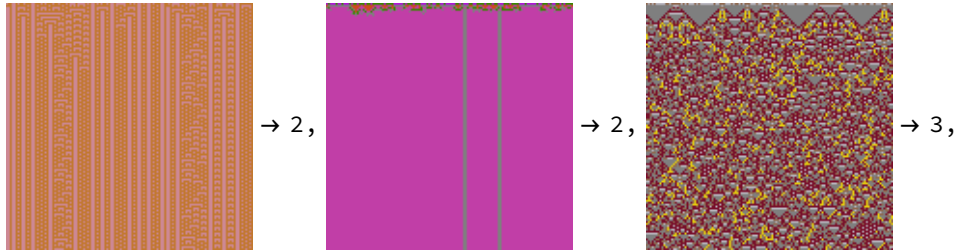
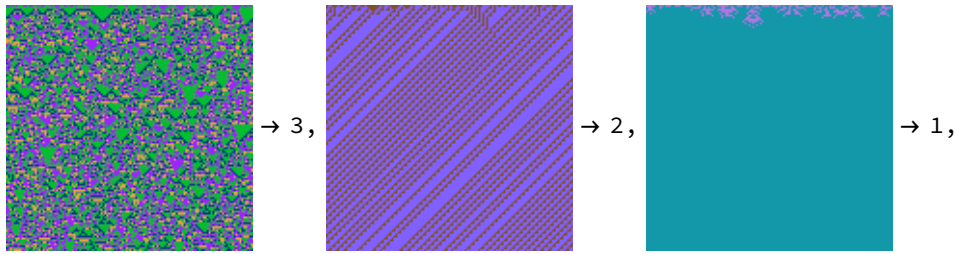
```
In[ ]:= dataTotalistic5BigC13 = genData5TCC[128, 128, 4096];
```

```
In[ ]:= fullTrainingBigC13 = Join[dataECA13, dataTotalistic2BigC13,  
    dataTotalistic3BigC13, dataTotalistic4BigC13, dataTotalistic5BigC13];  
Length[fullTrainingBigC13]
```

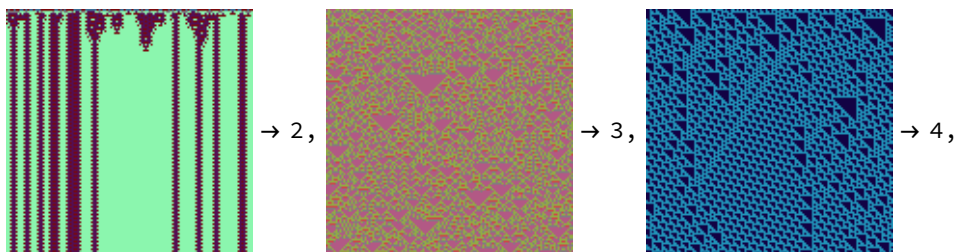
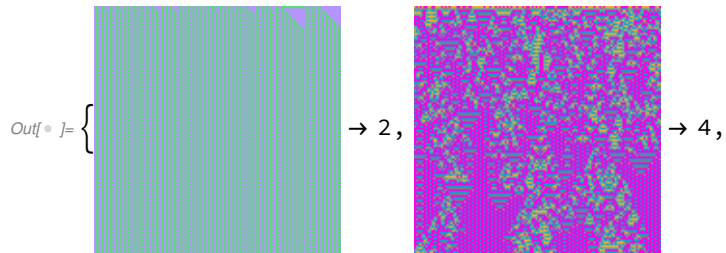
```
Out[ ]:= 26 624
```

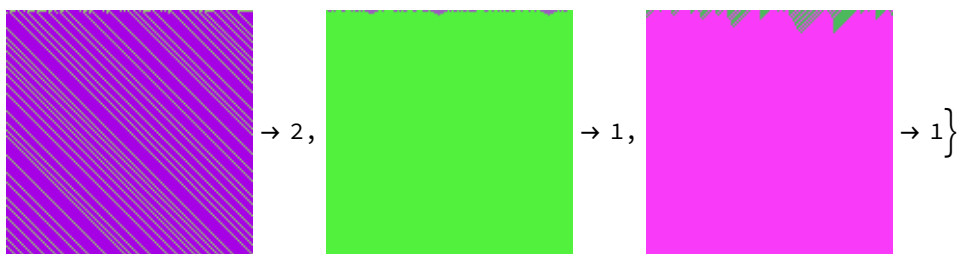
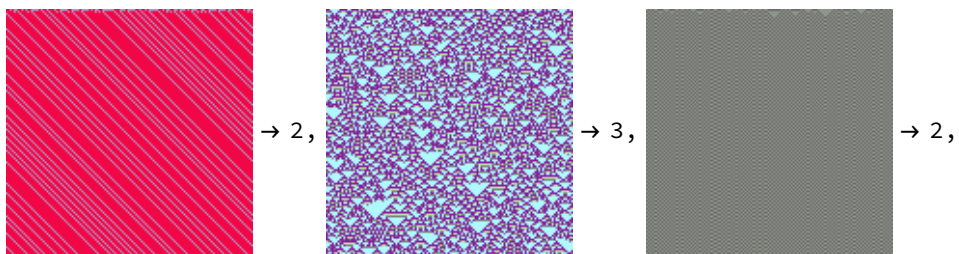
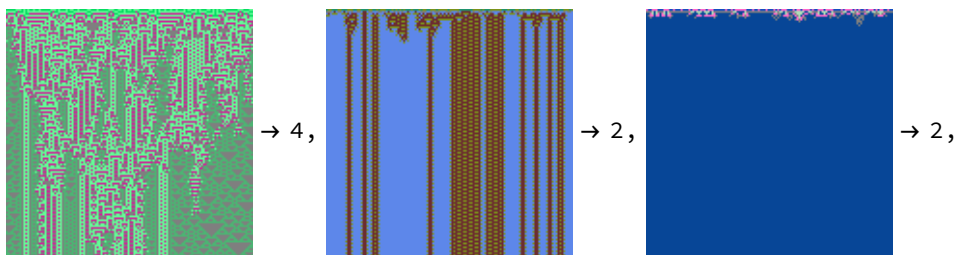
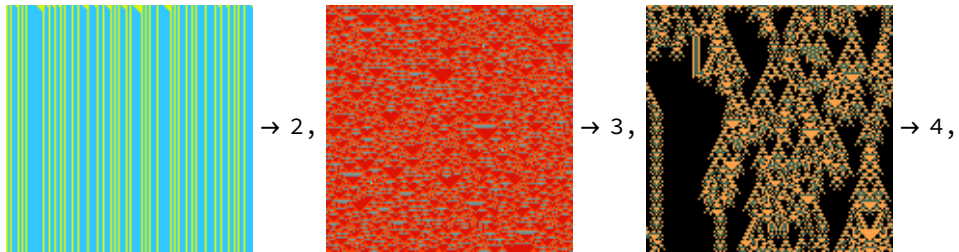
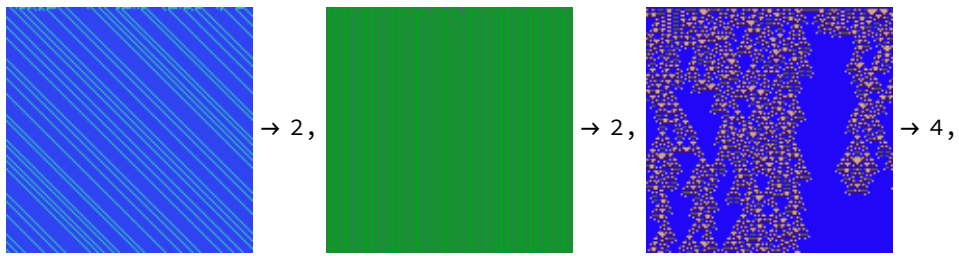
```
In[ ]:= RandomSample[fullTrainingBigC13, 20]
```





`In[ ] := RandomSample[fullTrainingBigC13, 20]`

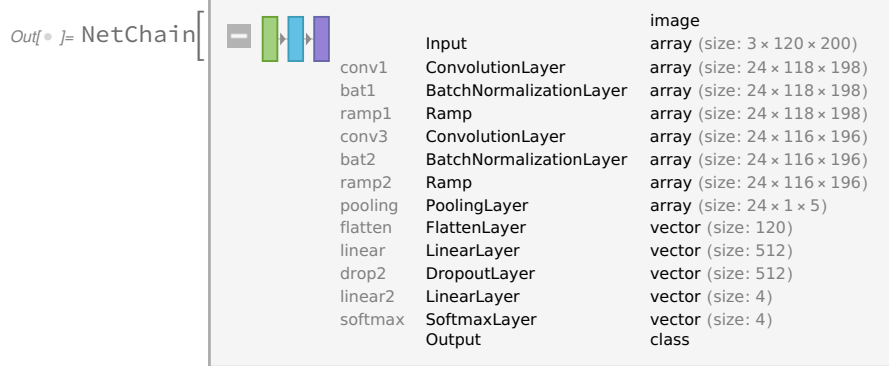




`In[ ]:= dir = SetDirectory[NotebookDirectory[]]`

`Out[ ]:= /Users/thorsilver/Downloads/Wolfram notebooks`

```
In[ ]:= netECA12 = Import["netECA12-r12.wlnet"]
```



```
In[ ]:= netECA13 =
```

```
NetTrain[netECA13, fullTrainingBigC13, MaxTrainingRounds → 20, BatchSize → 256 * 4,  
TargetDevice → "CPU", TrainingProgressCheckpointing → {"Directory", dir}]
```



```
In[ ]:= netECA13 = Import["netECA13-r20.wlnet"]
```



```
In[ ]:= netECA13 =
```

```
NetTrain[netECA13, fullTrainingBigC13, MaxTrainingRounds → 20, BatchSize → 256 * 4,  
TargetDevice → "CPU", TrainingProgressCheckpointing → {"Directory", dir}]
```



## Generate test data for Network XIII

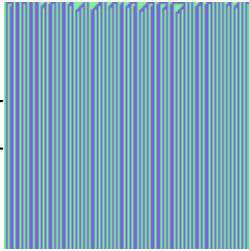
```
In[ ]:= testDataECABigC = dataC[128, 128, 1024];  
testData2TBigC = genData2r2C[128, 128, 1024];  
testData3TBigC = data3T2C[128, 128, 1024];  
testData4TBigC = data4TC[128, 128, 1024];  
testData5TBigC = genData5TCC[128, 128, 1024];  
fullTestSetBigC = Join[testDataECABigC,  
testData2TBigC, testData3TBigC, testData4TBigC, testData5TBigC];  
Length[fullTestSetBigC]
```

```
Out[ ]:= 10 240
```

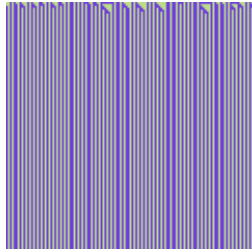


```
In[ ]:= RandomSample[fullTestSetBigC, 10]
```

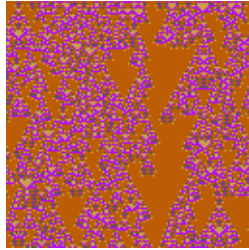
Out[ ]:= {



→ 2,



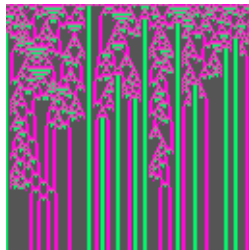
→ 2,



→ 4,



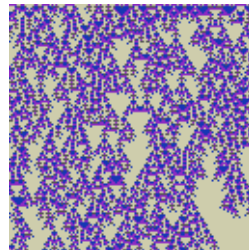
→ 3,



→ 4,



→ 4,



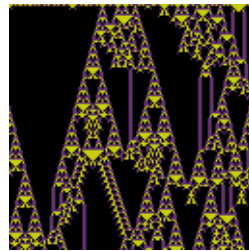
→ 4,



→ 4,



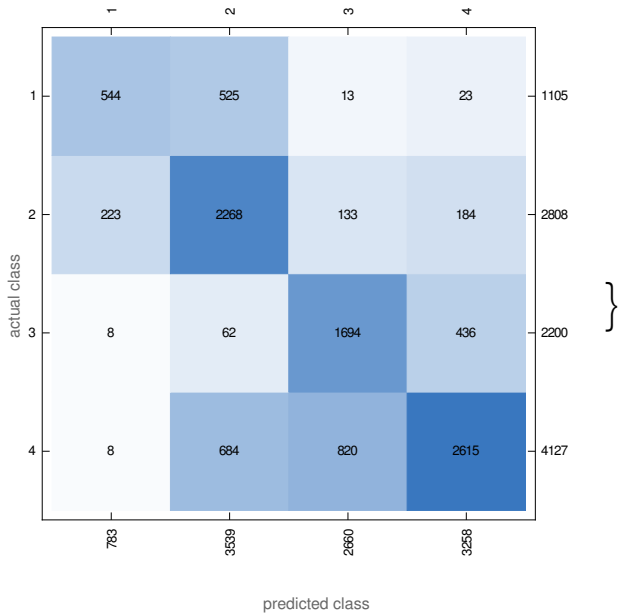
→ 4,



→ 4}

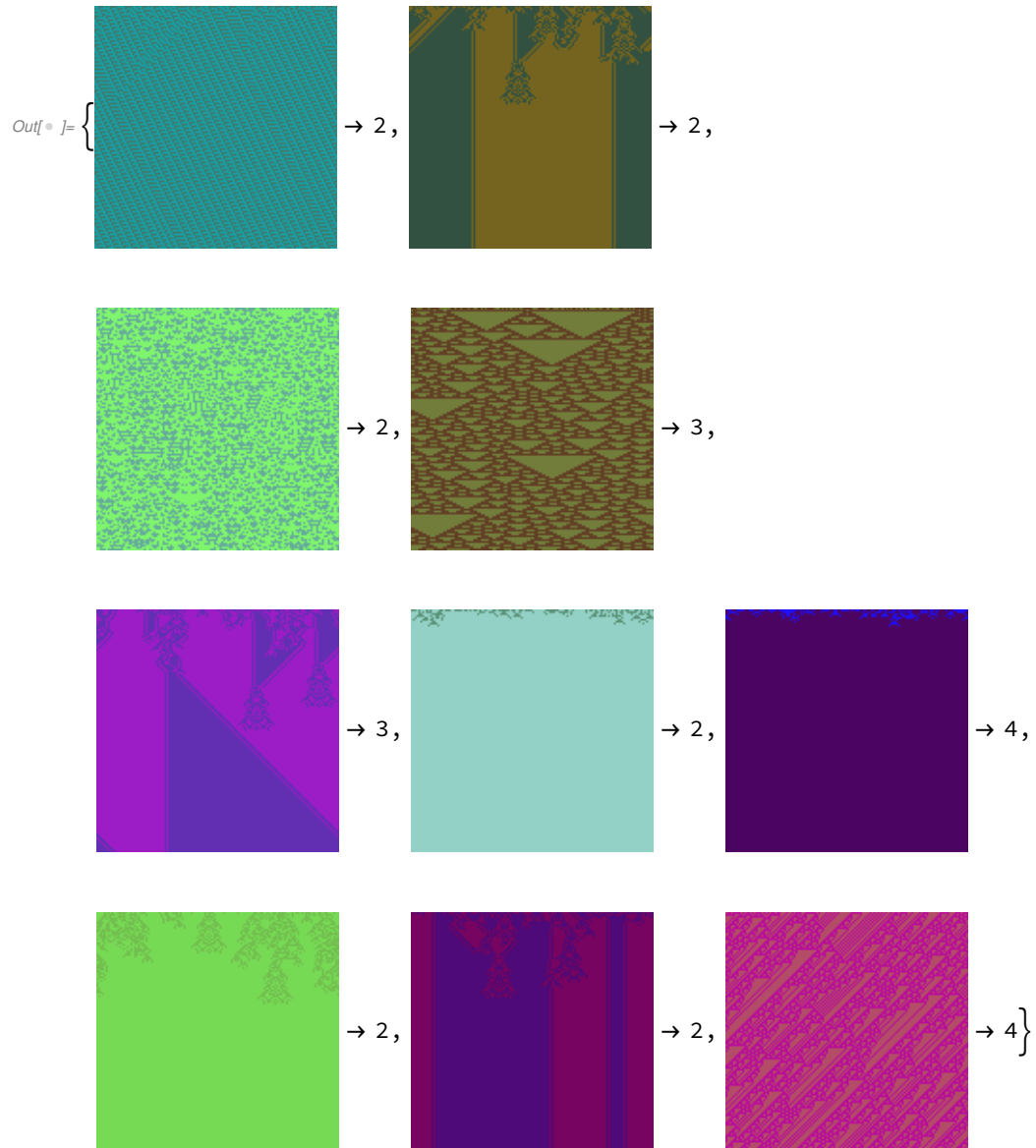
```
In[ ]:= NetMeasurements[netECA13, fullTestSetBigC,
  {"Accuracy", "Precision", "ConfusionMatrixPlot"}]
```

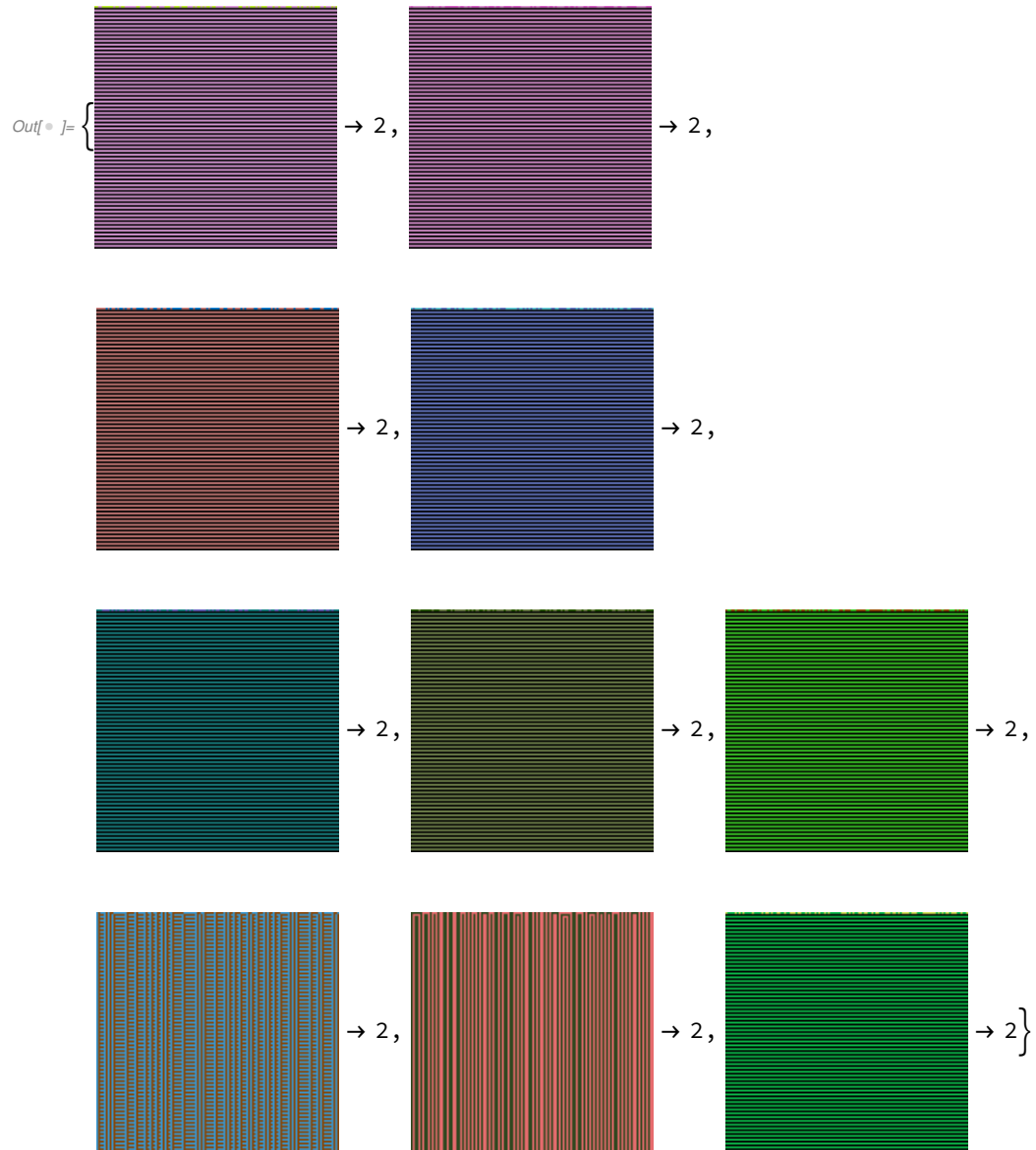
```
Out[ ]:= {0.69541, <| 1 → 0.694764, 2 → 0.640859, 3 → 0.636842, 4 → 0.80264 |>,
```



```
In[ ]:= entropyImagesBigC = RandomSample[Keys[fullTestSetBigC], 500];
  entropiesBigC = netECA13[entropyImagesBigC, "Entropy"];
  highEntBigC = entropyImagesBigC[[Ordering[entropiesBigC, -10]]];
  lowEntBigC = entropyImagesBigC[[Ordering[entropiesBigC, 10]]];
  Thread[highEntBigC → netECA13[highEntBigC]]
  Thread[lowEntBigC → netECA13[lowEntBigC]]
```

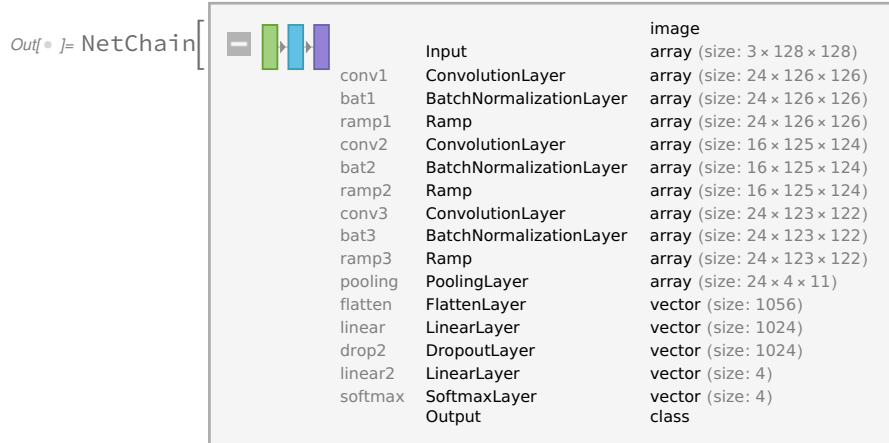






## Network XIV - BatchNorm, 1024 linear, dropout

```
In[ ]:= netECA14 = netEightCC512drop[128, 128]
```



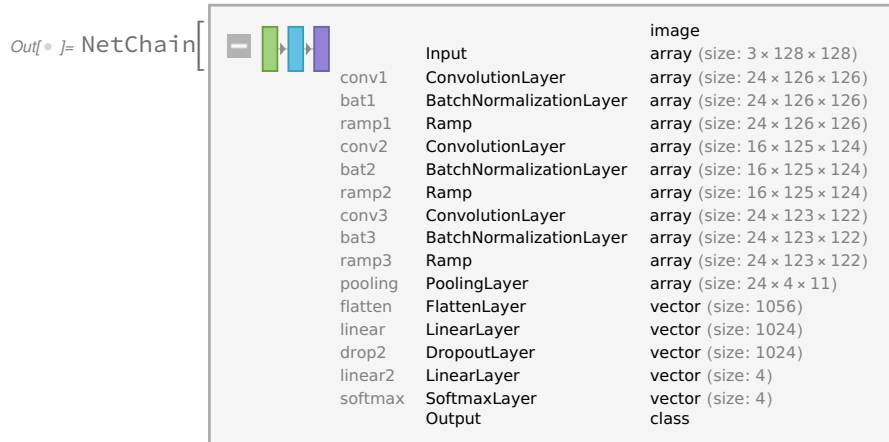
```
netECA14 =
```

```
NetTrain[netECA14, fullTrainingBigC13, MaxTrainingRounds → 20, BatchSize → 256 * 4,  
TargetDevice → "CPU", TrainingProgressCheckpointing → {"Directory", dir}]
```

```
In[ ]:= dir = SetDirectory[NotebookDirectory[]]
```

```
Out[ ]:= /Users/thorsilver/Downloads/Wolfram notebooks
```

```
In[ ]:= netECA14 = Import["netECA14-r20.wlnet"]
```

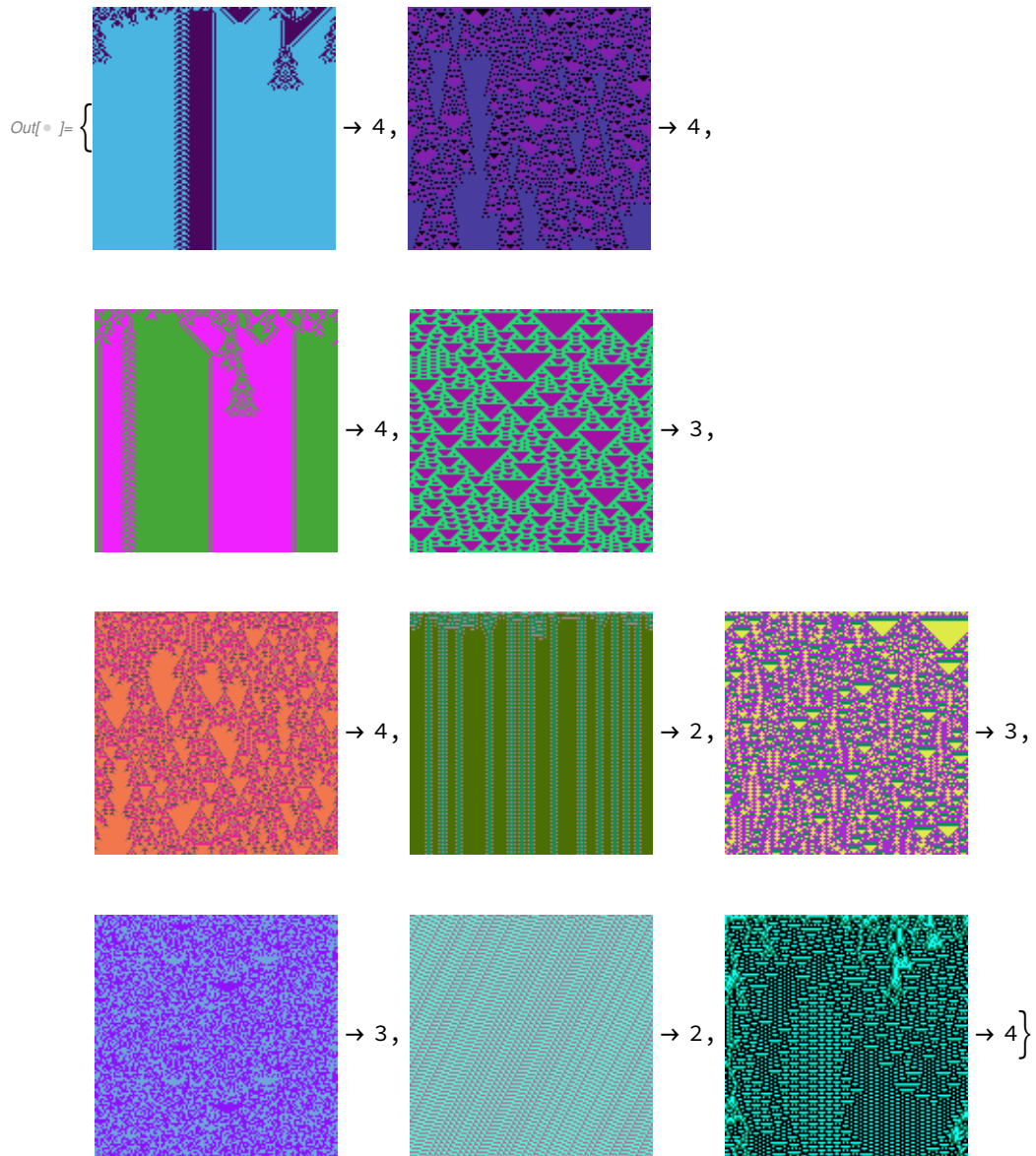


## Generating test data for Network XIV

```
In[ ]:= testDataECABigC = dataC[128, 128, 1024];  
testData2TBigC = genData2r2C[128, 128, 1024];  
testData3TBigC = data3T2C[128, 128, 1024];  
testData4TBigC = data4TC[128, 128, 1024];  
testData5TBigC = genData5TCC[128, 128, 1024];  
fullTestSetBigC = Join[testDataECABigC,  
  testData2TBigC, testData3TBigC, testData4TBigC, testData5TBigC];  
Length[fullTestSetBigC]
```

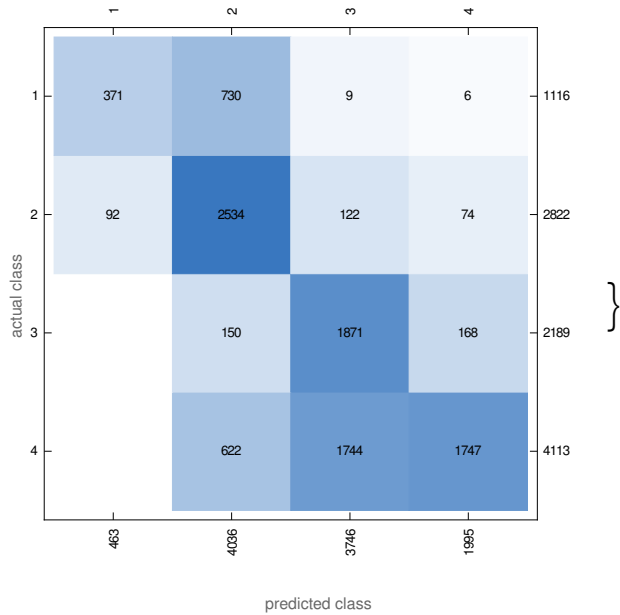
```
Out[ ]:= 10 240
```

```
In[ ]:= RandomSample[fullTestSetBigC, 10]
```

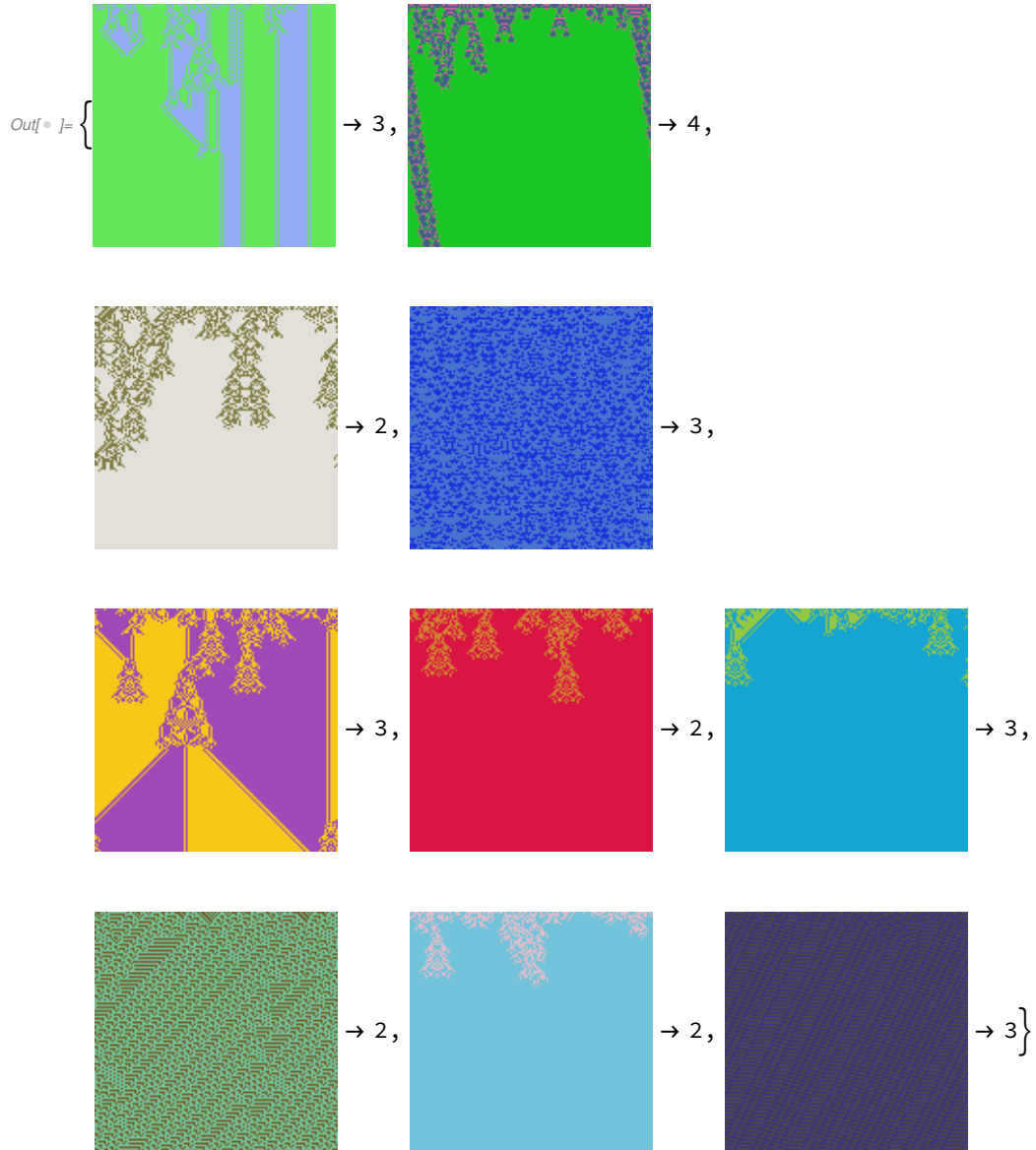


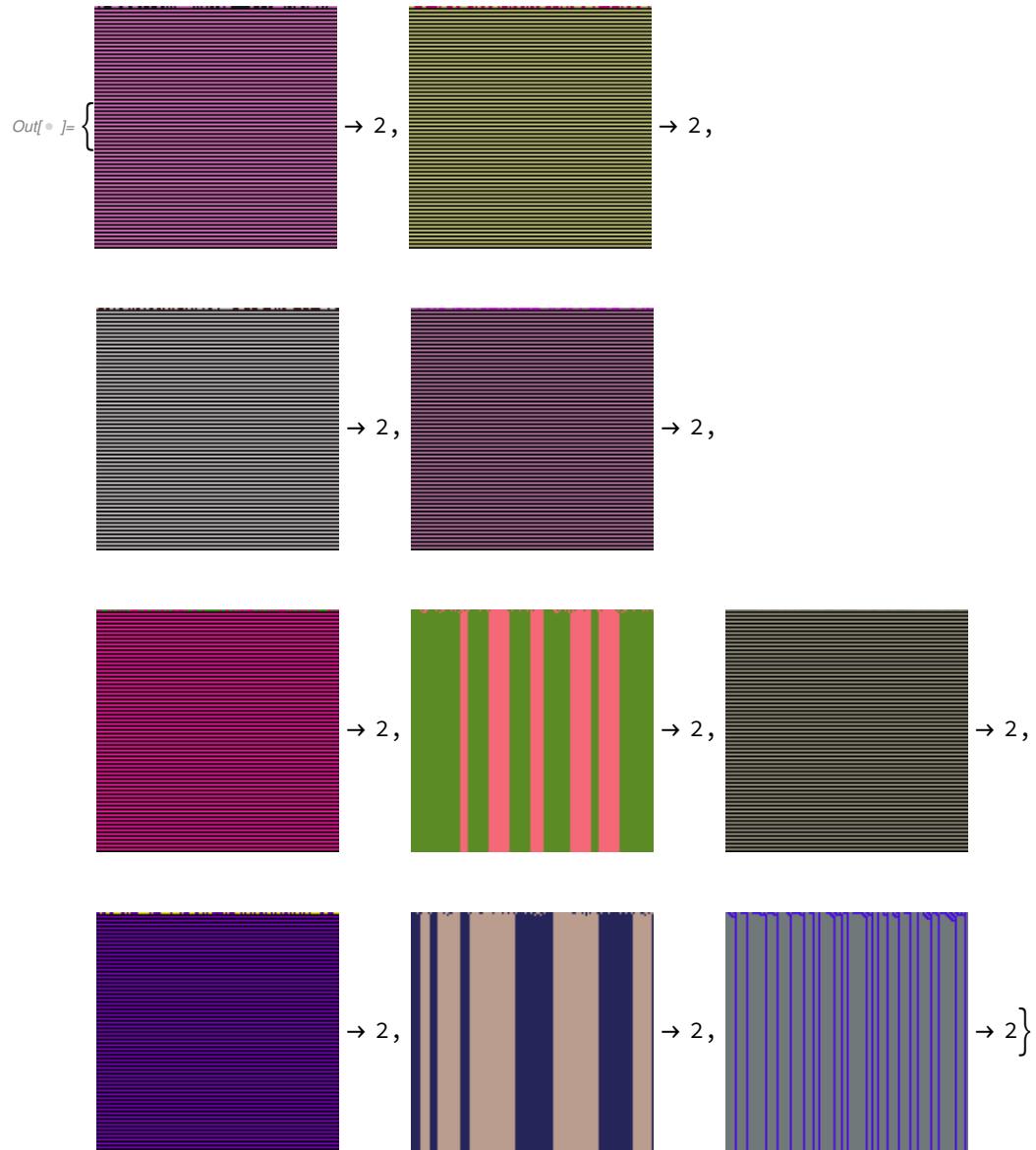
```
In[ ]:= NetMeasurements[netECA14, fullTestSetBigC,
  {"Accuracy", "Precision", "ConfusionMatrixPlot"}]
```

```
Out[ ]:= {0.637012, <| 1 → 0.801296, 2 → 0.627849, 3 → 0.499466, 4 → 0.875689 |>,
```



```
In[ ]:= entropyImagesBigC = RandomSample[Keys[fullTestSetBigC], 500];
  entropiesBigC = netECA14[entropyImagesBigC, "Entropy"];
  highEntBigC = entropyImagesBigC[[Ordering[entropiesBigC, -10]]];
  lowEntBigC = entropyImagesBigC[[Ordering[entropiesBigC, 10]]];
  Thread[highEntBigC → netECA14[highEntBigC]]
  Thread[lowEntBigC → netECA14[lowEntBigC]]
```





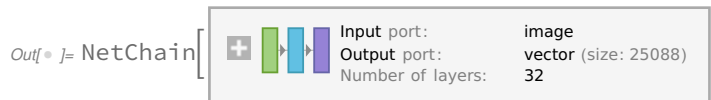


## Network XV - Transfer learning with pre-trained image recognition net (VGG-16)

```
In[ ]:= netECA15 = NetModel["VGG-16 Trained on ImageNet Competition Data"]
```



```
In[ ]:= subNet = NetTake[netECA15, {"conv1_1", "flatten_0"}]
```



```
In[ ]:= joinedNet = NetJoin[subNet,
  NetChain@<{"linear_new" → LinearLayer[1024], "linear_out" → LinearLayer[4],
    "prob" → SoftmaxLayer[]}>, "Output" → NetDecoder[{"Class", Range[1, 4]}]]
```



```
In[ ]:= netECA15final = NetPrepend[joinedNet, {"augment" → ImageAugmentationLayer[{224, 224}]},
  "Input" → NetExtract[joinedNet, "Input"]]
```

```
Out[ ]:= NetChain[
```

```
In[ ]:= dataECA15 = dataC[224, 224, 8192];
```

```
In[ ]:= dataTotalistic2BigC15 = genData2r2C[224, 224, 1024];
```

```
In[ ]:= dataTotalistic3BigC15 = data3T2C[224, 224, 512];
```

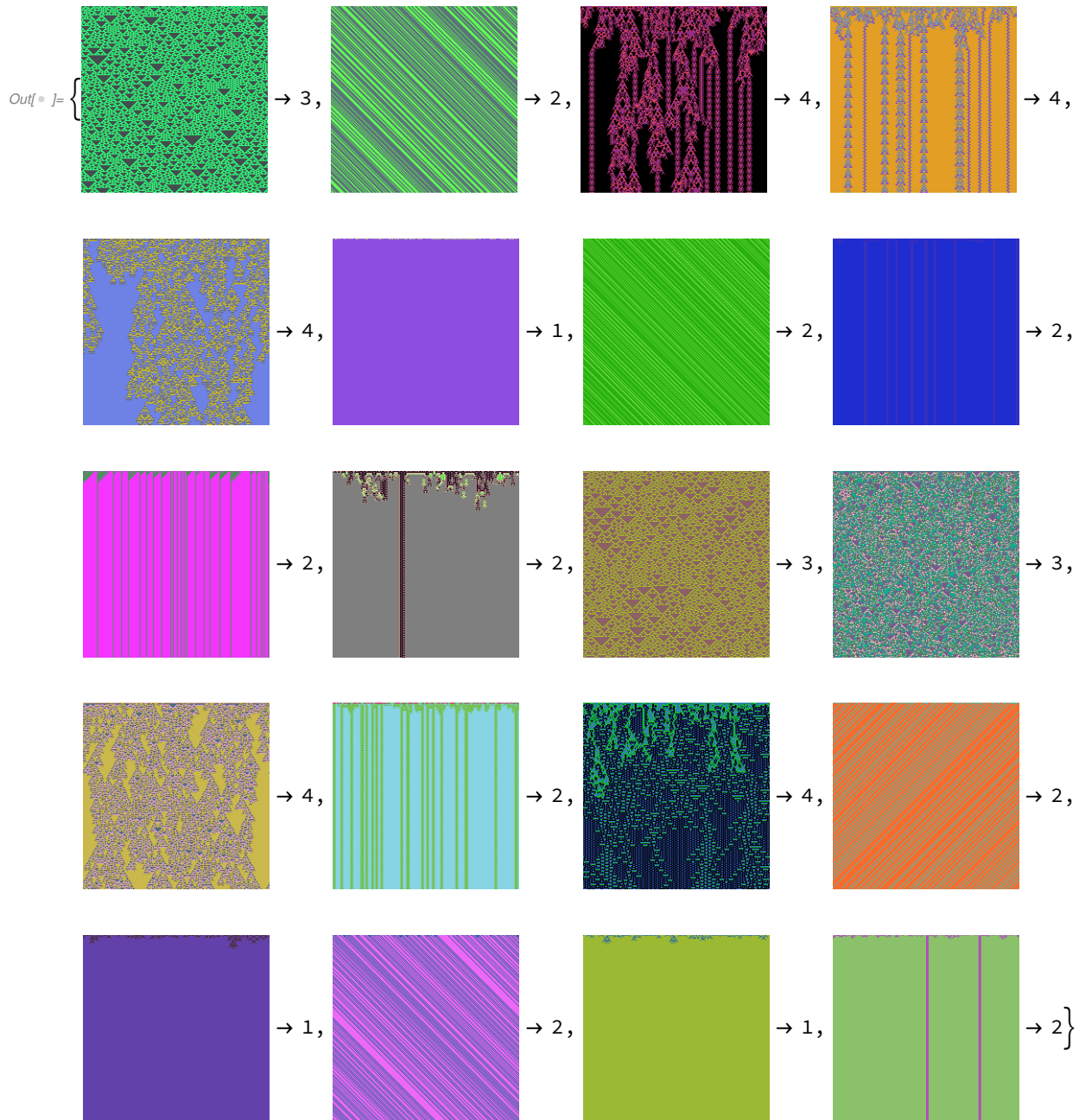
```
In[ ]:= dataTotalistic4BigC15 = data4TC[224, 224, 512];
```

```
In[ ]:= dataTotalistic5BigC15 = genData5TCC[224, 224, 1024];
```

```
In[ ]:= fullTrainingBigC15 = Join[dataECA15, dataTotalistic2BigC15,
  dataTotalistic3BigC15, dataTotalistic4BigC15, dataTotalistic5BigC15];
Length[fullTrainingBigC15]
```

```
Out[ ]:= 16 384
```

```
In[ ]:= RandomSample[fullTrainingBigC15, 20]
```



```
In[ ]:= netECA15final =
```

```
NetTrain[netECA15final, fullTrainingBigC15, MaxTrainingRounds → 5, BatchSize → 256 * 4,
  TargetDevice → "CPU", TrainingProgressCheckpointing → {"Directory", dir},
  LearningRateMultipliers → {"linear_new" → 1, "linear_out" → 1, _ → 0}]
```

## Network XVI - Three convolutions, dropout on linear only, BatchNorm

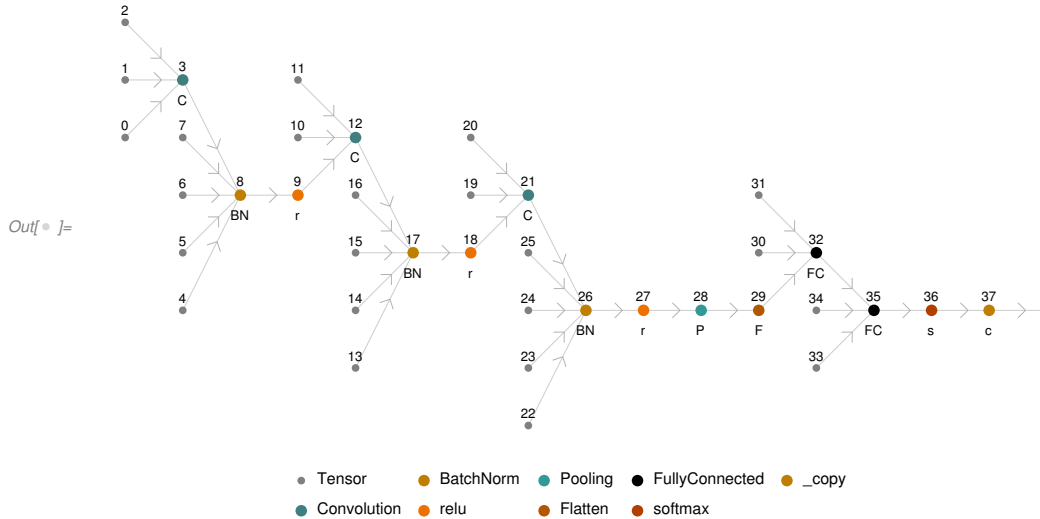
```
In[57]:= netECA16 = netNineCC512drop[128, 128]
```

```
Out[57]= NetChain[
```

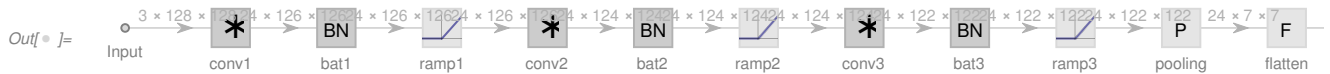


```
]
```

```
In[ ]:= NetInformation[netECA16, "MXNetNodeGraphPlot"]
```



```
In[ ]:= NetInformation[netECA16, "SummaryGraphic"]
```



```
In[58]:= dataECA16 = dataC[128, 128, 8192];
```

```
In[59]:= dataTotalistic2BigC16 = genData2r2C[128, 128, 1024];
```

```
In[60]:= dataTotalistic3BigC16 = data3T2C[128, 128, 1024];
```

```
In[61]:= dataTotalistic4BigC16 = data4TC[128, 128, 1024];
```

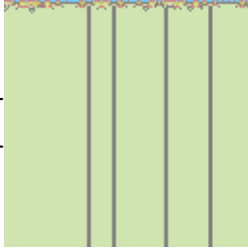
```
In[62]:= dataTotalistic5BigC16 = genData5TCC[128, 128, 4096];
```

```
In[63]:= fullTrainingBigC16 = Join[dataECA16, dataTotalistic2BigC16,  
    dataTotalistic3BigC16, dataTotalistic4BigC16, dataTotalistic5BigC16];  
Length[fullTrainingBigC16]
```

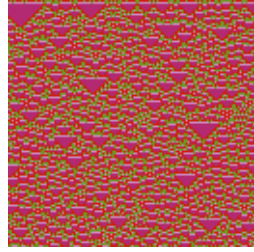
```
Out[64]= 26 624
```

```
In[65]:= RandomSample[fullTrainingBigC16, 20]
```

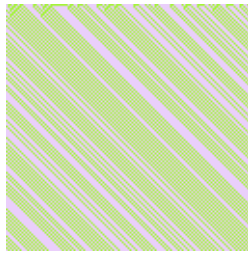
Out[65]=



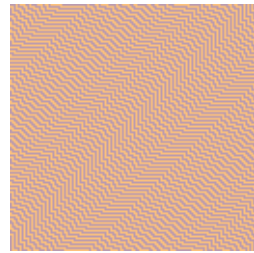
→ 2,



→ 3,



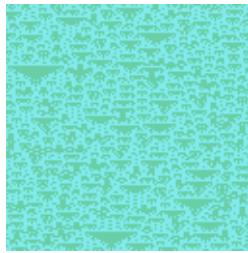
→ 2,



→ 2,



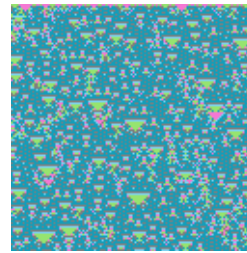
→ 1,



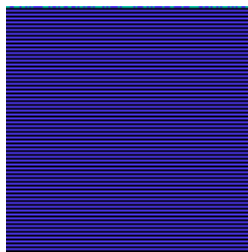
→ 3,



→ 2,



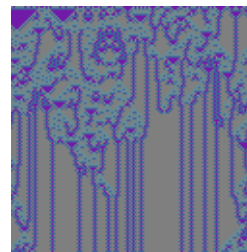
→ 3,



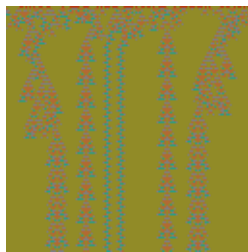
→ 2,



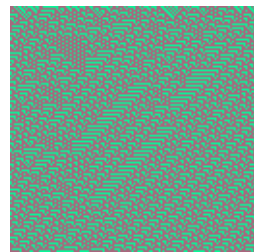
→ 1,



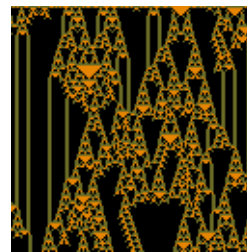
→ 4,



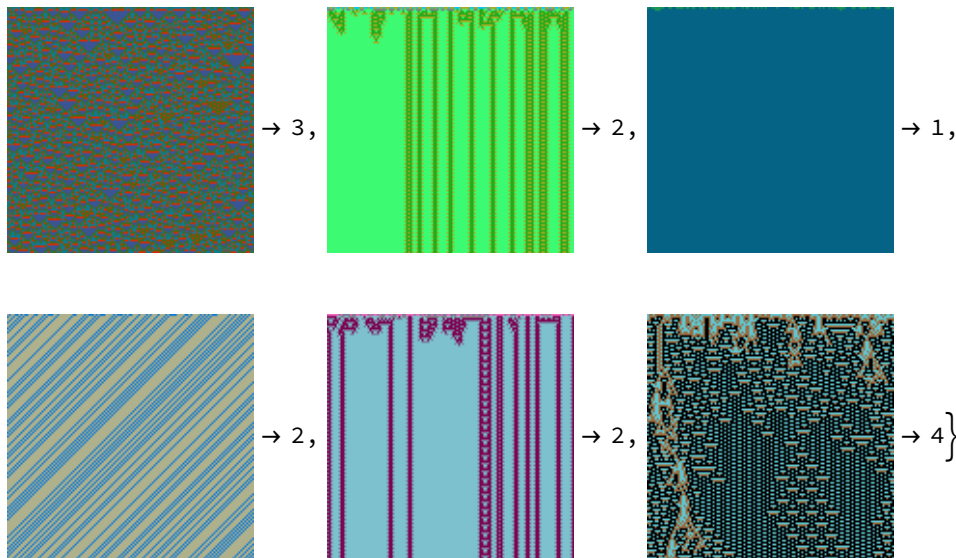
→ 4,



→ 3,



→ 4,



```
In[66]:= dir = SetDirectory[NotebookDirectory[]]
```

```
Out[66]= /home/esilverman/Documents
```

```
In[67]:= netECA16 =
```

```
NetTrain[netECA16, fullTrainingBigC16, MaxTrainingRounds → 200, BatchSize → 256,
TargetDevice → "GPU", TrainingProgressCheckpointing → {"Directory", dir}]
```

```
Out[67]= NetChain[
  {
    +,
    →,
    →,
    {
      Input port:
      Output port:
      Number of layers:
    },
    {
      image
      class
      15
    }
  ]
```

```
netECA16 = Import["netECA16-r20.wlnet"]
```

```
netECA16 =
```

```
NetTrain[netECA16, fullTrainingBigC16, MaxTrainingRounds → 20, BatchSize → 256 * 4,
TargetDevice → "CPU", TrainingProgressCheckpointing → {"Directory", dir}]
```

## Generate test data for Network XVI

```
In[ ]:= dir = SetDirectory[NotebookDirectory[]]
```

```
Out[ ]:= /Users/thorsilver/Downloads/Wolfram notebooks
```

```
In[ ]:= netECA16 = Import["netECA16-r20.wlnet"]
```

```
Out[ ]:= NetChain[
  {
    +,
    →,
    →,
    {
      Input port:
      Output port:
      Number of layers:
    },
    {
      image
      class
      15
    }
  ]
```



```

In[68]:= testDataECABigC = dataC[128, 128, 1024];
testData2TBigC = genData2r2C[128, 128, 1024];
testData3TBigC = data3T2C[128, 128, 1024];
testData4TBigC = data4TC[128, 128, 1024];
testData5TBigC = genData5TCC[128, 128, 1024];
fullTestSetBigC = Join[testDataECABigC,
  testData2TBigC, testData3TBigC, testData4TBigC, testData5TBigC];
Length[fullTestSetBigC]

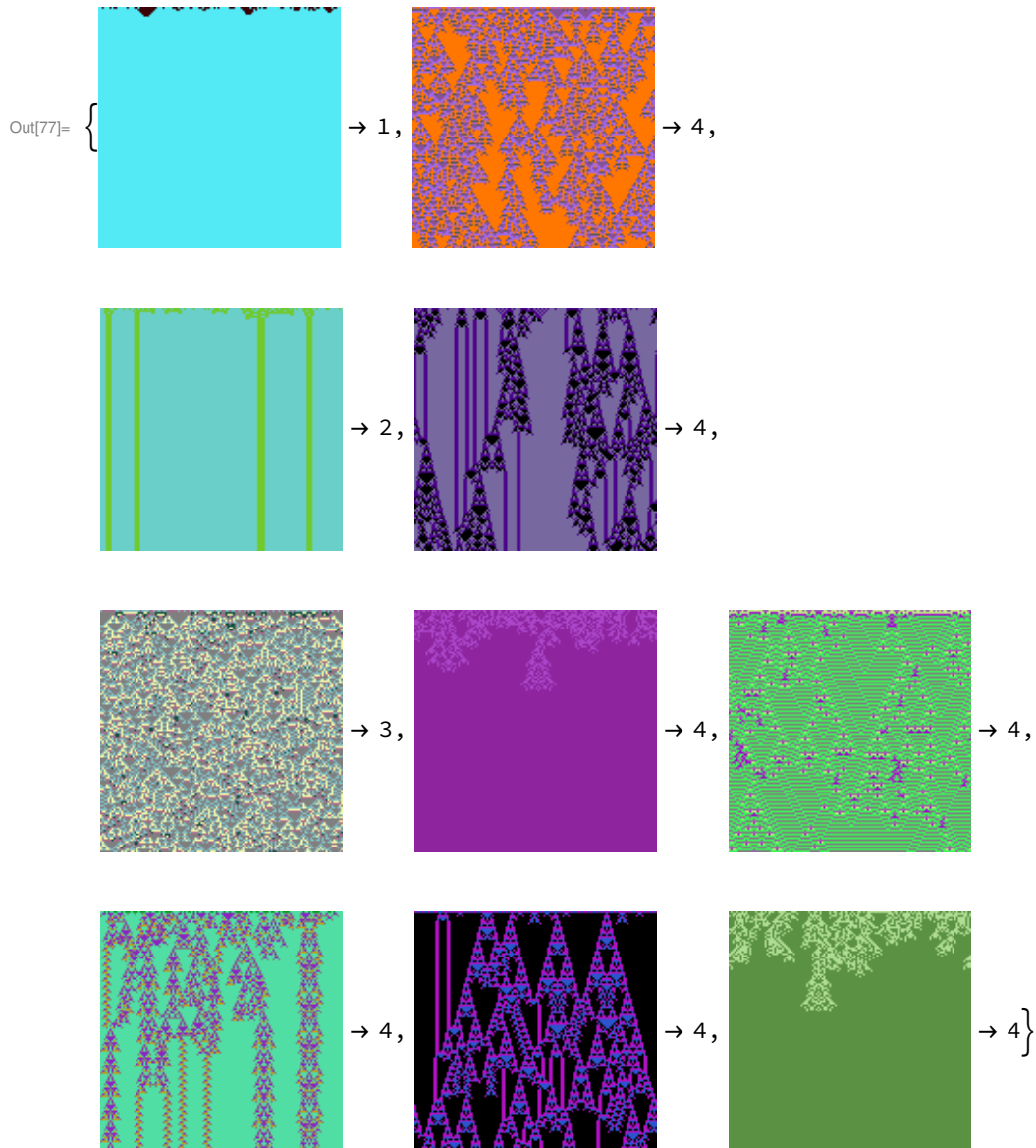
```

Out[74]= 10 240

```

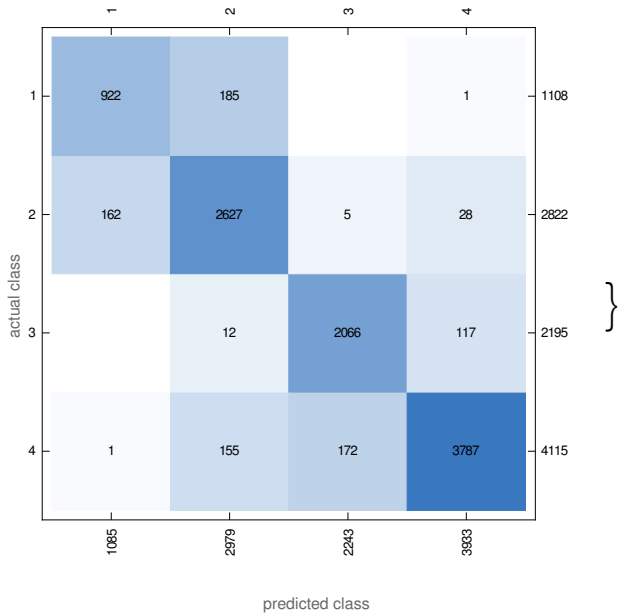
In[77]:= RandomSample[fullTestSetBigC, 10]

```



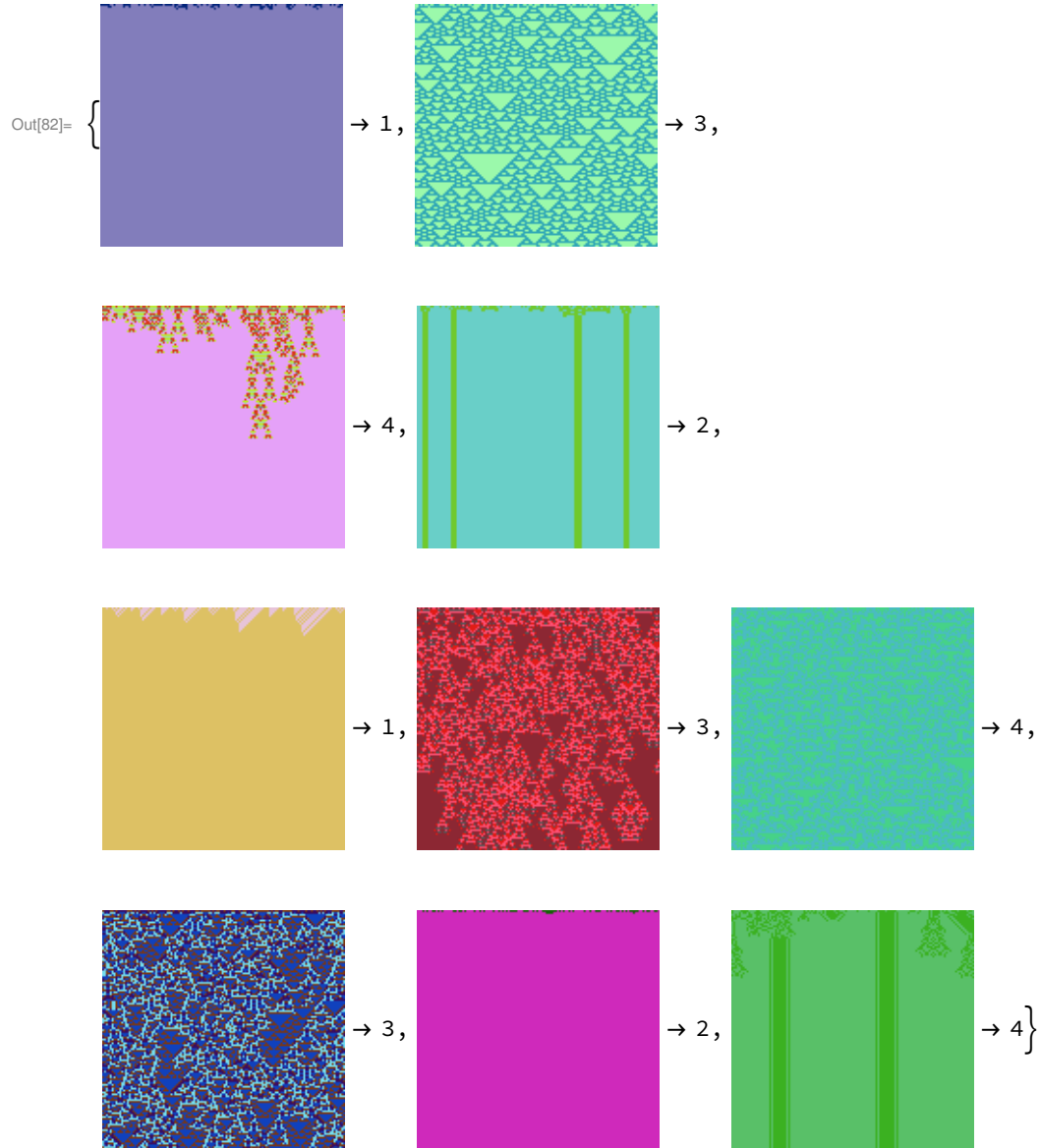
```
In[76]:= NetMeasurements[netECA16, fullTestSetBigC,
  {"Accuracy", "Precision", "ConfusionMatrixPlot"}]
```

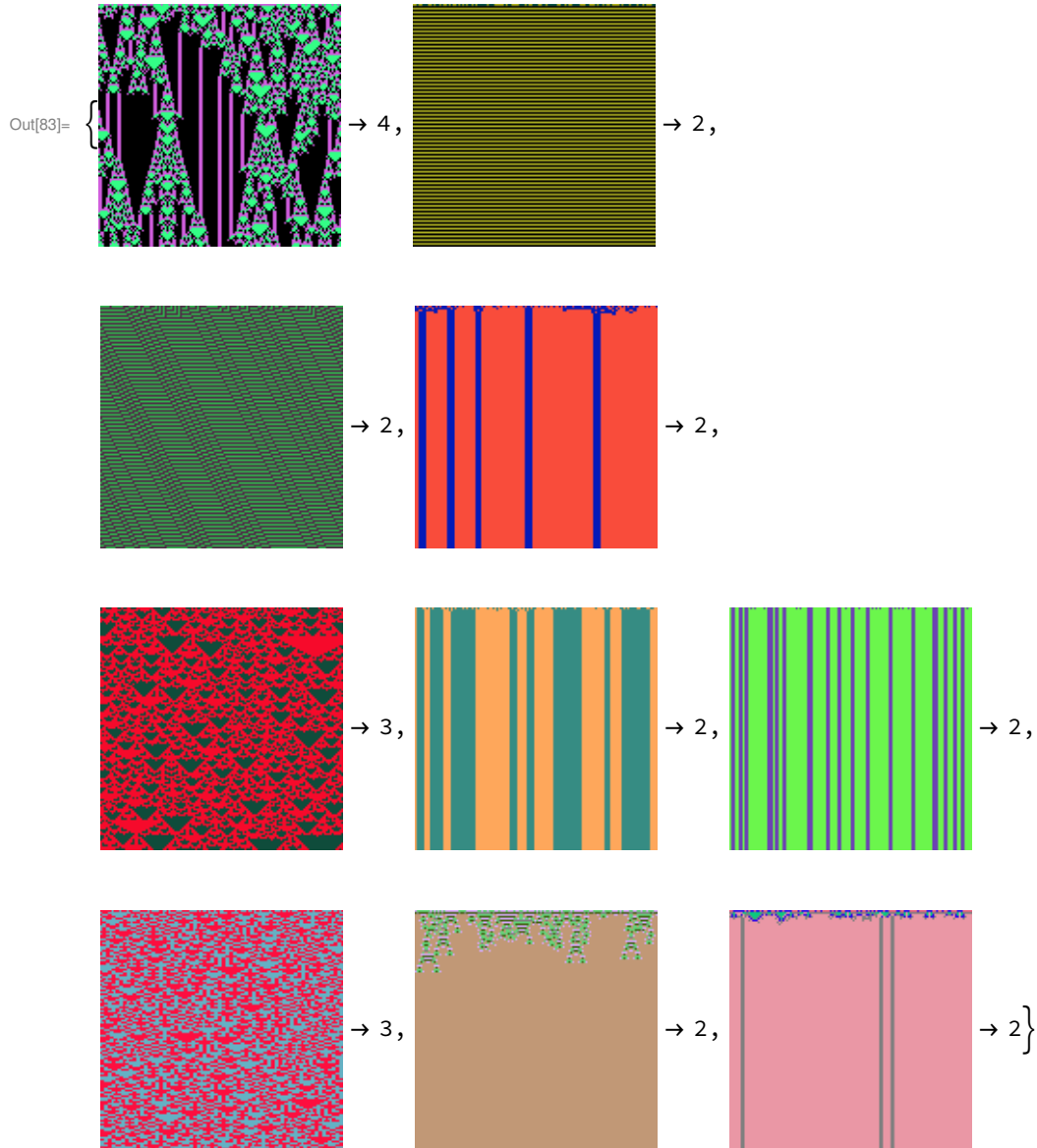
```
Out[76]= {0.918164, <| 1 → 0.84977, 2 → 0.88184, 3 → 0.921088, 4 → 0.962878 |>,
```



```
In[78]:= entropyImagesBigC = RandomSample[Keys[fullTestSetBigC], 500];
entropiesBigC = netECA16[entropyImagesBigC, "Entropy"];
highEntBigC = entropyImagesBigC[[Ordering[entropiesBigC, -10]]];
lowEntBigC = entropyImagesBigC[[Ordering[entropiesBigC, 10]]];
Thread[highEntBigC → netECA16[highEntBigC]]
Thread[lowEntBigC → netECA16[lowEntBigC]]
```







## Testing Network XVI on unseen CA rule spaces

### 2-colour non-totalistic, range 2

```
In[84]:= test4Data2kr2C16 = datak2r2C[128, 128, 8];
Thread[test4Data2kr2C16 → netECA16[Keys@test4Data2kr2C16, {"TopProbabilities", 2}]]
```

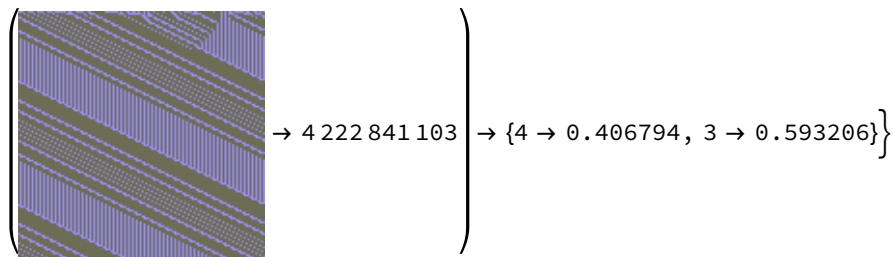
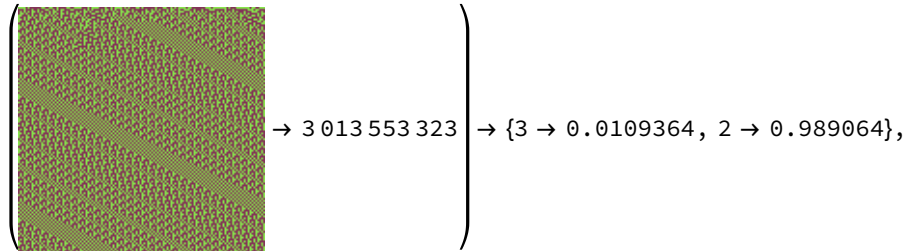
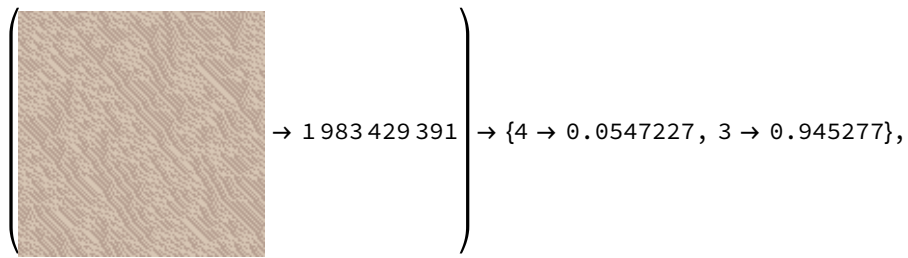
$$\text{Out[85]=} \left\{ \begin{array}{c} \text{[Image: A square plot with a green background and a dense, diagonal pattern of small, light green lines.]} \\ \rightarrow 142\,978\,078 \end{array} \right\} \rightarrow \{4 \rightarrow 0.0000385332, 3 \rightarrow 0.999961\},$$

$$\left\{ \begin{array}{c} \text{[Image: A square plot with a magenta background and a dense, diagonal pattern of small, light magenta lines.]} \\ \rightarrow 2\,651\,048\,833 \end{array} \right\} \rightarrow \{4 \rightarrow 8.69455 \times 10^{-12}, 2 \rightarrow 1.\},$$

$$\left\{ \begin{array}{c} \text{[Image: A square plot with a magenta background and a dense, diagonal pattern of small, light magenta lines.]} \\ \rightarrow 2\,132\,867\,963 \end{array} \right\} \rightarrow \{4 \rightarrow 2.86202 \times 10^{-17}, 2 \rightarrow 1.\},$$

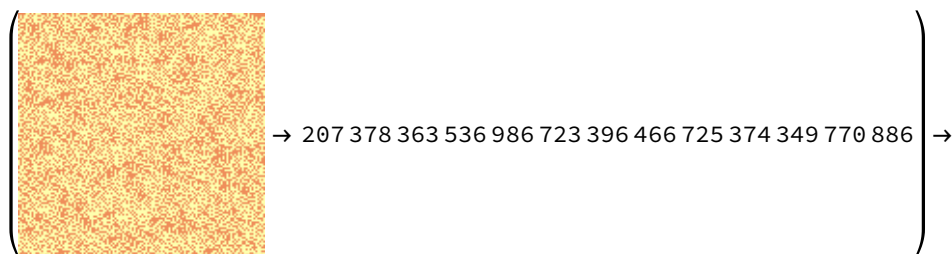
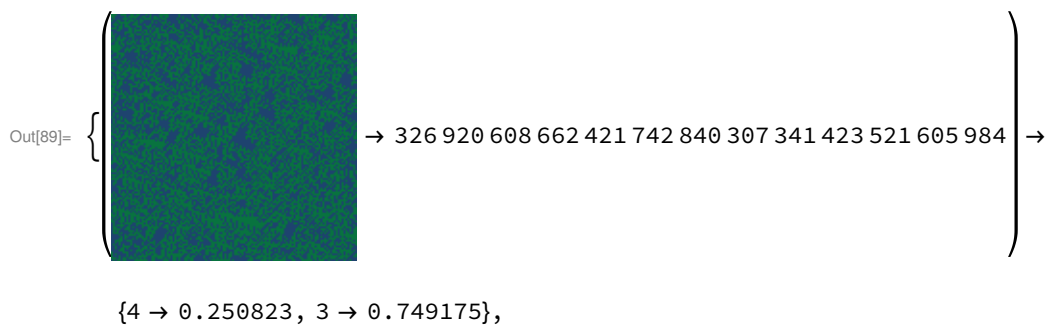
$$\left\{ \begin{array}{c} \text{[Image: A square plot with a brown background and a dense, diagonal pattern of small, light brown lines.]} \\ \rightarrow 3\,644\,758\,968 \end{array} \right\} \rightarrow \{4 \rightarrow 6.11899 \times 10^{-7}, 3 \rightarrow 0.999999\},$$

$$\left\{ \begin{array}{c} \text{[Image: A square plot with a blue background and a dense, diagonal pattern of small, light blue lines.]} \\ \rightarrow 1\,762\,420\,096 \end{array} \right\} \rightarrow \{1 \rightarrow 2.34707 \times 10^{-9}, 2 \rightarrow 1.\},$$

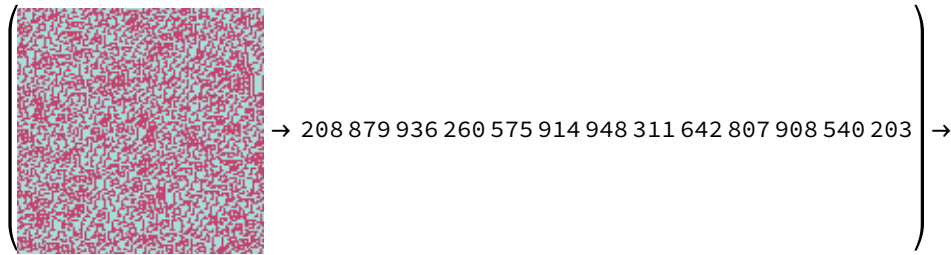


## 2-colour non-totalistic, range 3

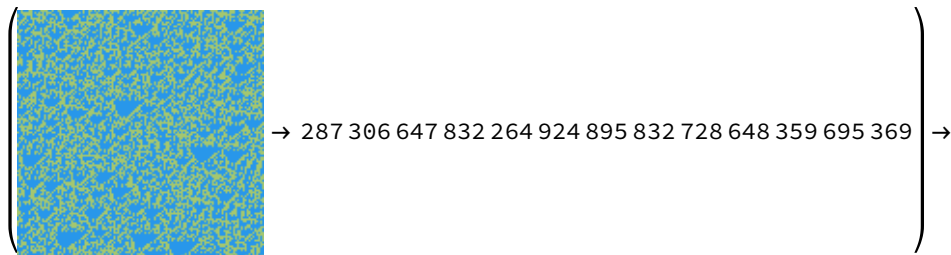
```
In[88]:= test4Data2kr3C16 = datak2r3NT[128, 128, 8];
Thread[test4Data2kr3C16 → netECA16[Keys@test4Data2kr3C16, {"TopProbabilities", 2}]]
```



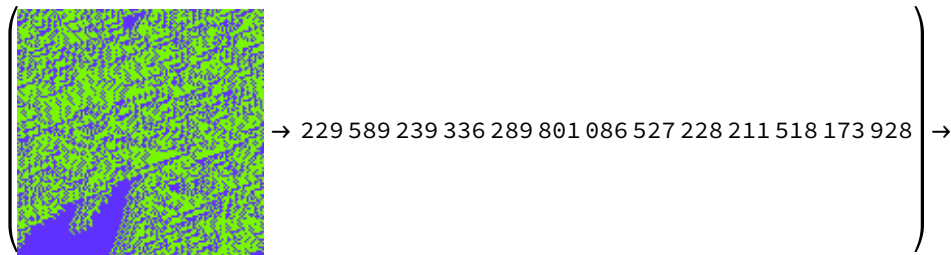
$$\{4 \rightarrow 3.99297 \times 10^{-14}, 3 \rightarrow 1.\},$$



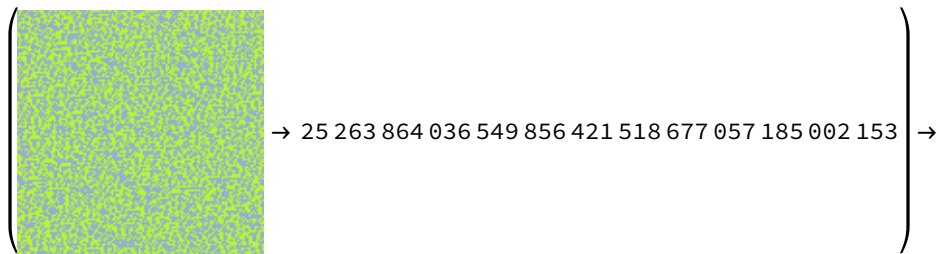
$$\{4 \rightarrow 1.58015 \times 10^{-11}, 3 \rightarrow 1.\},$$



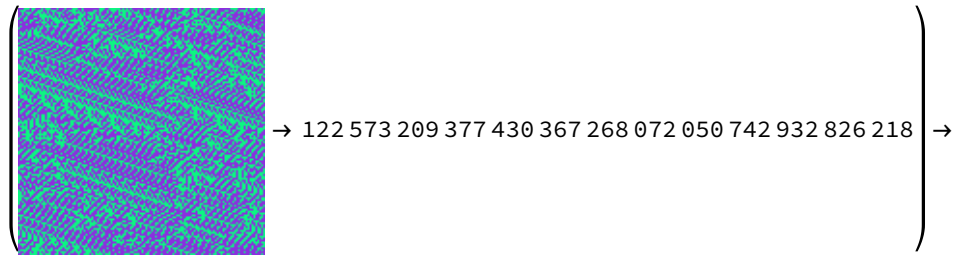
$$\{4 \rightarrow 1.21845 \times 10^{-8}, 3 \rightarrow 1.\},$$



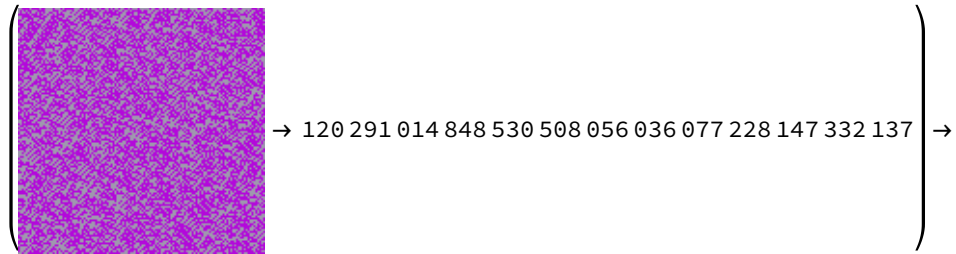
$$\{3 \rightarrow 0.0173989, 4 \rightarrow 0.982601\},$$



$$\{4 \rightarrow 2.486 \times 10^{-11}, 3 \rightarrow 1.\},$$



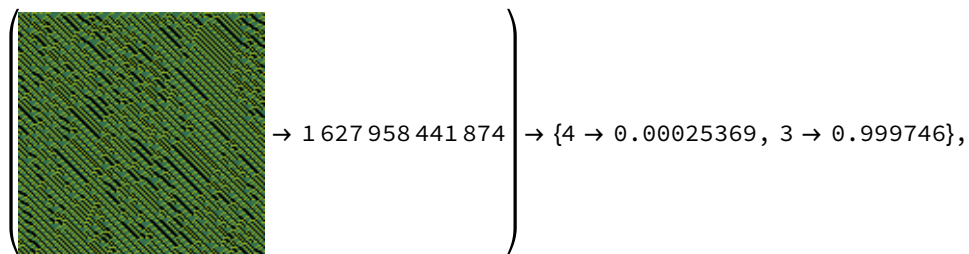
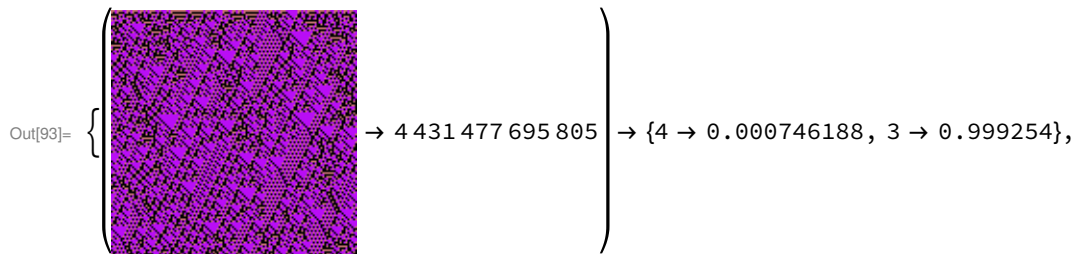
$\{4 \rightarrow 1.46881 \times 10^{-9}, 3 \rightarrow 1.\},$



$\{4 \rightarrow 0.00683298, 3 \rightarrow 0.993167\}$

### 3-colour non-totalistic, range 1

```
In[92]:= test4Data3kr1C16 = datak3r1NT[128, 128, 8];
Thread[test4Data3kr1C16 → netECA16[Keys@test4Data3kr1C16, {"TopProbabilities", 2}]]
```





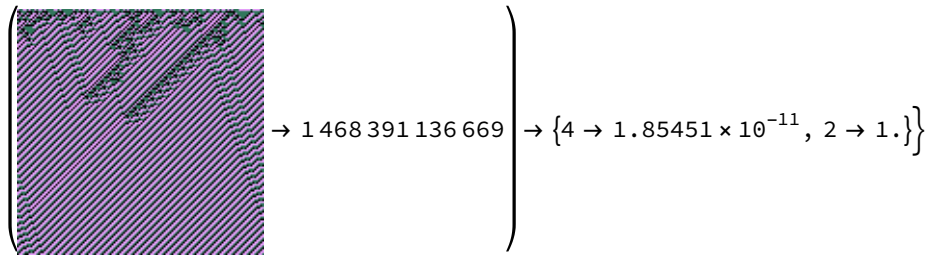
$$\left( \begin{array}{c} \text{Image 1} \\ \rightarrow 4\,241\,674\,451\,024 \end{array} \right) \rightarrow \{3 \rightarrow 0.194892, 2 \rightarrow 0.805108\},$$

$$\left( \begin{array}{c} \text{Image 2} \\ \rightarrow 4\,177\,916\,755\,057 \end{array} \right) \rightarrow \{3 \rightarrow 9.07174 \times 10^{-18}, 4 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image 3} \\ \rightarrow 2\,504\,235\,138\,103 \end{array} \right) \rightarrow \{4 \rightarrow 1.3375 \times 10^{-21}, 2 \rightarrow 1.\},$$

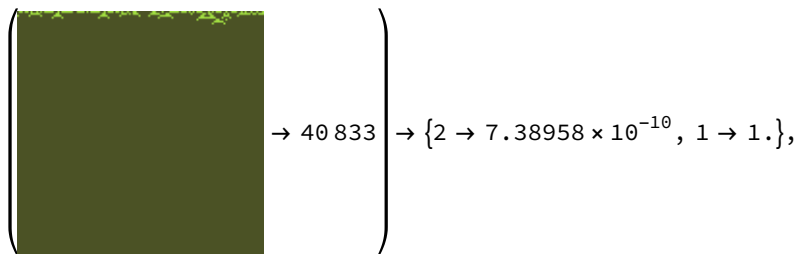
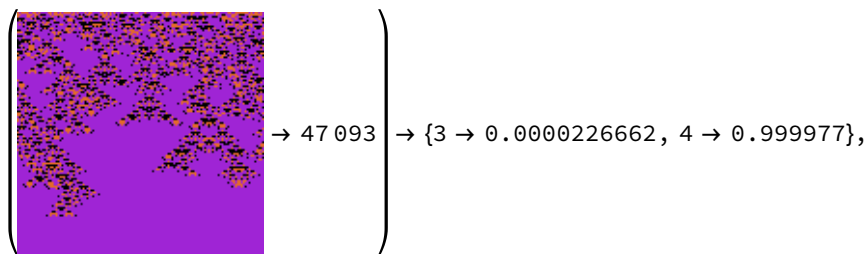
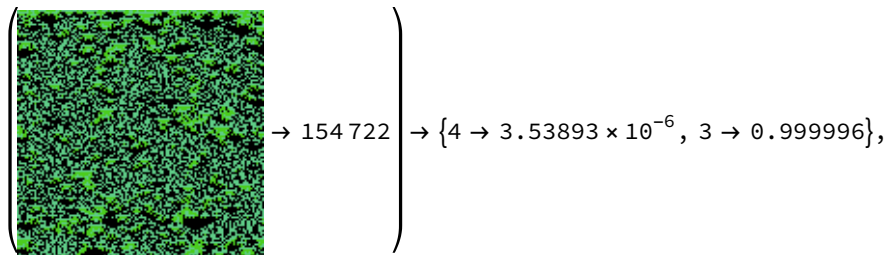
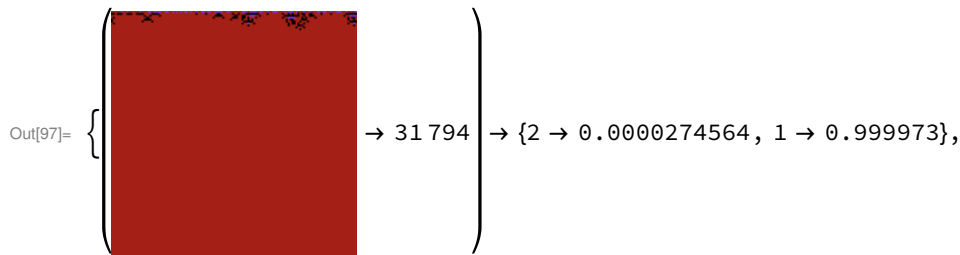
$$\left( \begin{array}{c} \text{Image 4} \\ \rightarrow 2\,281\,646\,033\,785 \end{array} \right) \rightarrow \{4 \rightarrow 0.164883, 3 \rightarrow 0.835117\},$$

$$\left( \begin{array}{c} \text{Image 5} \\ \rightarrow 886\,946\,817\,375 \end{array} \right) \rightarrow \{2 \rightarrow 0.000352038, 1 \rightarrow 0.999648\},$$

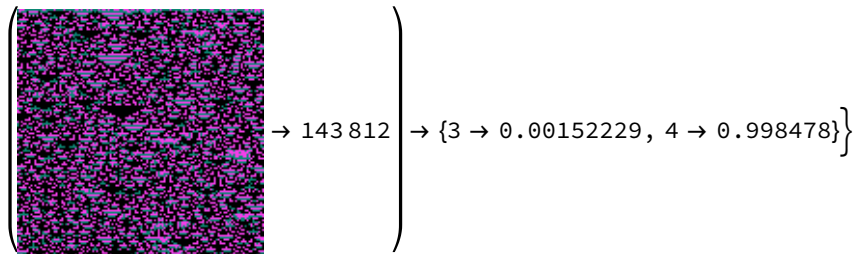
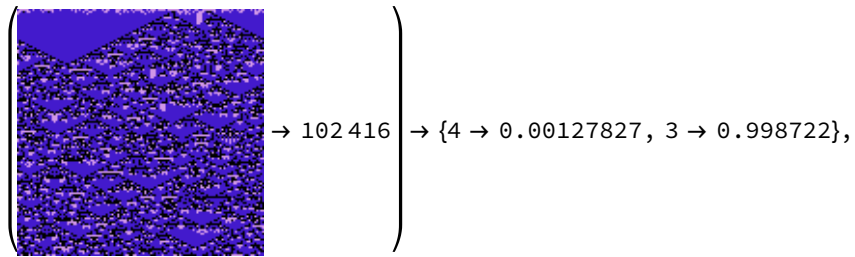
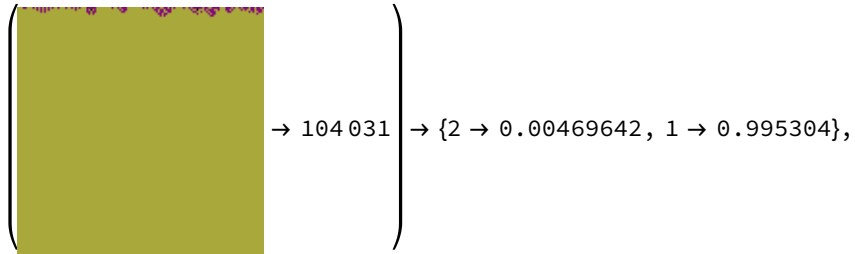
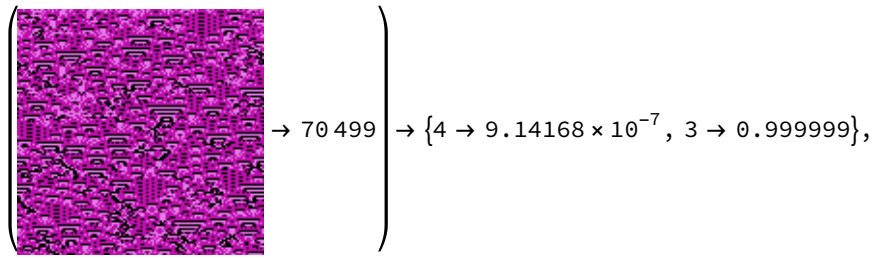


### 3-colour totalistic, range 2

```
In[96]:= test4Data3kr2C16 = datak3r2C[128, 128, 8];
Thread[test4Data3kr2C16 → netECA16[Keys@test4Data3kr2C16, {"TopProbabilities", 2}]]
```

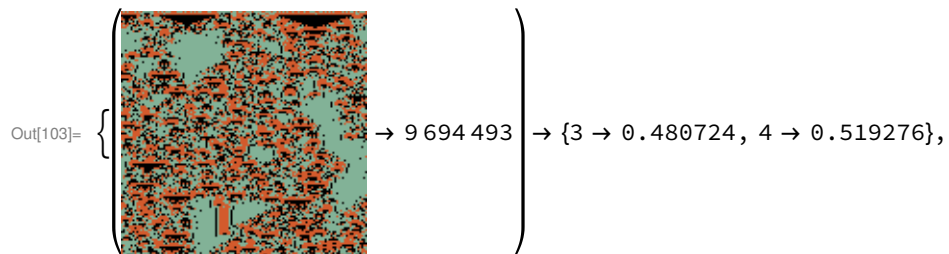






### 3-colour totalistic, range 3

```
In[102]:= test4Data3kr3C16 = datak3r3C[128, 128, 8];
Thread[test4Data3kr3C16 → netECA16[Keys@test4Data3kr3C16, {"TopProbabilities", 2}]]
```



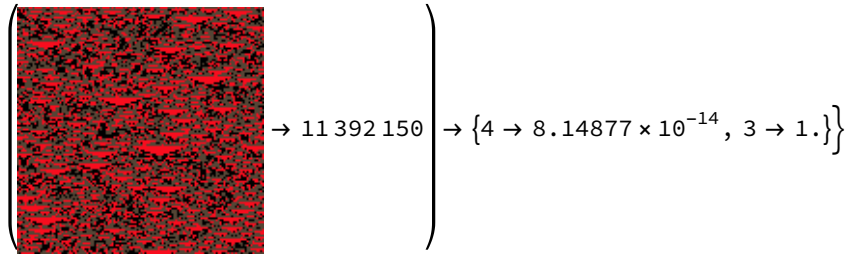
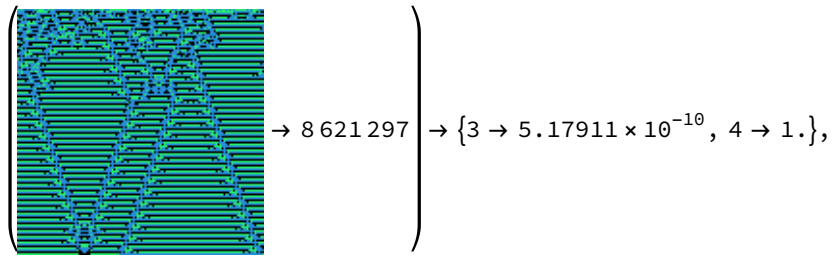
$$\left( \begin{array}{c} \text{Image 1} \\ \rightarrow 1\,266\,350 \end{array} \right) \rightarrow \{3 \rightarrow 2.07073 \times 10^{-17}, 4 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image 2} \\ \rightarrow 10\,922\,251 \end{array} \right) \rightarrow \{4 \rightarrow 0.0000302967, 3 \rightarrow 0.99997\},$$

$$\left( \begin{array}{c} \text{Image 3} \\ \rightarrow 10\,284\,081 \end{array} \right) \rightarrow \{4 \rightarrow 0.0000121386, 3 \rightarrow 0.999988\},$$

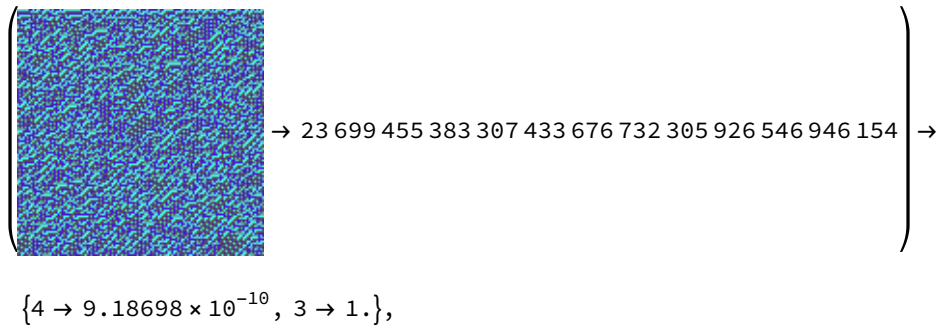
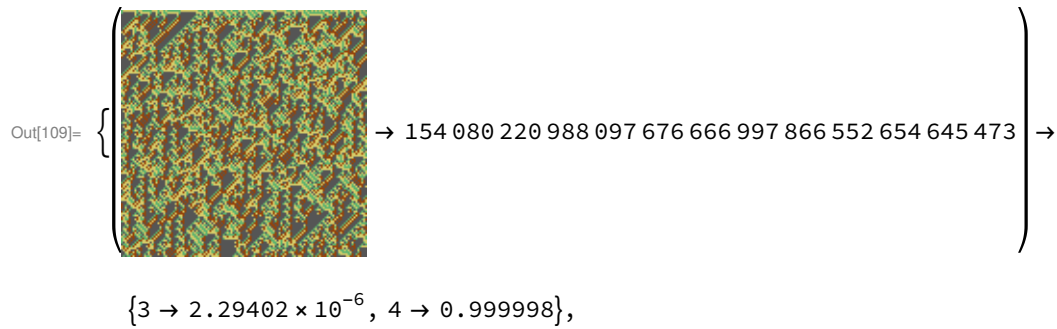
$$\left( \begin{array}{c} \text{Image 4} \\ \rightarrow 3\,664\,255 \end{array} \right) \rightarrow \{1 \rightarrow 0.0137727, 2 \rightarrow 0.986227\},$$

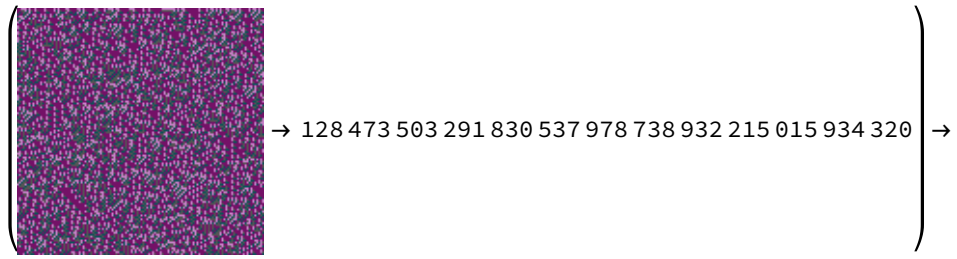
$$\left( \begin{array}{c} \text{Image 5} \\ \rightarrow 10\,298\,881 \end{array} \right) \rightarrow \{4 \rightarrow 0.000133186, 3 \rightarrow 0.999867\},$$



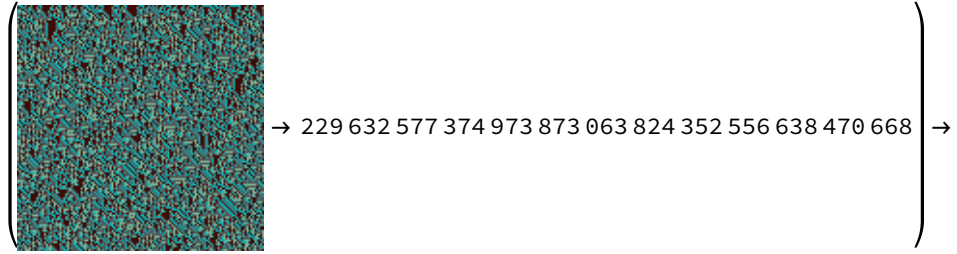
### 4-colour non-totalistic, range 1

```
In[108]:= test4Data4kr1C16 = datak4r1NT[128, 128, 8];
Thread[test4Data4kr1C16 → netECA16[Keys@test4Data4kr1C16, {"TopProbabilities", 2}]]
```

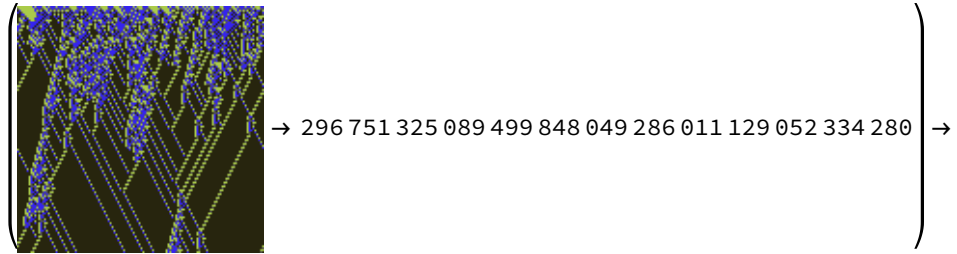




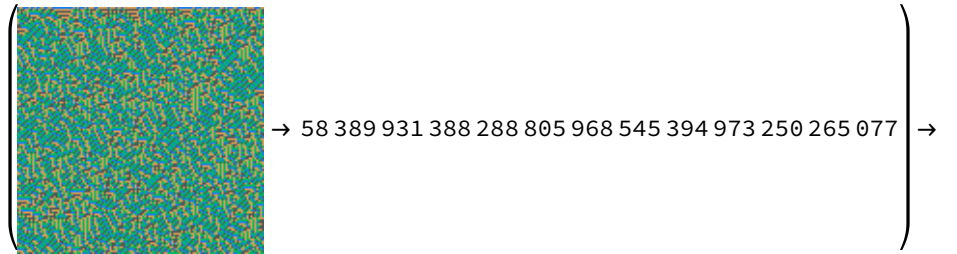
{4 → 0.016884, 3 → 0.983116},



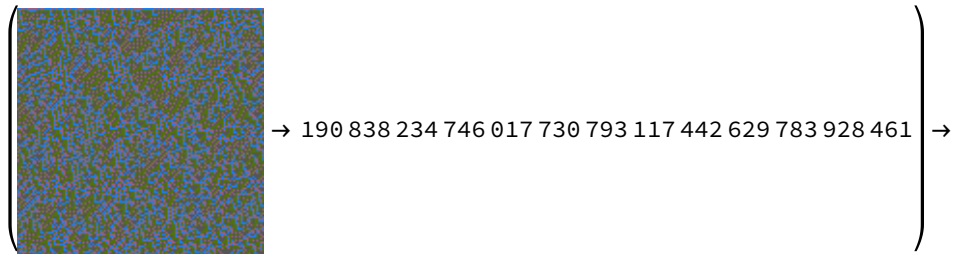
{3 →  $3.66751 \times 10^{-6}$ , 4 → 0.999996},



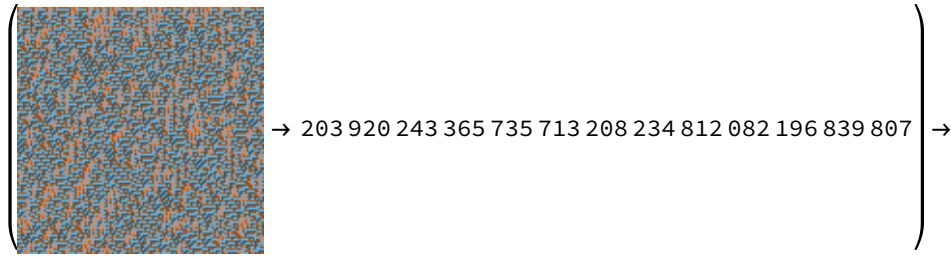
{2 → 0.0356663, 4 → 0.964334},



{4 → 0.392533, 3 → 0.607467},



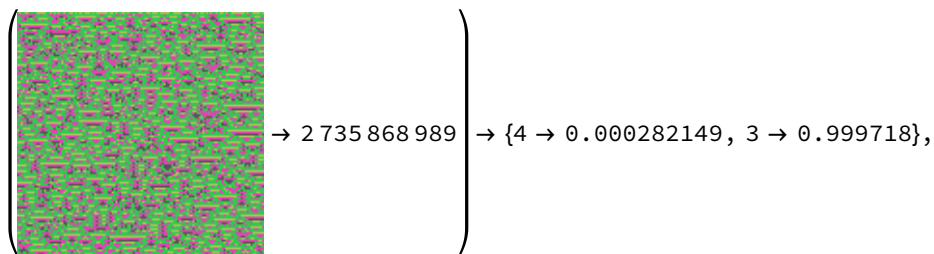
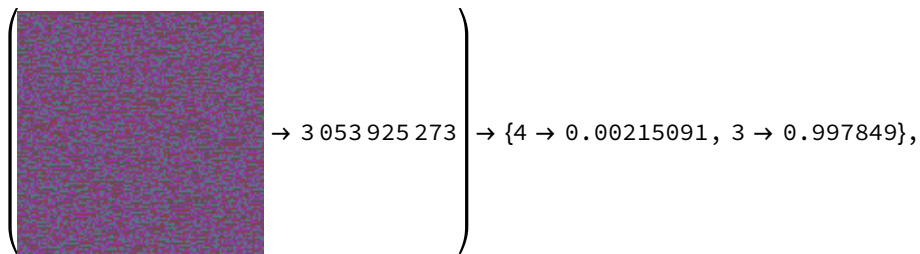
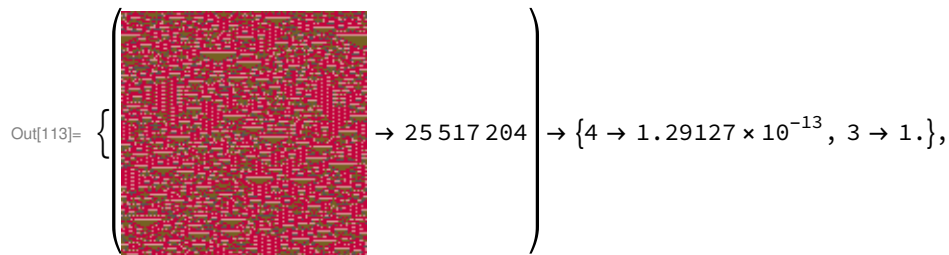
$\{3 \rightarrow 0.0000369307, 4 \rightarrow 0.999963\},$



$\{4 \rightarrow 0.00577653, 3 \rightarrow 0.994223\}$

## 4-colour totalistic, range 2

```
In[112]:= test4Data4kr2C16 = datak4r2C[128, 128, 8];
Thread[test4Data4kr2C16 → netECA16[Keys@test4Data4kr2C16, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 1\,440\,927\,950 \end{array} \right) \rightarrow \{4 \rightarrow 0.0889018, 3 \rightarrow 0.911098\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 3\,727\,816\,705 \end{array} \right) \rightarrow \{3 \rightarrow 2.78599 \times 10^{-7}, 4 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 818\,963\,457 \end{array} \right) \rightarrow \{4 \rightarrow 4.45422 \times 10^{-19}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 3\,948\,742\,311 \end{array} \right) \rightarrow \{4 \rightarrow 2.06369 \times 10^{-10}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 1\,159\,108\,912 \end{array} \right) \rightarrow \{4 \rightarrow 0.431018, 2 \rightarrow 0.568975\}$$

## 5-colour totalistic, range 1

```
In[116]:= test4Data5kr1C16 = data5T2C[8, 128, 128];
Thread[test4Data5kr1C16 → netECA16[Keys@test4Data5kr1C16, {"TopProbabilities", 2}]]
```



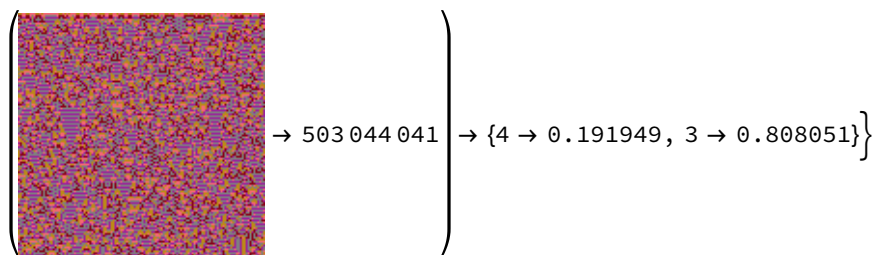
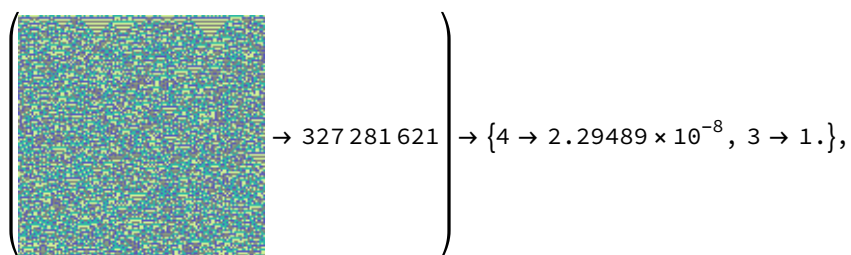
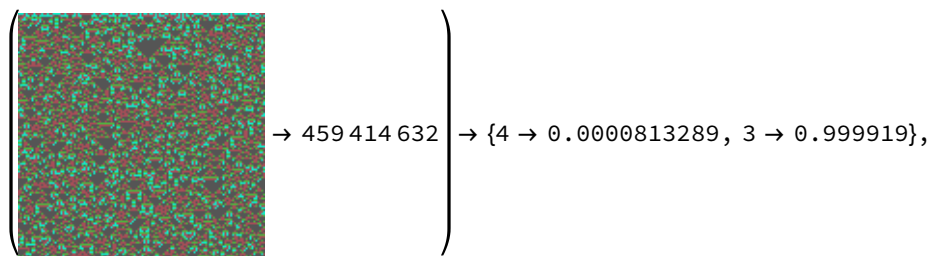
$$\text{Out}[117]= \left\{ \begin{array}{c} \left( \begin{array}{c} \text{[Image: A square plot with a green background and vertical, slightly wavy, lines of varying shades of green and yellow.]}\end{array} \right) \rightarrow 81\,353\,109 \rightarrow \{4 \rightarrow 4.44391 \times 10^{-7}, 2 \rightarrow 1.\}, \end{array} \right.$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a green background and horizontal, slightly wavy, lines of varying shades of green and yellow.]}\end{array} \right) \rightarrow 626\,536\,724 \rightarrow \{3 \rightarrow 3.46291 \times 10^{-8}, 4 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a dark purple background and a dense, noisy pattern of small, lighter purple and blue specks.]}\end{array} \right) \rightarrow 129\,595\,314 \rightarrow \{4 \rightarrow 0.00257287, 3 \rightarrow 0.997427\},$$

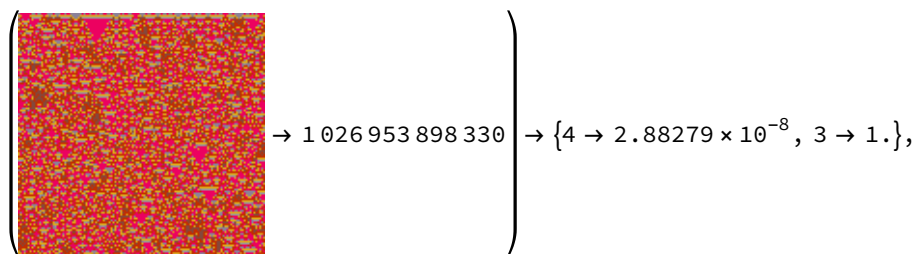
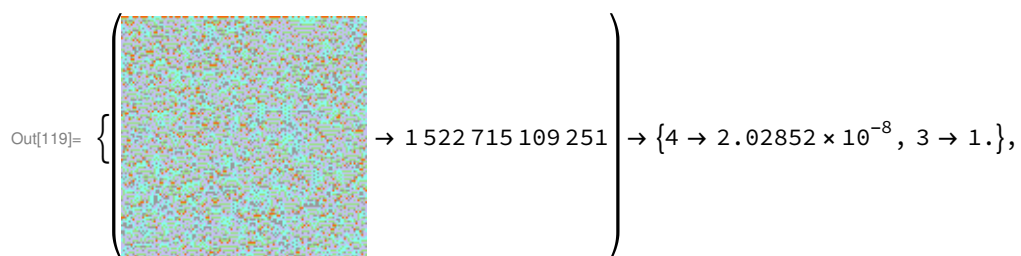
$$\left( \begin{array}{c} \text{[Image: A square plot with a light blue background and a vertical strip of dark blue and purple specks along the left edge.]}\end{array} \right) \rightarrow 513\,885\,470 \rightarrow \{1 \rightarrow 1.41572 \times 10^{-11}, 2 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a green background and a dense, noisy pattern of small, lighter green and yellow specks.]}\end{array} \right) \rightarrow 494\,894\,021 \rightarrow \{3 \rightarrow 0.0136503, 4 \rightarrow 0.98635\},$$



## 6-colour totalistic, range 1

```
In[118]:= test4Data6kr1C16 = data6TC[8, 128, 128];
Thread[test4Data6kr1C16 → netECA16[Keys@test4Data6kr1C16, {"TopProbabilities", 2}]]
```





$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 1\,583\,652\,682 \rightarrow \{3 \rightarrow 0.429972, 4 \rightarrow 0.570028\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 2\,123\,073\,201\,165 \rightarrow \{4 \rightarrow 6.23239 \times 10^{-10}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 341\,591\,565\,791 \rightarrow \{4 \rightarrow 0.00212154, 3 \rightarrow 0.997878\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 2\,568\,539\,246\,083 \rightarrow \{4 \rightarrow 0.00197237, 3 \rightarrow 0.998028\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 1\,213\,730\,554\,155 \rightarrow \{4 \rightarrow 0.0020281, 3 \rightarrow 0.997972\},$$

$$\left( \begin{array}{c} \text{[Random Colours Image]} \\ \rightarrow 389\,841\,312\,036 \end{array} \right) \rightarrow \{4 \rightarrow 2.91693 \times 10^{-11}, 3 \rightarrow 1.\}$$

## 6-colour totalistic, range 2

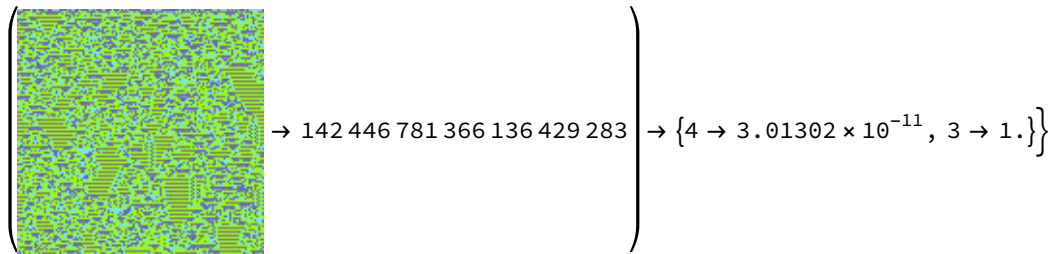
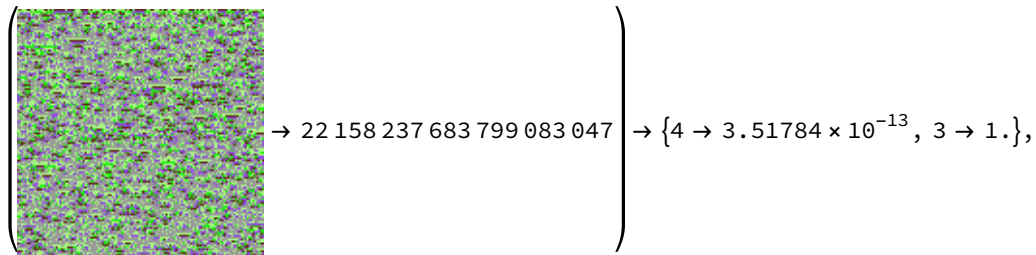
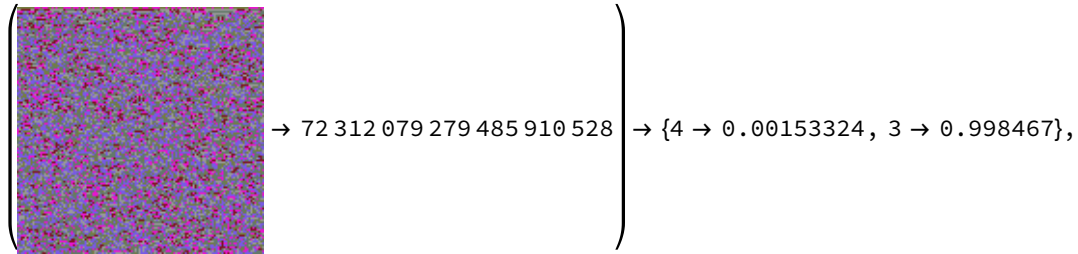
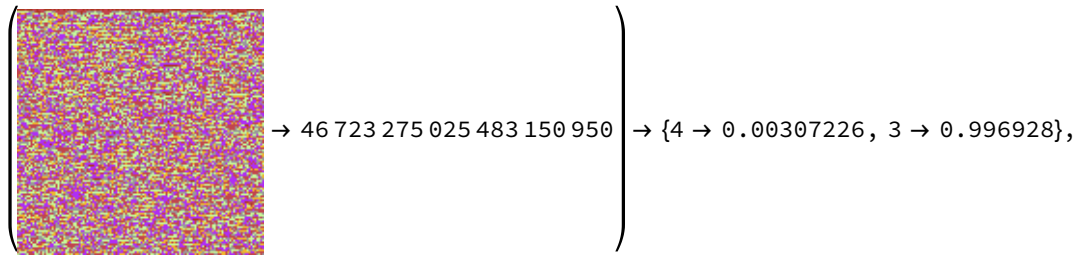
```
In[138]:= test4Data6kr2C16 = data6T2C[8, 128, 128];
Thread[test4Data6kr2C16 → netECA16[Keys@test4Data6kr2C16, {"TopProbabilities", 2}]]
```

$$\text{Out[139]=} \left\{ \begin{array}{c} \text{[Random Colours Image]} \\ \rightarrow 46\,177\,535\,535\,728\,053\,148 \end{array} \right\} \rightarrow \{4 \rightarrow 6.75757 \times 10^{-8}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Random Colours Image]} \\ \rightarrow 35\,643\,164\,656\,729\,746\,413 \end{array} \right) \rightarrow \{4 \rightarrow 4.68349 \times 10^{-15}, 3 \rightarrow 1.\},$$

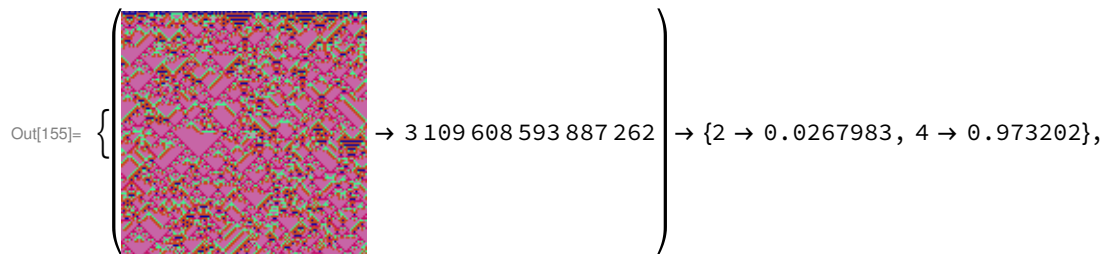
$$\left( \begin{array}{c} \text{[Random Colours Image]} \\ \rightarrow 151\,294\,335\,263\,255\,298\,785 \end{array} \right) \rightarrow \{4 \rightarrow 0.0673459, 3 \rightarrow 0.932654\},$$

$$\left( \begin{array}{c} \text{[Random Colours Image]} \\ \rightarrow 8\,803\,703\,818\,914\,948\,546 \end{array} \right) \rightarrow \{4 \rightarrow 0.00560205, 3 \rightarrow 0.994398\},$$



## 7-colour totalistic, range 1

```
In[154]:= test4Data7kr1C16 = data7TC[8, 128, 128];
Thread[test4Data7kr1C16 → netECA16[Keys@test4Data7kr1C16, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 10\,516\,337\,788\,191\,339 \rightarrow \{4 \rightarrow 0.202783, 3 \rightarrow 0.797217\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 10\,218\,434\,972\,470\,056 \rightarrow \{4 \rightarrow 2.59313 \times 10^{-9}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 11\,301\,098\,979\,433\,534 \rightarrow \{4 \rightarrow 5.31247 \times 10^{-20}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 4\,222\,218\,586\,098\,008 \rightarrow \{4 \rightarrow 2.3505 \times 10^{-8}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 6\,272\,918\,110\,265\,620 \rightarrow \{4 \rightarrow 2.82334 \times 10^{-12}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot filled with a dense, noisy pattern of red and purple pixels.]}\end{array} \right) \rightarrow 7\,237\,960\,144\,206\,103 \rightarrow \{4 \rightarrow 1.82804 \times 10^{-11}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot filled with a dense, noisy pattern of blue, green, and yellow pixels.]}\end{array} \right) \rightarrow 8\,157\,411\,943\,012\,914 \rightarrow \{4 \rightarrow 7.30163 \times 10^{-8}, 3 \rightarrow 1.\}$$

```
In[162]:= test4Data7kr1C16 = data7TC[8, 128, 128];
Thread[test4Data7kr1C16 → netECA16[Keys@test4Data7kr1C16, {"TopProbabilities", 2}]]
```

$$\text{Out[163]=} \left\{ \begin{array}{c} \text{[Image: A solid green square plot.]}\end{array} \right\} \rightarrow 2\,054\,187\,704\,193\,738 \rightarrow \{2 \rightarrow 1.00393 \times 10^{-7}, 1 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a dark brown background and two vertical lines of purple and blue pixels.]}\end{array} \right) \rightarrow 4\,502\,314\,670\,347\,259 \rightarrow \{1 \rightarrow 0.000272502, 2 \rightarrow 0.999727\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot filled with a dense, noisy pattern of purple and blue pixels.]}\end{array} \right) \rightarrow 6\,433\,286\,718\,439\,853 \rightarrow \{4 \rightarrow 3.57308 \times 10^{-13}, 3 \rightarrow 1.\},$$



$$\left( \begin{array}{c} \text{[Image: A square plot with a noisy, greenish background and scattered purple and red pixels.]}\end{array} \right) \rightarrow 10\,115\,271\,094\,201\,812 \rightarrow \{4 \rightarrow 1.83956 \times 10^{-14}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a green background, a vertical column of red dots on the left, and a cluster of red dots on the right.]}\end{array} \right) \rightarrow 2\,056\,629\,839\,849\,700 \rightarrow \{4 \rightarrow 7.03567 \times 10^{-6}, 2 \rightarrow 0.999993\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a noisy, multi-colored background.]}\end{array} \right) \rightarrow 6\,016\,684\,767\,156\,829 \rightarrow \{4 \rightarrow 0.0021258, 3 \rightarrow 0.997874\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a noisy, blue and purple background.]}\end{array} \right) \rightarrow 1\,150\,898\,749\,617\,983 \rightarrow \{4 \rightarrow 5.05985 \times 10^{-9}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a purple background and a diagonal pattern of red and blue lines.]}\end{array} \right) \rightarrow 3\,441\,885\,208\,643\,463 \rightarrow \{3 \rightarrow 1.57168 \times 10^{-8}, 2 \rightarrow 1.\}$$

```
In[170]:= test4Data7kr1C16 = data7TC[8, 128, 128];
Thread[test4Data7kr1C16 → netECA16[Keys@test4Data7kr1C16, {"TopProbabilities", 2}]]
```

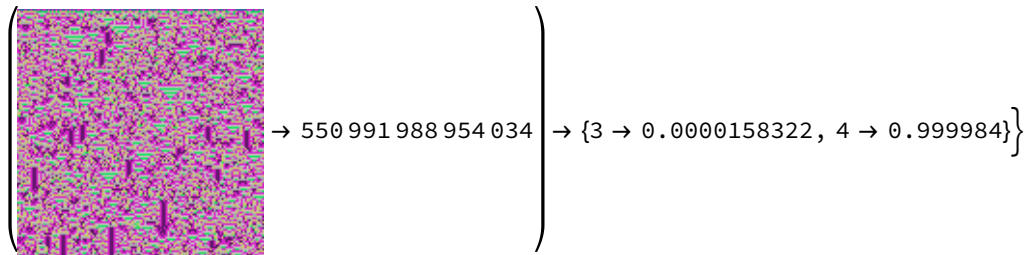
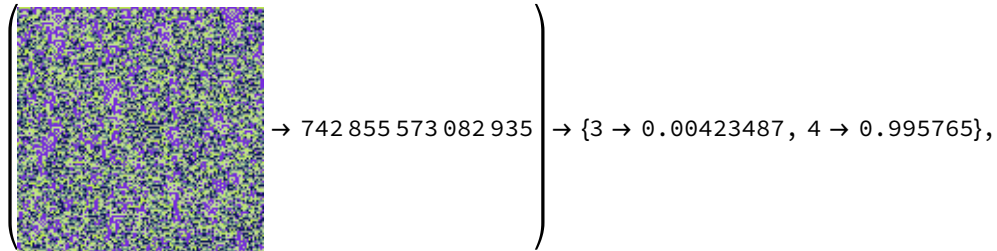
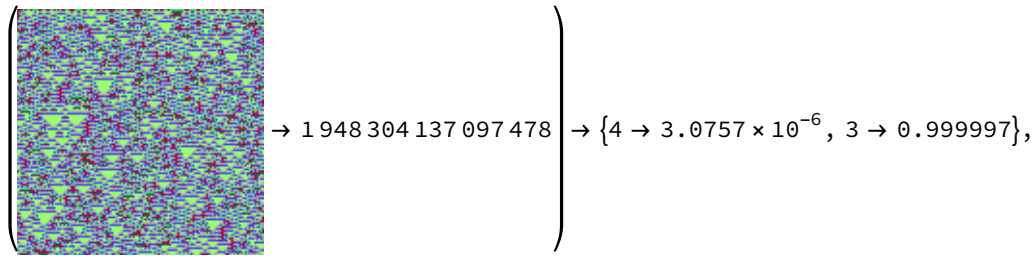
$$\text{Out}[171]= \left\{ \begin{array}{c} \text{Image} \\ \rightarrow 8\,718\,538\,805\,570\,808 \end{array} \right\} \rightarrow \{4 \rightarrow 0.0199047, 2 \rightarrow 0.980095\},$$

$$\left\{ \begin{array}{c} \text{Image} \\ \rightarrow 5\,687\,458\,247\,703\,346 \end{array} \right\} \rightarrow \{3 \rightarrow 3.931 \times 10^{-6}, 4 \rightarrow 0.999995\},$$

$$\left\{ \begin{array}{c} \text{Image} \\ \rightarrow 2\,004\,300\,484\,518\,722 \end{array} \right\} \rightarrow \{3 \rightarrow 0.0438658, 4 \rightarrow 0.956134\},$$

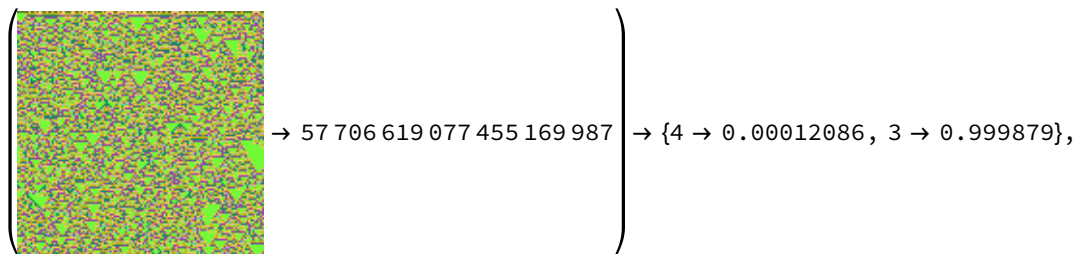
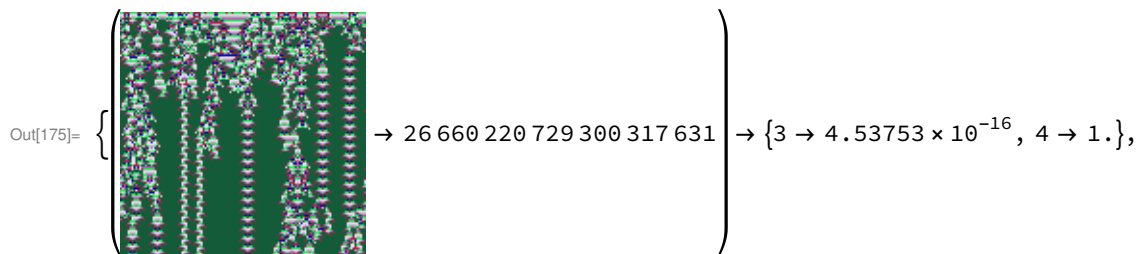
$$\left\{ \begin{array}{c} \text{Image} \\ \rightarrow 2\,106\,485\,862\,858\,275 \end{array} \right\} \rightarrow \{4 \rightarrow 3.36807 \times 10^{-10}, 3 \rightarrow 1.\},$$

$$\left\{ \begin{array}{c} \text{Image} \\ \rightarrow 10\,335\,102\,717\,390\,268 \end{array} \right\} \rightarrow \{4 \rightarrow 1.40275 \times 10^{-9}, 2 \rightarrow 1.\},$$



## 8-colour totalistic, range 1

```
In[174]:= test4Data8kr1C16 = data8TC[8, 128, 128];
Thread[test4Data8kr1C16 → netECA16[Keys@test4Data8kr1C16, {"TopProbabilities", 2}]]
```





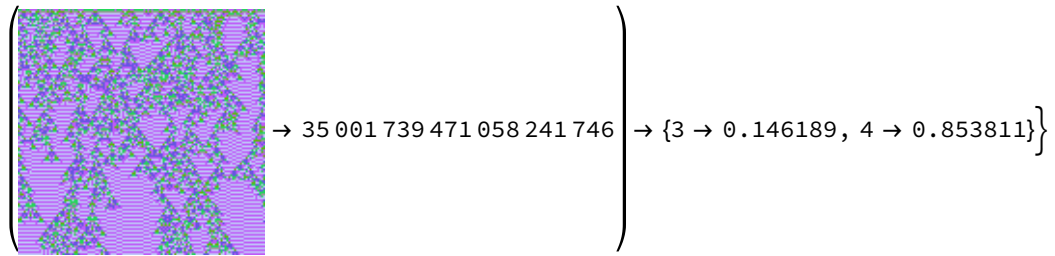
$$\left( \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of green and purple pixels.]}\end{array} \right) \rightarrow 64\,248\,301\,738\,433\,598\,883 \rightarrow \{4 \rightarrow 8.62498 \times 10^{-7}, 3 \rightarrow 0.999999\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of red and pink pixels.]}\end{array} \right) \rightarrow 38\,309\,191\,234\,358\,472\,181 \rightarrow \{3 \rightarrow 0.0920227, 4 \rightarrow 0.907977\},$$

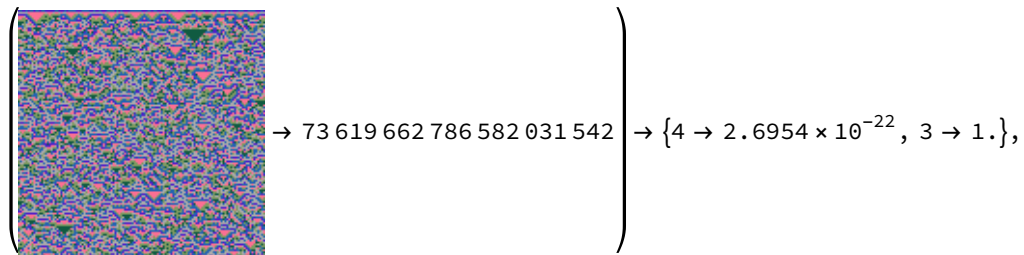
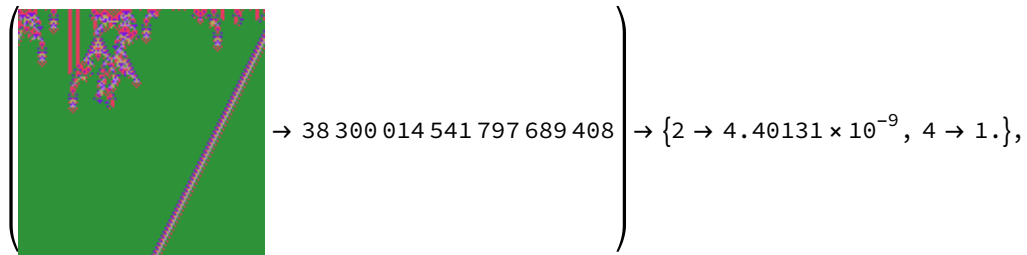
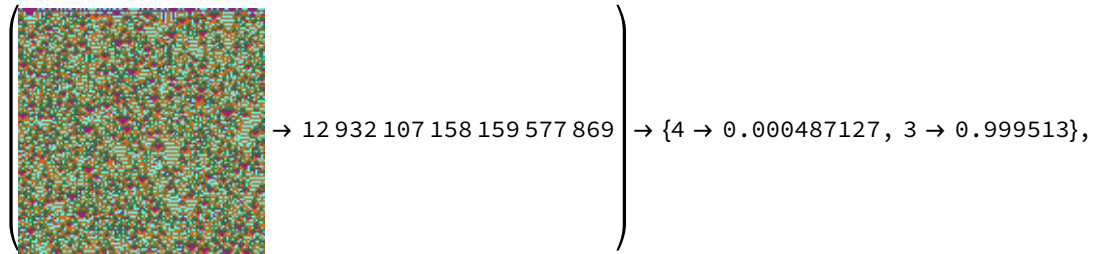
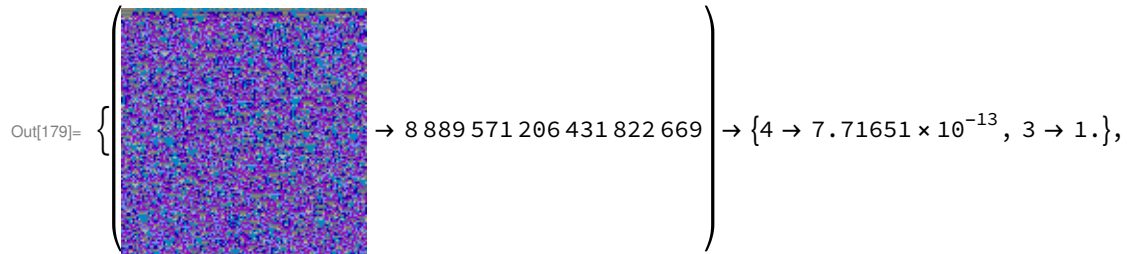
$$\left( \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of green and blue pixels.]}\end{array} \right) \rightarrow 10\,057\,418\,236\,647\,939\,786 \rightarrow \{3 \rightarrow 0.00153869, 4 \rightarrow 0.998461\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of blue and purple pixels.]}\end{array} \right) \rightarrow 55\,038\,816\,396\,722\,824\,044 \rightarrow \{4 \rightarrow 7.93818 \times 10^{-11}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of red and grey pixels.]}\end{array} \right) \rightarrow 13\,857\,790\,822\,319\,662\,750 \rightarrow \{4 \rightarrow 1.6375 \times 10^{-9}, 2 \rightarrow 1.\},$$



```
In[178]:= test4Data8kr1C16 = data8TC[8, 128, 128];
Thread[test4Data8kr1C16 → netECA16[Keys@test4Data8kr1C16, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{[Image: A square image with a noisy, greenish-brown pattern.]}\end{array} \right) \rightarrow 25\,075\,664\,454\,379\,326\,631 \rightarrow \{4 \rightarrow 2.7484 \times 10^{-6}, 3 \rightarrow 0.999997\},$$

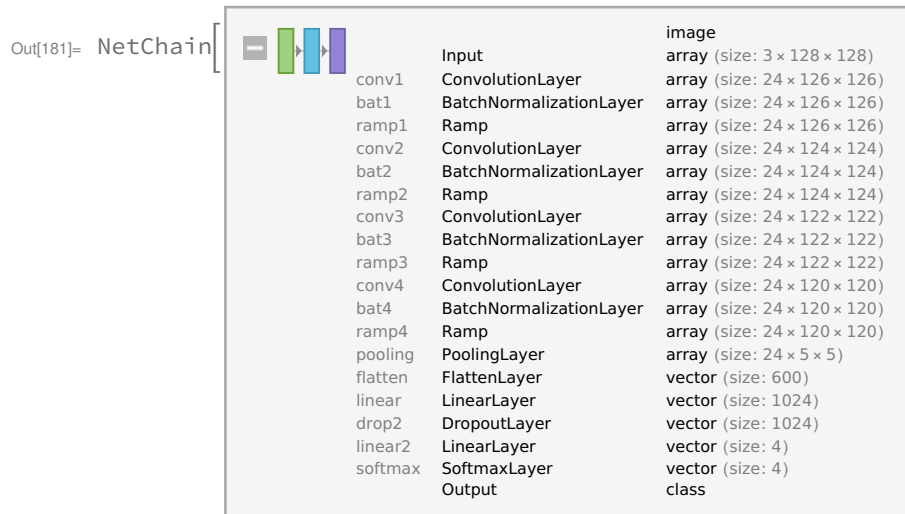
$$\left( \begin{array}{c} \text{[Image: A square image with a blue background and a repeating pattern of small white triangles.]}\end{array} \right) \rightarrow 66\,251\,301\,754\,800\,867\,134 \rightarrow \{4 \rightarrow 0.045885, 3 \rightarrow 0.954115\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a noisy, pinkish-purple pattern.]}\end{array} \right) \rightarrow 45\,939\,929\,171\,629\,412\,480 \rightarrow \{3 \rightarrow 0.0015837, 4 \rightarrow 0.998416\},$$

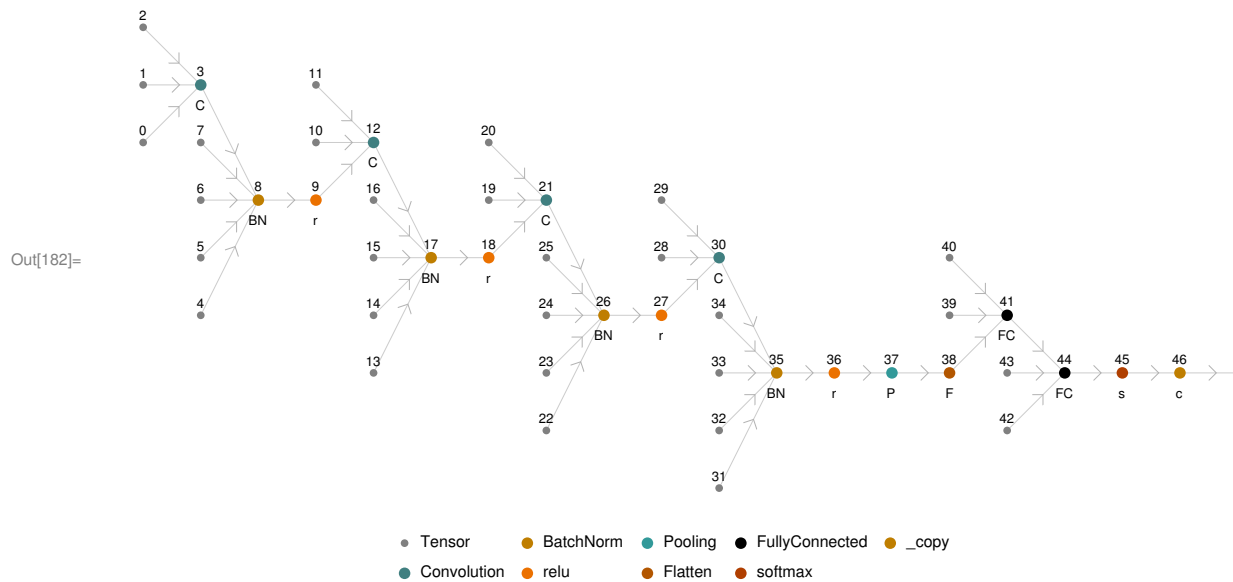
$$\left( \begin{array}{c} \text{[Image: A square image with a noisy, greenish-brown pattern, similar to the first one.]}\end{array} \right) \rightarrow 48\,918\,067\,825\,593\,479\,863 \rightarrow \{4 \rightarrow 0.000601606, 3 \rightarrow 0.999398\}$$

## Network XVII - Four convolutions, dropout on linear only, BatchNorm

```
ln[181]:= netECA17 = netTenCC1024drop[128, 128]
```



In[182]:= NetInformation[netECA17, "MXNetNodeGraphPlot"]



In[183]:= NetInformation[netECA17, "SummaryGraphic"]



In[184]:= dataECA17 = dataC[128, 128, 16 384];

In[185]:= dataTotalistic2BigC17 = genData2r2C[128, 128, 2048];

In[186]:= dataTotalistic3BigC17 = data3T2C[128, 128, 2048];

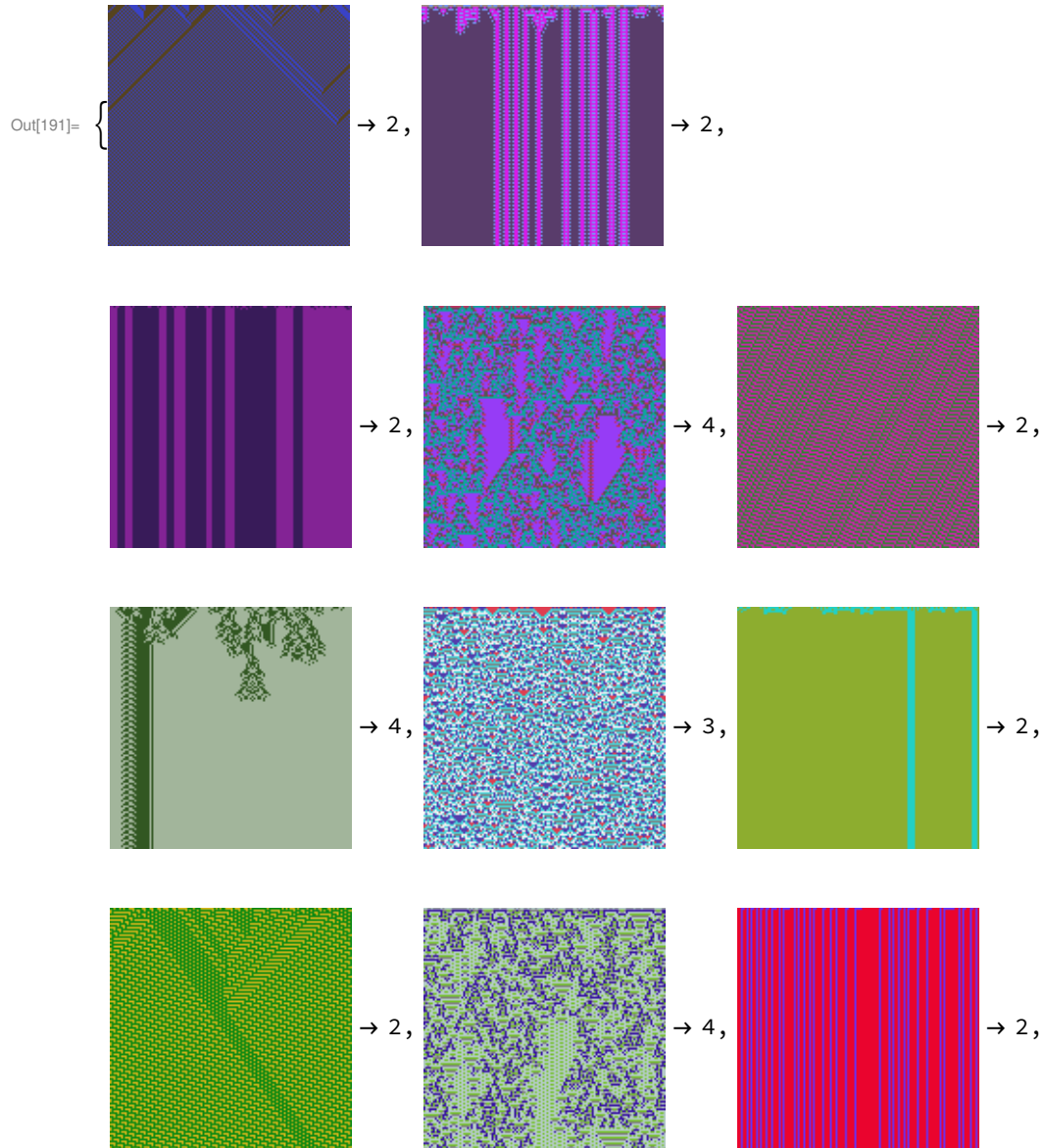
In[187]:= dataTotalistic4BigC17 = data4TC[128, 128, 2048];

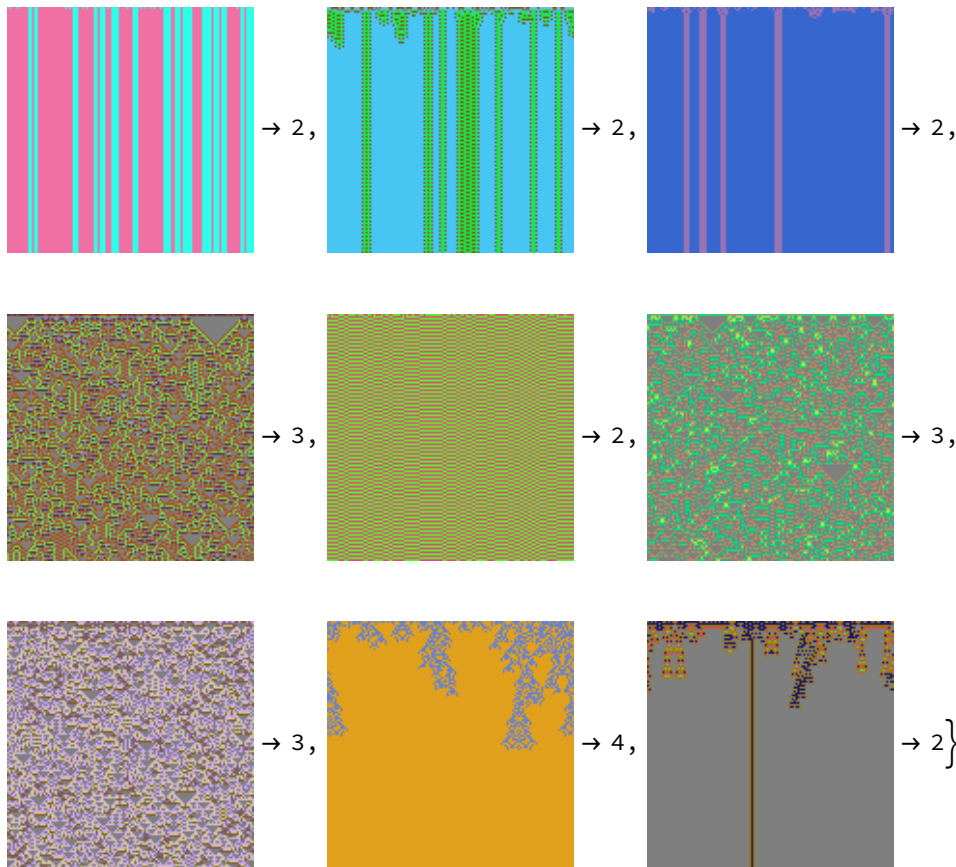
In[188]:= dataTotalistic5BigC17 = genData5TCC[128, 128, 8192];

```
In[189]:= fullTrainingBigC17 = Join[dataECA17, dataTotalistic2BigC17,
    dataTotalistic3BigC17, dataTotalistic4BigC17, dataTotalistic5BigC17];
Length[fullTrainingBigC17]
```

Out[190]= 53 248

```
In[191]:= RandomSample[fullTrainingBigC17, 20]
```





```
In[ ]:= dir = SetDirectory[NotebookDirectory[]]
```

```
In[192]:= "/home/esilverman/Documents"
```

```
Out[192]= /home/esilverman/Documents
```

```
In[193]:= netECA17 =
```

```
NetTrain[netECA17, fullTrainingBigC17, MaxTrainingRounds → 200, BatchSize → 256,  
TargetDevice → "GPU", TrainingProgressCheckpointing → {"Directory", dir}]
```

```
Out[193]= NetChain[  
  {  
    +,   
    →,   
    →,   
    Input port: image  
    Output port: class  
    Number of layers: 18  
  }  
]
```

```
netECA17 = Import["netECA17-r200.wlnet"]
```

## Generate test data for Network XVII (200 epochs)

```
In[ ]:= dir = SetDirectory[NotebookDirectory[]]
```

```
Out[ ]:= /Users/thorsilver/Downloads/Wolfram notebooks
```



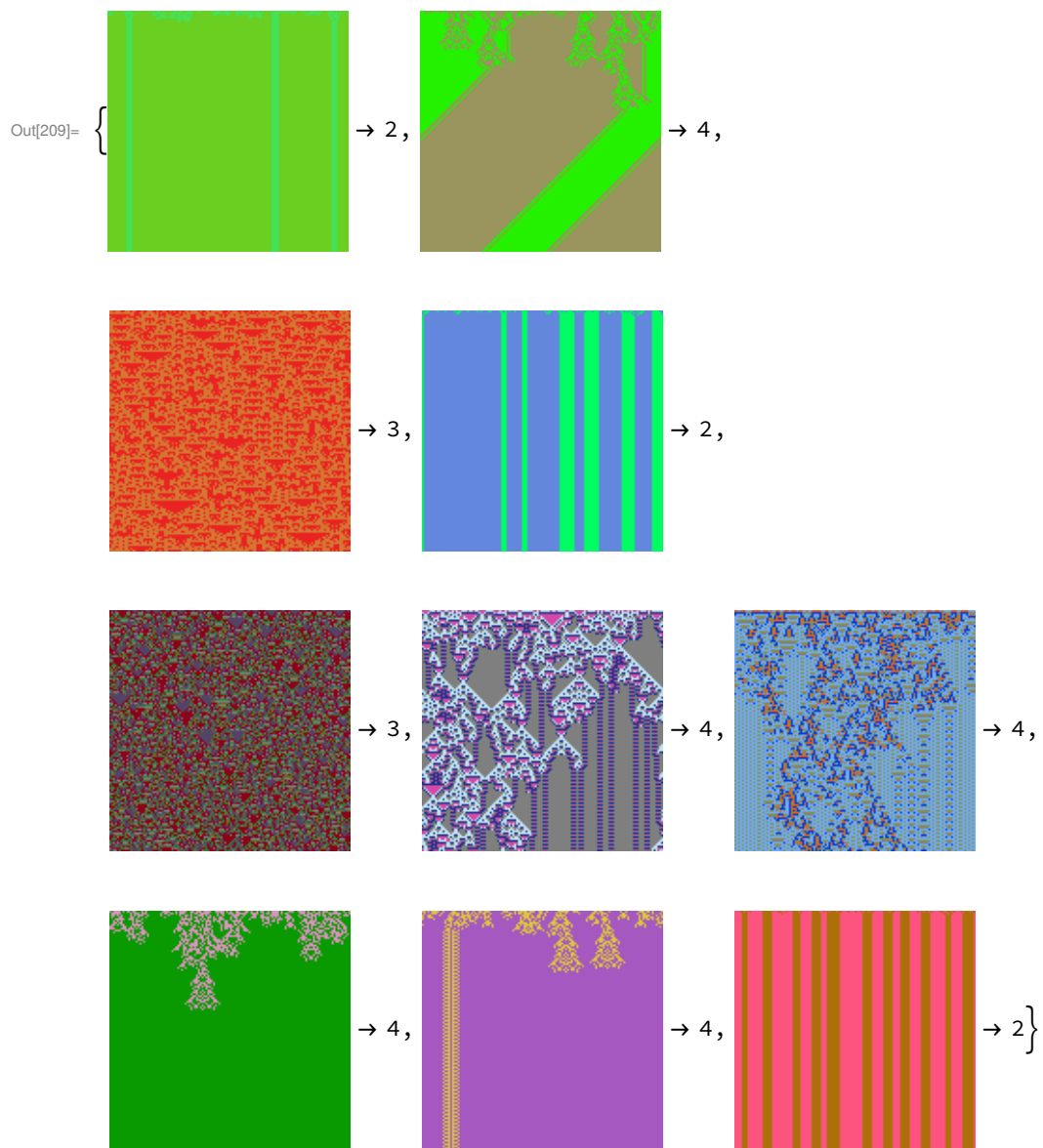
```
netECA17 = Import["netECA17-r200.wlnet"]
```

Out[ ] = J= NetChain[

```
In[202]:= testDataECABigC = dataC[128, 128, 1024];
testData2TBigC = genData2r2C[128, 128, 1024];
testData3TBigC = data3T2C[128, 128, 1024];
testData4TBigC = data4TC[128, 128, 1024];
testData5TBigC = genData5TCC[128, 128, 1024];
fullTestSetBigC = Join[testDataECABigC,
  testData2TBigC, testData3TBigC, testData4TBigC, testData5TBigC];
Length[fullTestSetBigC]
```

Out[208]= 10 240

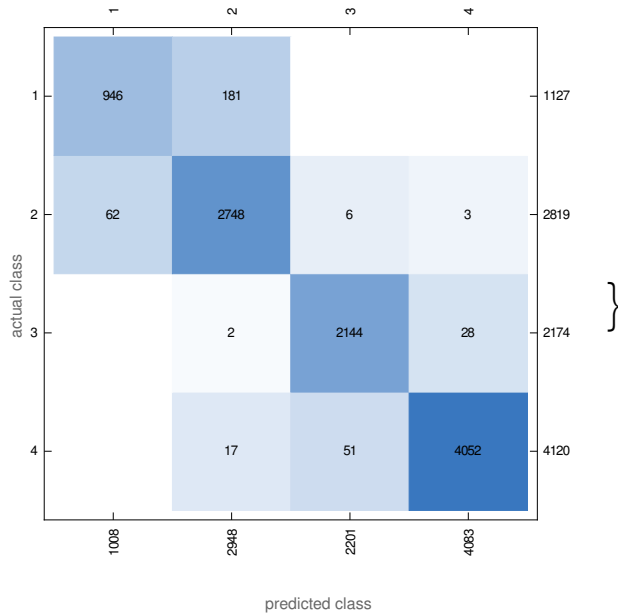
In[209]:= RandomSample[fullTestSetBigC, 10]



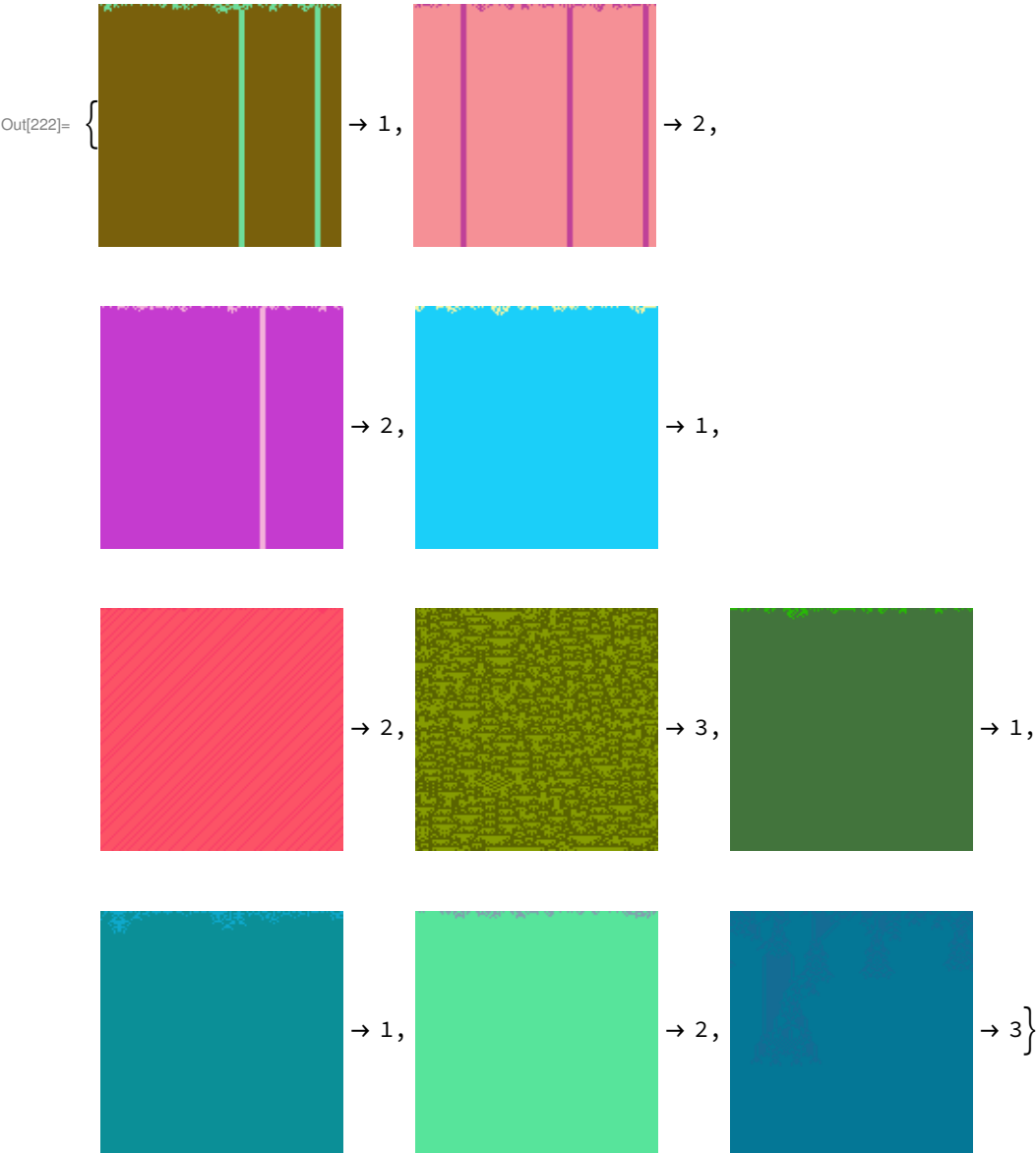


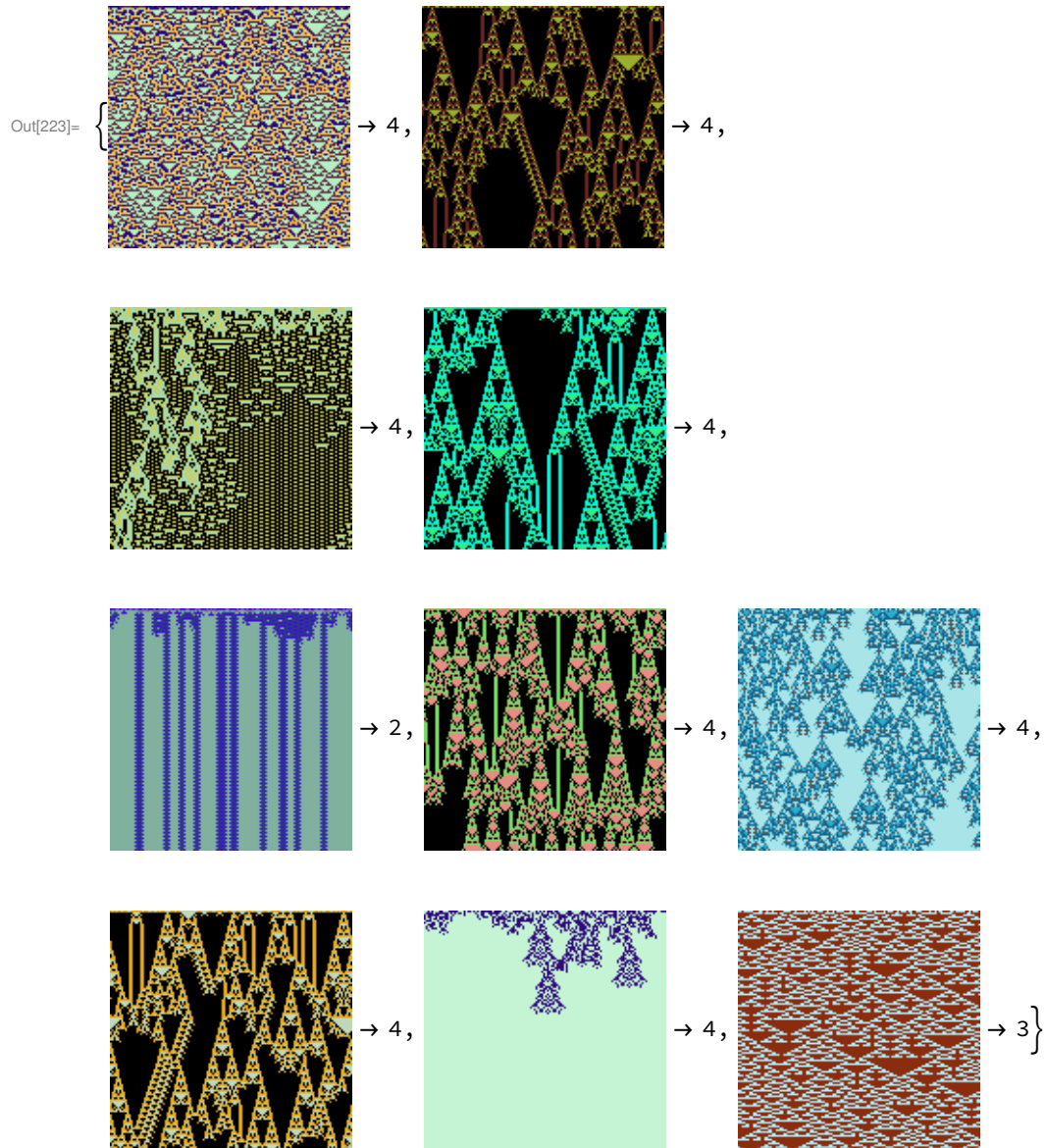
```
In[217]:= NetMeasurements[netECA17, fullTestSetBigC,
  {"Accuracy", "Precision", "ConfusionMatrixPlot"}]
```

```
Out[217]= {0.96582, <| 1 → 0.938492, 2 → 0.932157, 3 → 0.974103, 4 → 0.992408 |>,
```



```
In[218]:= entropyImagesBigC = RandomSample[Keys[fullTestSetBigC], 500];
entropiesBigC = netECA17[entropyImagesBigC, "Entropy"];
highEntBigC = entropyImagesBigC[[Ordering[entropiesBigC, -10]]];
lowEntBigC = entropyImagesBigC[[Ordering[entropiesBigC, 10]]];
Thread[highEntBigC → netECA17[highEntBigC]]
Thread[lowEntBigC → netECA17[lowEntBigC]]
```





Testing Network XVII (200 epochs) on unseen CA rule spaces

2-colour non-totalistic, range 2

```
In[224]:= test4Data2kr2C17 = datak2r2C[128, 128, 8];
Thread[test4Data2kr2C17 → netECA17[Keys@test4Data2kr2C17, {"TopProbabilities", 2}]]
```

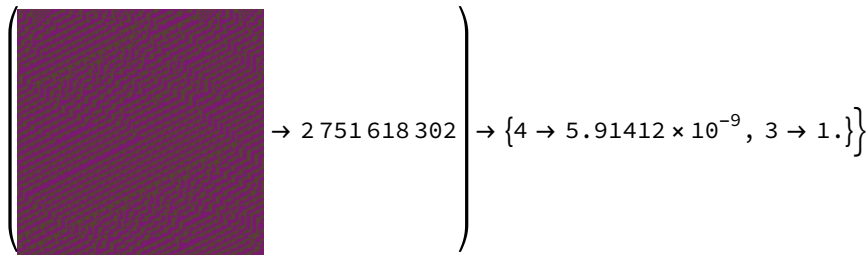
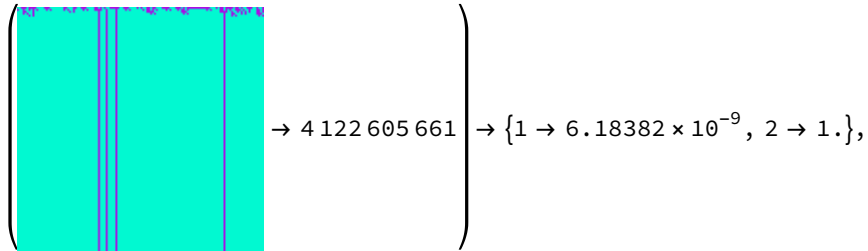
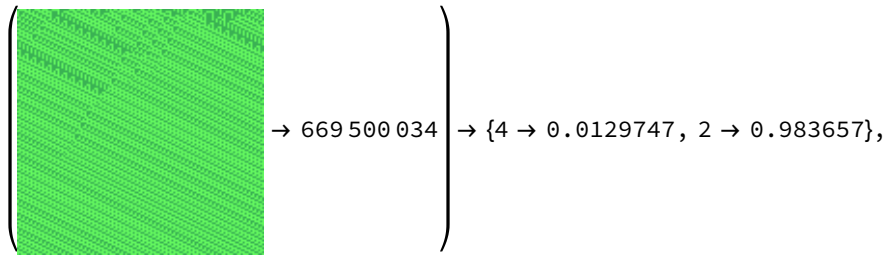
$$\text{Out}[225]= \left\{ \begin{array}{c} \left( \begin{array}{c} \text{[Orange Noise Pattern]} \end{array} \right) \end{array} \right\} \rightarrow 3\,594\,886\,935 \rightarrow \{3 \rightarrow 1.19587 \times 10^{-7}, 2 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Green/Yellow Noise Pattern]} \end{array} \right) \rightarrow 4\,012\,014\,789 \rightarrow \{4 \rightarrow 0.00317589, 3 \rightarrow 0.996824\},$$

$$\left( \begin{array}{c} \text{[Purple/Pink Noise Pattern]} \end{array} \right) \rightarrow 736\,342\,145 \rightarrow \{4 \rightarrow 0.000138652, 3 \rightarrow 0.999861\},$$

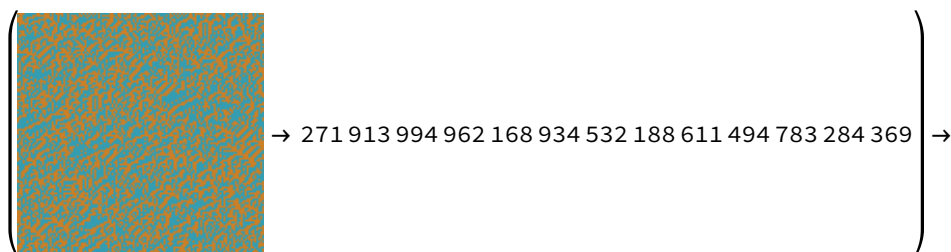
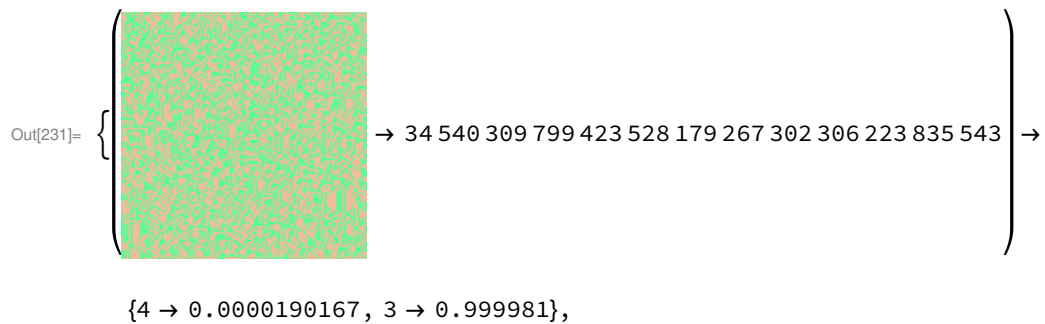
$$\left( \begin{array}{c} \text{[Vertical Green/Yellow Stripes]} \end{array} \right) \rightarrow 3\,597\,938\,931 \rightarrow \{4 \rightarrow 5.42024 \times 10^{-16}, 2 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Vertical Blue Stripes]} \end{array} \right) \rightarrow 49\,406\,137 \rightarrow \{1 \rightarrow 4.03179 \times 10^{-30}, 2 \rightarrow 1.\},$$



## 2-colour non-totalistic, range 3

```
In[230]:= test4Data2kr3C17 = datak2r3NT[128, 128, 8];
Thread[test4Data2kr3C17 → netECA17[Keys@test4Data2kr3C17, {"TopProbabilities", 2}]]
```



$$\{4 \rightarrow 8.79258 \times 10^{-15}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a green and yellow pixelated pattern]} \end{array} \rightarrow 328\,899\,203\,265\,424\,040\,719\,063\,648\,351\,222\,904\,052 \right) \rightarrow$$

$$\{3 \rightarrow 0.000609094, 4 \rightarrow 0.999391\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a magenta and black pixelated pattern]} \end{array} \rightarrow 193\,952\,196\,319\,600\,964\,880\,345\,641\,761\,505\,849\,023 \right) \rightarrow$$

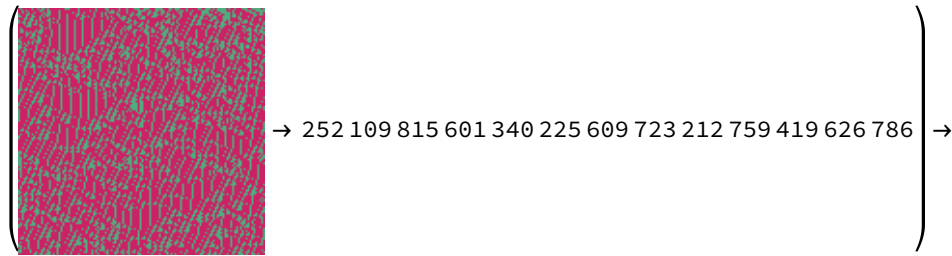
$$\{4 \rightarrow 8.96571 \times 10^{-10}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a teal and black pixelated pattern]} \end{array} \rightarrow 196\,656\,308\,173\,162\,097\,107\,061\,836\,715\,379\,270\,739 \right) \rightarrow$$

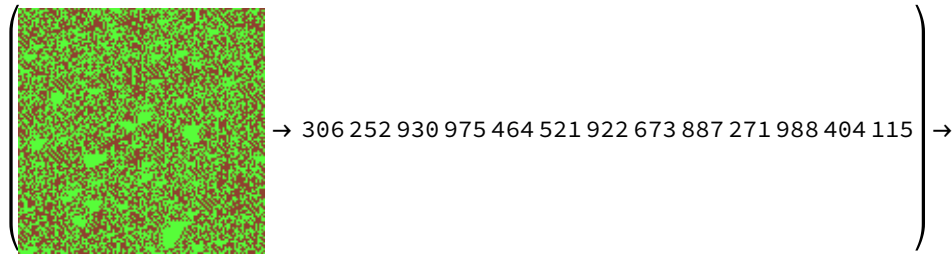
$$\{4 \rightarrow 3.36397 \times 10^{-6}, 3 \rightarrow 0.999997\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a yellow and black pixelated pattern]} \end{array} \rightarrow 26\,038\,517\,878\,084\,050\,104\,120\,010\,197\,636\,533\,172 \right) \rightarrow$$

$$\{3 \rightarrow 5.4757 \times 10^{-7}, 4 \rightarrow 0.999999\},$$



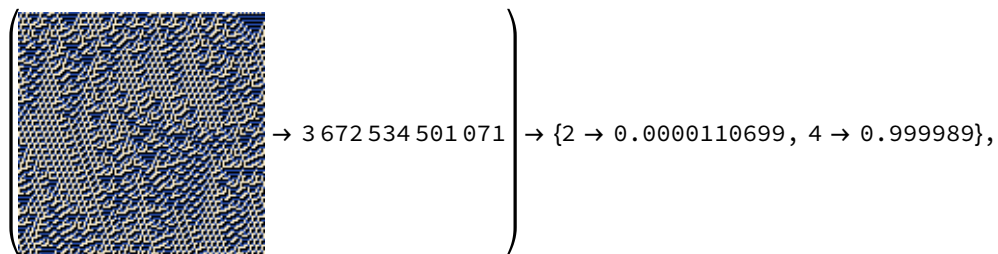
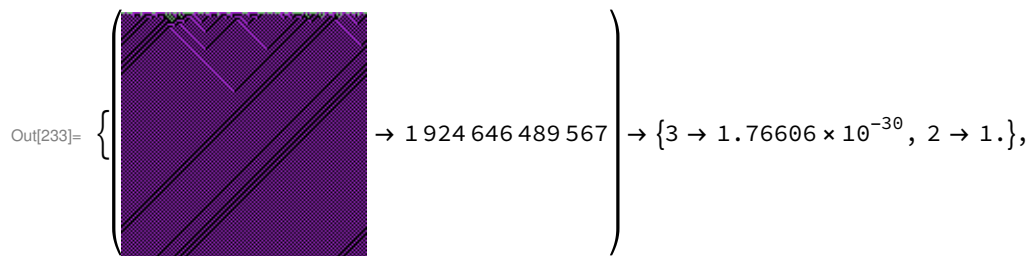
$\{4 \rightarrow 1.35911 \times 10^{-8}, 3 \rightarrow 1.\},$



$\{4 \rightarrow 5.68649 \times 10^{-7}, 3 \rightarrow 0.999999\}$

### 3-colour non-totalistic, range 1

```
In[232]:= test4Data3kr1C17 = datak3r1NT[128, 128, 8];
Thread[test4Data3kr1C17 → netECA17[Keys@test4Data3kr1C17, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{Image 1: A square image with a complex, fractal-like pattern in shades of pink, red, and black.} \\ \rightarrow 5\,833\,330\,297\,781 \end{array} \right) \rightarrow \{2 \rightarrow 0.000232935, 4 \rightarrow 0.999767\},$$

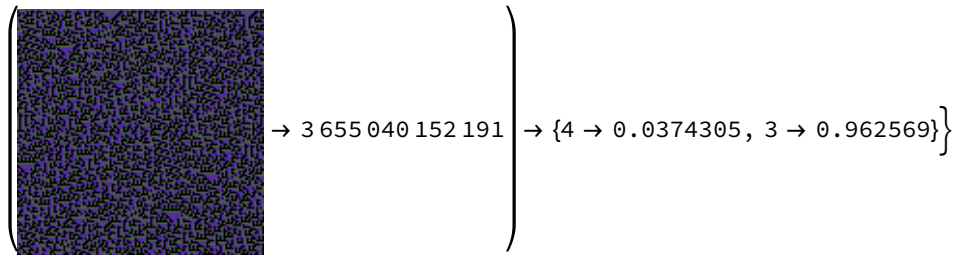
$$\left( \begin{array}{c} \text{Image 2: A square image that is almost entirely black, with a thin horizontal band of yellow and green at the top.} \\ \rightarrow 7\,606\,192\,973\,798 \end{array} \right) \rightarrow \{2 \rightarrow 6.802 \times 10^{-10}, 1 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image 3: A square image with a complex, fractal-like pattern in shades of blue, green, and black.} \\ \rightarrow 7\,622\,301\,560\,954 \end{array} \right) \rightarrow \{3 \rightarrow 0.0391643, 2 \rightarrow 0.960836\},$$

$$\left( \begin{array}{c} \text{Image 4: A square image with a complex, fractal-like pattern in shades of red, orange, and black.} \\ \rightarrow 3\,685\,910\,174\,297 \end{array} \right) \rightarrow \{3 \rightarrow 2.7602 \times 10^{-8}, 4 \rightarrow 1.\},$$

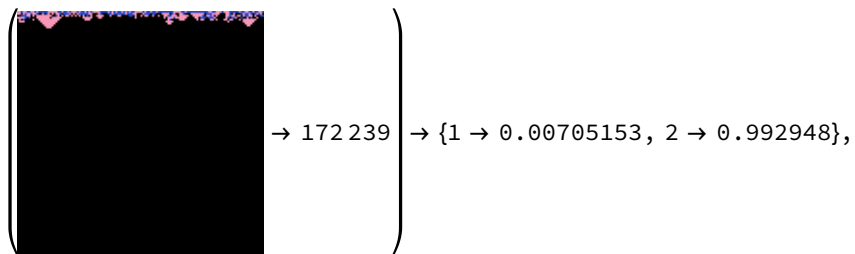
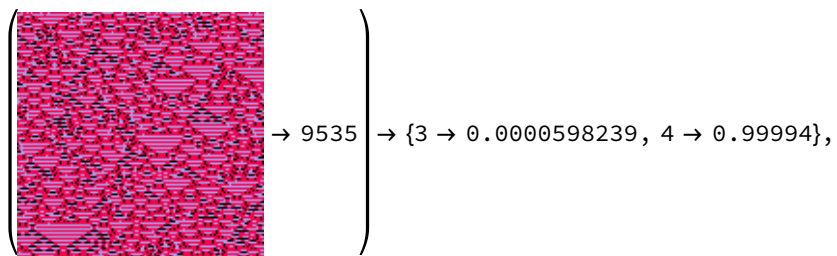
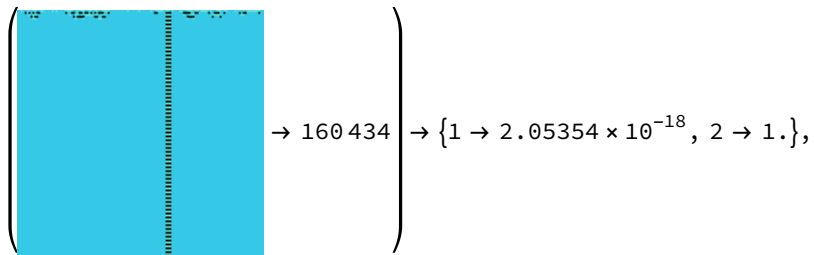
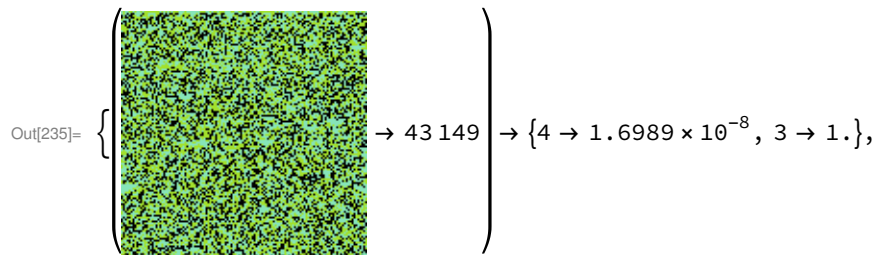
$$\left( \begin{array}{c} \text{Image 5: A square image with a complex, fractal-like pattern in shades of green, yellow, and black.} \\ \rightarrow 5\,743\,838\,876\,456 \end{array} \right) \rightarrow \{1 \rightarrow 6.15406 \times 10^{-23}, 2 \rightarrow 1.\},$$





### 3-colour totalistic, range 2

```
In[234]:= test4Data3kr2C17 = datak3r2C[128, 128, 8];
Thread[test4Data3kr2C17 → netECA17[Keys@test4Data3kr2C17, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{[Image: A dark, noisy square with a thin green border at the top]} \\ \rightarrow 174\,680 \end{array} \right) \rightarrow \{2 \rightarrow 5.824 \times 10^{-11}, 1 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A noisy square with blue and white pixels]} \\ \rightarrow 55\,945 \end{array} \right) \rightarrow \{4 \rightarrow 0.0138349, 3 \rightarrow 0.986165\},$$

$$\left( \begin{array}{c} \text{[Image: A noisy square with blue and black pixels]} \\ \rightarrow 113\,483 \end{array} \right) \rightarrow \{4 \rightarrow 3.72822 \times 10^{-6}, 3 \rightarrow 0.999996\},$$

$$\left( \begin{array}{c} \text{[Image: A noisy square with green and black pixels]} \\ \rightarrow 67\,810 \end{array} \right) \rightarrow \{1 \rightarrow 6.91386 \times 10^{-17}, 2 \rightarrow 1.\}$$

### 3-colour totalistic, range 3

```
In[236]:= test4Data3kr3C17 = datak3r3C[128, 128, 8];
Thread[test4Data3kr3C17 → netECA17[Keys@test4Data3kr3C17, {"TopProbabilities", 2}]]
```

$$\text{Out[237]=} \left\{ \begin{array}{c} \text{[Image: A noisy square with green and black pixels]} \\ \rightarrow 3\,046\,610 \end{array} \right\} \rightarrow \{4 \rightarrow 7.58312 \times 10^{-7}, 3 \rightarrow 0.999999\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a blue background and a vertical spiral binding on the left side.]}\end{array} \right) \rightarrow 7\,801\,434 \rightarrow \{1 \rightarrow 1.19167 \times 10^{-14}, 2 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a green and blue pixelated pattern.]}\end{array} \right) \rightarrow 5\,445\,843 \rightarrow \{4 \rightarrow 1.60992 \times 10^{-19}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a purple and black pixelated pattern.]}\end{array} \right) \rightarrow 1\,451\,413 \rightarrow \{4 \rightarrow 0.144413, 3 \rightarrow 0.855587\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a green and black pixelated pattern.]}\end{array} \right) \rightarrow 10\,676\,790 \rightarrow \{3 \rightarrow 0.0738921, 4 \rightarrow 0.926108\},$$

$$\left( \begin{array}{c} \text{[Image: A square image with a red and black pixelated pattern.]}\end{array} \right) \rightarrow 10\,375\,449 \rightarrow \{4 \rightarrow 1.04031 \times 10^{-17}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 5\,181\,761 \rightarrow \{4 \rightarrow 1.75908 \times 10^{-8}, 3 \rightarrow 1.\},$$

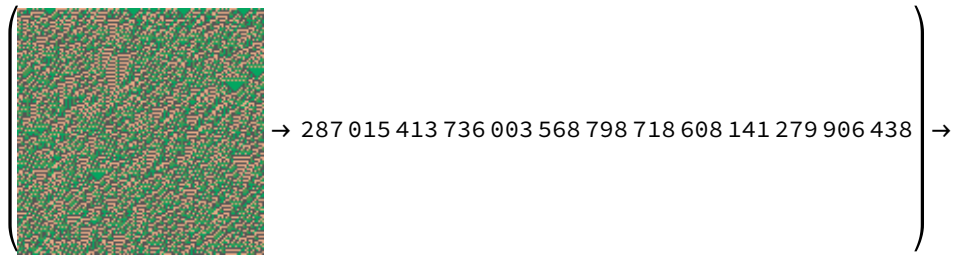
$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 8\,884\,285 \rightarrow \{4 \rightarrow 7.91486 \times 10^{-6}, 3 \rightarrow 0.999992\}$$

### 4-colour non-totalistic, range 1

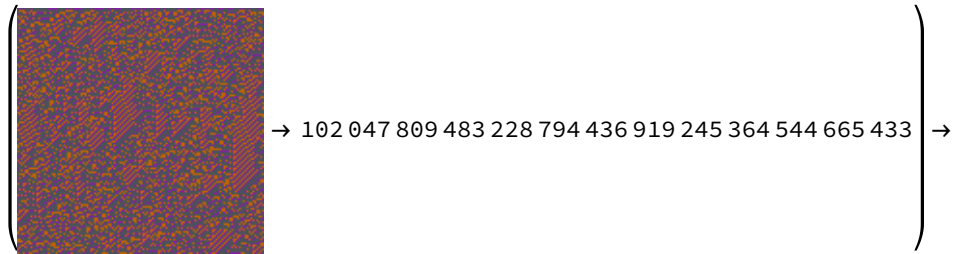
```
In[238]:= test4Data4kr1C17 = datak4r1NT[128, 128, 8];
Thread[test4Data4kr1C17 → netECA17[Keys@test4Data4kr1C17, {"TopProbabilities", 2}]]
```

$$\text{Out[239]=} \left\{ \begin{array}{c} \text{Image} \end{array} \right\} \rightarrow 255\,219\,118\,556\,246\,495\,764\,448\,982\,135\,818\,252\,673 \rightarrow \{3 \rightarrow 2.62807 \times 10^{-6}, 4 \rightarrow 0.999997\},$$

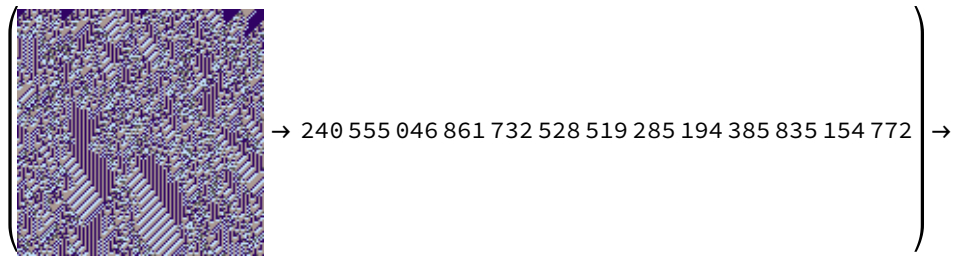
$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 256\,372\,744\,774\,750\,994\,462\,308\,116\,064\,689\,670\,029 \rightarrow \{4 \rightarrow 1.66442 \times 10^{-17}, 3 \rightarrow 1.\},$$



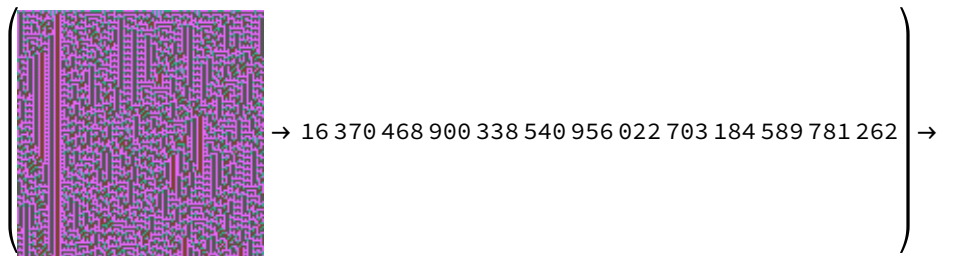
$\{4 \rightarrow 2.56385 \times 10^{-7}, 3 \rightarrow 1.\},$



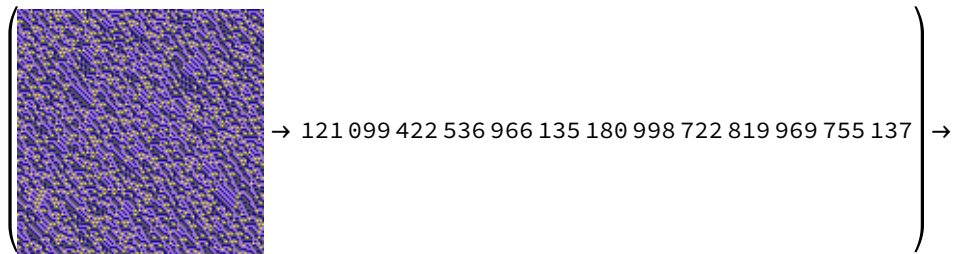
$\{4 \rightarrow 0.000696463, 3 \rightarrow 0.999304\},$



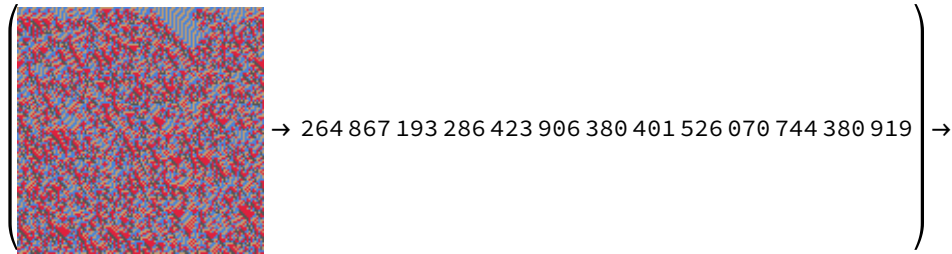
$\{4 \rightarrow 3.62136 \times 10^{-16}, 3 \rightarrow 1.\},$



$\{3 \rightarrow 1.45252 \times 10^{-15}, 4 \rightarrow 1.\},$



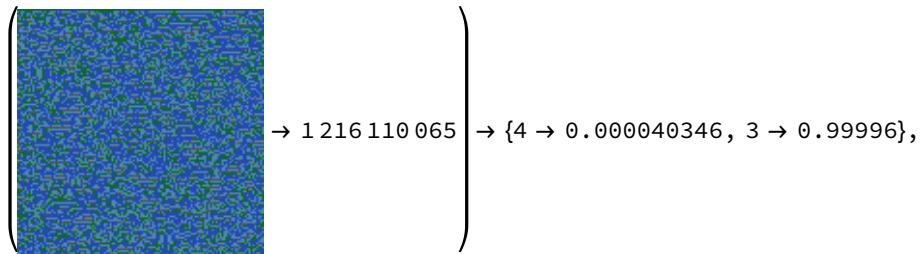
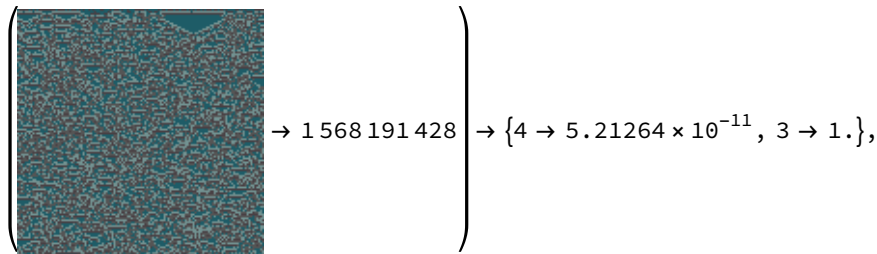
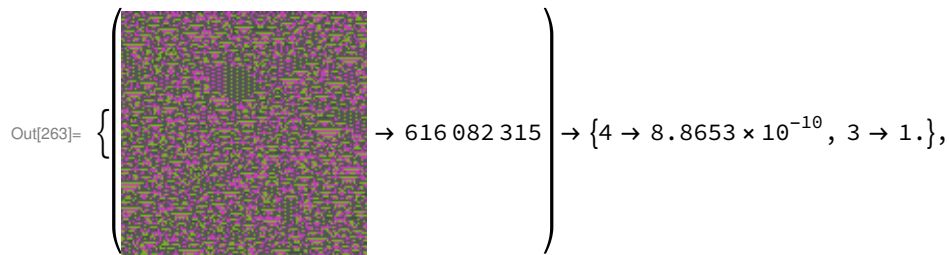
$\{4 \rightarrow 0.0000270873, 3 \rightarrow 0.999973\},$



$\{4 \rightarrow 0.105214, 3 \rightarrow 0.894786\}$

## 4-colour totalistic, range 2

```
In[262]:= test4Data4kr2C17 = datak4r2C[128, 128, 8];
Thread[test4Data4kr2C17 → netECA17[Keys@test4Data4kr2C17, {"TopProbabilities", 2}]]
```





$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 2\,419\,903\,949 \end{array} \right) \rightarrow \{4 \rightarrow 3.69897 \times 10^{-10}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 453\,961\,055 \end{array} \right) \rightarrow \{4 \rightarrow 3.89961 \times 10^{-8}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 3\,969\,029\,930 \end{array} \right) \rightarrow \{2 \rightarrow 0.0000237283, 4 \rightarrow 0.999976\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 3\,970\,208\,335 \end{array} \right) \rightarrow \{4 \rightarrow 4.03487 \times 10^{-9}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 3\,418\,384\,250 \end{array} \right) \rightarrow \{4 \rightarrow 9.0126 \times 10^{-15}, 3 \rightarrow 1.\}$$

## 5-colour totalistic, range 1

```
In[242]:= test4Data5kr1C17 = data5T2C[8, 128, 128];
Thread[test4Data5kr1C17 → netECA17[Keys@test4Data5kr1C17, {"TopProbabilities", 2}]]
```



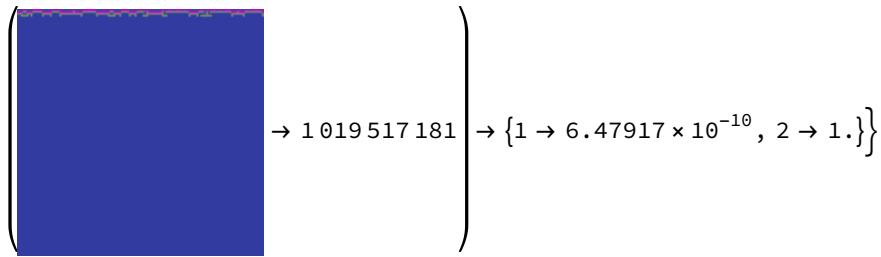
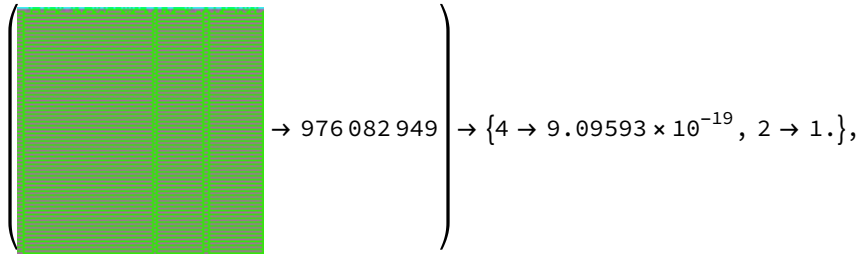
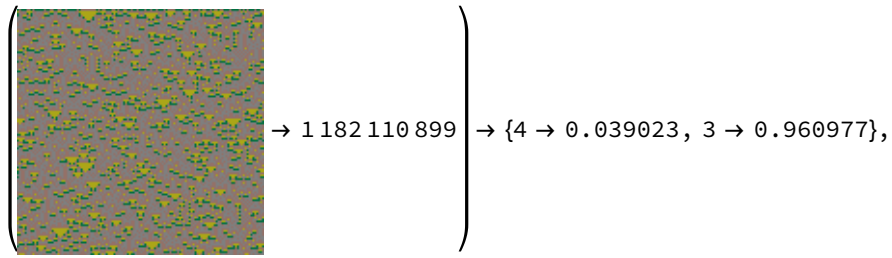
$$\text{Out}[243]= \left( \begin{array}{c} \text{[Red and Green Noise Pattern]} \end{array} \right) \rightarrow 720\,503\,516 \rightarrow \{2 \rightarrow 0.000105936, 4 \rightarrow 0.999894\},$$

$$\left( \begin{array}{c} \text{[Blue and Purple Noise Pattern]} \end{array} \right) \rightarrow 771\,013\,684 \rightarrow \{4 \rightarrow 1.94282 \times 10^{-8}, 2 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Green and Yellow Noise Pattern]} \end{array} \right) \rightarrow 543\,872\,434 \rightarrow \{4 \rightarrow 6.11423 \times 10^{-7}, 3 \rightarrow 0.999999\},$$

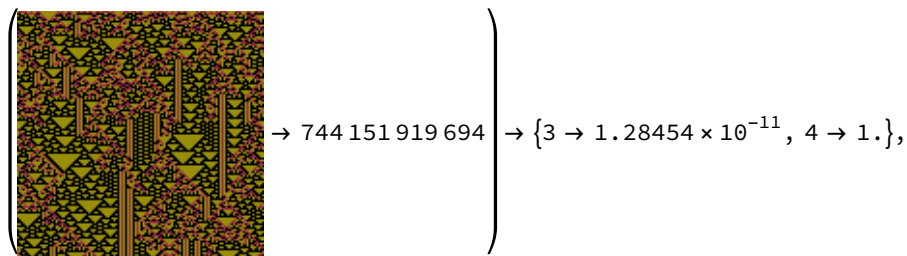
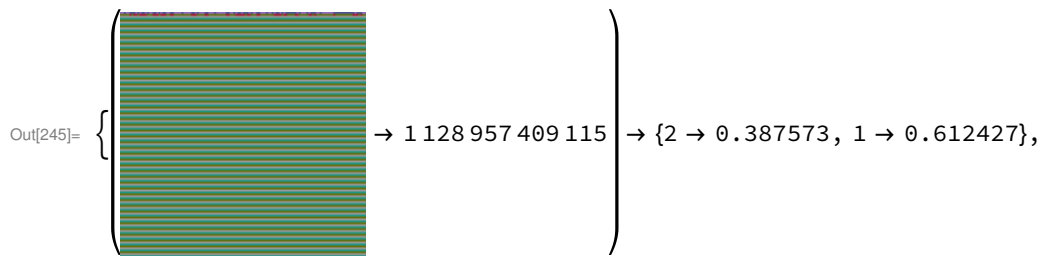
$$\left( \begin{array}{c} \text{[Blue and Green Noise Pattern]} \end{array} \right) \rightarrow 341\,908\,586 \rightarrow \{4 \rightarrow 0.310854, 3 \rightarrow 0.689146\},$$

$$\left( \begin{array}{c} \text{[Grey and Black Noise Pattern]} \end{array} \right) \rightarrow 664\,036\,861 \rightarrow \{2 \rightarrow 0.00511847, 4 \rightarrow 0.994882\},$$



## 6-colour totalistic, range 1

```
In[244]:= test4Data6kr1C17 = data6TC[8, 128, 128];
Thread[test4Data6kr1C17 → netECA17[Keys@test4Data6kr1C17, {"TopProbabilities", 2}]]
```



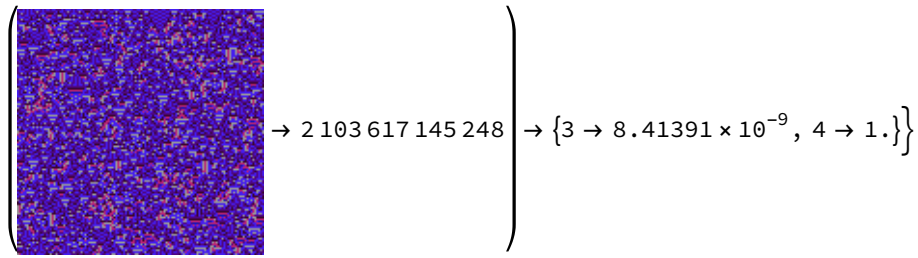
$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 411\,482\,269\,593 \end{array} \right) \rightarrow \{4 \rightarrow 9.50671 \times 10^{-6}, 3 \rightarrow 0.99999\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 2\,122\,826\,252\,429 \end{array} \right) \rightarrow \{4 \rightarrow 4.58698 \times 10^{-10}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 2\,443\,710\,325\,124 \end{array} \right) \rightarrow \{4 \rightarrow 5.97811 \times 10^{-9}, 3 \rightarrow 1.\},$$

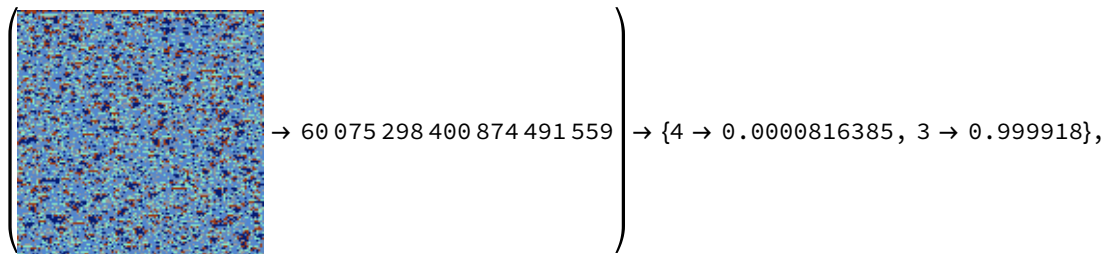
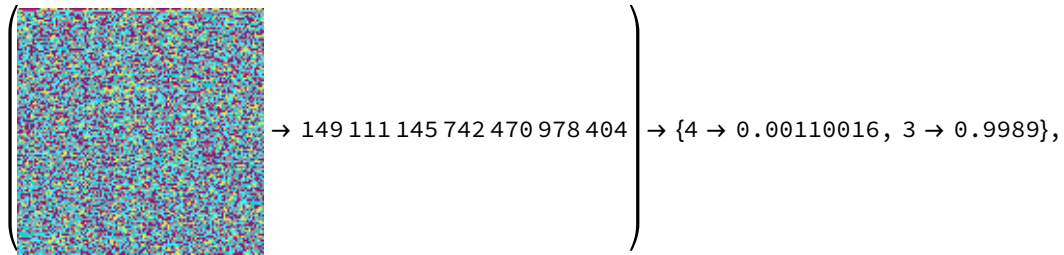
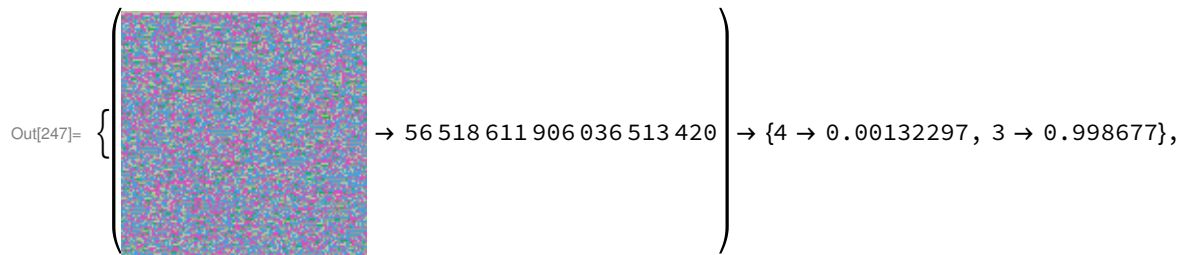
$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 2\,519\,595\,515\,832 \end{array} \right) \rightarrow \{2 \rightarrow 0.396179, 4 \rightarrow 0.603821\},$$

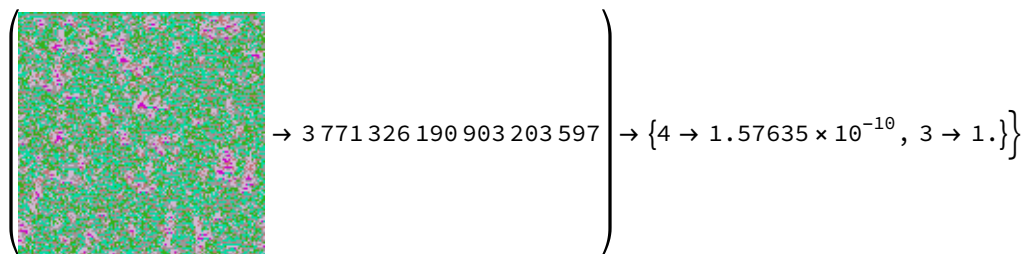
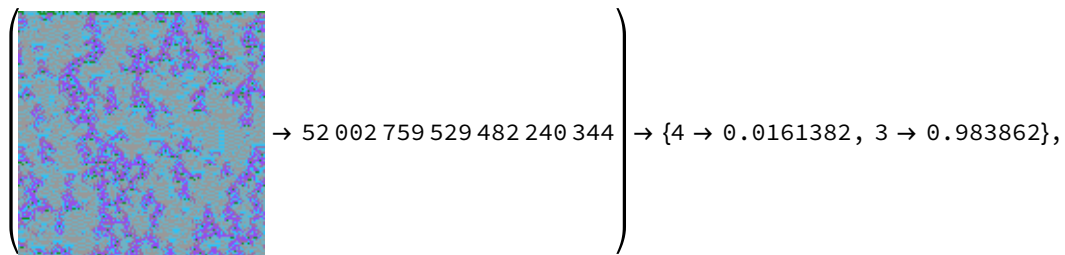
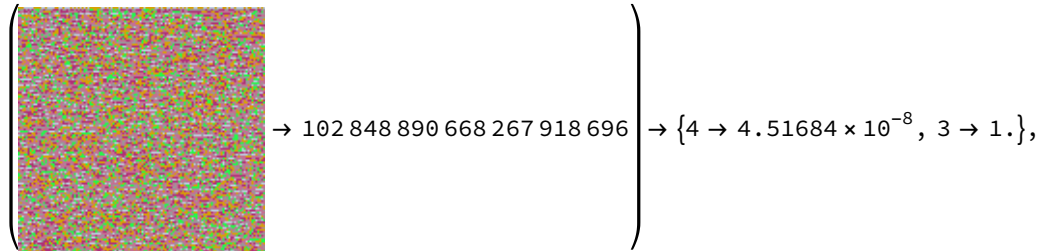
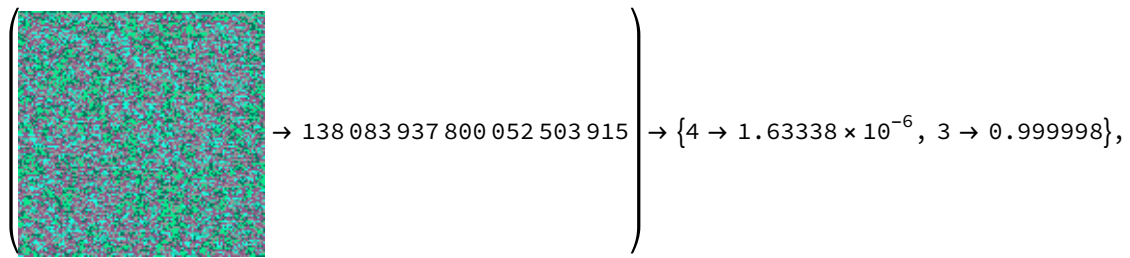
$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 572\,558\,234\,379 \end{array} \right) \rightarrow \{4 \rightarrow 1.63969 \times 10^{-11}, 3 \rightarrow 1.\},$$



## 6-colour totalistic, range 2

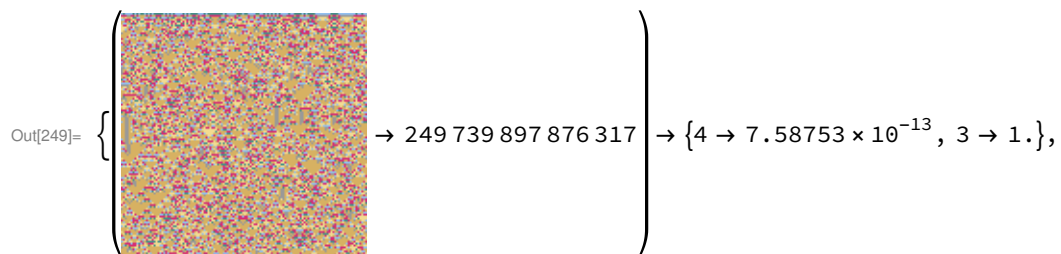
```
In[246]:= test4Data6kr2C17 = data6T2C[8, 128, 128];
Thread[test4Data6kr2C17 → netECA17[Keys@test4Data6kr2C17, {"TopProbabilities", 2}]]
```





## 7-colour totalistic, range 1

```
In[248]:= test4Data7kr1C17 = data7TC[8, 128, 128];
Thread[test4Data7kr1C17 → netECA17[Keys@test4Data7kr1C17, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{Image 1} \\ \rightarrow 6\,589\,873\,174\,284\,234 \end{array} \right) \rightarrow \{4 \rightarrow 3.70203 \times 10^{-21}, 3 \rightarrow 1.\},$$

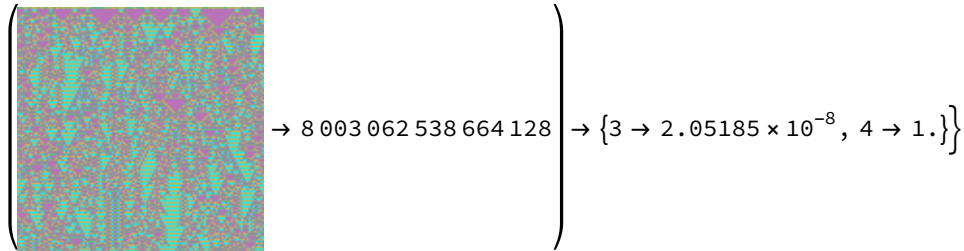
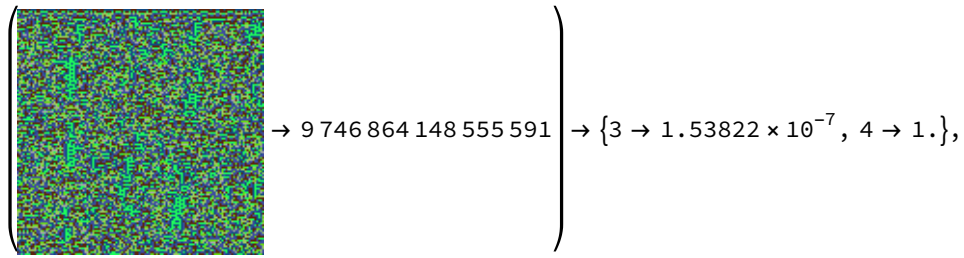
$$\left( \begin{array}{c} \text{Image 2} \\ \rightarrow 2\,838\,251\,451\,633\,386 \end{array} \right) \rightarrow \{3 \rightarrow 0.0000362001, 4 \rightarrow 0.999964\},$$

$$\left( \begin{array}{c} \text{Image 3} \\ \rightarrow 3\,069\,021\,856\,393\,877 \end{array} \right) \rightarrow \{4 \rightarrow 4.6982 \times 10^{-6}, 3 \rightarrow 0.999995\},$$

$$\left( \begin{array}{c} \text{Image 4} \\ \rightarrow 10\,282\,712\,720\,317\,214 \end{array} \right) \rightarrow \{3 \rightarrow 4.14045 \times 10^{-19}, 4 \rightarrow 1.\},$$

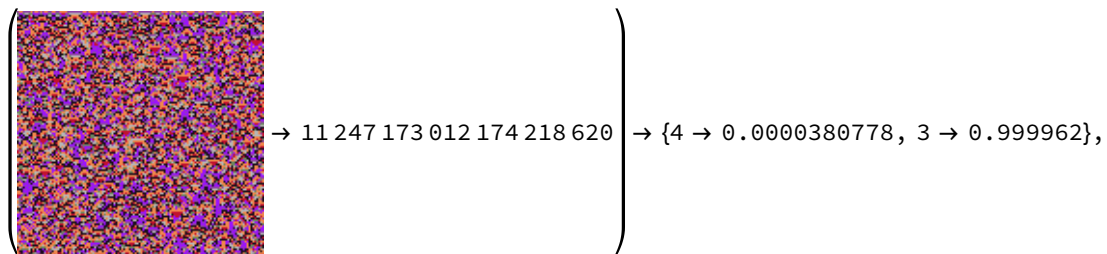
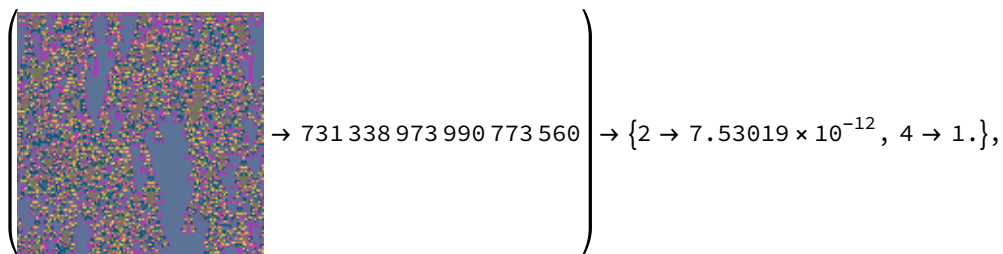
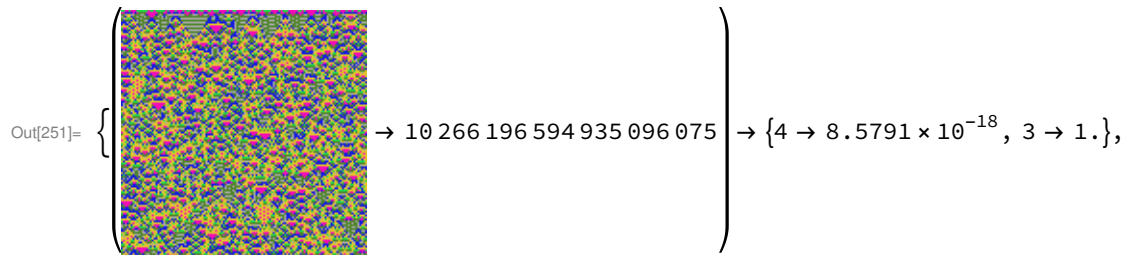
$$\left( \begin{array}{c} \text{Image 5} \\ \rightarrow 203\,015\,413\,423\,084 \end{array} \right) \rightarrow \{4 \rightarrow 2.87431 \times 10^{-9}, 3 \rightarrow 1.\},$$





### 8-colour totalistic, range 1

```
In[250]:= test4Data8kr1C17 = data8TC[8, 128, 128];
Thread[test4Data8kr1C17 → netECA17[Keys@test4Data8kr1C17, {"TopProbabilities", 2}]]
```





$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 63\,742\,472\,032\,617\,219\,918 \end{array} \right) \rightarrow \{4 \rightarrow 0.0000371126, 3 \rightarrow 0.999963\},$$

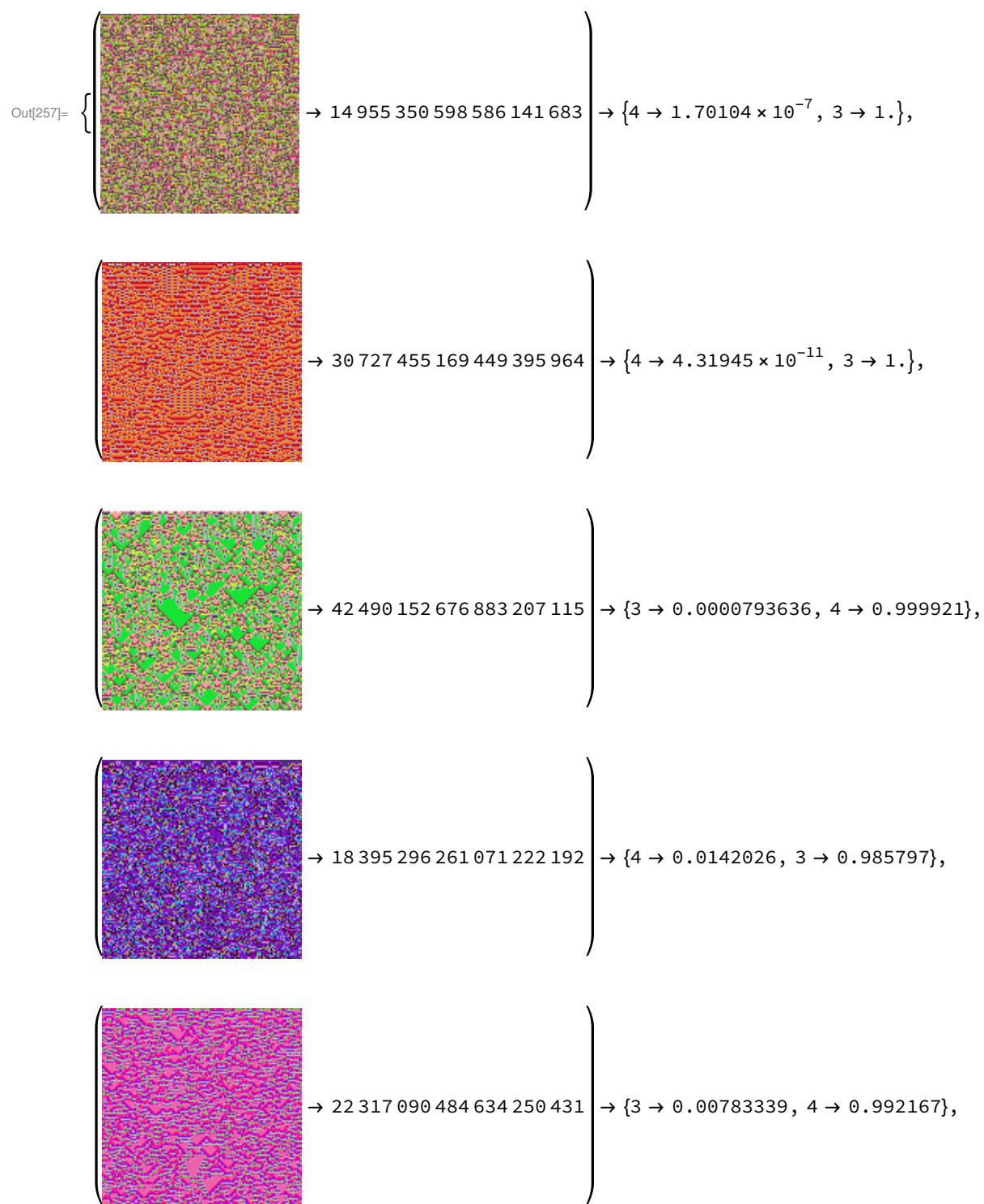
$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 7\,382\,455\,380\,800\,363\,015 \end{array} \right) \rightarrow \{4 \rightarrow 8.07468 \times 10^{-15}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 59\,100\,651\,667\,569\,734\,000 \end{array} \right) \rightarrow \{4 \rightarrow 1.27228 \times 10^{-11}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 24\,971\,306\,247\,396\,766\,335 \end{array} \right) \rightarrow \{4 \rightarrow 0.0333734, 3 \rightarrow 0.966627\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 45\,946\,581\,080\,593\,555\,746 \end{array} \right) \rightarrow \{4 \rightarrow 1.08598 \times 10^{-15}, 3 \rightarrow 1.\}$$

```
In[256]:= test4Data8kr1C17 = data8TC[8, 128, 128];
Thread[test4Data8kr1C17 → netECA17[Keys@test4Data8kr1C17, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{Image 1} \end{array} \right) \rightarrow 33\ 329\ 414\ 465\ 629\ 594\ 174 \rightarrow \{4 \rightarrow 0.00851294, 3 \rightarrow 0.991487\},$$

$$\left( \begin{array}{c} \text{Image 2} \end{array} \right) \rightarrow 68\ 439\ 232\ 681\ 205\ 604\ 962 \rightarrow \{2 \rightarrow 1.97568 \times 10^{-13}, 4 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image 3} \end{array} \right) \rightarrow 53\ 049\ 830\ 479\ 062\ 864\ 751 \rightarrow \{4 \rightarrow 3.6815 \times 10^{-10}, 3 \rightarrow 1.\}$$

### Network XVIII- Four convolutions, dropout on linear only, BatchNorm

```
In[264]:= netECA18 = netTenCC1024drop[128, 128]
```

```
Out[264]= NetChain[
```



Input port:  
 Output port:  
 Number of layers:

image  
 class  
 18

```
]
```

```
In[266]:= NetInformation[netECA18, "MXNetNodeGraphPlot"]
```



```
In[267]:= NetInformation[netECA18, "SummaryGraphic"]
```



```
In[268]:= dataECA18 = dataC[128, 128, 16 384];
```

```
In[269]:= dataTotalistic2BigC18 = genData2r2C[128, 128, 4096];
```

```
In[270]:= dataTotalistic3BigC18 = data3T2C[128, 128, 4096];
```

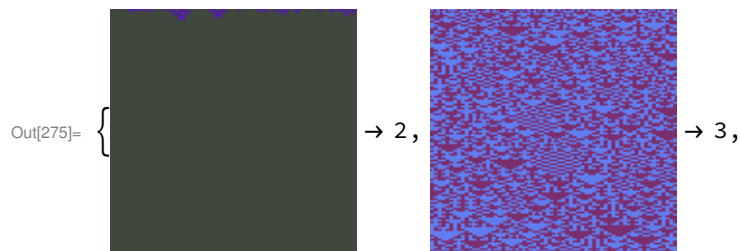
```
In[271]:= dataTotalistic4BigC18 = data4TC[128, 128, 4096];
```

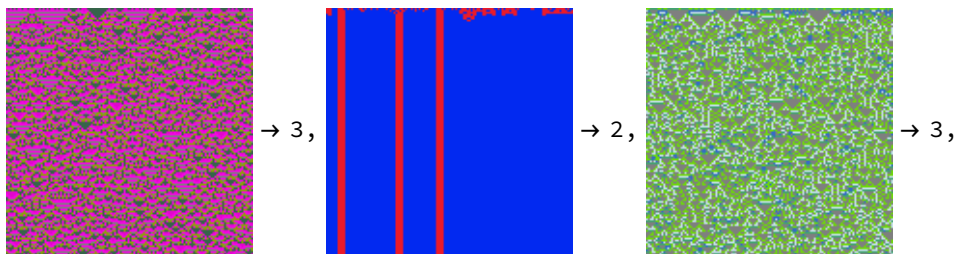
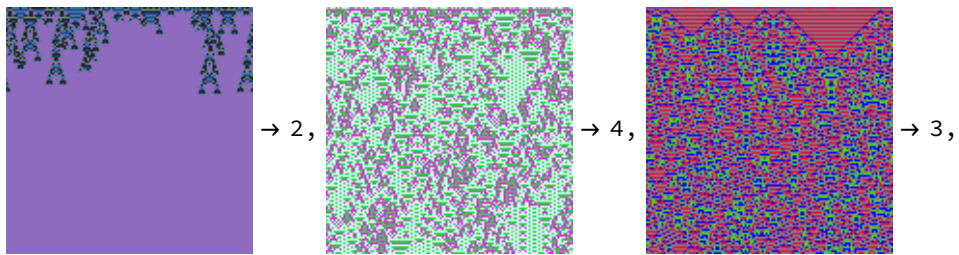
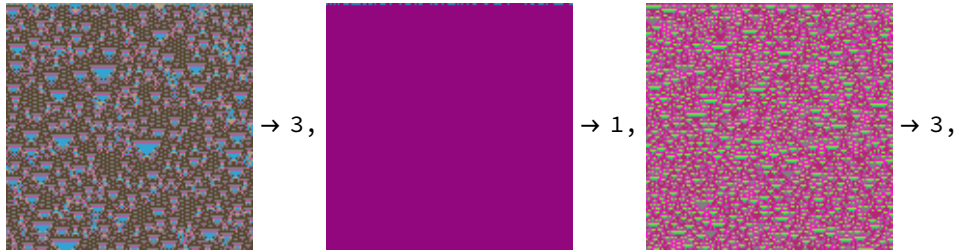
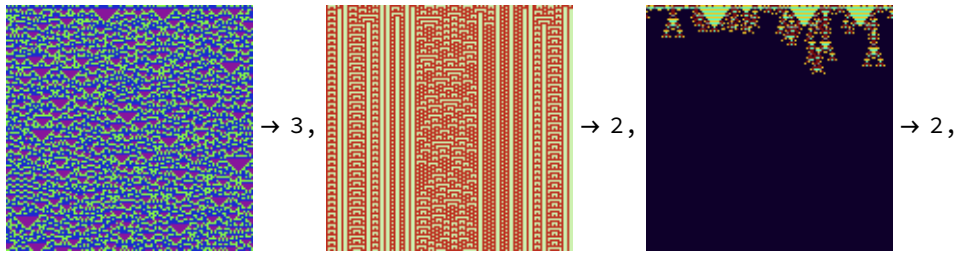
```
In[272]:= dataTotalistic5BigC18 = genData5TCC[128, 128, 16 384];
```

```
In[273]:= fullTrainingBigC18 = Join[dataECA18, dataTotalistic2BigC18,
  dataTotalistic3BigC18, dataTotalistic4BigC18, dataTotalistic5BigC18];
Length[fullTrainingBigC18]
```

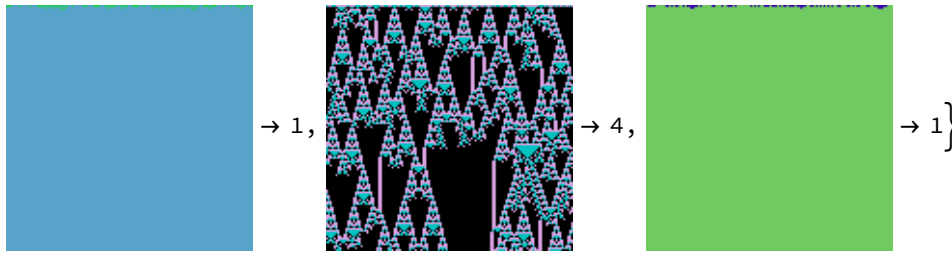
```
Out[274]= 90 112
```

```
In[275]:= RandomSample[fullTrainingBigC18, 20]
```









```
In[276]:= dir = SetDirectory[NotebookDirectory[]]
```

```
Out[276]= /home/esilverman/Documents
```

```
In[277]:= "/home/esilverman/Documents"
```

```
Out[277]= /home/esilverman/Documents
```

```
In[287]:= netECA18 =
```

```
NetTrain[netECA18, fullTrainingBigC18, MaxTrainingRounds → 200, BatchSize → 256,
  TargetDevice → "GPU", TrainingProgressCheckpointing → {"Directory", dir}]
```

Out[287]= NetChain[ Input port: image  
Output port: class  
Number of layers: 18

```
In[294]:= netECA18 = Import["netECA18-r200.wl\net"]
```

Out[294]= NetChain[ Input port: image  
Output port: class  
Number of layers: 18

## Testing Network XVIII (200 epochs) on unseen CA rule spaces

## 2-colour non-totalistic, range 2

```
In[295]:= test4Data2kr2C18 = datak2r2C[128, 128, 8];
```

```
Thread[test4Data2kr2C18 → netECA18[Keys@test4Data2kr2C18, {"TopProbabilities", 2}]]
```

$$\text{Out}[296]=\left\{\begin{array}{c} \text{[Redacted Matrix]} \\ \rightarrow 2\,381\,133\,287 \end{array}\right\} \rightarrow \{4 \rightarrow 1.78812 \times 10^{-7}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 2\,956\,498\,967 \rightarrow \{2 \rightarrow 0.00630379, 3 \rightarrow 0.992493\},$$

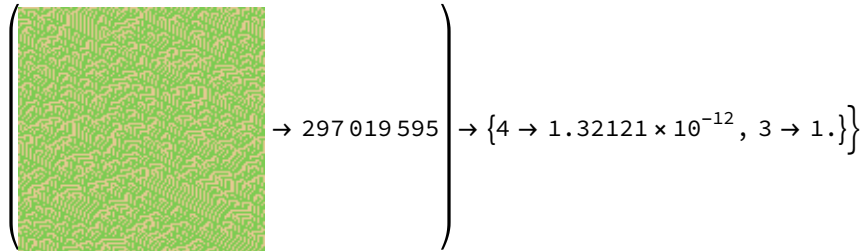
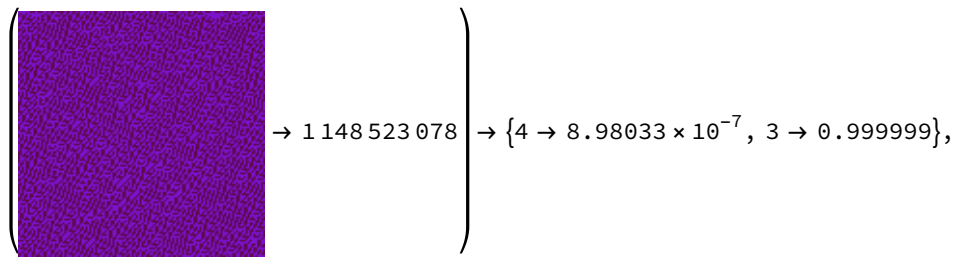
$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 677\,038\,591 \rightarrow \{4 \rightarrow 1.21175 \times 10^{-11}, 2 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 3\,273\,379\,163 \rightarrow \{4 \rightarrow 0.34545, 3 \rightarrow 0.65455\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 2\,209\,740\,989 \rightarrow \{2 \rightarrow 1.84144 \times 10^{-11}, 3 \rightarrow 1.\},$$

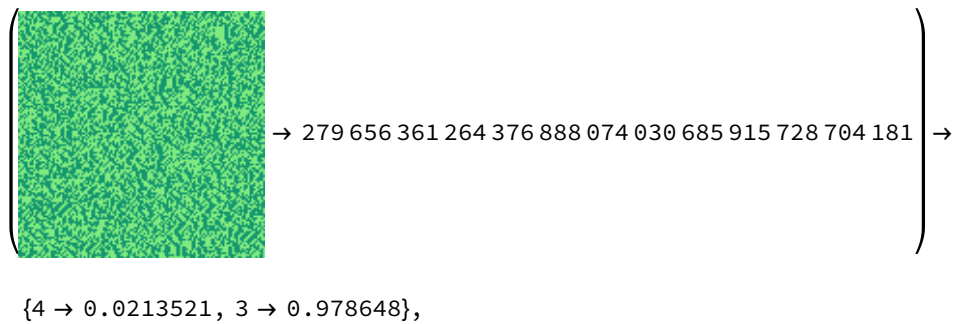
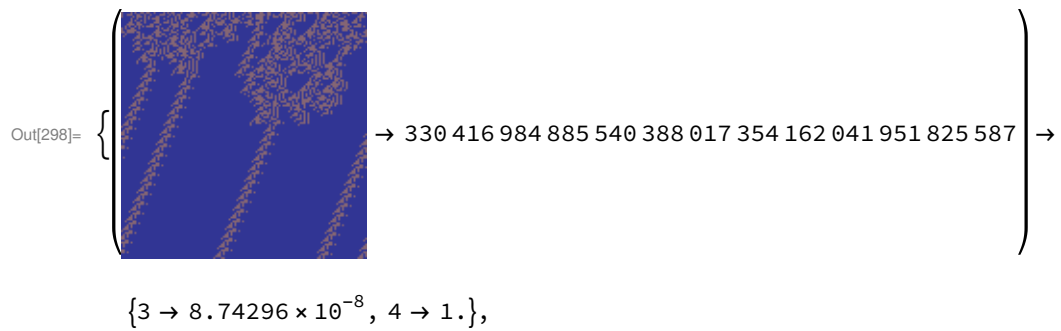
$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 939\,607\,251 \rightarrow \{3 \rightarrow 8.18048 \times 10^{-7}, 4 \rightarrow 0.999999\},$$

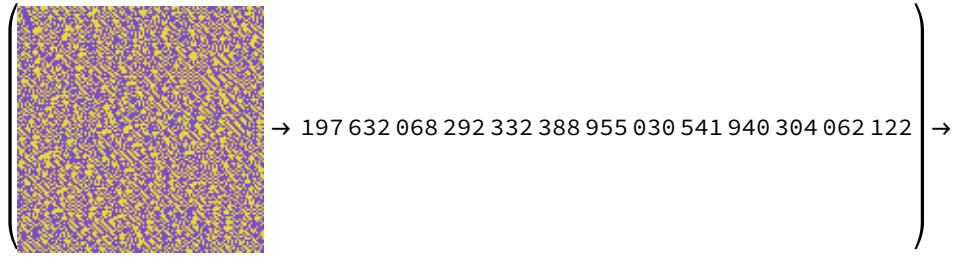




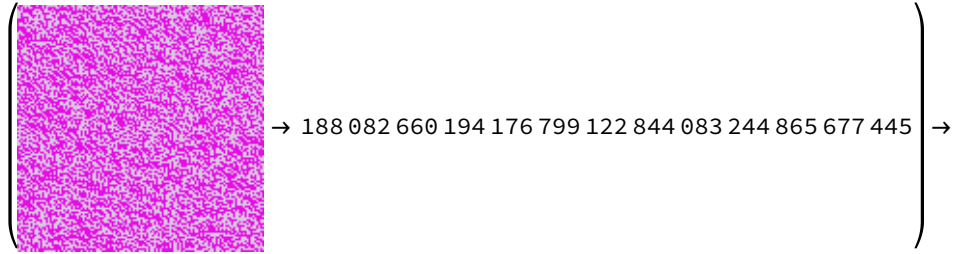
## 2-colour non-totalistic, range 3

```
In[297]:= test4Data2kr3C18 = datak2r3NT[128, 128, 8];
Thread[test4Data2kr3C18 → netECA18[Keys@test4Data2kr3C18, {"TopProbabilities", 2}]]
```

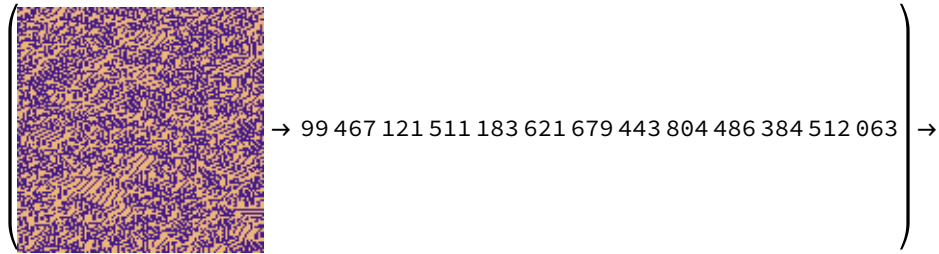




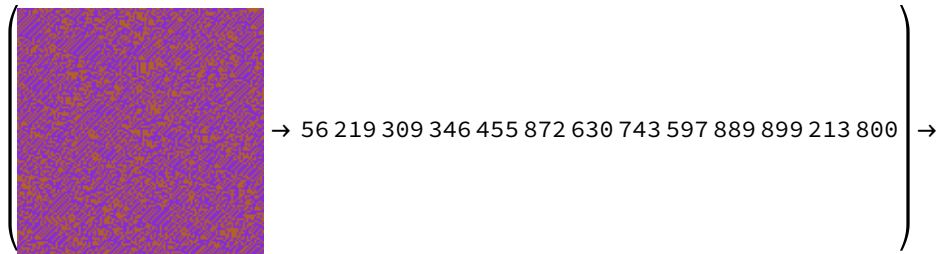
$\{4 \rightarrow 5.0499 \times 10^{-16}, 3 \rightarrow 1.\},$



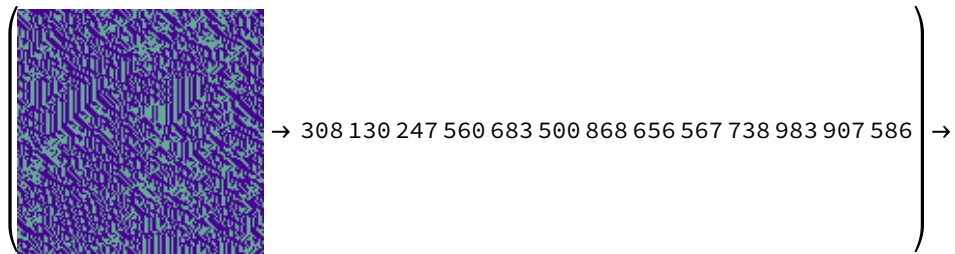
$\{4 \rightarrow 2.34238 \times 10^{-6}, 3 \rightarrow 0.999998\},$



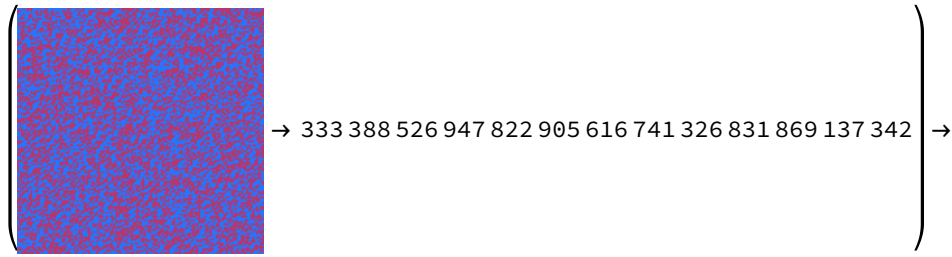
$\{4 \rightarrow 0.00329566, 3 \rightarrow 0.996704\},$



$\{4 \rightarrow 1.38574 \times 10^{-10}, 3 \rightarrow 1.\},$



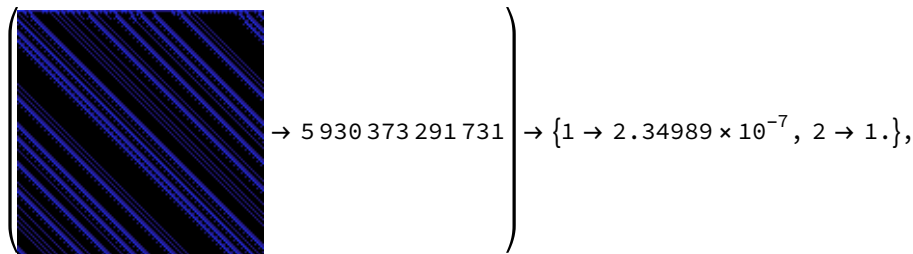
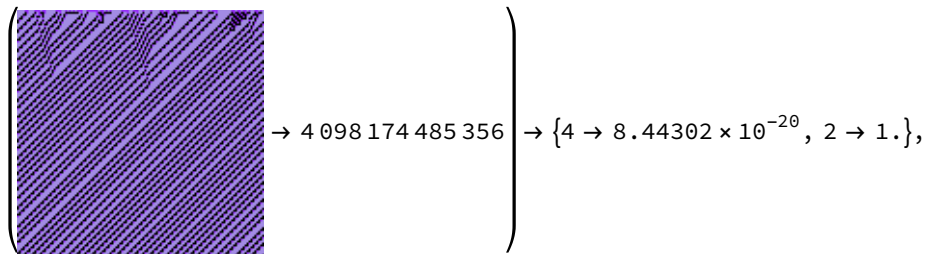
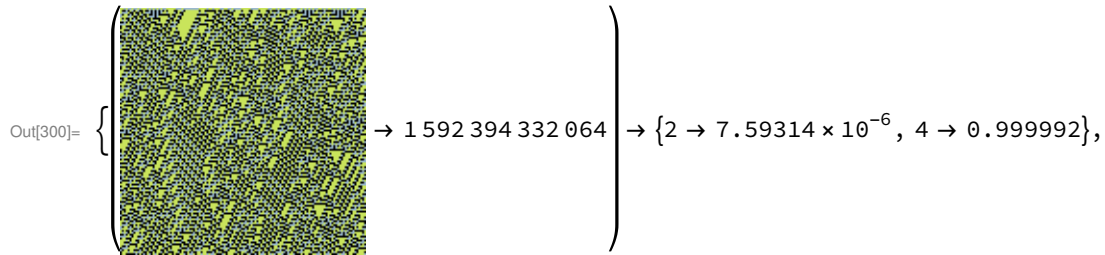
$$\{4 \rightarrow 5.1263 \times 10^{-8}, 3 \rightarrow 1.\},$$

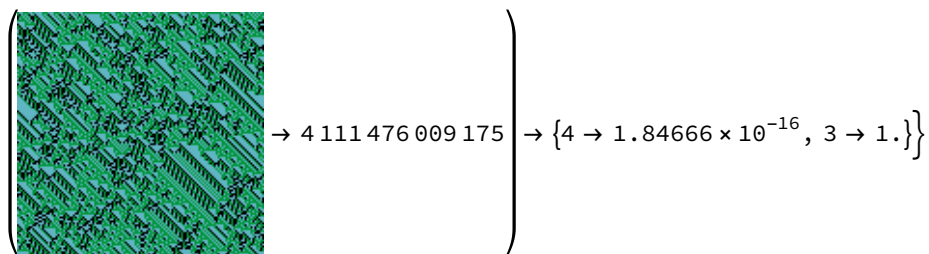
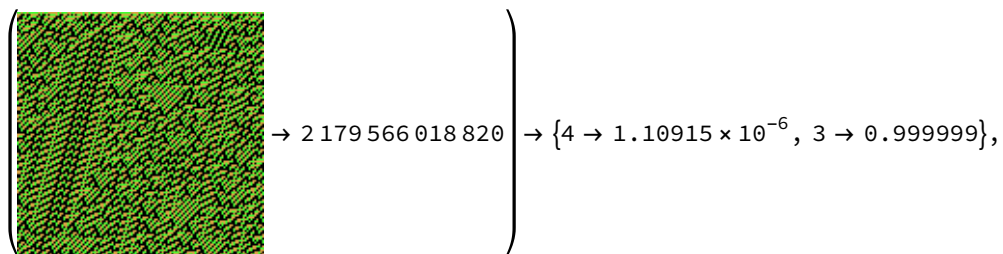
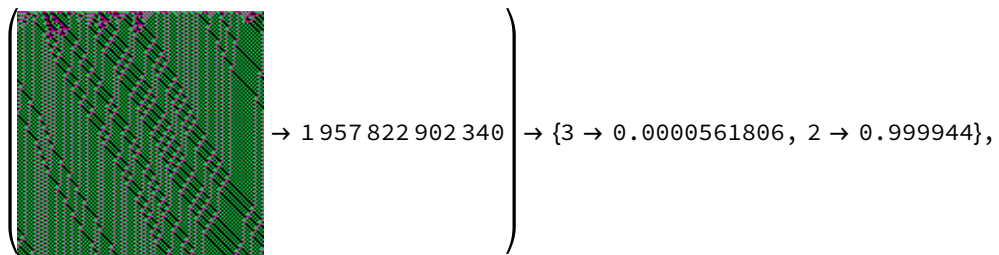
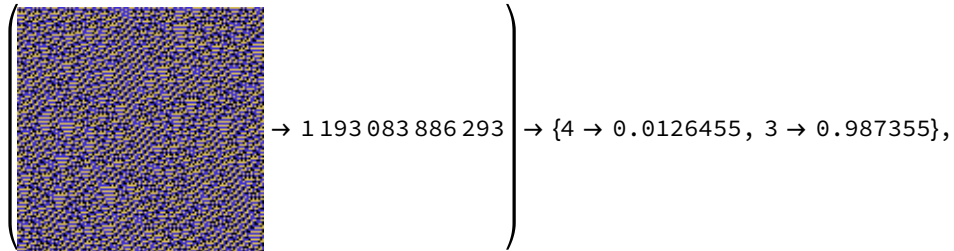
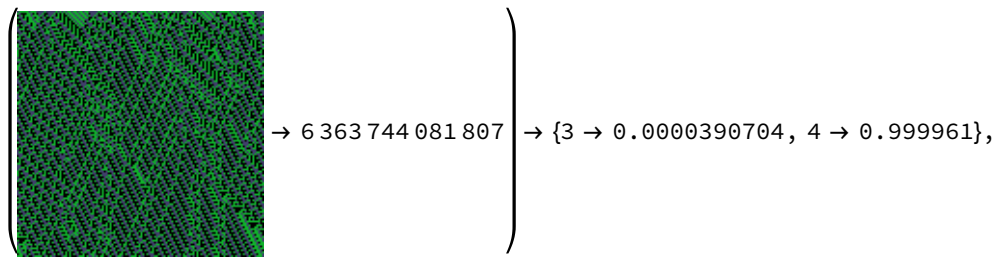


$$\{4 \rightarrow 7.10494 \times 10^{-8}, 3 \rightarrow 1.\}$$

### 3-colour non-totalistic, range 1

```
In[299]:= test4Data3kr1C18 = datak3r1NT[128, 128, 8];
Thread[test4Data3kr1C18 → netECA18[Keys@test4Data3kr1C18, {"TopProbabilities", 2}]]
```





### 3-colour totalistic, range 2

```
In[301]:= test4Data3kr2C18 = datak3r2C[128, 128, 8];
Thread[test4Data3kr2C18 → netECA18[Keys@test4Data3kr2C18, {"TopProbabilities", 2}]]
```

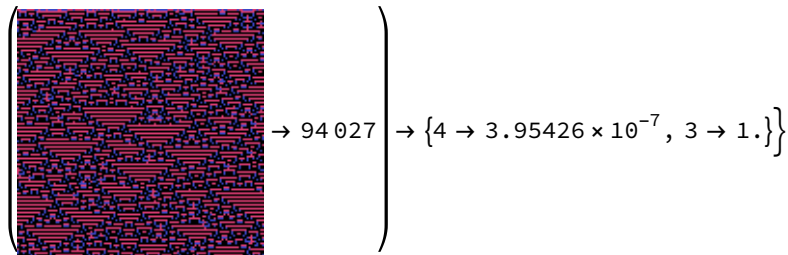
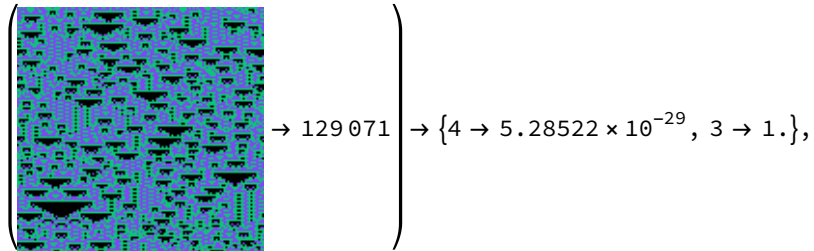
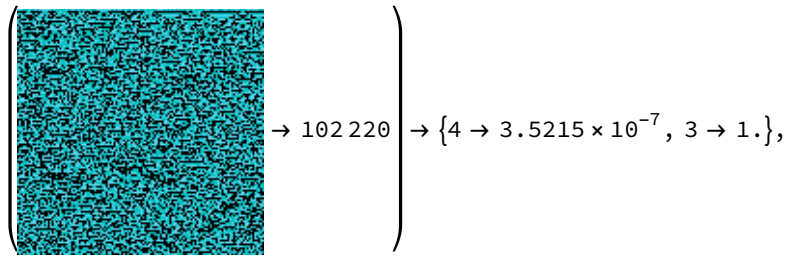
$$\text{Out[302]=} \left\{ \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 101\,105 \end{array} \right\} \rightarrow \{4 \rightarrow 1.49159 \times 10^{-7}, 3 \rightarrow 1.\},$$

$$\left\{ \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 48\,212 \end{array} \right\} \rightarrow \{4 \rightarrow 3.39887 \times 10^{-8}, 3 \rightarrow 1.\},$$

$$\left\{ \begin{array}{c} \text{[Image with cyan triangle]} \\ \rightarrow 81\,243 \end{array} \right\} \rightarrow \{2 \rightarrow 6.61823 \times 10^{-15}, 1 \rightarrow 1.\},$$

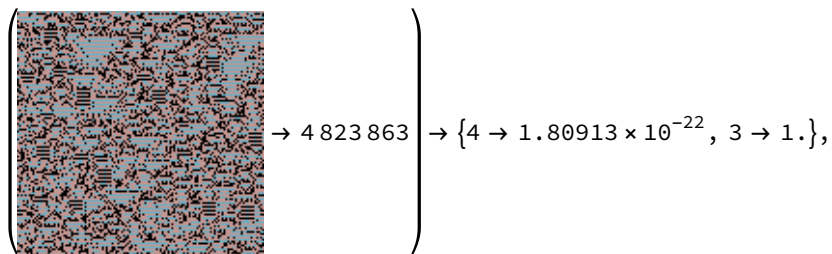
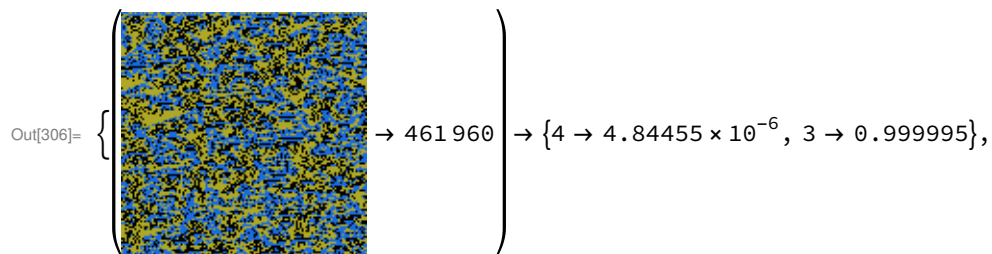
$$\left\{ \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 144\,952 \end{array} \right\} \rightarrow \{4 \rightarrow 5.58692 \times 10^{-14}, 3 \rightarrow 1.\},$$

$$\left\{ \begin{array}{c} \text{[Image with vertical lines]} \\ \rightarrow 167\,730 \end{array} \right\} \rightarrow \{1 \rightarrow 3.79715 \times 10^{-7}, 2 \rightarrow 1.\},$$



### 3-colour totalistic, range 3

```
In[305]:= test4Data3kr3C18 = datak3r3C[128, 128, 8];
Thread[test4Data3kr3C18 → netECA18[Keys@test4Data3kr3C18, {"TopProbabilities", 2}]]
```





$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 7\,272\,180 \end{array} \right) \rightarrow \{3 \rightarrow 3.43734 \times 10^{-9}, 4 \rightarrow 1.\},$$

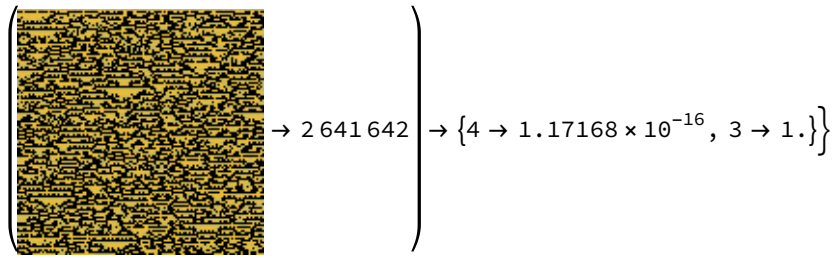
$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 8\,672\,980 \end{array} \right) \rightarrow \{4 \rightarrow 6.70981 \times 10^{-6}, 3 \rightarrow 0.999993\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 254\,357 \end{array} \right) \rightarrow \{4 \rightarrow 2.17773 \times 10^{-6}, 3 \rightarrow 0.999998\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 9\,226\,537 \end{array} \right) \rightarrow \{4 \rightarrow 1.70317 \times 10^{-12}, 3 \rightarrow 1.\},$$

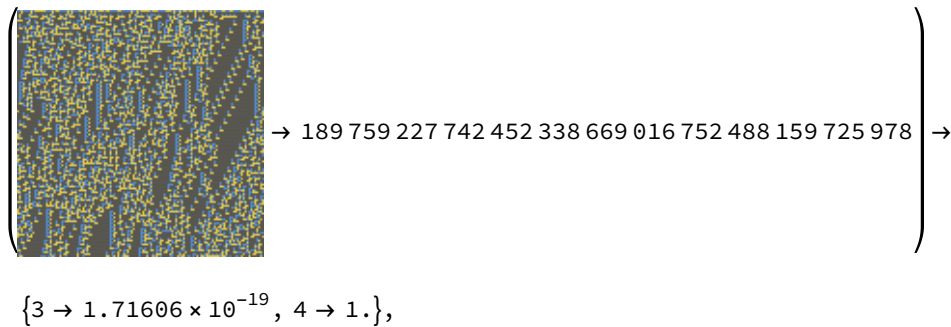
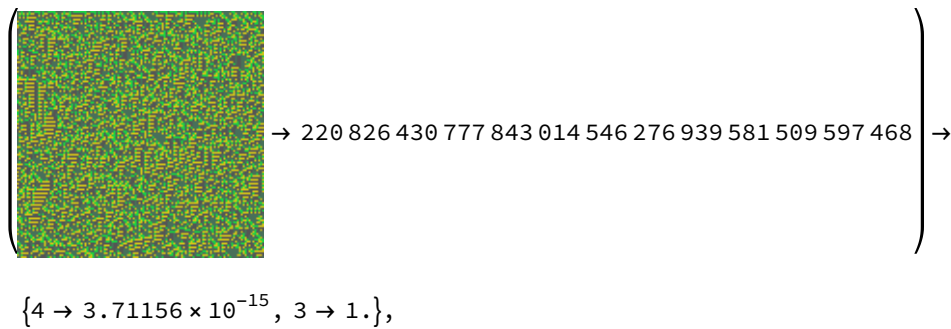
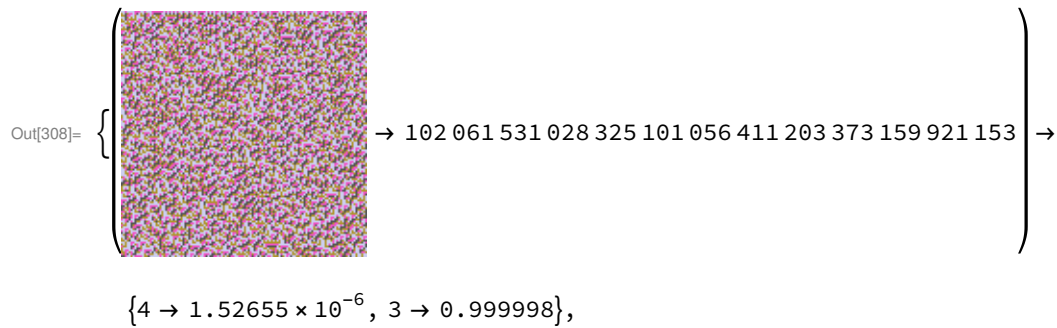
$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 13\,913\,899 \end{array} \right) \rightarrow \{4 \rightarrow 1.88245 \times 10^{-14}, 3 \rightarrow 1.\},$$





### 4-colour non-totalistic, range 1

```
In[307]:= test4Data4kr1C18 = datak4r1NT[128, 128, 8];
Thread[test4Data4kr1C18 → netECA18[Keys@test4Data4kr1C18, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 39\,086\,343\,891\,936\,177\,646\,298\,077\,938\,677\,400\,893 \rightarrow \right.$$

$$\{4 \rightarrow 0.0000617923, 3 \rightarrow 0.999938\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 325\,561\,419\,730\,770\,765\,208\,110\,988\,081\,624\,688\,453 \rightarrow \right.$$

$$\{4 \rightarrow 4.25321 \times 10^{-7}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 247\,405\,682\,417\,519\,114\,838\,497\,454\,468\,289\,242\,544 \rightarrow \right.$$

$$\{4 \rightarrow 3.94091 \times 10^{-12}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 173\,148\,209\,233\,830\,655\,560\,355\,066\,654\,224\,721\,166 \rightarrow \right.$$

$$\{4 \rightarrow 1.19081 \times 10^{-14}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 272\,790\,974\,570\,863\,467\,026\,653\,486\,918\,315\,112\,723 \rightarrow \right.$$

$$\{3 \rightarrow 2.26679 \times 10^{-10}, 4 \rightarrow 1.\}$$

## 4-colour totalistic, range 2

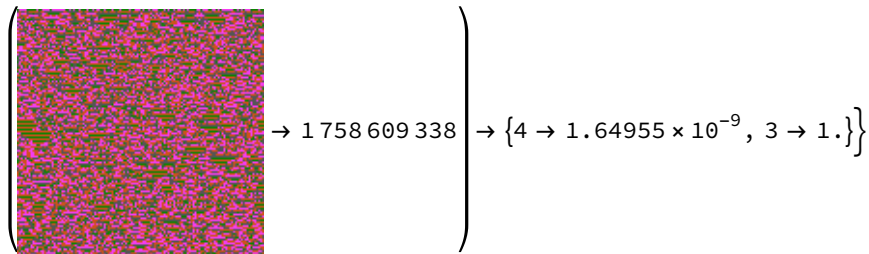
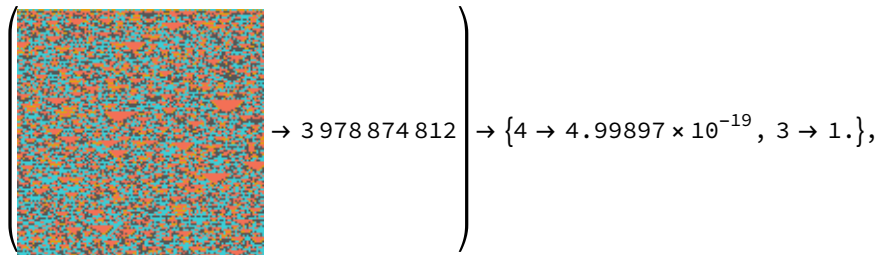
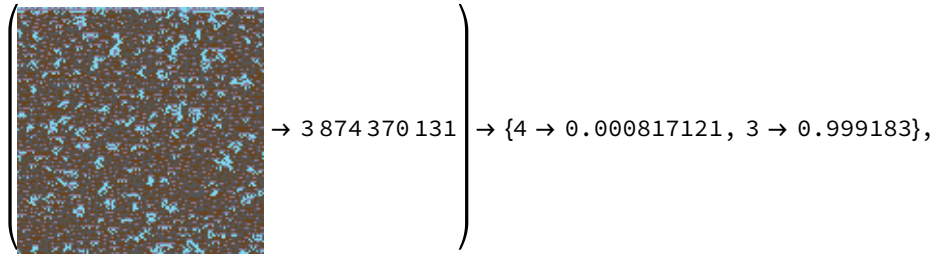
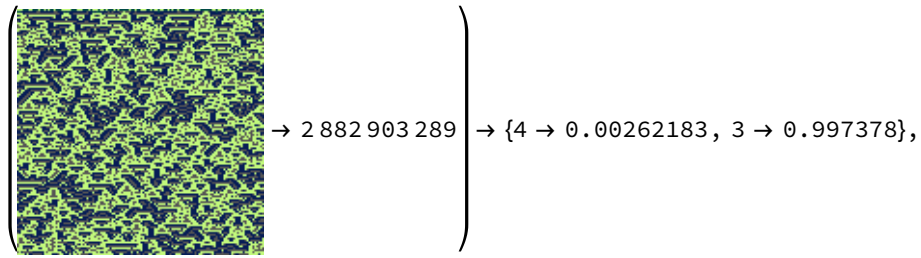
```
In[309]:= test4Data4kr2C18 = datak4r2C[128, 128, 8];
Thread[test4Data4kr2C18 → netECA18[Keys@test4Data4kr2C18, {"TopProbabilities", 2}]]
```

$$\left\{ \begin{array}{c} \text{Out[310]=} \left\{ \begin{array}{c} \text{Image} \\ \rightarrow 3\,511\,876\,239 \end{array} \right\} \rightarrow \{2 \rightarrow 1.5807 \times 10^{-10}, 4 \rightarrow 1.\}, \end{array} \right.$$

$$\left\{ \begin{array}{c} \text{Image} \\ \rightarrow 1\,629\,765\,289 \end{array} \right\} \rightarrow \{4 \rightarrow 1.84811 \times 10^{-17}, 3 \rightarrow 1.\},$$

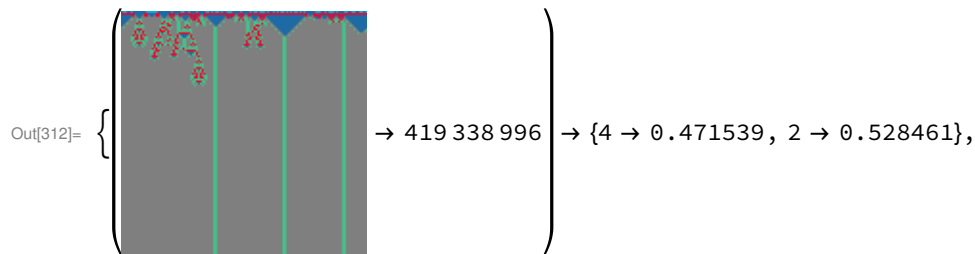
$$\left\{ \begin{array}{c} \text{Image} \\ \rightarrow 3\,309\,785\,711 \end{array} \right\} \rightarrow \{4 \rightarrow 5.75659 \times 10^{-20}, 3 \rightarrow 1.\},$$

$$\left\{ \begin{array}{c} \text{Image} \\ \rightarrow 521\,880\,538 \end{array} \right\} \rightarrow \{4 \rightarrow 2.42952 \times 10^{-8}, 3 \rightarrow 1.\},$$



## 5-colour totalistic, range 1

```
In[311]:= test4Data5kr1C18 = data5T2C[8, 128, 128];
Thread[test4Data5kr1C18 → netECA18[Keys@test4Data5kr1C18, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 884\,788\,964 \rightarrow \{4 \rightarrow 0.0217201, 3 \rightarrow 0.97828\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 743\,542\,029 \rightarrow \{3 \rightarrow 1.08355 \times 10^{-9}, 4 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 782\,108\,342 \rightarrow \{4 \rightarrow 3.73846 \times 10^{-6}, 3 \rightarrow 0.999996\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 785\,621\,045 \rightarrow \{4 \rightarrow 1.13554 \times 10^{-10}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \end{array} \right) \rightarrow 540\,834\,160 \rightarrow \{4 \rightarrow 0.0000100212, 3 \rightarrow 0.99999\},$$



$$\left( \begin{array}{c} \text{[Image: A square plot with a purple background and green vertical streaks.]}\end{array} \right) \rightarrow 1\,180\,125\,611 \rightarrow \{3 \rightarrow 8.69272 \times 10^{-10}, 4 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a noisy, multi-colored background.]}\end{array} \right) \rightarrow 604\,699\,906 \rightarrow \{4 \rightarrow 5.02809 \times 10^{-11}, 3 \rightarrow 1.\}$$

### 6-colour totalistic, range 1

```
In[313]:= test4Data6kr1C18 = data6TC[8, 128, 128];
Thread[test4Data6kr1C18 → netECA18[Keys@test4Data6kr1C18, {"TopProbabilities", 2}]]
```

$$\text{Out[314]=} \left\{ \begin{array}{c} \text{[Image: A square plot with a noisy, multi-colored background.]}\end{array} \right\} \rightarrow 1\,598\,104\,240\,744 \rightarrow \{4 \rightarrow 0.385354, 3 \rightarrow 0.614646\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a noisy, multi-colored background.]}\end{array} \right) \rightarrow 2\,744\,610\,103\,617 \rightarrow \{4 \rightarrow 4.14684 \times 10^{-12}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a noisy, multi-colored background.]}\end{array} \right) \rightarrow 2\,679\,723\,007\,553 \rightarrow \{4 \rightarrow 0.0146554, 3 \rightarrow 0.985345\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 333\,206\,194\,422 \end{array} \right) \rightarrow \{4 \rightarrow 1.77212 \times 10^{-6}, 3 \rightarrow 0.999998\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 385\,745\,608\,648 \end{array} \right) \rightarrow \{4 \rightarrow 4.96414 \times 10^{-18}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 1\,376\,556\,791\,013 \end{array} \right) \rightarrow \{4 \rightarrow 1.60883 \times 10^{-6}, 3 \rightarrow 0.999998\},$$

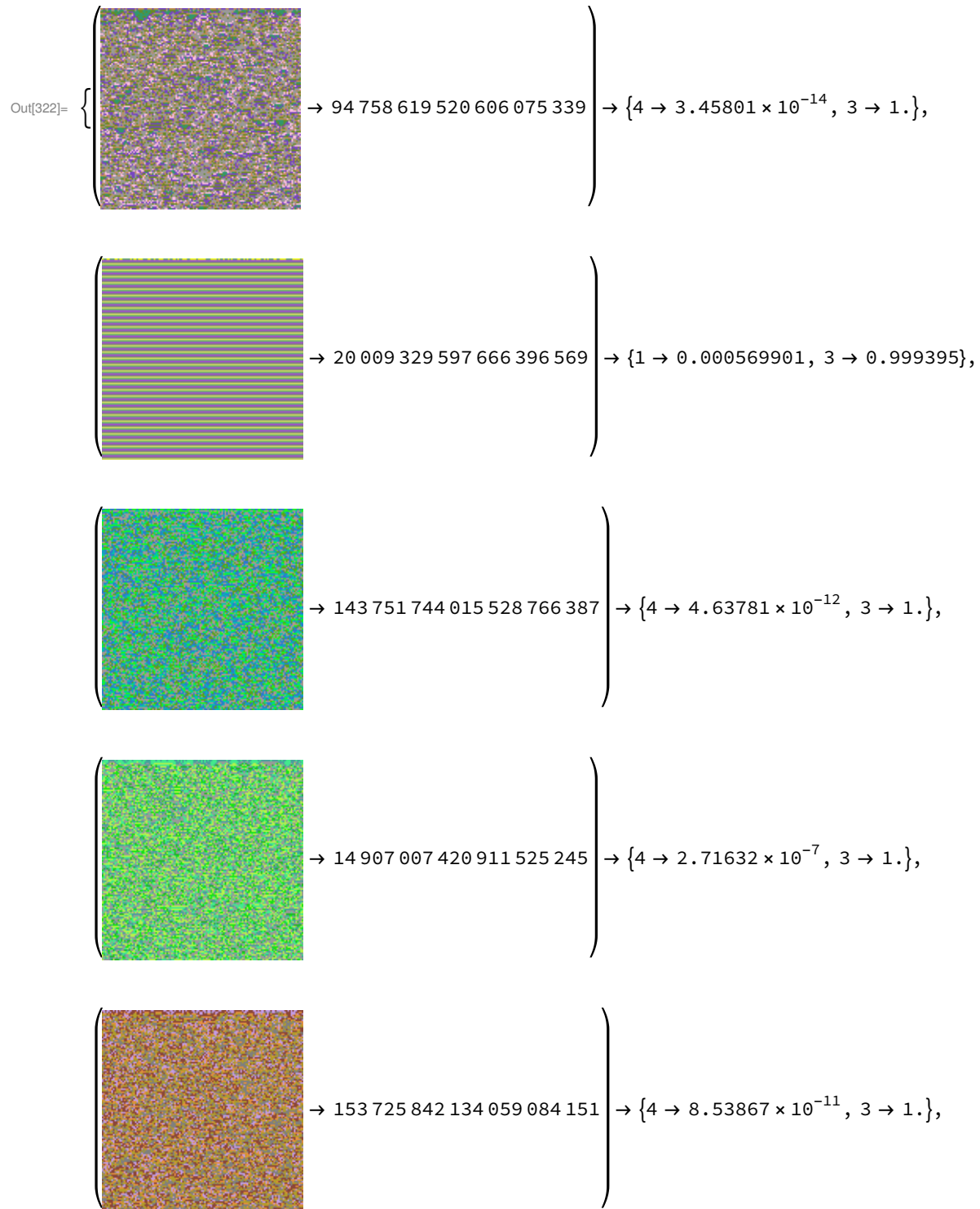
$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 1\,497\,722\,744\,619 \end{array} \right) \rightarrow \{2 \rightarrow 5.2004 \times 10^{-22}, 4 \rightarrow 1.\},$$

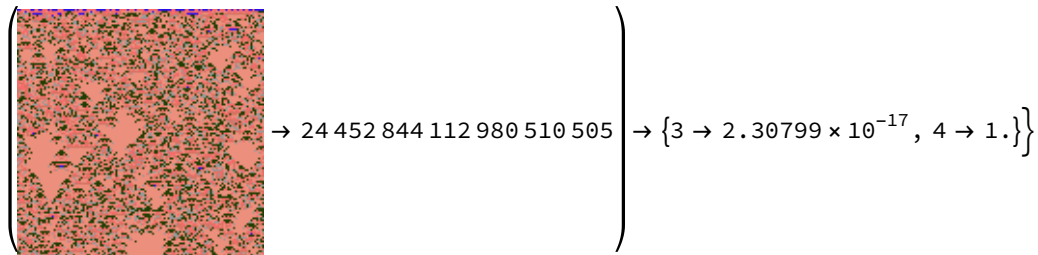
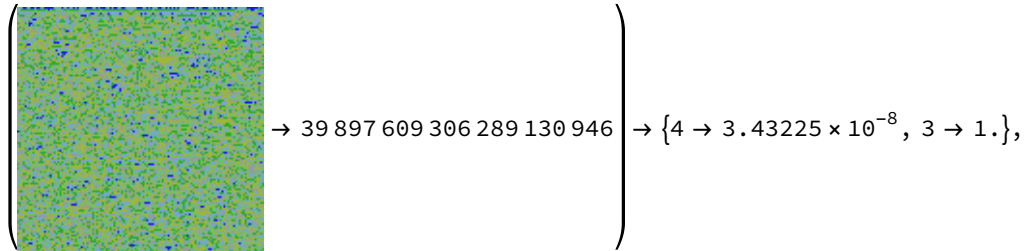
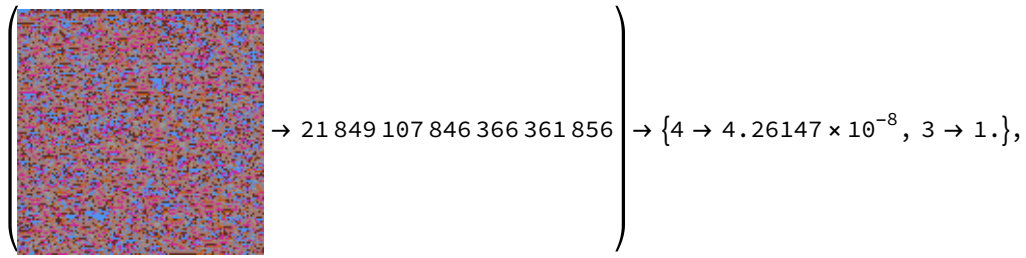
$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 1\,461\,122\,595\,667 \end{array} \right) \rightarrow \{4 \rightarrow 0.0117104, 3 \rightarrow 0.98829\}$$

## 6-colour totalistic, range 2

```
In[321]:= test4Data6kr2C18 = data6T2C[8, 128, 128];
Thread[test4Data6kr2C18 → netECA18[Keys@test4Data6kr2C18, {"TopProbabilities", 2}]]
```

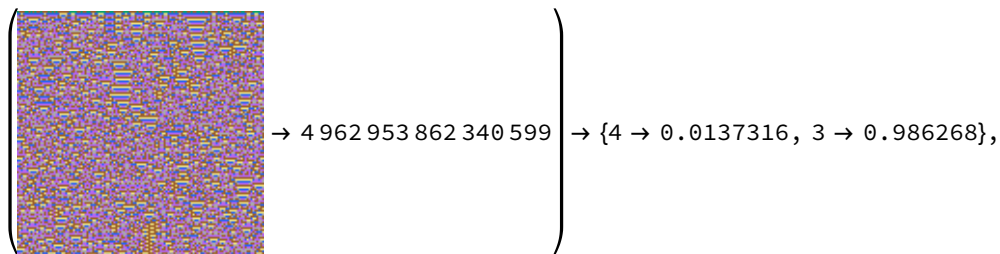
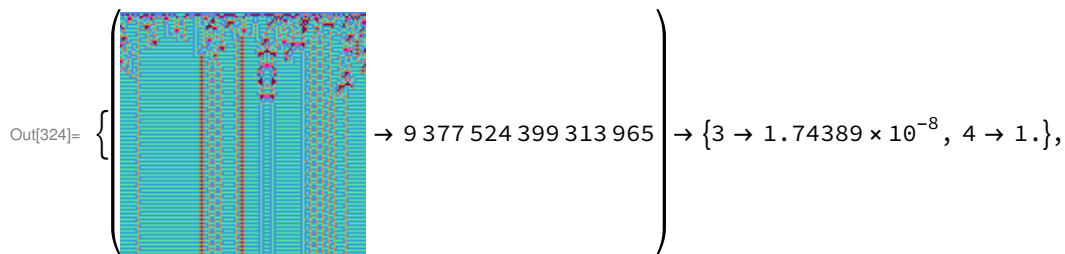






## 7-colour totalistic, range 1

```
In[323]:= test4Data7kr1C18 = data7TC[8, 128, 128];
Thread[test4Data7kr1C18 → netECA18[Keys@test4Data7kr1C18, {"TopProbabilities", 2}]]
```



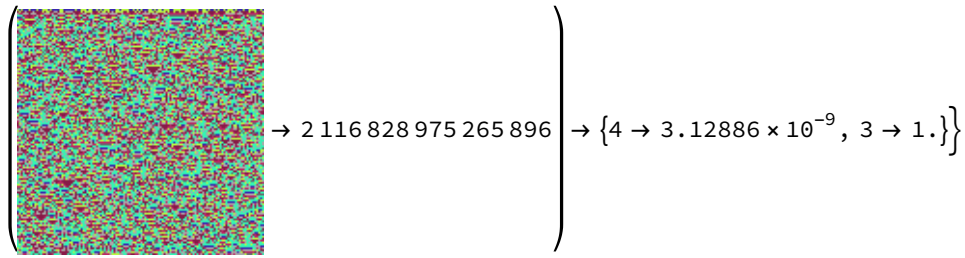
$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 8\,745\,570\,953\,687\,246 \end{array} \right) \rightarrow \{4 \rightarrow 2.19284 \times 10^{-7}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 5\,868\,018\,872\,447\,407 \end{array} \right) \rightarrow \{4 \rightarrow 0.000111761, 3 \rightarrow 0.999888\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 4\,309\,418\,628\,605\,253 \end{array} \right) \rightarrow \{4 \rightarrow 1.75407 \times 10^{-6}, 3 \rightarrow 0.999998\},$$

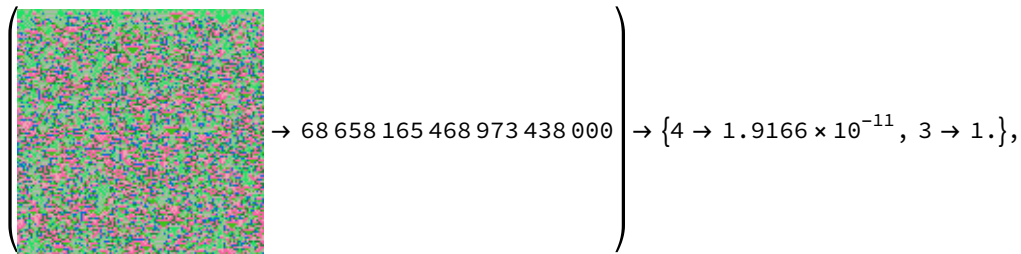
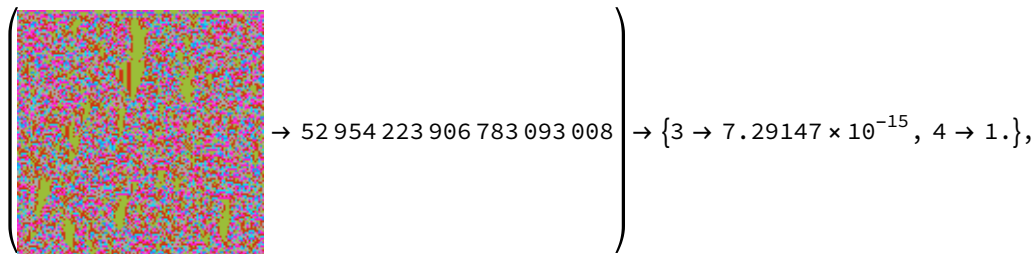
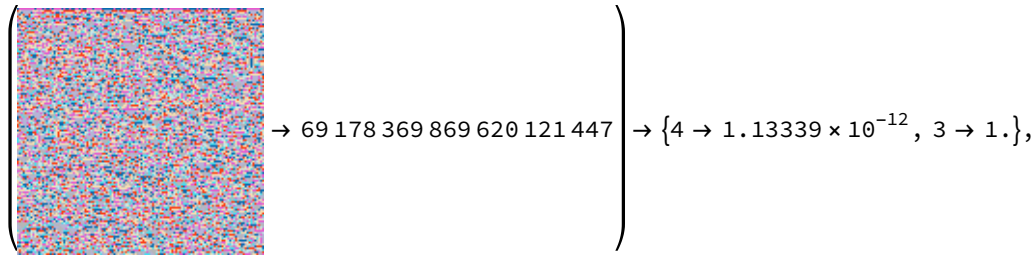
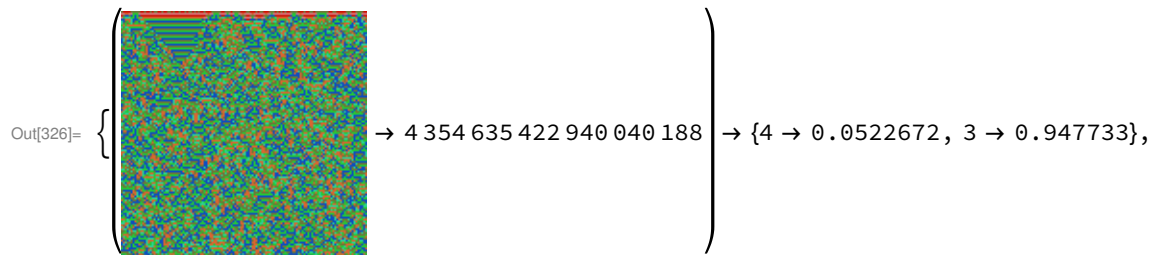
$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 602\,881\,578\,564\,447 \end{array} \right) \rightarrow \{3 \rightarrow 1.85106 \times 10^{-9}, 4 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{Image} \\ \rightarrow 2\,664\,890\,136\,425\,923 \end{array} \right) \rightarrow \{4 \rightarrow 3.52309 \times 10^{-12}, 3 \rightarrow 1.\},$$



## 8-colour totalistic, range 1

```
In[325]:= test4Data8kr1C18 = data8TC[8, 128, 128];
Thread[test4Data8kr1C18 → netECA18[Keys@test4Data8kr1C18, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 40\,882\,704\,313\,683\,534\,715 \end{array} \right) \rightarrow \{4 \rightarrow 0.0000183002, 3 \rightarrow 0.999982\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 4\,334\,236\,228\,138\,547\,400 \end{array} \right) \rightarrow \{4 \rightarrow 1.8216 \times 10^{-12}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 38\,056\,813\,477\,139\,716\,563 \end{array} \right) \rightarrow \{4 \rightarrow 0.025224, 3 \rightarrow 0.974776\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 17\,144\,034\,197\,046\,476\,300 \end{array} \right) \rightarrow \{4 \rightarrow 1.1918 \times 10^{-10}, 3 \rightarrow 1.\}$$

```
In[331]:= test4Data8kr1C18 = data8TC[8, 128, 128];
Thread[test4Data8kr1C18 → netECA18[Keys@test4Data8kr1C18, {"TopProbabilities", 2}]]
```

$$\text{Out[332]=} \left\{ \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 27\,295\,602\,810\,117\,462\,452 \end{array} \right\} \rightarrow \{4 \rightarrow 1.93716 \times 10^{-14}, 3 \rightarrow 1.\},$$



$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 68\,187\,226\,482\,692\,112\,227 \rightarrow \{4 \rightarrow 1.97888 \times 10^{-15}, 3 \rightarrow 1.\}, \right.$$

$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 26\,338\,422\,679\,712\,858\,793 \rightarrow \{2 \rightarrow 1.54265 \times 10^{-15}, 4 \rightarrow 1.\}, \right.$$

$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 20\,106\,191\,194\,925\,098\,456 \rightarrow \{4 \rightarrow 1.32784 \times 10^{-9}, 3 \rightarrow 1.\}, \right.$$

$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 27\,427\,530\,853\,867\,733\,909 \rightarrow \{3 \rightarrow 7.69696 \times 10^{-6}, 4 \rightarrow 0.999992\}, \right.$$

$$\left( \begin{array}{c} \text{Image} \end{array} \rightarrow 67\,626\,281\,665\,658\,424\,537 \rightarrow \{4 \rightarrow 1.31383 \times 10^{-8}, 3 \rightarrow 1.\}, \right.$$

$$\left( \begin{array}{c} \text{[Image: 128x128 random noise with all colors]} \\ \rightarrow 25\,326\,375\,293\,896\,897\,208 \end{array} \right) \rightarrow \{4 \rightarrow 9.39517 \times 10^{-8}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: 128x128 random noise with all colors]} \\ \rightarrow 17\,284\,363\,590\,068\,343\,962 \end{array} \right) \rightarrow \{4 \rightarrow 0.000164327, 3 \rightarrow 0.999836\}$$

## 8-colour totalistic, range 2

```
In[348]:= test4Data8kr2C18 = data8T2C[8, 128, 128];
Thread[test4Data8kr2C18 → netECA18[Keys@test4Data8kr2C18, {"TopProbabilities", 2}]]
```

$$\text{Out[349]=} \left( \begin{array}{c} \text{[Image: 128x128 random noise with all colors]} \\ \rightarrow 91\,605\,229\,994\,459\,866\,473\,701\,555\,510\,459 \end{array} \right) \rightarrow \{4 \rightarrow 8.95721 \times 10^{-9}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: 128x128 random noise with all colors]} \\ \rightarrow 148\,194\,329\,210\,486\,766\,360\,332\,149\,681\,908 \end{array} \right) \rightarrow$$

$\{4 \rightarrow 0.000259168, 3 \rightarrow 0.999741\},$

$$\left( \begin{array}{c} \text{[Image: 128x128 random noise with all colors]} \\ \rightarrow 312\,941\,122\,843\,853\,005\,350\,894\,081\,598\,045 \end{array} \right) \rightarrow$$



$$\{4 \rightarrow 3.01437 \times 10^{-25}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 218\,758\,040\,018\,464\,082\,211\,209\,760\,600\,342 \end{array} \right) \rightarrow$$

$$\{4 \rightarrow 1.84707 \times 10^{-6}, 3 \rightarrow 0.999998\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 1\,618\,101\,134\,718\,809\,353\,656\,768\,953\,028 \end{array} \right) \rightarrow \{4 \rightarrow 8.91462 \times 10^{-12}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 117\,778\,321\,396\,037\,048\,003\,724\,837\,215\,261 \end{array} \right) \rightarrow$$

$$\{4 \rightarrow 1.56349 \times 10^{-7}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 158\,334\,416\,764\,854\,082\,261\,529\,458\,487\,247 \end{array} \right) \rightarrow$$

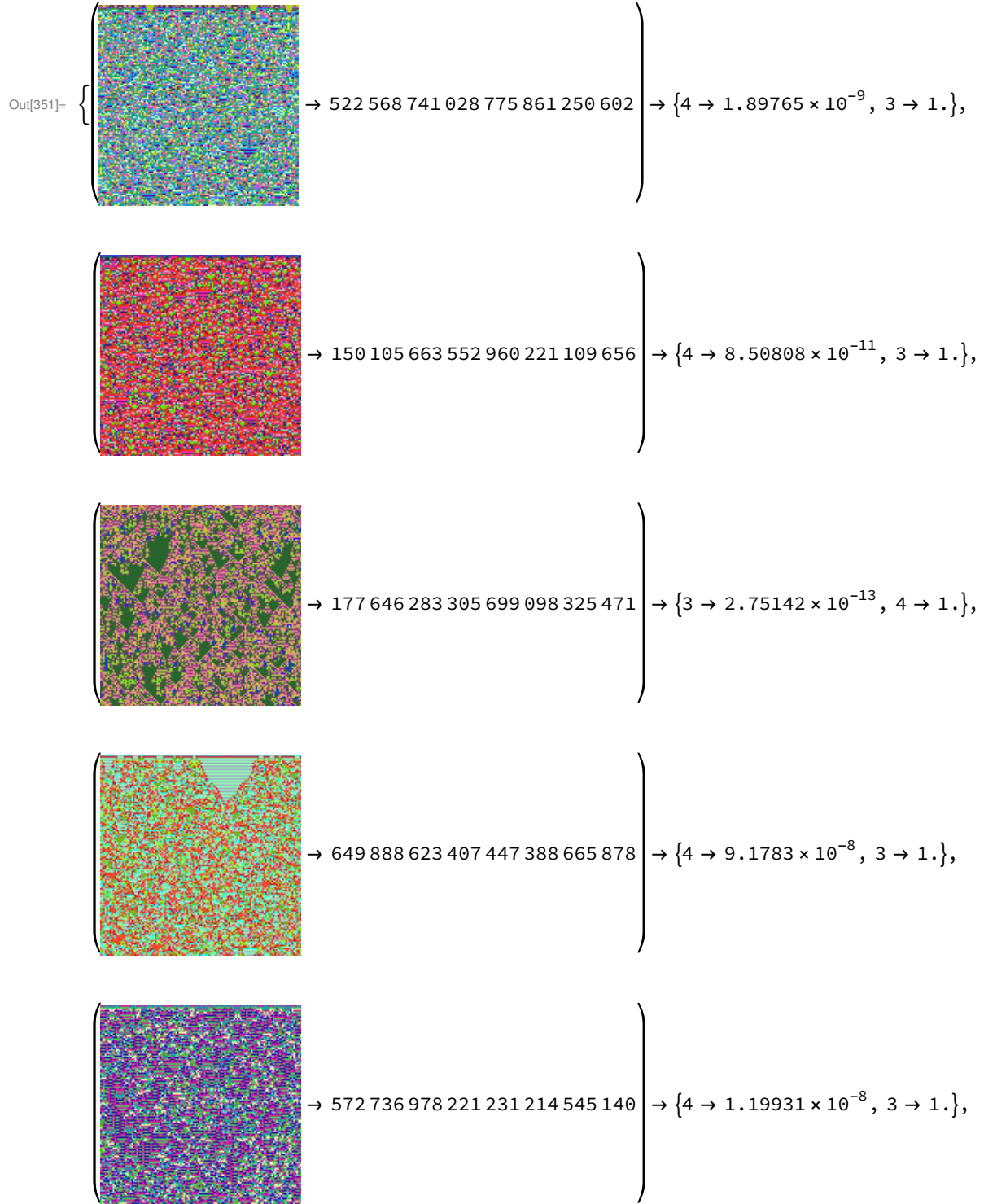
$$\{3 \rightarrow 0.381514, 4 \rightarrow 0.618486\},$$

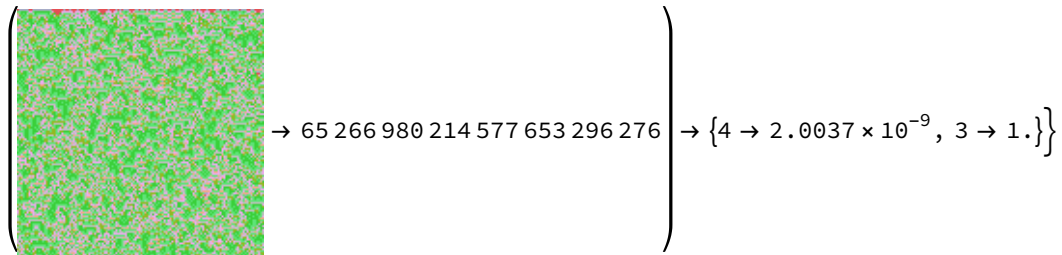
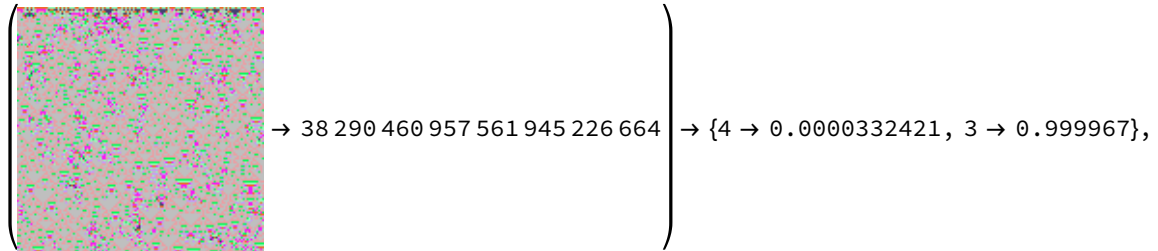
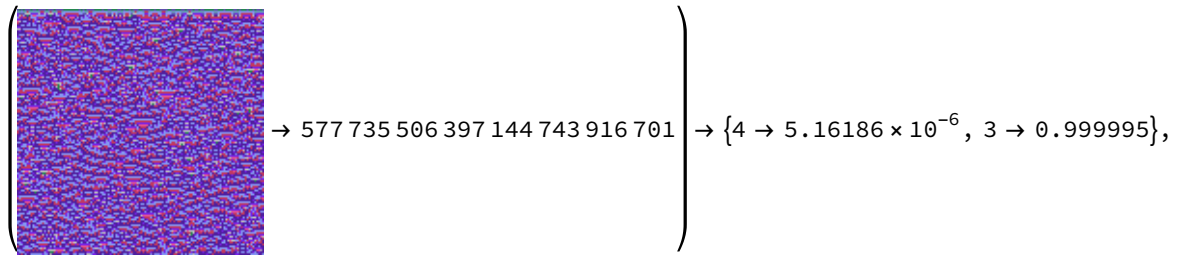
$$\left( \begin{array}{c} \text{[Noise Image]} \\ \rightarrow 64\,962\,204\,199\,289\,039\,065\,352\,424\,560\,944 \end{array} \right) \rightarrow \{4 \rightarrow 3.15489 \times 10^{-10}, 3 \rightarrow 1.\}$$

## 9-colour totalistic, range 1

```
In[350]:= test4Data9kr1C18 = data9TC[8, 128, 128];
```

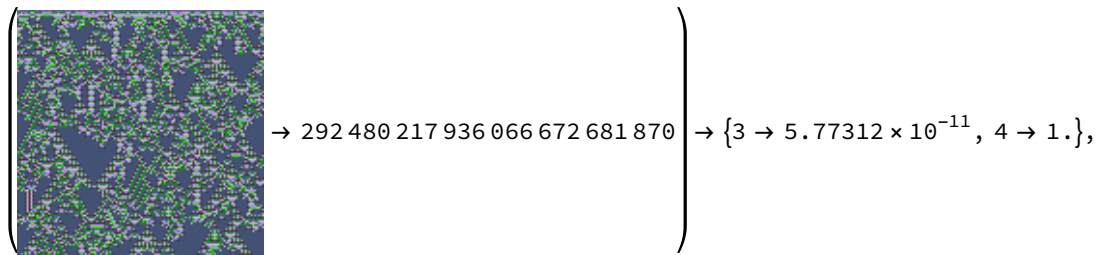
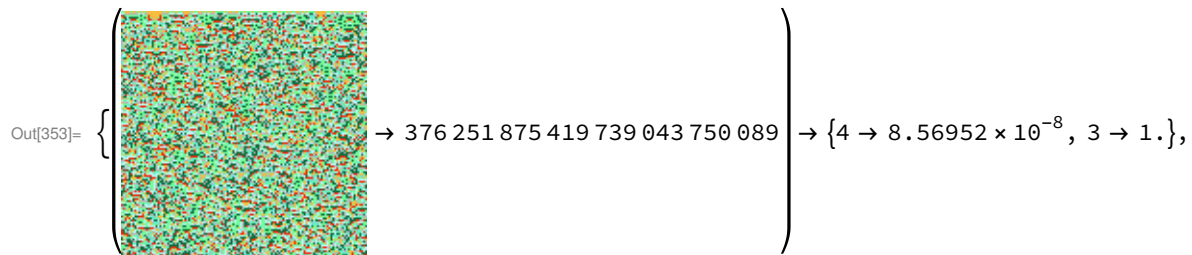
```
Thread[test4Data9kr1C18 → netECA18[Keys@test4Data9kr1C18, {"TopProbabilities", 2}]]
```





```
In[352]:= test4Data9kr1C18 = data9TC[8, 128, 128];
```

```
Thread[test4Data9kr1C18 → netECA18[Keys@test4Data9kr1C18, {"TopProbabilities", 2}]]
```



$$\left( \begin{array}{c} \text{[Image: A square image filled with random noise in shades of purple, magenta, and pink.]}\end{array} \rightarrow 641\,652\,971\,419\,956\,634\,593\,698 \rightarrow \{4 \rightarrow 9.34668 \times 10^{-11}, 3 \rightarrow 1.\}, \right.$$

$$\left( \begin{array}{c} \text{[Image: A square image filled with random noise in shades of green and cyan.]}\end{array} \rightarrow 560\,384\,664\,222\,257\,507\,134\,238 \rightarrow \{4 \rightarrow 1.28321 \times 10^{-8}, 3 \rightarrow 1.\}, \right.$$

$$\left( \begin{array}{c} \text{[Image: A square image filled with random noise in shades of purple and blue.]}\end{array} \rightarrow 431\,262\,759\,200\,417\,990\,085\,248 \rightarrow \{4 \rightarrow 0.000429963, 3 \rightarrow 0.99957\}, \right.$$

$$\left( \begin{array}{c} \text{[Image: A square image filled with random noise in shades of green and yellow.]}\end{array} \rightarrow 349\,851\,539\,333\,502\,282\,320\,618 \rightarrow \{4 \rightarrow 5.50927 \times 10^{-7}, 3 \rightarrow 0.999999\}, \right.$$

$$\left( \begin{array}{c} \text{[Image: A square image filled with random noise in shades of blue and purple, with some green and red specks.]}\end{array} \rightarrow 141\,618\,270\,878\,027\,879\,702\,319 \rightarrow \{4 \rightarrow 0.0000610287, 3 \rightarrow 0.999939\}, \right.$$



$$\left( \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of green, blue, and red pixels.]}\end{array} \right) \rightarrow 401\,516\,309\,538\,894\,848\,288\,118 \rightarrow \{4 \rightarrow 6.7734 \times 10^{-12}, 3 \rightarrow 1.\}$$

```
In[360]:= test4Data9kr1C18 = data9TC[8, 128, 128];
Thread[test4Data9kr1C18 → netECA18[Keys@test4Data9kr1C18, {"TopProbabilities", 2}]]
```

$$\text{Out[361]= } \left\{ \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of green, blue, and red pixels.]}\end{array} \right\} \rightarrow 102\,484\,955\,339\,910\,707\,201\,065 \rightarrow \{4 \rightarrow 2.85292 \times 10^{-12}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of green, blue, and red pixels.]}\end{array} \right) \rightarrow 225\,104\,493\,515\,167\,213\,968\,116 \rightarrow \{4 \rightarrow 2.59132 \times 10^{-19}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of green, blue, and red pixels.]}\end{array} \right) \rightarrow 82\,955\,736\,870\,484\,114\,072\,206 \rightarrow \{4 \rightarrow 7.69447 \times 10^{-10}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square plot with a dense, noisy pattern of green, blue, and red pixels.]}\end{array} \right) \rightarrow 671\,885\,429\,563\,685\,913\,440\,220 \rightarrow \{3 \rightarrow 0.0000324644, 4 \rightarrow 0.999968\},$$

$$\left( \begin{array}{c} \text{[Image: A square image filled with random noise in shades of green, yellow, and blue.]}\end{array} \right) \rightarrow 319\,540\,285\,313\,900\,469\,182\,543 \rightarrow \{4 \rightarrow 1.25701 \times 10^{-7}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square image showing a repeating pattern of purple and yellow triangles.]}\end{array} \right) \rightarrow 472\,683\,159\,387\,864\,865\,560\,932 \rightarrow \{4 \rightarrow 0.0000136606, 3 \rightarrow 0.999986\},$$

$$\left( \begin{array}{c} \text{[Image: A square image filled with random noise in shades of green, yellow, and blue.]}\end{array} \right) \rightarrow 522\,719\,371\,598\,196\,985\,234\,941 \rightarrow \{4 \rightarrow 5.12875 \times 10^{-11}, 3 \rightarrow 1.\},$$

$$\left( \begin{array}{c} \text{[Image: A square image showing a repeating pattern of purple and yellow triangles.]}\end{array} \right) \rightarrow 135\,699\,531\,967\,322\,696\,819\,064 \rightarrow \{4 \rightarrow 4.81907 \times 10^{-12}, 3 \rightarrow 1.\}\}$$