4. Are these building blocks good enough?

Diagrams, formally

Three definitions

- A **diagram** in a category $\mathscr{C}$ is a functor $D\colon \mathscr{J} \to \mathscr{C}$ where the **shape** $\mathscr{J}$ is a small category
  - for our purposes, $\mathscr{C}$ is usually some category of "geometric sheaves" on a space, e.g. wedge products of the (co)tangent bundle of a manifold (so sections are differential forms and vector fields)
- Given a category $\mathscr{C}$ and an object $S \in \mathscr{C}$, the category $\mathrm{El}_S(\mathscr{C})$ of **generalised elements of shape** S is the coslice category $\mathrm{El}_S(\mathscr{C}) = S/\mathscr{C}$
- A **lift** of a diagram $D\colon \mathscr{J} \to \mathscr{C}$ through a functor $\pi\colon \mathscr{E} \to \mathscr{C}$ is a functor $\overline{D}\colon \mathscr{J} \to \mathscr{E}$ such that $\pi \circ \overline{D} = D$
  - we generally take π to be a discrete opfibration