BVPs (and IVPs)

Pretty important for actually solving and modelling stuff

- An **extension** of a diagram $D_0\colon \mathscr{J}_0 \to \mathscr{C}$ along a functor $R\colon \mathscr{J}_0 \to \mathscr{J}$ is a diagram $D\colon \mathscr{J} \to \mathscr{C}$ and a (backwards) morphism $(R,\rho)\colon (\mathscr{J},D) \to (\mathscr{J}_0, D_0)$

$$\begin{tikzcd} \mathsf{J}_0 \arrow[r, "D_0"] \arrow[d, "R"'] & \mathsf{C} \\ \mathsf{J} \arrow[ur, dashed, "D"'] \arrow[u, phantom, "\rho"] & \end{tikzcd}$$

- Given

    1. an extension of D_0 along R

    2. a lift $\overline{D_0}$ of D_0 through some functor $\pi\colon \mathscr{E} \to \mathscr{C}$

$$\begin{tikzcd} \mathsf{J}_0 \arrow[r, "\overline{D}_0"] \arrow[d, "R"'] \arrow[dr, "D_0"] & \mathsf{E} \arrow[d, "\pi"] \\ \mathsf{J} \arrow[r, "D"'] \arrow[u, phantom, "\rho"] & \mathsf{C} \end{tikzcd}$$

the **extension lifting problem** is to find an extension $(R,\overline{\rho})\colon (\mathscr{J},\overline{D}) \to (\mathscr{J}_0, \overline{D_0})$ of $\overline{D_0}$ along R such that $\overline{D}$ is a lift of D through π in a “2-compatible way”

$$\begin{tikzcd} \mathsf{J}_0 \arrow[r, "\overline{D}_0"] \arrow[d, "R"'] & \mathsf{E} \arrow[d, "\pi"] \\ \mathsf{J} \arrow[r, "D"'] \arrow[ur, dashed, "\overline{D}"'] \arrow[u, phantom, "\overline{\rho}"] & \mathsf{C} \end{tikzcd} = \begin{tikzcd} \mathsf{J}_0 \arrow[d, "R"'] \arrow[dr, "D_0"] & \\ \mathsf{J} \arrow[r, "D"'] \arrow[u, phantom, "\rho"] & \mathsf{C} \end{tikzcd}$$

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- D represents the whole system
- D_0 represents the boundary of the system
- $D \to D_0$ projects the system onto its boundary
- A lift $\overline{D_0}$ of D_0 is a choice of boundary data
- N.B. unlike in “classical” algebraic topology, we allow extension-lifting problems to be non-strict, i.e. to have non-trivial 2-cells