

Solutions

Not limits, but lifts

$$C : \Omega_t^0 \rightrightarrows^{\partial_t}_{k\Delta} \dot{C} : \Omega_t^0$$

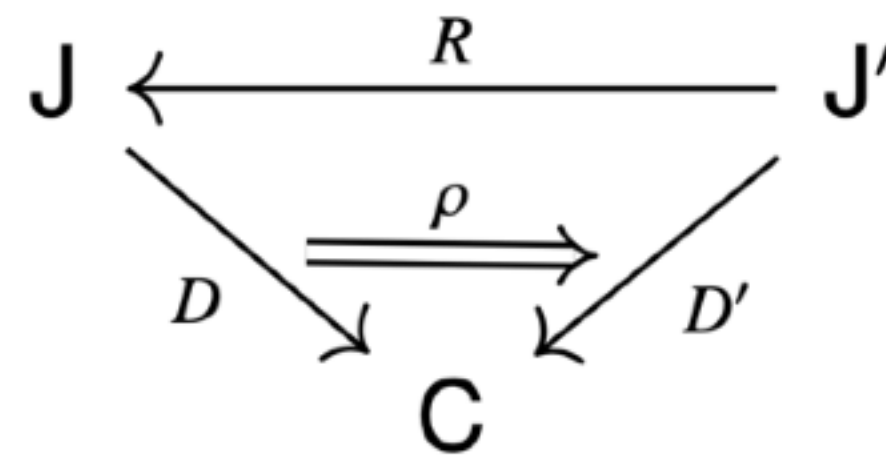
The heat equation: $\frac{\partial C}{\partial t} = k \nabla C$

- Instead, solutions correspond to lifts of the diagram D through the codomain functor $\text{cod} : \text{El}_S(\mathcal{C}) \rightarrow \mathcal{C}$
- when working with (P)DEs, we usually take $S = \underline{\mathbb{R}}$ (the constant sheaf of \mathbb{R} on M), since morphisms $\underline{\mathbb{R}} \rightarrow \mathcal{F}$ of sheaves (of real vector spaces on M) are in bijection with global sections of \mathcal{F}
- Note that we can recover limits from lifts, since a lift through cod is exactly a cone over D with apex S

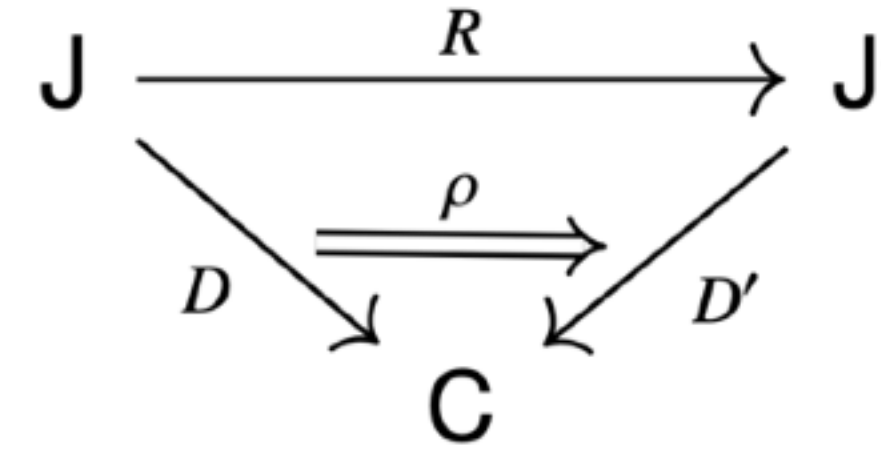
Morphisms of diagrams

Two constructions

- Given a category \mathcal{C} , the **backwards category of diagrams in \mathcal{C}** has objects $(\mathcal{J}, D: \mathcal{J} \rightarrow \mathcal{C})$ and morphisms $(R: \mathcal{J}' \rightarrow \mathcal{J}, \rho: D \circ R \Rightarrow D')$; the **forwards** category has morphisms $(R: \mathcal{J} \rightarrow \mathcal{J}', \rho: D \Rightarrow D' \circ R)$



Forwards



Backwards

- If ρ is the identity, then we recover (the opposite of) the slice category Cat/\mathcal{C}
- Morphisms in the *backwards* category send solutions to solutions
 - thus these tend to go in the direction of *increasing generality*, e.g. from static Maxwell–Faraday *with* potentials to static Maxwell–Faraday *without* potentials