Morphisms of diagrams

Two constructions

- Given a category $\mathscr{C}$, the **backwards category of diagrams in** $\mathscr{C}$ has objects $(\mathscr{J}, D\colon \mathscr{J} \to \mathscr{C})$ and morphisms $(R\colon \mathscr{J}' \to \mathscr{J}, \rho\colon D \circ R \Rightarrow D')$; the **forwards** category has morphisms $(R\colon \mathscr{J} \to \mathscr{J}', \rho\colon D \Rightarrow D' \circ R)$

$$\begin{tikzcd}
\mathbf{J} \arrow[dr, "D"'] & & \mathbf{J}' \arrow[ll, "R"'] \arrow[dl, "D'"] \\
& \mathbf{C} \arrow[u, phantom, "\overset{\rho}{\Rightarrow}"] &
\end{tikzcd}$$

Forwards

$$\begin{tikzcd}
\mathbf{J} \arrow[rr, "R"] \arrow[dr, "D"'] & & \mathbf{J}' \arrow[dl, "D'"] \\
& \mathbf{C} \arrow[u, phantom, "\overset{\rho}{\Rightarrow}"] &
\end{tikzcd}$$

Backwards

- If ρ is the identity, then we recover (the opposite of) the slice category $\mathsf{Cat}/\mathscr{C}$

- Morphisms in the *backwards* category send solutions to solutions

    - thus these tend to go in the direction of *increasing generality*, e.g. from static Maxwell–Faraday *with* potentials to static Maxwell–Faraday *without* potentials

Purpose of morphisms

Why care?

- We can use morphisms for a few things, e.g.
  - steady states of diffusion processes
  - different presentations of the same physics (but this is subtle!)
  - boundary (and initial) value problems
    - this is the really nice one!