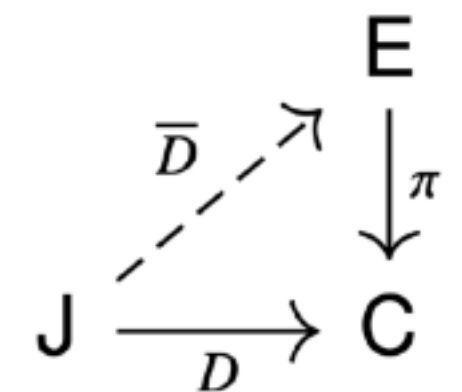


# Diagrams, formally

## Three definitions

- A **diagram** in a category  $\mathcal{C}$  is a functor  $D: \mathcal{J} \rightarrow \mathcal{C}$  where the **shape**  $\mathcal{J}$  is a small category
  - for our purposes,  $\mathcal{C}$  is usually some category of “geometric sheaves” on a space, e.g. wedge products of the (co)tangent bundle of a manifold (so sections are differential forms and vector fields)
- Given a category  $\mathcal{C}$  and an object  $S \in \mathcal{C}$ , the category  $\text{El}_S(\mathcal{C})$  of **generalised elements of shape**  $S$  is the coslice category  $\text{El}_S(\mathcal{C}) = S/\mathcal{C}$
- A **lift** of a diagram  $D: \mathcal{J} \rightarrow \mathcal{C}$  through a functor  $\pi: \mathcal{E} \rightarrow \mathcal{C}$  is a functor  $\bar{D}: \mathcal{J} \rightarrow \mathcal{E}$  such that  $\pi \circ \bar{D} = D$ 
  - we generally take  $\pi$  to be a discrete opfibration



# Solutions

Not limits, but lifts

$$C : \Omega_t^0 \xrightleftharpoons[k\Delta]{\partial_t} \dot{C} : \Omega_t^0$$

The heat equation:  $\frac{\partial C}{\partial t} = k \nabla C$

- Note that the diagram presenting an equation does **not** commute
  - if it did, then this would say that **everything** is a solution to the equation!
- Maybe surprisingly, we do **not** just take the limit (e.g. equaliser) of a diagram
  - this would give us **all** solutions — in (P)DEs, we're often just interested in single solutions! (we would also have to leave the categories of classical differential geometry in order for the relevant limits to exist)