Solutions

Not limits, but lifts

$$C : \Omega_t^0 \underset{k\Delta}{\overset{\partial_t}{\rightrightarrows}} \dot{C} : \Omega_t^0$$

The heat equation: $\frac{\partial C}{\partial t} = k \nabla C$

- Note that the diagram presenting an equation does ***not*** commute
  - if it did, then this would say that ***everything*** is a solution to the equation!
- Maybe surprisingly, we do ***not*** just take the limit (e.g. equaliser) of a diagram
  - this would give us ***all*** solutions — in (P)DEs, we're often just interested in single solutions! (we would also have to leave the categories of classical differential geometry in order for the relevant limits to exist)

Solutions

Not limits, but lifts

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- Instead, *solutions correspond to lifts of the diagram* D *through the codomain functor* $\mathrm{cod}\colon \mathsf{El}_S(\mathscr{C}) \to \mathscr{C}$
  - when working with (P)DEs, we usually take $S = \underline{\mathbb{R}}$ (the constant sheaf of $\mathbb{R}$ on M), since morphisms $\underline{\mathbb{R}} \to \mathscr{F}$ of sheaves (of real vector spaces on M) are in bijection with global sections of $\mathscr{F}$
- Note that we can recover limits from lifts, since a lift through cod is exactly a cone over D with apex S