AUTO-ENCODING VARIATIONAL BAYES

AUTHORS:
DIEDERIK P. KINGMA, MAX WELLING

GROUP 8
RACHITHA CHANDRA BHANU
THOTA BHUVANA CHANDRA
SHEHARIYAR FIRDOUS SHAIKH
SAI SURYA MADA

THE PROBLEM

- Efficient inference and learning in probabilistic models with:
  - Intractable posterior distributions
  - Large datasets
  - Continuous latent variables
- Traditional variational inference requires model-specific derivations
- Monte Carlo methods are often too slow for large-scale data

KEY CONTRIBUTIONS

- The Variational Autoencoder (VAE) framework
- The reparameterization trick for backpropagation through stochastic nodes
- Stochastic Gradient Variational Bayes (SGVB) estimator
- Auto-Encoding Variational Bayes (AEVB) algorithm
- Scalable inference for continuous latent variable models

VARIATIONAL INFERENCE BACKGROUND

- Goal: Approximate intractable posterior $p(z|x)$ with tractable $q(z|x)$
- Minimize KL divergence: $KL(q(z|x) \,||\, p(z|x))$
- Equivalent to maximizing the Evidence Lower Bound (ELBO)
- $ELBO = E_{q_\phi(z \mid x)}[\log p_\theta(x \mid z)] - KL(q_\phi(z \mid x) \,||\, p(z))$
- Traditional methods: mean-field approximations, require conjugacy

THE REPARAMETERIZATION TRICK

- Problem: Can't backpropagate through sampling operation
- Solution: Reparameterize random variable z
- Instead of: $z \sim q(z|x)$
- Write as: $z = g(\varepsilon, x)$ where $\varepsilon \sim p(\varepsilon)$
- Example: $z \sim N(\mu, \sigma^2)$ becomes $z = \mu + \sigma \cdot \varepsilon$ where $\varepsilon \sim N(0,1)$
- Now gradients can flow through μ and σ.

VAE ARCHITECTURE

Encoder (Recognition Model): $q=\varphi(z|x)$

- Neural network that maps input x to latent distribution parameters
- Outputs mean μ and variance σ^2 of approximate posterior

Decoder (Generative Model): $p=\theta(x|z)$

- Neural network that maps latent z to data distribution parameters
- Reconstructs input from latent representation
- Both trained jointly to maximize ELBO

THE LOWER BOUND DECOMPOSITION

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}; \mathbf{x}^{(i)}) = -D_{KL}(q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{x}^{(i)})||p_{\boldsymbol{\theta}}(\mathbf{z})) + \mathbb{E}_{q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{x}^{(i)})}\left[\log p_{\boldsymbol{\theta}}(\mathbf{x}^{(i)}|\mathbf{z})\right]$$

- First term: Regularization (KL divergence between approximate and prior)
- Second term: Reconstruction loss (negative expected log-likelihood)
- Trade-off between reconstruction quality and latent space regularity

SGVB ESTIMATOR

- Stochastic Gradient Variational Bayes
- Unbiased gradient estimator
- Can be computed efficiently with automatic differentiation

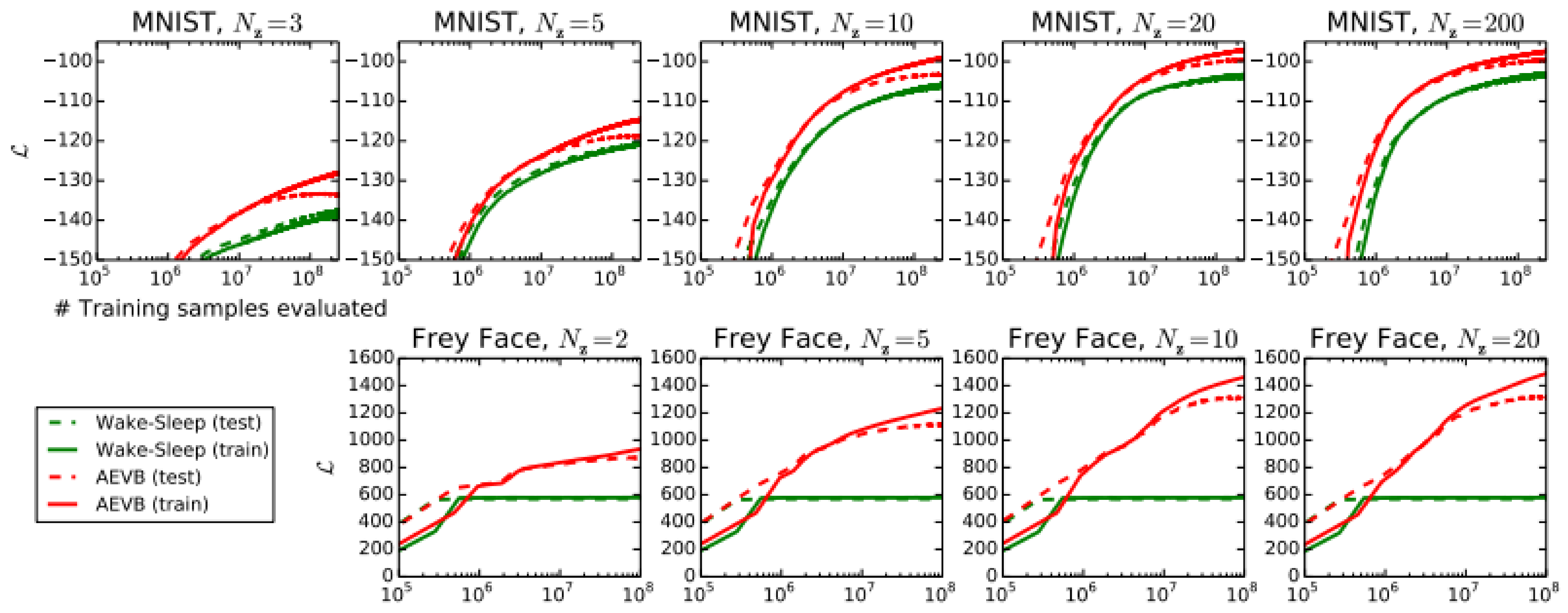
$$\widetilde{\mathcal{L}}^A(\boldsymbol{\theta}, \phi; \mathbf{x}^{(i)}) = \frac{1}{L} \sum_{l=1}^{L} \log p_{\boldsymbol{\theta}}(\mathbf{x}^{(i)}, \mathbf{z}^{(i,l)}) - \log q_{\boldsymbol{\phi}}(\mathbf{z}^{(i,l)} | \mathbf{x}^{(i)})$$

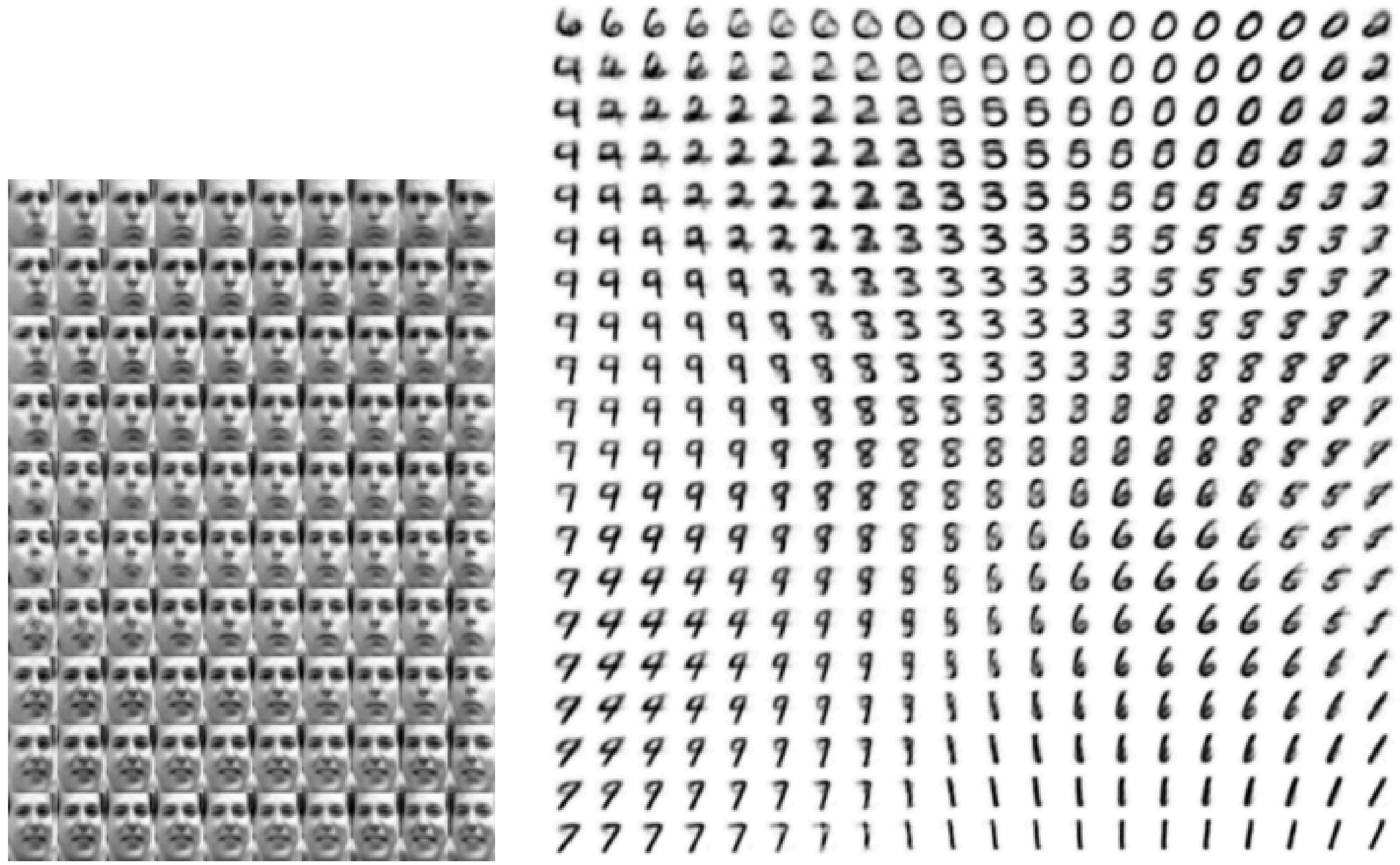
$$\text{where} \quad \mathbf{z}^{(i,l)} = g_{\boldsymbol{\phi}}(\boldsymbol{\epsilon}^{(i,l)}, \mathbf{x}^{(i)}) \quad \text{and} \quad \boldsymbol{\epsilon}^{(l)} \sim p(\boldsymbol{\epsilon})$$

AEVB ALGORITHM

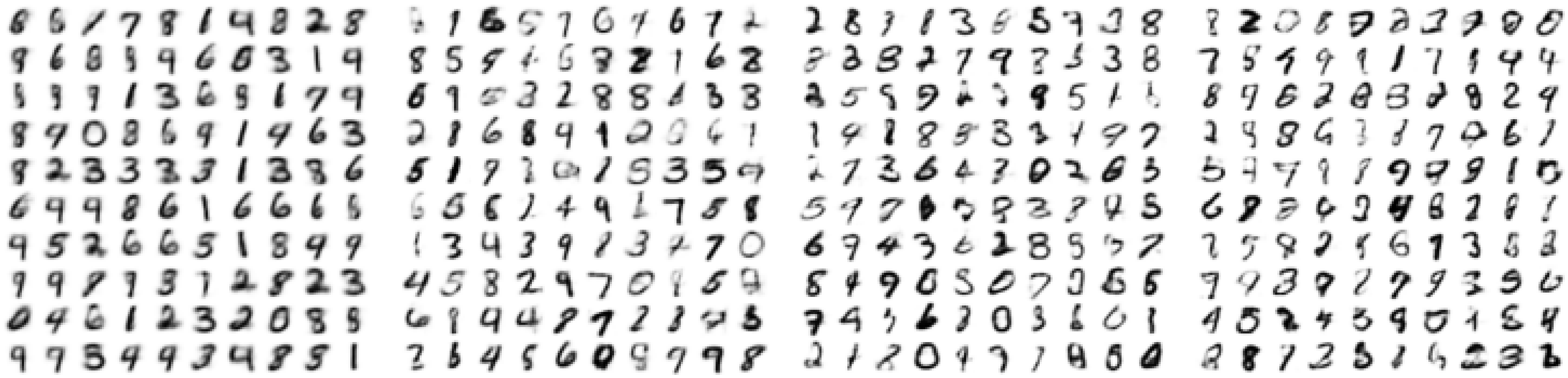
- Initialize parameters θ, φ
- For each iteration:
- Sample minibatch of datapoints
- Sample ε from p(ε) for each datapoint
- Compute gradients using SGVB estimator
- Update θ, φ using gradient.
- Works with any continuous latent variable model

EXPERIMENTS





(a) Learned Frey Face manifold (b) Learned MNIST manifold



(a) 2-D latent space (b) 5-D latent space (c) 10-D latent space (d) 20-D latent space

CONCLUSION

- Auto-Encoding Variational Bayes revolutionized variational inference
- Elegant solution to intractable inference problem
- Enabled scalable learning of complex generative models
- Continues to influence modern machine learning research
- Foundational work with thousands of citations

THANK YOU