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Home-work-9 1

(1)

①. $\cdot(x_i, y_i)$

ⓐ $(x_1, y_1) = (1, 1)$

$(x_2, y_2) = (2, 1)$

$(x_3, y_3) = (3, 4)$

$$y_1 = \alpha x + \beta$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \quad x = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$A^T A x = A^T y$$

$$x = (A^T A)^{-1} \cdot A^T y$$

$$|A^T A| \Rightarrow A^T A = \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}} = \begin{bmatrix} 1+4+9 & 1+2+3 \\ 1+2+3 & 1+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix}$$

$$|A^T A| = 42 - 36 = 6 \neq 0$$

$$(A^T A)^{-1} = \frac{1}{6} \begin{bmatrix} 3 & -6 \\ -6 & 14 \end{bmatrix}$$

$$x = \frac{1}{6} \underset{2\times 2}{\begin{bmatrix} 3 & -6 \\ -6 & 14 \end{bmatrix}} \left(\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}} \right)$$

$$= \frac{1}{6} \begin{bmatrix} 3 & -6 \\ -6 & 14 \end{bmatrix} \begin{bmatrix} 1+2+12 \\ 1+1+4 \end{bmatrix}$$

$$= \frac{1}{6} \underset{2\times 2}{\begin{bmatrix} 3 & -6 \\ -6 & 14 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 15 \\ 6 \end{bmatrix}}$$

$$= \frac{1}{6} \begin{bmatrix} 45 - 36 \\ -90 + 84 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 9 \\ -6 \end{bmatrix}$$

$$x \Rightarrow \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 3/2 \\ -1 \end{bmatrix}$$

(b) $x = \alpha y + \beta$

~~$(y_1, x_1), (y_2, x_2), (y_3, x_3)$~~

$x \in \{ \ldots$

$(x_1, y_1) = (1, 1)$

$(x_2, y_2) = (2, 1)$

$(x_3, y_3) = (3, 4)$

$$x_1 = \alpha y_1 + \beta + \delta_1$$

$$x_2 = \alpha y_2 + \beta + \delta_2$$

$$\vdots$$

$$x_m = \alpha y_m + \beta + \delta_m$$

$$x = Ay + \delta$$

$$\cancel{A^T A y = A^T x}$$

$$\cancel{y = (A^T A)^{-1} A^T x}$$

$$\cancel{= \frac{1}{6}\begin{bmatrix} 3 & -6 \\ -6 & 14 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ \end{bmatrix}}$$

$$y = (A^T A)^{-1} A^T x$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} ; \quad A^T A = \underset{2\times 3}{\begin{bmatrix} 1 & 1 & 4 \\ 1 & 1 & 1 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 4 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 1+1+16 & 1+1+4 \\ 1+1+4 & 1+1+1 \end{bmatrix} = \begin{bmatrix} 18 & 6 \\ 6 & 3 \end{bmatrix}$$

$$|A^T A| = 54 - 36 = 18 \neq 0$$

$$(A^T A)^{-1} = \frac{1}{18}\begin{bmatrix} 3 & -6 \\ -6 & 18 \end{bmatrix}$$

$$\Rightarrow \quad y = \frac{1}{18}\begin{bmatrix} 3 & -6 \\ -6 & 18 \end{bmatrix} \underset{2\times 3}{\begin{bmatrix} 1 & 1 & 4 \\ 1 & 1 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}$$

$$\frac{1}{18}\begin{bmatrix} 3 & -6 \\ -6 & 18 \end{bmatrix}\begin{bmatrix} 1+2+12 \\ 1+2+3 \end{bmatrix} = \frac{1}{18}\underset{2\times 2}{\begin{bmatrix} 3 & -6 \\ -6 & 18 \end{bmatrix}}\underset{2\times 1}{\begin{bmatrix} 15 \\ 6 \end{bmatrix}}$$

$$= \frac{1}{18}\begin{bmatrix} 45-36 \\ -90+108 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 9 \\ -18 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1 \end{bmatrix}$$

(2) ② $A^H = -A$

$$A = \begin{bmatrix} ib & ia \\ ia & ic \end{bmatrix} = \begin{bmatrix} -ib & -ia \\ -ia & -ic \end{bmatrix}$$

$$\det(A-\lambda I) = \begin{bmatrix} ib-\lambda & +ia \\ ia & ic-\lambda \end{bmatrix}$$

$$= (ib-\lambda)(ic-\lambda) - (ia)^2$$

$$= i^2bc - ib\lambda - ic\lambda + \lambda^2 + a^2$$

$$= \lambda^2 + a^2 - bc - i\lambda(b+c)$$

$$\Rightarrow \lambda^2 - \lambda(ib+ic) + (a^2-bc)$$

The roots of the above quadriatic equation gives us the eigen values of A

For $b^2-4ac<0 \Rightarrow$ Then eigen values are pure imaginary

$$(-(ib+ic))^2 - 4(1)(a^2-bc)$$

$$= (ib)^2 + (ic)^2 + 2(ib)(ic) - 4a^2 + 4bc$$

$$= -b^2 - c^2 - 2bc - 4a^2 + 4bc$$

$$= -4a^2 - b^2 - c^2 + 2bc$$

$$= -4(a^2+b^2+c^2) + 2bc$$

It is obvious that the above result <0

$\therefore$ the Eigen values are pure imaginary

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③. Given $A = \begin{bmatrix} 1 & 1+j \\ 2-2j & 4 \end{bmatrix}$

$$|A-\lambda I| = \left| \begin{bmatrix} 1-\lambda & 1+j \\ 2-2j & 4-\lambda \end{bmatrix} \right| \Rightarrow \lambda^2 - 5\lambda = 0$$

$$\lambda = 0, \lambda = 5$$

Right Eigen vectors

For $\lambda = 0$, $N(A-\lambda I)$

$$A - \lambda I = \begin{bmatrix} 1 & 1+j \\ 2-2j & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2j & 4 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \text{Span} \begin{bmatrix} -(1+j) \\ 1 \end{bmatrix}$$

For $\lambda = 5$, $N(A-\lambda I)$

$$N(A-5I) = \begin{bmatrix} -4 & 1+j \\ 2-2j & -1 \end{bmatrix}$$

$$= \text{Span} \begin{bmatrix} 1 \\ 2-2j \end{bmatrix}$$

$$V = \begin{bmatrix} -1-j & 1 \\ 1 & 2-2j \end{bmatrix} \Rightarrow V^{-1} \Rightarrow |V| = (-1-j)(2-2j) - 1$$

$$= -2 + \cancel{2j} - \cancel{2j} + 2j^2 - 1$$

$$= -2 - 2 - 1$$

$$= -5$$

$$V^{-1} = \frac{1}{-5} \begin{bmatrix} 2-2j & -1 \\ -1 & -1-j \end{bmatrix}$$

$$V^{-1}AV = \frac{1}{-5} \begin{bmatrix} 2-2j & -1 \\ -1 & -1-j \end{bmatrix} \begin{bmatrix} 1 & 1+j \\ 2-2j & 4 \end{bmatrix} \begin{bmatrix} -1-j & 1 \\ 1 & 2-2j \end{bmatrix}$$

$$= \frac{-1}{5} \begin{bmatrix} \cancel{2} - \cancel{2j} - \cancel{2} + \cancel{2j} & (2-2j)(1+j) - 4 \\ -1 + (-1-j)(2-2j) & -1-j + (4)(-1-j) \end{bmatrix}$$

$(2-2j)(1+j)$

$2 + \cancel{2j} - \cancel{2j} - 2j^2$

$2 + 2 = 4$

$$= \frac{-1}{5} \begin{bmatrix} 0 & 0 \\ -5 & -5-5j \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ +1 & +1+j \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 1 & 1+j \end{bmatrix} \begin{bmatrix} -1-j & 1 \\ 1 & 2-2j \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ -1-j+1+j & 1+(2-2j)(1+j) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$$

(*) **Left Eigen vectors:**

To get left Eigen

$$y_1^T (A-\lambda I) = 0$$

For $\lambda = 0$

$$y_1^T((A)) = 0$$

$$\underset{1\times 2}{\begin{bmatrix} x & y \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 1+j \\ 2-2j & 4 \end{bmatrix}} = \cancel{\begin{bmatrix} x+(2-2j)y & (1+j)(x)+4y \end{bmatrix}}$$

$$y_1^T = \cancel{\begin{bmatrix} -2+2j & 1 \end{bmatrix}} \quad \begin{bmatrix} 1 & \frac{1+j}{-4} \end{bmatrix}$$

$$y_1 = \begin{bmatrix} \cancel{-2+2j} \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ \frac{1+j}{-4} \end{bmatrix}$$

For $\lambda = 5$

$$y_2^T (A-5\lambda) 0$$

$$\underset{1\times 2}{\begin{bmatrix} x & y \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} -4 & 1+j \\ 2-2j & -1 \end{bmatrix}} = [$$

$$= \begin{bmatrix} -4x+(2-2j)y & (1+j)(x)-y \end{bmatrix}$$

$$y_2^T = \begin{bmatrix} 1 & (1+j) \end{bmatrix}$$

$$y_2 = \begin{bmatrix} 1 \\ 1+j \end{bmatrix}$$

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$$W = \begin{bmatrix} 1 & 1 \\ \frac{1+j}{-4} & 1+j \end{bmatrix}$$

$$W^H = \begin{bmatrix} 1 & \frac{1-j}{-4} \\ 1 & 1-j \end{bmatrix}$$

$$W^H A = \begin{bmatrix} 1 & \frac{1-j}{-4} \\ 1 & 1-j \end{bmatrix} \begin{bmatrix} 1 & 1+j \\ 2-2j & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{(1-j)(2-2j)}{-4} & +(1+j)+(1-j) \\ 1+(2-2j)(1-j) & 1+j+4-4j \end{bmatrix}$$

$$= \begin{bmatrix} 1+j & 2j \\ 1-4j & 5-3j \end{bmatrix} \begin{bmatrix} -1-j & 1 \\ 1 & 2-2j \end{bmatrix}$$

$$= \begin{bmatrix} (1+j)(-1-j)+2j & 1+j+(2j)(2-2j) \\ (1-4j)(-1-j)+5-3j & 1-4j+(5-3j)(2-2j) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$$

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$$B = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B - \lambda I = \begin{bmatrix} 2-\lambda & 3 & 1 \\ 0 & 2-\lambda & -2 \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$$= (2-\lambda)(2-\lambda)(1-\lambda)$$

$$= (2-\lambda)^2(1-\lambda) = 0$$

$1-\lambda = 0$; $(2-\lambda)(2-\lambda) = 0$

$\lambda = 1$

$2-\lambda = 0$

$-\lambda = -2$

$\lambda = 2, 2$

$$N(B-\lambda I) = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_2 - 2x_3 = 0$

$x_2 = 2x_3$

$x_1 + 3x_2 + x_3 = 0$

$x_1 + 6x_3 + x_3 = 0$

$x_1 = -7x_3$

$$x_3 \begin{bmatrix} -7 \\ 2 \\ 1 \end{bmatrix}$$

IAS

$$N(B-\lambda I) = \begin{bmatrix} 0 & 3 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$JCF = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(5) $A = u \cdot v^H$ be a rank 1 matrix

u and v are column vectors

To prove

$v^H u$ as an eigen value

$\Rightarrow \quad Ax = \lambda x$

$Ax = (v^H u) x$

$\Rightarrow u \cdot v^H x = v^H \cdot u x$

$\therefore \quad v^H \cdot u$ is an eigen value

0 is an eigen value of multipicity $(n-1)$

$\Rightarrow \quad A = u \cdot v^H$

$\det(A - \lambda I) = |u \cdot v^H - \lambda I|$

JCF of A

⑥. $Ax = \lambda x$.

$A^2 x = \cancel{A}\ \lambda A x = \lambda^2 x$

$A^3 x = \lambda^2 A x = \lambda^3 x$

$\Rightarrow A^4 x = \lambda^4 x$

$\Rightarrow A^4 = 0 \Rightarrow \lambda^4 = 0$

$\Rightarrow$ Eigen values are zero

$\boxed{\lambda = 0}$

$\therefore$ A is nilpotent if $\lambda = 0$

$\Rightarrow$ A upper (or) lower triangle is such that

$tr(A) = 0$

$\lambda \in \sigma(A)$

$\lambda^k \in \sigma(A^k)$

$\Rightarrow \lambda^k = 0$

$\Rightarrow \lambda = 0$

therefore $A^k = 0$

A is nilpotent matrix

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(7)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & -3-\lambda & -2 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)(-3-\lambda)(1-\lambda) - 2(0) + 3(0)$$

$$\Rightarrow (1-\lambda)^2(-3-\lambda)$$

$$\Rightarrow -\lambda^3 - \lambda^2 + 5\lambda - 3 = 0$$

By cayley Hamilton theorem

$$-A^3 - A^2 + 5A - 3I = 0$$

$$-A^3 - A^2 + 5A - 3 \cdot A \cdot A^{-1} = 0$$

$$-3A \cdot A^{-1} = \frac{A^3 + A^2 - 5A}{-3}$$

$$A^{-1}AA^{-1} = A^{-1}A^3 + A^{-1}A^2 \cancel{+} 5A^{-1}A$$

$$\boxed{A^{-1} = \frac{1}{3}(A^2 + A - 5)}$$