Assigment - 6

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1 ⓐ. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 2 & 4 \end{bmatrix}$

Basis for N(A) ?

$R_2 \to 2R_1 - R_2$
$R_3 \to R_1 - R_3$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} =$

$R_3 \to R_2 + R_1$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$z \qquad \Rightarrow 0$

$z = 0$
$x + 2y + 3z = 0$ ✓
$x + 2y = 0$
$x = -2y.$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = y \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

Null space of Matrix = Basis $\left\{ x \in R^4 \;\middle|\; \text{span} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

R

$\Rightarrow$ Here the Basis for R(A) = Pivot columns

$\Rightarrow R(A) = \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix} \right\}$ is the basis for N(A)

$N(A)^{\perp} = R(A^T)$

$A^T = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 5 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

$R_2 \to 2R_1 - R_2$
$R_3 \to 3R_1 - R_3$

$R_2 \leftrightarrow R_3 \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad R_2 \to R_1 + R_2$

$\uparrow \quad \uparrow$

$\Rightarrow R(A^T) = N(A)^{\perp}$

$\therefore \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$

$\therefore$ Span of $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \right\}$ is the basis for $N(A)^{\perp}$

$R(A)^{\perp} = N(A^T)$

$\Rightarrow \cancel{z = 0}$

$y - z = 0$ $\quad \boxed{\cancel{y=0}}$ $\quad \boxed{y = z}$

$x + 2y + z = 0$ $\quad x + 2z + z = 0$

$x = -3z$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = z \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$R(A)^{\perp}$ $\therefore$ $N(A^T)$ is spanned by $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$ which is the basis

⑥ $\phi : R^2 \to R$

$$\begin{bmatrix} x \\ y \end{bmatrix} \to [x]$$

$$T(x+y) = T(x)+T(y)$$

$$T(x_1+y_1) = \begin{bmatrix} x_1+y_1 \\ x_2+y_2 \end{bmatrix} = [x_1+y_1]$$

$$T(x) = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = [x_1]$$

$$\Rightarrow [x_1+y_1]$$

$$T(y) = \begin{bmatrix} y_1 \\ x_1 \end{bmatrix} = [y_1]$$

$$T(\alpha x) = \alpha \cdot T(x)$$

$$T(\alpha x) = \begin{bmatrix} \alpha x \\ \alpha y \end{bmatrix} = [\alpha x], \quad \alpha \cdot T(x) = \alpha \begin{pmatrix} x \\ y \end{pmatrix} = \alpha x$$

$$T(\alpha x) = \alpha \cdot T(x)$$

$\therefore$ It is a linear transformation

$N(A) =$ $T(x) = \underline{0}$

$\{x \in R \mid (x, y) = 0\}$

$(x = 0)$

span(e^2).

$R(A)$ $T(x) = \underline{y}$

$[x \ y] \Rightarrow (x = y)$ $\quad R(A) = [1]$

span(e^1)

(b) $\phi : R^2 \to R$

$$\begin{bmatrix} x \\ y \end{bmatrix} \to [x-y]$$

$$T(x+y) \Rightarrow \begin{bmatrix} x_1+y_1 \\ x_2+y_2 \end{bmatrix} \Rightarrow [(x_1+y_1)-(x_2+y_2)]$$

$$= [(x_1-x_2)+(y_1-y_2)]$$

$T(x) =$ ~~$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow [x_1 - x_2]$; $T\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = [x_2 - y_2]$~~

$T(x) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [x_1 - x_2]$; $T(y) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = (y_1 - y_2]$

$T(x) + T(y) = [x_1 - x_2 + y_1 - y_2]$ $\quad T\begin{pmatrix} \alpha x \\ \alpha y \end{pmatrix} = [\alpha x - \alpha y] = \alpha [x - y]$

$\alpha \cdot T\begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore \boxed{T(x+y) = T(x) + T(y)}$ $\quad \boxed{T(\alpha x) = \alpha \cdot T(x)}$

$\therefore$ T is linear transformation

$N(A) \Rightarrow T(x) = 0$

$T\begin{pmatrix} x \\ y \end{pmatrix} = (x - y) = 0$ $\quad$ span $\left\{ \begin{matrix} 1 \\ 1 \end{matrix} \right\}$

$x = y$

$R(A) = T(x) = \cancel{y}\,(x - y) = y_1$

$\cancel{x = y + y}$ span $\left\{ \begin{matrix} 1 \\ -1 \end{matrix} \right\}$

$y = x - y$

(c) $\cdot\ \phi : R^2 \rightarrow R^3$

$[x, y]^T \rightarrow [x+y,\ x-y,\ 2x+3y]^T$

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \mapsto \begin{bmatrix} x+y \\ x-y \\ 2x+3y \end{bmatrix}$

$T\begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix} = \begin{bmatrix} x_1 + x_2 + y_1 + y_2 \\ x_1 - x_2 + y_1 - y_2 \\ 2x_1 + 2y_1 + 3x_2 + 3y_2 \end{bmatrix}$

$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \\ 2x_1 + 3x_2 \end{pmatrix}$ $\quad T\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{pmatrix} y_1 + y_2 \\ y_1 - y_2 \\ 2y_1 + 3y_2 \end{pmatrix}$

$\Rightarrow T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + T\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 + y_1 - y_2 \\ x_1 - x_2 + y_1 - y_2 \\ 2x_1 + 3x_2 + 2y_1 + 3y_2 \end{pmatrix}$

$T(x+y) = T$

$N(A) = \{ x \in R^2 \mid (x_1 + y,\ x - y,\ 2x + 3y) = (0, 0, 0) \}$

$\Rightarrow \begin{matrix} x + y = 0 \\ x - y = 0 \\ 2x + 3y = 0 \end{matrix} \Rightarrow x = y = 0 \Rightarrow \{0\}$

$R(A) = \{x \in R^2 \mid (x+y, x-y, 2x+3y) = (y_1, y_2, y_3)\}$

$x+y = y_1$
$x-y = y_2$
$2x+3y = y_2$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 3 \end{bmatrix} \Rightarrow \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} \right\}$$

(4) $A : R^3 \to R^2$

$$A[x_1 \; x_2 \; x_3]^T = [x_1 + 2x_2, x_1 - x_2]^T$$

Given $\{a_1, a_2, a_3\}$ is the basis of R^3

$\{b_1, b_2\}$ of R^2

$$a_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, a_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, a_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T(a_1) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, T(a_2) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, T(a_3) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$L[\cancel{x}] = [L[a_1] \quad L[a_2] \quad L[a_3]]$$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$a_1 = \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{22} \end{bmatrix}$$

$a_{11} + a_{22} = 3$

$a_{11} = 0 \Rightarrow a_{22} = 3$

$$a_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_{12} \\ a_{21} \end{bmatrix}$$

$\Rightarrow a_{12} = -1$

$a_{21} = 3$

$$a_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_{13} \\ a_{33} \end{bmatrix}$$

$a_{13} = 0, a_{23} = 0$

Matrix representation of A

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 3 & 3 & 0 \end{bmatrix}$$

7. K is set of linear transformation

$$\begin{bmatrix}1\\0\\0\end{bmatrix} \to \begin{bmatrix}1\\2\end{bmatrix}, \quad \begin{bmatrix}0\\1\\0\end{bmatrix} \to \begin{bmatrix}0\\1\end{bmatrix}$$

let

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}$$

$$A\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix} \Rightarrow \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$$

$$\begin{bmatrix}a_1\\a_4\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix} \Rightarrow \boxed{\begin{matrix}a_1=1\\a_4=2\end{matrix}}$$

$$A\begin{bmatrix}0\\1\\0\end{bmatrix} \to \begin{bmatrix}0\\1\end{bmatrix}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$$

$$\boxed{a_2=0 \ \& \ a_5=1}$$

$$\therefore A = \begin{bmatrix} 1 & 0 & a_3 \\ 2 & 1 & a_6 \end{bmatrix}$$

$\therefore$ K is the set of linear transformations

$$\left\{ \begin{bmatrix} 1 & 0 & K_1 \\ 2 & 1 & K_2 \end{bmatrix}_{2\times 3} \middle/ \forall \ K_1 K_2 \in R \right\}$$

To Find the Subspace of $Hom_R(R^3, R^2)$

$\Rightarrow$ It should contain '0' vector

Here by altering the K-set, we cannot obtain

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ which maps } \begin{bmatrix}0\\0\\0\end{bmatrix} \to \begin{bmatrix}0\\0\end{bmatrix}$$

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⑤ $\phi : R^3 \to R$ is a surjective ~~linear~~ linear transformation

ⓐ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to [R]$

Here $R^3 \to R$

$\therefore$ In this case we are losing two dimensions meaning there is only one dimension left for the image space.

then A condeses an 3-dimensions to 2-dimens

It is clear that the nullspace of ~~$[x, y, 0]$~~ ~~$(x,y) \in R$~~

$N(\phi) = (x, y, z) \Rightarrow$

$\text{Span}(y, z)$

$\alpha_1 y_1 + \alpha_2 z \quad (\alpha_1, \alpha_2 \in R)$

whish means that it is a plane containg origin

For $(y, z) = (0, 0)$ it is origin

ⓑ

Given

$x \in R, \ \phi^{\leftarrow}(x) = \{ z \in R^3 \mid \phi(z) = x \}$

Here same, we are losing two dimensions,

$\phi(x) = (x, 0, 0)$.

$\Rightarrow \text{span}(x)$

$\therefore$ Therefore it is a plane that is parallel to W

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③. V is a linear vector space over F

$a_1, a_2, a_3, \ldots, a_n$ constitute a basis of V

$$V = \alpha_1 a_1 + \alpha_2 a_2 + \cdots + \alpha_n a_n$$

where $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$

$$A = \begin{array}{cccc} a_1 & a_2 & & a_n \\ \end{array}$$
$$\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & & \vdots \\ \vdots & \vdots & & \vdots \\ a_{1m} & a_{2m} & \cdots & a_{mn} \end{bmatrix}$$

Given $Aa_i = a_{i+1}, 1 \le i \le n-1, An = 0.$

$$L(Z) = \{L[a_1] \quad L[a_2] \cdots L[a_n]\}$$

$$Aa_1 = a_2$$
$$Aa_3 = a_4$$
$$Aa_2 = a_3$$

$$[a_2] = \left[A \right] [a_1]$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{n1} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{mn} \end{bmatrix}$$

$$Aa_i = A\alpha_1 a_1 + A\alpha_2 a_2 + A\alpha_3 a_3 + \cdots + A\alpha_n a_n$$

$$A^n = Aa_n(\alpha_1 + \alpha_2 + \cdots + \alpha_n)$$

$$= \underline{0} \quad (\because Aa_n = 0)$$

$$A^{n-1} = Aa_{n-1}(\alpha_1 + \alpha_2 + \cdots + \alpha_n)$$

which is not equal to zero

② . (a).

Given $V = W_1 \oplus W_2 \cdots \oplus W_s$

Linear Transformation $\mathcal{E}_1, \mathcal{E}_2, \cdots, \mathcal{E}_s$ of V such that

$\Rightarrow \quad V = \alpha_1 W_1 + \alpha_2 W_2 + \cdots + \alpha_s W_s$

$\cancel{\mathcal{E}_1(V) = \mathcal{E}_1(\alpha_1 W_1 + \alpha_2 W_2 + \cdots + \alpha_s W_s)}$

$\Rightarrow$ Here

$V = \alpha_1 W_1 + \alpha_2 W_2 + \cdots + \alpha_s W_s$

$V = \beta_1 W_1 + \beta_2 W_2 + \cdots + \beta_s W_s$

$\Rightarrow (\alpha_1 W_1 + \alpha_2 W_2 + \cdots + \alpha_s W_s) = (\beta_1 W_1 + \beta_2 W_2 + \cdots + \beta_s W_s)$

$\Rightarrow (\alpha_1 - \beta_1) W_1 + (\alpha_2 - \beta_2) W_2 + \cdots + (\alpha_s - \beta_s) W_s = 0$

$\alpha_1 = \beta_1, \alpha_2 = \beta_2 \cdots \alpha_s = \beta_s$

$\therefore$ 1

$\cancel{\mathcal{E}_1(V)}$ $\mathcal{E}_1, \mathcal{E}_2, \cdots, \mathcal{E}_s$ of V

(i). $\mathcal{E}_i^2 = \mathcal{E}_i$

$\mathcal{E}_1(\mathcal{E}_1(V)) \Rightarrow \mathcal{E}_1(V) = 1 W_1 + 0 \cdot W_2 + \cdots + 0 \cdot W_s$

$= W_1$

$\mathcal{E}_1(\mathcal{E}_1(V)) = 1 \cdot W_1$

$= W_1$

$$\boxed{\mathcal{E}_i^2 = \mathcal{E}_i}$$

② . $\mathcal{E}_i \mathcal{E}_j = 0$ for $j \neq i$

e. For $i = 1, j = 2$

$\mathcal{E}_1 \mathcal{E}_2 = 0 \Rightarrow \mathcal{E}_1(\mathcal{E}_2(V)) = \mathcal{E}_1(0 \cdot W_1 + 1 \cdot W_2 + \cdots + 0 W_s)$

$= \mathcal{E}_1(1 \cdot W_2)$

$= 0$

$\therefore \mathcal{E}_i \mathcal{E}_j = 0$

(3) $\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_s = I$

Here in order to prove as Identity transformation

$N(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_s) = \{0\}$

$R(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_s) = V$

$\Rightarrow N(W_1 + W_2 + W_3 + \cdots + W_s) = \{0\}$ ✓

$R(W_1 + W_2 + W_3 + \cdots + W_s) = V$ it gives us the entire V

$\therefore$ $\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_s$ is a identity transformaton

(4). $\varepsilon_i(V) = W_i$

$\varepsilon_1(V) = 1 \cdot W_1 + 0 \cdot W_2 + \cdots + 0 W_s$

$\Rightarrow \varepsilon_2(V) = 0 \cdot W_1 + 1 \cdot W_2 + \cdots + 0 W_s$

$\therefore \boxed{\varepsilon_i(V) = W_i}$

(5). Given

$\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_s$ of V,

$V = W_1 \oplus W_2 \oplus \cdots \oplus W_s$

$W_i = \varepsilon_i V$

$W_1 = \varepsilon_1 V$

$W_2 = \varepsilon_2 V$

$\vdots$

$W_s = \varepsilon_s(V)$

Image(V) $\Rightarrow$ Here all the Vectors are mapped to that with ε

therefore

$(W_1, W_2, \ldots, W_s)$

Range (ε_i) :-

Range of ε_i is $(W_1, W_2, \ldots, W_s)$