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Home-Work- 10

(1) For every

$$e^{A} = e^{S^{-1}JS}$$
$$= S^{-1}e^{J}S$$
$$\det(e^{A}) = \det(e^{SJS^{-1}})$$
$$= \det(Se^{J}\cdot S^{-1})$$
$$= \det(S)\det(e^{J})(\det(S^{-1}))$$
$$= \det(e^{J})$$
$$= \prod_{i=1}^{n} e^{J_{ii}}$$
$$= \exp\left(\sum_{i=1}^{n} J_{ii}\right)$$
$$\boxed{\det e^{A} = \exp(\mathrm{Tra}(A))}$$

. Since $\det(e^{A})$ is non singular

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$$A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

3

$\boxed{\sin A}$ $(A-\lambda I) = \begin{vmatrix} -1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$

$(-1-\lambda)(1-\lambda) - 1 = 0$

$-(1+\lambda)(1-\lambda) - 1 = 0$

$-(1-\lambda^2) - 1 = 0$

$1-\lambda^2 = -1$

$\cancel{+}\lambda^2 = \cancel{+}2$

$\lambda = \pm\sqrt{2}, \; \lambda_1 = \sqrt{2}, \lambda_2 = -\sqrt{2}$

$\sin(A) = \alpha_0 I + \alpha_1 A$

$\sin(\lambda_1) = \alpha_0 + \alpha_1\lambda_1 \Rightarrow \sin(\sqrt{2}) = \alpha_0 + \sqrt{2}\alpha_1 \rightarrow ①$

$\sin(-\sqrt{2}) = \alpha_0 - \sqrt{2}\alpha_1 \rightarrow ②$

we know $\sin(-\theta) = -\sin\theta$

$\alpha_0 + \cancel{\sqrt{2}\alpha_1} = \sin(\sqrt{2})$

$\alpha_0 - \cancel{\sqrt{2}\alpha_1} = -\sin(\sqrt{2})$

$2\alpha_0 = 0$

$\boxed{\alpha_0 = 0}$

$\sqrt{2}\alpha_1 = \sin(\sqrt{2})$

$\alpha_1 = \dfrac{\sin(\sqrt{2})}{\sqrt{2}} = 0.9877$

$\sin A = 0.698 \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$

$\boxed{\cos A}$:- $\cos(\sqrt{2}) = \alpha_0 + \alpha_1\sqrt{2}$

$\cos(-\sqrt{2}) = \alpha_0 - \alpha_1\sqrt{2} \Rightarrow$

$\alpha_0 + \cancel{\alpha_1\sqrt{2}} = \cos\sqrt{2}$

$\alpha_0 - \cancel{\alpha_1\sqrt{2}} = \cos\sqrt{2}$

$\cancel{2}\alpha_0 = \cancel{2}\cos\sqrt{2}$

$\alpha_0 = 0.155$

$\alpha_1\sqrt{2} = \cos\sqrt{2} - \alpha_0$

$\boxed{\alpha_1 = 0}$

$\cos A = 0.155 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\tan A = \alpha_0 I + \alpha_1 A$

$\tan(\sqrt{2}) = \alpha_0 + \alpha_1(\sqrt{2})$

$\tan(-\sqrt{2}) = \alpha_0 + \alpha_1(-\sqrt{2})$

$$\Rightarrow \quad \begin{array}{r} \alpha_0 + \cancel{\alpha_1\sqrt{2}} = \cancel{\tan(\sqrt{2})} \\ \alpha_0 - \cancel{\alpha_1\sqrt{2}} = -\cancel{\tan(\sqrt{2})} \\ \hline \boxed{\alpha_0 = 0} \end{array}$$

4.47

$$\alpha_1 = \frac{\tan(\sqrt{2})}{\sqrt{2}}$$

$$\tan A = 4.478 \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

(b) $\tan A = (\sin A)(\cos A)^{-1}$

$$\cos A = \begin{bmatrix} 0.155 & 0 \\ 0 & 0.155 \end{bmatrix} \Rightarrow \frac{1}{0.024}\begin{bmatrix} 0.155 & 0 \\ 0 & 0.155 \end{bmatrix}$$

$$= \begin{bmatrix} 6.3737 & 0 \\ 0 & 6.377 \end{bmatrix}$$

$$(\sin A)(\cos A)^{-1} = \begin{bmatrix} -0.698 & 0.698 \\ 0.698 & 0.698 \end{bmatrix}\begin{bmatrix} 6.3737 & 0 \\ 0 & 6.377 \end{bmatrix}$$

$$= \begin{bmatrix} (-0.698)(6.3737) & (0.698)(6.3737) \\ (0.698)(6.3737) & (0.698)(6.317) \end{bmatrix}$$

$$= 4.48 \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

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4. Given - e^{At}

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

λ. Here the $\pi(\lambda) =$

$$(1-\lambda^2)+1=0$$

$$\lambda_1 = +\sqrt{2}, \; \lambda_2 = -\sqrt{2}$$

$$e^{At} = \alpha_0 I + \alpha_1 A \;, \; e^{\lambda t} = \alpha_0 I + \alpha_1 \lambda$$

λ

$$e^{\sqrt{2}t} = \alpha_0 + \alpha_1(\sqrt{2}) \longrightarrow ①$$

$$- \; e^{-\sqrt{2}t} = \alpha_0 - \alpha_1\sqrt{2} \longrightarrow ②$$

$$2\alpha_1\sqrt{2} = e^{\sqrt{2}t} - e^{-\sqrt{2}t}$$

$$\alpha_1 = \frac{e^{\sqrt{2}t} - e^{-\sqrt{2}t}}{2\sqrt{2}}$$

From ①

$$\Rightarrow e^{\sqrt{2}t} = \alpha_0 + \left(\frac{e^{\sqrt{2}t} - e^{-\sqrt{2}t}}{2\cancel{\sqrt{2}}}\right)(\cancel{\sqrt{2}})$$

$$= 2e^{\sqrt{2}t} = 2\alpha_0 + (e^{\sqrt{2}t} - e^{-\sqrt{2}t})$$

$$\Rightarrow 2\alpha_0 = 2e^{\sqrt{2}t} - e^{\sqrt{2}t} + e^{-\sqrt{2}t}$$

$$\alpha_0 = \frac{e^{\sqrt{2}t} + e^{-\sqrt{2}t}}{2}$$

$$e^{At} = \left(\frac{e^{\sqrt{2}t} + e^{-\sqrt{2}t}}{2}\right) + \left(\frac{e^{\sqrt{2}t} - e^{-\sqrt{2}t}}{2\sqrt{2}}\right)\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

(6). Given $M = \begin{bmatrix} 4 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & \cancel{1} \end{bmatrix}$

$(\lambda I - A)$

$$\Rightarrow \begin{bmatrix} \lambda-4 & -1 & 0 & 0 & 0 \\ 0 & \lambda-4 & -1 & 0 & 0 \\ 0 & 0 & \lambda-4 & 0 & 0 \\ 0 & 0 & 0 & \lambda-4 & -1 \\ 0 & 0 & 0 & 0 & \lambda-4 \end{bmatrix}$$

$$\pi(\lambda) = (\lambda-4)^5$$

minimal polynomial =

we know

$\operatorname{rank}(A-\lambda I) + n = \dim(N(A-\lambda I))$

$$\alpha(\lambda) = (\lambda-4)^3$$

$$M' = \begin{bmatrix} 2 & 7 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{bmatrix}$$

$$\lambda I - A = \left| \begin{bmatrix} \lambda-2 & 7 & 0 & 0 \\ 0 & \lambda-2 & 0 & 0 \\ 0 & 0 & \lambda-1 & -1 \\ 0 & 0 & \lambda+2 & -4 \end{bmatrix} \right|$$

$$= (\lambda-2) \begin{vmatrix} \lambda-2 & 0 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & \lambda+2 & -4 \end{vmatrix} \cdot \lambda$$

$$= (\lambda-2)^2 \begin{vmatrix} \lambda-1 & -1 \\ \lambda+2 & -4 \end{vmatrix}$$

$$= (\lambda-2)^2 \left[(-4)(\lambda-1) + (\lambda+2) \right]$$

$$= (\lambda-2)^2 \left[-4\lambda+4+\lambda+2 \right]$$

$$= (\lambda-2)^2 (-3\lambda+6)$$

$$= (\lambda^2 - 4\lambda+4)(-3\lambda+6)$$

$$\Rightarrow -3\lambda^3+6\lambda^2+12\lambda^2-24\lambda-12\lambda+2$$

$$\Rightarrow -3\lambda^3+18\lambda^2-36\lambda+2 = 0$$

$$\pi(\lambda) \quad -\lambda^3+6\lambda^2-12\lambda+\frac{2}{3}=0$$

$$\lambda = 2, 2, 2$$

$$\alpha(\lambda) = (\lambda-2)^2$$

7. Determining JCF of matrix

$$\pi(\lambda) = (\lambda-2)^3(\lambda-5)^2$$

$$m_A(x) = (x-2)(x-5) \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$m_A(x) = (x-5)^2(x-2) = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$m_A(x) = (x-2)^2(x-5) = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$m_A(x) = (x-2)^2(x-5)^2 = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$m_A(x) = (x-2)^3(x-5) = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$m_A(x) = (x-2)^3(x-5)^2 \quad \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

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(8) $\pi(\lambda) = (\lambda-2)^5, \; m(\lambda) = (\lambda-2)^2$

$$\pi(\lambda) = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

JCF

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

(9) Compute A^K

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

By Cayley Hamilton

$$|\lambda I - A| = 0$$

$$\Rightarrow \left|\begin{bmatrix} \lambda-1 & 1 & -1 \\ 0 & \lambda-1 & -1 \\ 0 & 0 & \lambda-1 \end{bmatrix}\right|$$

$$(\lambda-1)\left[(\lambda-1)^2\right] - 1(0) - 1(0)$$

$$(\lambda-1)^3 = 0$$

$$\lambda = 1, 1, 1$$

$$(A-1)^3 = 0$$

$$A^3 - 1 - 3A(A-1) = 0$$

$$A^3 - 3A^2 + 3A - I = 0$$

$$f(A) = A^K$$

$$f(A) = \gamma(A)$$

$\gamma(A)$ degree less than 3

$$\alpha_0 A + \alpha_1 A + \alpha_2 A^2$$

$$f(\lambda) = \gamma(\lambda) = \alpha_0 + \alpha_1 + \alpha_2 = 1^K$$

$$\Rightarrow \alpha_0 + \alpha_1 + \alpha_2 = 1$$

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$\therefore$ For any $\alpha_0 + \alpha_1 + \alpha_2 = 1$

$$A^k = \alpha_0 + \alpha_1 A + \alpha_2 A^2$$

(5) Given

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -27 & 54 & -36 & 10 \end{bmatrix}$$

$$(\lambda I - A) = \lambda^4 - 10\lambda^3 + 36\lambda^2 - 54\lambda + 27$$

$$(\lambda - 1)(\lambda - 3)^2$$

$$f(A) = \alpha_0 I + \alpha_1 A$$

$$e^{At} = f(\lambda) = \alpha_0 + \alpha_1 A$$

$$\Rightarrow e^{t} = \alpha_0 + \alpha_1$$

$$\Rightarrow e^{3t} = \alpha_0 + 3\alpha_1$$

$$\Rightarrow -2\alpha_1 = e^t - e^{3t}$$

$$\alpha_1 = \frac{e^{3t} - e^t}{2}$$

$$\Rightarrow \alpha_0 = e^t - \frac{e^{3t} - e^t}{2}$$

$$\alpha_0 = \frac{e^t - e^{3t}}{2}$$

$$\Rightarrow e^{At} = \left(\frac{e^t - e^{3t}}{2}\right) + \left(\frac{e^{3t} - e^t}{2}\right) \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -27 & 54 & -36 & 10 \end{bmatrix}$$

(2) We Given :-

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$e^{JH} \Rightarrow$

For any nxn matrix

$$e^{A} = \sum_{n=0}^{\infty} \frac{A^{n}}{n!} = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$e^{JH} = \sum_{n=0}^{\infty} \frac{(JH)^n}{n!} = I + JH + \frac{(JH)^2}{2!} + \frac{(JH)^3}{3!} + \cdots$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = I + JH - \frac{H^2}{2} - \frac{JH^3}{6} + \cdots$$

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