数学.

1 重. 20分

(1) $\vec{e_1} = \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}, \vec{e_2} = \begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}, \cdots \vec{e_5} = \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ ととって.

写像後の関係を確かめる.

$\begin{pmatrix} f(\vec{e_1}) & f(\vec{e_2}) & \cdots & f(\vec{e_5}) \end{pmatrix} = A$ だから. Aの各列の一次関係を調べることで. $f(\vec{e_1}), \cdots, f(\vec{e_5})$ の1次関係を調べられる

$$A = \begin{pmatrix} -2 & -5 & -1 & 8 & -3 \\ 3 & 3 & -3 & 1 & -8 \\ 1 & 3 & 1 & -2 & -4 \\ -1 & 3 & 5 & -1 & a \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 1 & 4 & -11 \\ 0 & -6 & -6 & 7 & 4 \\ 1 & 3 & 1 & -2 & -4 \\ 0 & 6 & 6 & -3 & a-4 \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 1 & 4 & -11 \\ 0 & 0 & 0 & 31 & -62 \\ 1 & 3 & 1 & -2 & -4 \\ 0 & 0 & 0 & -27 & a+62 \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -2 \\ 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & a+8 \end{pmatrix}$$

よって. $f(\vec{e_1}), f(\vec{e_2}), f(\vec{e_4})$ は線型独立で. $a=-8$ のとき. $f(\vec{e_5})$ は線型従属となるので. $\mathrm{Im} f = \mathbb{R}^3 \neq \mathbb{R}^4$

従って. $a=-8$.

(2)
$$\begin{pmatrix} -2 & -5 & -1 & 8 & -3 \\ 3 & 3 & -3 & 1 & -8 \\ 1 & 3 & 1 & -2 & -4 \\ -1 & 3 & 5 & -1 & -8 \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -2 \\ 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{array}{c} \begin{matrix} x & y & z & w & u \end{matrix} \\ \begin{pmatrix} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

よって.
$x = 2z - u$
$y = -z + 3u$
$w = 2u$.

従って. $\begin{pmatrix}x\\y\\z\\w\\u\end{pmatrix} = a\begin{pmatrix}2\\-1\\1\\0\\0\end{pmatrix} + b\begin{pmatrix}-1\\3\\0\\2\\1\end{pmatrix}$. $(a, b$ は任意$)$.

従って. $\dim(\ker f) = 2$. 基底は $\left\{ \begin{pmatrix}2\\-1\\1\\0\\0\end{pmatrix}, \begin{pmatrix}-1\\3\\0\\2\\1\end{pmatrix} \right\}$ がとれる.

これについては. $5-3=2$ で出る.

(3). $v = \begin{pmatrix}-8\\16\\7\\b\end{pmatrix} = a\overset{f(\vec{e_1})}{\begin{pmatrix}-2\\3\\1\\-1\end{pmatrix}} + b\overset{f(\vec{e_2})}{\begin{pmatrix}-5\\3\\3\\3\end{pmatrix}} + c\overset{f(\vec{e_4})}{\begin{pmatrix}8\\1\\-2\\-1\end{pmatrix}}$ とおく.

$$\left(\begin{array}{ccc|c} -2 & -5 & 8 & -8 \\ 3 & 3 & 1 & 16 \\ 1 & 3 & -2 & 7 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & 1 & 4 & 6 \\ 0 & -6 & 7 & -5 \\ 1 & 3 & -2 & 7 \end{array}\right) \to \left(\begin{array}{cccc} 0 & 1 & 4 & 6 \\ 0 & 0 & 31 & 31 \\ 1 & 3 & -2 & 7 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 3 \end{array}\right).$$

従って. $b = \underset{-3}{3\times(-1)} + \underset{6}{2\times 3} + \underset{-1}{1\times(-1)} = 2$. $\therefore b = 2$.

$\lambda = 1 \pm \sqrt{1-2}$

2. 普 13分.

(1) $|A-\lambda E| = \begin{vmatrix} 2-\lambda & -2 & 2 \\ 0 & -2-\lambda & 4 \\ 1 & -3 & 4-\lambda \end{vmatrix} = \begin{vmatrix} 2-\lambda & -2 & 2 \\ 2-\lambda & -2-\lambda & 4 \\ 2-\lambda & -3 & 4-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1 & -2 & 2 \\ 1 & -2-\lambda & 4 \\ 1 & -3 & 4-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1 & -2 & 2 \\ 0 & -\lambda & 2 \\ 0 & -1 & 2-\lambda \end{vmatrix}$

$= (2-\lambda)\{\lambda(\lambda-2)+2\} = (2-\lambda)(\lambda^2-2\lambda+2) = 0.$

実数解は. $\lambda = 2$.

$A-2E = \begin{pmatrix} 0 & -2 & 2 \\ 0 & -4 & 4 \\ 1 & -3 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ より. $\lambda=2$の固有ベクトルは. $a\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (a:任意).

(2) $A\vec{v} = \begin{pmatrix} 2 & -2 & 2 \\ 0 & -2 & 4 \\ 1 & -3 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2-4+4 \\ -4+8 \\ 1-6+8 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}.$

$A^2\vec{v} = \begin{pmatrix} 2 & -2 & 2 \\ 0 & -2 & 4 \\ 1 & -3 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 4-8+6 \\ -8+12 \\ 2-12+12 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}.$

$(\vec{v}\ \ A\vec{v}\ \ A^2\vec{v}) = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 2 & 3 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & -1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

行基本変形で各列の1次関係は変わらない.

従って. $A^2\vec{v}$が$\vec{v}$と$A\vec{v}$の線形結合で表せるので. 1次独立でない.

(3) $A^3\vec{v} = \begin{pmatrix} 2 & -2 & 2 \\ 0 & -2 & 4 \\ 1 & -3 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4-8+4 \\ -8+8 \\ 2-12+8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$

$A^4\vec{v} = \begin{pmatrix} 2 & -2 & 2 \\ 0 & -2 & 4 \\ 1 & -3 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \\ -8 \end{pmatrix}$

$(\vec{v}\ \ A\vec{v}\ \ A^3\vec{v}\ \ A^4\vec{v}) = \begin{pmatrix} 1 & 2 & 0 & -4 \\ 2 & 4 & 0 & -8 \\ 2 & 3 & -2 & -8 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -4 & -4 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

行基本変形で各列の1次関係は変化しないので.

$A^3\vec{v} = -4\vec{v} + 2A\vec{v}$, $A^4\vec{v} = -4\vec{v}$

③ 17分.

(1) $f(x,y) = \frac{\pi}{4} - \mathrm{Tan}^{-1}\sqrt{x^2+y^2}$

$$f_x = \frac{\frac{x}{\sqrt{x^2+y^2}}}{1+x^2+y^2}$$

$$f_y = \frac{\frac{y}{\sqrt{x^2+y^2}}}{1+x^2+y^2}$$

(2). $f\left(\frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}\right) = \frac{\pi}{4} - \mathrm{Tan}^{-1}\sqrt{\frac{6}{4}+\frac{6}{4}} = \frac{\pi}{4} - \mathrm{Tan}^{-1}\sqrt{3} = \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$

$$f_x\left(\frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}\right) = \frac{\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{6}}{2}}{1+3} = \frac{\frac{\sqrt{2}}{2}}{4} = \frac{\sqrt{2}}{8}$$

$$f_y\left(\frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}\right) = -\frac{\sqrt{2}}{8}.$$

従って. 求める方程式は. $\frac{\sqrt{2}}{8}\left(x-\frac{\sqrt{6}}{2}\right) - \frac{\sqrt{2}}{8}\left(y+\frac{\sqrt{6}}{2}\right) + \left(z+\frac{\pi}{12}\right) = 0.$

$$\frac{\sqrt{2}}{8}x - \frac{\sqrt{3}}{8} - \frac{\sqrt{2}}{8}y - \frac{\sqrt{3}}{8} + z + \frac{\pi}{12} = 0.$$

$$\underline{\frac{\sqrt{2}}{8}x - \frac{\sqrt{2}}{8}y + z + \frac{\pi}{12} - \frac{\sqrt{3}}{4} = 0.}$$ //

(3) S と $z=0$ の交線は. $\frac{\pi}{4} - \mathrm{Tan}^{-1}\sqrt{x^2+y^2} = 0$ を満たす.

$$\sqrt{x^2+y^2} = 1 \qquad x^2+y^2 = 1.$$

領域 $D = \{(x,y) \mid x^2+y^2 \leqq 1\}$ とすると. $V = \iint_D f(x,y)\cdot dxdy$ で求められる.

$x = r\cos\theta$, $y = r\sin\theta$ とおくと. Dは $E = \{(r,\theta) \mid 0 \leqq r \leqq 1,\ 0 \leqq \theta \leqq 2\pi\}$ に移るので.

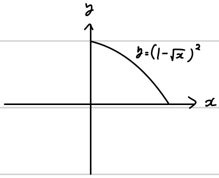
$$V = \int_0^{2\pi}\int_0^1 \left(\frac{\pi}{4}r - r\cdot\mathrm{Tan}^{-1}r\right)\cdot dr\, d\theta = 2\pi\int_0^1\left(\frac{\pi}{4}\cdot r - r\cdot\mathrm{Tan}^{-1}r\right)dr$$

$$\int_0^1 r\,\mathrm{Tan}^{-1}r = \left[\frac{1}{2}r^2\cdot\mathrm{Tan}^{-1}r\right]_0^1 - \frac{1}{2}\int_0^1\left(1-\frac{1}{1+r^2}\right)dr = \frac{\pi}{8} - \frac{1}{2}\left[r - \mathrm{Tan}^{-1}r\right]_0^1 = \frac{\pi}{8} - \frac{1}{2}\left(1-\frac{\pi}{4}\right)$$

また. $\int_0^1 \frac{\pi}{4}\cdot r\cdot dr = \frac{\pi}{4}\left[\frac{1}{2}r^2\right]_0^1 = \frac{\pi}{8}$ なので.

$$V = 2\pi\cdot\frac{1}{2}\left(1-\frac{\pi}{4}\right) = \pi\left(1-\frac{\pi}{4}\right).$$

$\boxed{4}$.



(1) $\iint_D xy \cdot dxdy$

$= \int_0^1 \int_0^{(1-\sqrt{x})^2} xy \cdot dy \cdot dx$

$= \int_0^1 x \left[\frac{1}{2}y^2\right]_0^{(1-\sqrt{x})^2} \cdot dx$

$= \int_0^1 \frac{x}{2} \cdot (1-\sqrt{x})^4 \cdot dx$

$\sqrt{x} = t$ とおくと. $\frac{1}{2\sqrt{x}} \cdot dx = dt$. $\therefore dx = 2t\,dt$. より.

$\int_0^1 \frac{t^2}{2} \cdot (1-t)^4 \cdot 2t \cdot dt = \int_0^1 t^3 \cdot (1-t)^4 \cdot dt.$

$\left(\begin{array}{l} {}_4C_0 = 1 \\ {}_4C_1 = 4 \\ {}_4C_2 = \frac{4\cdot 3}{2\cdot 1} = 6 \\ {}_4C_3 = 4 \\ {}_4C_4 = 1. \end{array} \right.$

$= \int_0^1 t^3 (1 - 4t + 6t^2 - 4t^3 + t^4) \cdot dt$

$= \int_0^1 (t^7 - 4t^6 + 6t^5 - 4t^4 + t^3) \cdot dt$

$= \left[\frac{1}{8}t^8 - \frac{4}{7}t^7 + t^6 - \frac{4}{5}t^5 + \frac{1}{4}t^4\right]_0^1$

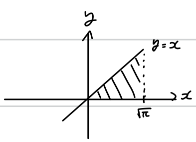
$= \frac{1}{8} - \frac{4}{7} + 1 - \frac{4}{5} + \frac{1}{4}$

$\begin{array}{r} 56 \\ \times 4 \\ \hline 224 \end{array}$

$5 \times 8 \times 7$

$= \frac{35 - 160 + 280 - 224 + 70}{280} = \frac{1}{280}$

こんな小っちゃいことある?



(2). $\iint_D \sin(x^2) \cdot dxdy.$

$= \int_0^{\sqrt{\pi}} \int_0^{x} \sin(x^2) \cdot dy \cdot dx$

$= \int_0^{\sqrt{\pi}} x \sin(x^2) \cdot dx$

$= \left[-\frac{1}{2}\cos(x^2)\right]_0^{\sqrt{\pi}}$

$= \frac{1}{2} + \frac{1}{2} = \underline{1}$

$\boxed{5}$.

(1) $z^4 = -1$

$z = re^{i\theta}$ とおくと.

$r^4 e^{i4\theta} = -1.$ $\quad r = 1.$

$4\theta = \pi.$

$\theta = \frac{\pi}{4},\ \frac{3}{4}\pi,\ \frac{5}{4}\pi,\ \frac{7}{4}\pi.$

従って.

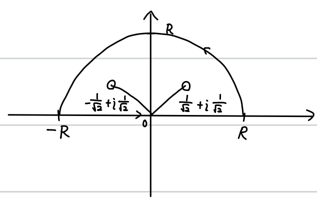
$$z = e^{i\frac{\pi}{4}},\ e^{i\frac{3}{4}\pi},\ e^{i\frac{5}{4}\pi},\ e^{i\frac{7}{4}\pi}$$

$$= \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$$

(2) $x = \sqrt{\tan\theta}$

$$\frac{dx}{d\theta} = \frac{\frac{1}{\cos^2\theta}}{2\sqrt{\tan\theta}} = \frac{1+\tan^2\theta}{2\sqrt{\tan\theta}} = \frac{1+x^4}{2x}$$

(3). $I = \int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\cdot d\theta$



$x = \sqrt{\tan\theta}$ とおくと. $d\theta = \frac{2x}{1+x^4}\cdot dx.$ $\quad \theta:\ 0 \to \frac{\pi}{2}$, $x:\ 0 \to \infty$ なので.

$$I = \int_0^\infty \frac{2x^2}{1+x^4}\cdot dx = \int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\,dx$$

ここで、図のような経路をとった積分 $\oint \frac{z^2}{z^4+1}\,dz = \oint \frac{z^2}{\left(z-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)\left(z+\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)\left(z+\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)\left(z-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)}\cdot dz$ を考える.

$$\mathrm{Res}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = \frac{\frac{1}{2}+i-\frac{1}{2}}{\sqrt{2}\cdot\sqrt{2}(1+i)\cdot\sqrt{2}i} = \frac{1}{2\sqrt{2}(1+i)} = \frac{1-i}{4\sqrt{2}}$$

$$\mathrm{Res}\left[-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = \frac{\frac{1}{2}-i-\frac{1}{2}}{-\sqrt{2}\cdot\sqrt{2}i\cdot\sqrt{2}(-1+i)} = \frac{1}{2\sqrt{2}(-1+i)} = \frac{-1-i}{4\sqrt{2}}.$$

従って. $\oint \frac{z^2}{z^4+1}\,dz = 2\pi i\cdot\frac{-2i}{4\sqrt{2}} = \frac{\pi}{\sqrt{2}}$

上半円周に沿った積分は. $R \to \infty$ で $\left|\frac{z\cdot z^2}{z^4+1}\right| \to 0$ より. 0 になるので.

$$I = \frac{\pi}{\sqrt{2}}$$

物理.

1

(1) $M \cdot \dfrac{d^2x}{dt^2} = -k \cdot x - \lambda \cdot \dfrac{dx}{dt}$

(2) 特性方程式 $Ms^2 + \lambda s + k = 0$ の解. $s = \dfrac{-\lambda \pm \sqrt{\lambda^2 - 4Mk}}{2M}$ より. $\lambda^2 > 4Mk$. 負を除いて. $\lambda > 2\sqrt{Mk}$.

(3) (1)の式を満たす一般解は.

$$x = e^{-\frac{\lambda}{2M}t}\left(A\cos\frac{\sqrt{\lambda^2-4Mk}}{2M}t + B\sin\frac{\sqrt{\lambda^2-4Mk}}{2M}t.\right) \quad (A, B : 任意定数).$$

条件より. $x(0) = A = x_0$, $\dfrac{dx}{dt}(0) = -\dfrac{\lambda}{2M}A + \dfrac{\sqrt{\lambda^2-4Mk}}{2M}B = 0$ だから.

$$A = x_0, \quad B = \frac{2M}{\sqrt{\lambda^2-4Mk}} \cdot \frac{\lambda}{2M} = \frac{\lambda}{\sqrt{\lambda^2-4Mk}}$$

従って. $x(t) = e^{-\frac{\lambda}{2M}t}\left(x_0\cos\dfrac{\sqrt{\lambda^2-4Mk}}{2M}t + \dfrac{\lambda}{\sqrt{\lambda^2-4Mk}}\sin\dfrac{\sqrt{\lambda^2-4Mk}}{2M}t\right).$

(4) 小物体Bの加速度が0のとき. 離れるため. (これを境に. $a_A < a_B$ だから)

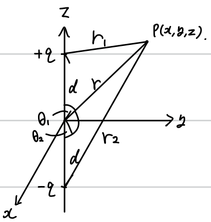
$$0 = -kx_a - \lambda \cdot V_a$$

$$kx_a = -\lambda V_a.$$

(5) ばねが伸びる途中でBは離れるため. $V_a > 0$.

$k > 0$, $\lambda > 0$ より. $x_a < 0$ 従って. x_aは負.

$\boxed{2}$. 自信ない.



(1) $$\phi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+d)^2}}\right).$$

(2) 余弦定理より.

$$r_1^2 = r^2+d^2-2rd\cdot\frac{z}{r} \fallingdotseq r^2\left(1-\frac{2z}{r^2}d\right).$$

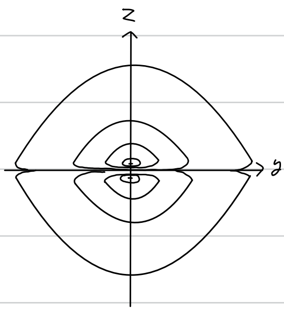
$$r_2^2 = r^2+d^2+2rd\cdot\frac{z}{r} \fallingdotseq r^2\left(1+\frac{2z}{r^2}d\right)$$ だから.

$$\frac{1}{r_1} = \frac{1}{r}\left(1-\frac{2z}{r^2}d\right)^{-\frac{1}{2}} \fallingdotseq \frac{1}{r}\left(1+\frac{z}{r^2}d\right).$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1+\frac{2z}{r^2}d\right)^{-\frac{1}{2}} \fallingdotseq \frac{1}{r}\left(1-\frac{z}{r^2}d\right).$$

$$\phi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \fallingdotseq \frac{q}{4\pi\varepsilon_0}\cdot 2\cdot\frac{z}{r^3}d = \underline{\frac{q\cdot z\cdot d}{2\pi\varepsilon_0\cdot r^3}}$$ //

(3).



(4) $\vec{E} = -\nabla\phi$

$$\frac{\partial}{\partial x}\left(\frac{q\cdot z\cdot d}{2\pi\varepsilon_0\cdot r^3}\right) = \frac{q\cdot z\cdot d}{2\pi\varepsilon_0}\cdot\left(-\frac{3}{r^4}\right)\cdot\frac{x}{r} = -\frac{3qzd\cdot x}{2\pi\varepsilon_0 r^5},$$ $\frac{\partial\phi}{\partial y}$ は同様.

$$\frac{\partial}{\partial z}\left(\frac{qdz}{2\pi\varepsilon_0\cdot r^3}\right) = \frac{qd}{2\pi\varepsilon_0}\left\{\frac{1}{r^3}+z\cdot\left(-\frac{3}{r^4}\right)\cdot\frac{z}{r}\right\} = \frac{qd}{2\pi\varepsilon_0}\frac{r^2-3z^2}{r^5}$$

従って.

$$\vec{E} = \left(\frac{3q\cdot z\cdot d\cdot x}{2\pi\varepsilon_0\cdot r^5}, \frac{3q\cdot z\cdot d\cdot y}{2\pi\varepsilon_0\cdot r^5}, \frac{qd\cdot(3z^2-r^2)}{2\pi\varepsilon_0 r^5}\right)$$

3.

ファンデルワールスの状態方程式 : $\left(p+\frac{a}{V^2}\right)(V-b) = 1\cdot RT$ …⊛. (1mol.)

(1) 気体分子間に働く引力 と 気体分子の持つ体積 を 考慮に入れている.

(2). ⊛を pについて整理すると. $p = \frac{RT}{V-b} - \frac{a}{V^2}$

$\left(\frac{\partial p}{\partial V}\right)_T = -\frac{RT}{(V-b)^2} + \frac{3a}{V^3} = 0$ より. $RT = \frac{3a}{V^3}(V-b)^2$

$\left(\frac{\partial^2 p}{\partial V^2}\right)_T = \frac{2RT}{(V-b)^3} - \frac{9a}{V^4} = 0$ $\rightarrow \frac{6a}{V^3(V-b)} = \frac{9a}{V^4}$

$6aV = 9a(V-b)$. $-3aV = -9ab$, $\therefore V_c = 3b$.

従って. $RT_c = \frac{3a}{27b^3}\cdot 4b^2 = \frac{4a}{9b}$ $\therefore T_c = \frac{4a}{9Rb}$

また. $P_c = \frac{\frac{4a}{9b}}{2b} - \frac{a}{9b^2} = \frac{2a}{9b^2} - \frac{a}{9b^2} = \frac{a}{9b^2}$ である.

(3). $T_c = \frac{4a}{9Rb} = \frac{a}{9b^2}\cdot\frac{4}{R}\cdot 3b\cdot\frac{1}{3} = \frac{4}{3R}\cdot P_c\cdot V_c$ である.

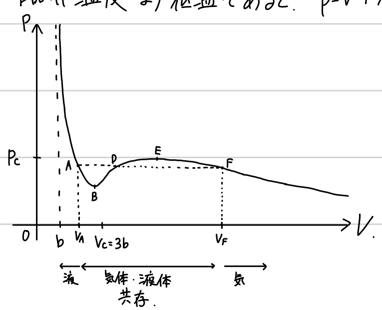
$J = N\cdot m$, $Pa = N/m^2$ だから. $J = Pa\cdot m^3$ であることに注意して.

$T_c = \frac{4}{3\times 8.3\,[J/(K\cdot mol)]}\times 2.3\times 10^5\,[Pa]\times 63\,[cm^3/mol]$

$= \frac{4}{3\times 8.3}\times 2.3\times 10^5\times 63\times 10^{-6}\ [K\cdot mol\cdot \underbrace{Pa\cdot m^3}/(J\cdot mol)] \rightarrow [K]$.

$\fallingdotseq 2.3\,[K]$ 電卓計算なので誤差アリ

(4) 臨界温度より低温であると. p-Vグラフで極小点をとる.



B, Eをそれぞれ極小, 極大点とし.
面積ABD, DEF が等しくなるような
点A, D, Fをとった時.
点A, Fでの体積 V_A から V_F が
気体と液体の共存区間である. (マクスウェルの等面積則).