数学　120分.

1. 普

(1) $\overrightarrow{AB}$, $\overrightarrow{AC}$を求め. 外積により. Hの法線ベクトルを求め. Hを求める.

$$\overrightarrow{AB}=\begin{pmatrix}-1\\0\\-1\end{pmatrix},\quad \overrightarrow{AC}=\begin{pmatrix}1\\-1\\-2\end{pmatrix}$$ より. $$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\-1&0&-1\\1&-1&-2\end{vmatrix}=\begin{pmatrix}-1\\-3\\1\end{pmatrix}=-\begin{pmatrix}1\\3\\-1\end{pmatrix}.$$

従って. Hの方程式は. $(x-2)+3(y-1)-(z-3)=0.$

$$\underline{x+3y-z-2=0.}$$

(2). ~~媒介~~ 変数tを用いて.

$$(2+t)+3(\overset{-1}{-1}+3t)-(2-t)-2=0.$$

$$11t=5.\qquad t=\frac{5}{11}$$

$$d=\sqrt{11}\times\frac{5}{11}=\underline{\frac{5}{\sqrt{11}}}$$

(3). $$\overrightarrow{OA}\times\overrightarrow{OB}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\2&1&3\\1&1&2\end{vmatrix}=\begin{pmatrix}-1\\-1\\1\end{pmatrix}.$$

(4) $$S=\frac{|\overrightarrow{OA}\times\overrightarrow{OB}|}{2}=\frac{\sqrt{3}}{2}$$

(5). $$V=\frac{1}{3}\times S\times\frac{1}{|\overrightarrow{OA}\times\overrightarrow{OB}|}\times\left|(\overrightarrow{OA}\times\overrightarrow{OB})\cdot\overrightarrow{OC}\right|.$$

$$=\frac{1}{6}\left|(\overrightarrow{OA}\times\overrightarrow{OB})\cdot\overrightarrow{OC}\right|.$$

$$=\frac{1}{6}\left|-3+0+1\right|=\frac{1}{3}$$

2. 重.

(1) $\begin{pmatrix} -1 & -2 & 2 \\ 2 & 4 & -4 \\ 1 & 2 & -2 \end{pmatrix}\begin{pmatrix} -2 & -2 & 2 \\ 2 & 3 & -4 \\ 1 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 2-4+2 & 2-6+4 & -2+8-6 \\ -4+8-4 & -4+12-8 & 4-16+12 \\ -2+4-2 & -2+6-4 & 2-8+6 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

(2). $|A-\lambda E| = \begin{vmatrix} 1-\lambda & -2 & 2 \\ 2 & 6-\lambda & -4 \\ 1 & 2 & -\lambda \end{vmatrix} = \begin{vmatrix} 2-\lambda & 0 & 2-\lambda \\ 2 & 6-\lambda & -4 \\ 1 & 2 & -\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1 & 0 & 1 \\ 2 & 6-\lambda & -4 \\ 1 & 2 & -\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 2 & 6-\lambda & -6 \\ 1 & 2 & -1-\lambda \end{vmatrix}$.

$= (2-\lambda)\begin{vmatrix} 6-\lambda & -6 \\ 2 & -1-\lambda \end{vmatrix} = (2-\lambda)\{(\lambda-6)(\lambda+1)+12\} = (2-\lambda)(\lambda^2-5\lambda+6) = (2-\lambda)(\lambda-2)(\lambda-3)$.

$|A-\lambda E| = 0$ を解いて. 固有値 λ は. $\underline{\lambda = 2\text{(重解)}, 3}$ //

(3)

$A-2E = \begin{pmatrix} -1 & -2 & 2 \\ 2 & 4 & -4 \\ 1 & 2 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ より. $\lambda = 2$ の. 固有ベクトルは. $a\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ (a, b は任意).

$A-3E = \begin{pmatrix} -2 & -2 & 2 \\ 2 & 3 & -4 \\ 1 & 2 & -3 \end{pmatrix} \to \begin{pmatrix} 0 & 1 & -2 \\ 0 & -1 & 2 \\ 1 & 2 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ より.

$\lambda = 3$ の 固有ベクトルは. $c\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ (c は任意).

従って. $P = \begin{pmatrix} 2 & -2 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ ととれば $P^{-1}A^mP = \begin{pmatrix} 2^m & 0 & 0 \\ 0 & 2^m & 0 \\ 0 & 0 & 3^m \end{pmatrix}$.

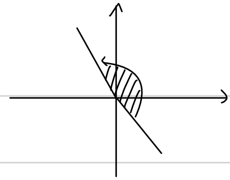
$A^m = P\begin{pmatrix} 2^m & 0 & 0 \\ 0 & 2^m & 0 \\ 0 & 0 & 3^m \end{pmatrix}P^{-1}$.

P^{-1} を求めると. $\left(\begin{array}{ccc|ccc} 2 & -2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 0 & -2 & -3 & 1 & 0 & -2 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 2 & -2 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 2 & -2 \\ 0 & 1 & 0 & -2 & -3 & 4 \\ 1 & 0 & 0 & -1 & -2 & 3 \end{array}\right)$.

$P^{-1} = \begin{pmatrix} -1 & -2 & 3 \\ -2 & -3 & 4 \\ 1 & 2 & -2 \end{pmatrix}$ だから.

$A^m = \begin{pmatrix} 2 & -2 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2^m & 0 & 0 \\ 0 & 2^m & 0 \\ 0 & 0 & 3^m \end{pmatrix}\begin{pmatrix} -1 & -2 & 3 \\ -2 & -3 & 4 \\ 1 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 2\cdot 2^m & -2\cdot 2^m & -3^m \\ 0 & 2^m & 2\cdot 3^m \\ 2^m & 0 & 3^m \end{pmatrix}\begin{pmatrix} -1 & -2 & 3 \\ -2 & -3 & 4 \\ 1 & 2 & -2 \end{pmatrix}$

$= \begin{pmatrix} -2\cdot 2^m+4\cdot 2^m-3^m & -4\cdot 2^m+6\cdot 2^m-2\cdot 3^m & 6\cdot 2^m-8\cdot 2^m+2\cdot 3^m \\ -2\cdot 2^m+2\cdot 3^m & -3\cdot 2^m+4\cdot 3^m & 4\cdot 2^m-4\cdot 3^m \\ -2^m+3^m & -2\cdot 2^m+2\cdot 3^m & 3\cdot 2^m-2\cdot 3^m \end{pmatrix} = \underline{2^m\begin{pmatrix} 2 & 2 & -2 \\ -2 & -3 & 4 \\ -1 & -2 & 3 \end{pmatrix} + 3^m\begin{pmatrix} -1 & -2 & 2 \\ 2 & 4 & -4 \\ 1 & 2 & -2 \end{pmatrix}}$ //



3. 普. $-\frac{\pi}{3} < x, y < \frac{2}{3}\pi$.

(1) $f_x = 4\sin x\cos x - 2\cos x\sin y. = 2\cos x(2\sin x - \sin y)$

$f_y = -2\sin x\cos y - 2\sin y\cos y. = -2\cos y(\sin x + \sin y).$

(2). $f_x = 0$ のとき. $\cos x = 0$ または. $\sin y = 2\sin x$.

(i) $\cos x = 0$ つまり. $x = \frac{\pi}{2}$ のとき.

$f_y = -2\cos y(1 + \sin y) = 0.$ $y = \frac{\pi}{2}$

(ii) $\sin y = 2\sin x$ のとき.

$f_y = -2\cos y \cdot 3\sin x.$ $\left(y = \frac{\pi}{2}, x = \frac{\pi}{3}\right)$ $(x = 0, y = 0).$

従って. $(a, b) = (0, 0), \left(\frac{\pi}{3}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

(3) $f_{xx} = 4\cos^2 x - 4\sin^2 x + 2\sin x\sin y.$

$f_{yy} = 2\sin x\sin y - 2\cos^2 y + 2\sin^2 y.$

$f_{xy} = -2\cos x\cos y.$

より. ヘッシアン $H(x, y) = 4(2\cos^2 x - 2\sin^2 x + \sin x\sin y)(\sin x\sin y - \cos^2 y + \sin^2 y) - 4(\cos x \cdot \cos y)^2$

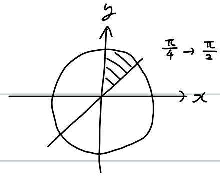
$H(0, 0) = 4 \cdot 2 \cdot (-1) - 4 < 0$ より. $(0, 0)$ で極値をとらない.

$H\left(\frac{\pi}{3}, \frac{\pi}{2}\right) = 4\left(\frac{3}{2} - \frac{1}{2} + \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2} + 1\right) - 0 > 0.$ $f_{xx}\left(\frac{\pi}{3}, \frac{\pi}{2}\right) > 0$ より.

$\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ で極小値. $\frac{3}{2} - \sqrt{3} - 1 = -\frac{1}{2} - \sqrt{3}$ をとる.

$H\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = 4(-2+1)(1+1) - 0 < 0$ より. $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ で極値はとらない.

④. 普 : 17分



(1) $\iint_D x^2 y \cdot dxdy$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^1 r^4 \cos^2\theta \cdot \sin\theta \cdot dr \cdot d\theta$

$= \frac{1}{5} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta \cdot \sin\theta \cdot d\theta$

$= \frac{1}{5} \left[-\frac{1}{3} \cos^3\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \frac{1}{15} \cdot \left(\frac{1}{\sqrt{2}} \right)^3 = \underline{\frac{1}{30\sqrt{2}}}$ //

(2). $\iint_D xy \cdot \sin(xy) \cdot dxdy = \int_1^{\frac{\pi}{2}} \int_{-\frac{\pi}{2x}}^{\frac{\pi}{2x}} xy \cdot \sin(xy) \cdot dy \cdot dx.$

$\int xy \cdot \sin(xy) \cdot dy = y \cdot (-\cos(xy)) + \int \cos(xy) \cdot dy.$

$= -y\cos(xy) + \frac{1}{x} \sin(xy) + C.$ より.

$\int_1^{\frac{\pi}{2}} \left[-y\cos(xy) + \frac{1}{x} \sin(xy) \right]_{y=-\frac{\pi}{2x}}^{y=\frac{\pi}{2x}} \cdot dx = \int_1^{\frac{\pi}{2}} \left(\frac{1}{x} + \frac{1}{x} \right) \cdot dx.$

$= 2 \left[\log|x| \right]_1^{\frac{\pi}{2}} = \underline{2\log\left(\frac{\pi}{2}\right)}$ //

(II). $y'' + 2y' + y = \sin 2x. \cdots$ ⊛

$y'' + 2y' + y = 0$ の一般解は $y = Ae^{-x} + Bxe^{-x}$ (A,B: 任意定数)

⊛の1つの解を. $y = a\cos 2x + b\sin 2x$ とおいて.

$y' = 2b\cos 2x - 2a\sin 2x$, $y'' = -4a\cos 2x - 4b\sin 2x$ を ⊛ に代入して.

$(-4a + 4b + a)\cos 2x + (-4b - 4a + b)\sin 2x = \sin 2x.$

$$\begin{cases} -3a + 4b = 0. \\ -4a - 3b = 1. \end{cases} \quad \begin{pmatrix} -3 & 4 & 0 \\ -4 & -3 & 1 \end{pmatrix} \to \begin{pmatrix} 12 & -16 & 0 \\ -12 & -9 & 3 \end{pmatrix} \to \begin{pmatrix} 12 & -16 & 0 \\ 0 & -25 & 3 \end{pmatrix} \to \begin{pmatrix} 12 & 0 & -\frac{4}{25} \\ 0 & 1 & -\frac{3}{25} \end{pmatrix}$$

㊍の1つの解として $y=-\frac{4}{25}\cos 2x-\frac{3}{25}\sin 2x$ がとれるので、

㊍の一般解は $\underline{y=Ae^{-x}+Bxe^{-x}-\frac{4}{25}\cos 2x-\frac{3}{25}\sin 2x}$.

5. 普 : 15分.

(1) $\cos\theta=\frac{e^{i\theta}+e^{-i\theta}}{2}=\frac{z+\frac{1}{z}}{2}=\frac{z^2+1}{2z}$

(2) $f(z)=\frac{z}{(z^2+4z+1)^2}=\frac{z}{(z+2-\sqrt{3})^2(z+2+\sqrt{3})^2}$

$z=-2+\sqrt{3}$ で $\frac{z}{(z+2+\sqrt{3})^2}$ は正則なので、$z=-2+\sqrt{3}$ は2位の極。

$z=-2-\sqrt{3}$ で $\frac{z}{(z+2-\sqrt{3})^2}$ は正則なので、$z=-2-\sqrt{3}$ は2位の極。

$|-2+\sqrt{3}|<1$ より、$\mathrm{Res}[-2+\sqrt{3}]=\frac{-2+\sqrt{3}}{12}$

(3). $I=\int_0^{2\pi}\frac{d\theta}{(2+\cos\theta)^2}$

$z=e^{i\theta}$ とおくと、$\cos\theta=\frac{z^2+1}{2z}$, $dz=i\,e^{i\theta}\cdot d\theta$ より $d\theta=-\frac{i}{z}dz$.

積分経路は $|z|=1$ に変わるので、

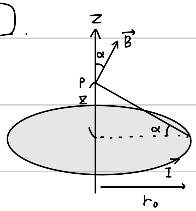
$$I=\oint\frac{-\frac{i}{z}dz}{\left(\frac{z^2+4z+1}{2z}\right)^2}=\oint\frac{4z^2}{(z^2+4z+1)^2}\cdot\left(-\frac{i}{z}\right)dz=-4i\cdot\oint\frac{z}{(z^2+4z+1)^2}dz=-4i\cdot 2\pi i\cdot\frac{-2+\sqrt{3}}{12}$$

$$=2\pi\cdot\frac{-2+\sqrt{3}}{3}$$

$$\underline{I=-\frac{2\pi(2-\sqrt{3})}{3}}$$

物理 ： 90分.

Ⅱ.



(1) $dB(0) = \dfrac{\mu_0}{4\pi} \cdot \dfrac{I \cdot ds}{r_0{}^2}$

(2) $B(0) = \dfrac{\mu_0}{4\pi} \cdot \dfrac{I \times 2\pi r_0}{r_0{}^2} = \dfrac{\mu_0 I}{2\pi r_0}$

(3) $\cos\alpha = \dfrac{r_0}{\sqrt{r_0{}^2 + z^2}}$

(4) $dB_z(z) = \dfrac{\mu_0}{4\pi} \cdot \dfrac{I \cdot ds}{r_0{}^2 + z^2} \cdot \dfrac{r_0}{\sqrt{r_0{}^2 + z^2}}$

$B(z) = \dfrac{\mu_0}{4\pi} \cdot \dfrac{I \cdot 2\pi r_0 \cdot r_0}{(r_0{}^2 + z^2)^{\frac{3}{2}}} = \dfrac{\mu_0 I \cdot r_0{}^2}{2\pi (r_0{}^2 + z^2)^{\frac{3}{2}}}$

(5) $50 \times 10^{-6} = \dfrac{\cancel{4\pi} \times 10^{-7}}{\cancel{2\pi}} \cdot I \cdot \dfrac{(3600 \times 10^3)^2}{\{(3600 \times 10^3)^2 + (7200 \times 10^3)^2\}^{\frac{3}{2}}}$

$I = \dfrac{1}{500} \times \dfrac{(5 \times 3600^2 \times 10^6)^{\frac{3}{2}}}{3600^2 \times 10^6}$

$= \dfrac{1}{100} \times \overset{2.236}{\sqrt{5}} \times 3600 \times 10^3$

$\fallingdotseq 36000 \times 2.236$

$$
\begin{array}{r}
36 \\
\times 2236 \\
\hline
15416 \\
6708 \\
\hline
82496
\end{array}
$$

$I \fallingdotseq 8.2 \times 10^4$ [A]. //

2

(1) $m x^2 = -kx$

(2) 方程式の一般解は. $x = a\cos\sqrt{\frac{k}{m}}t + b\sin\sqrt{\frac{k}{m}}t$. ($a, b$：任意定数)

微分して. $\frac{dx}{dt} = \sqrt{\frac{k}{m}}\left(-a\sin\sqrt{\frac{k}{m}}t + b\cos\sqrt{\frac{k}{m}}t\right)$.

条件 $x(0) = A$, $\frac{dx}{dt}(0) = 0$ より. $a = A$, $b = 0$ より.

$x = A\cos\sqrt{\frac{k}{m}}t$.

(3). $m\frac{d^2x}{dt^2} = -\lambda\cdot\frac{dx}{dt} - kx$

(4) $m\cdot\frac{d^2x}{dt^2} + \lambda\frac{dx}{dt} + kx = 0$ を特性方程式 $ms^2 + \lambda s + k = 0$ を解いて.

$s = \frac{-\lambda \pm \sqrt{\lambda^2 - 4mk}}{2}$ ここで. $k > \frac{\lambda^2}{4m}$ より. $\lambda^2 - 4mk < 0$ であるから.

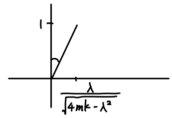
方程式の一般解は. $x = e^{-\frac{\lambda}{2}t}\left(a\cos\frac{\sqrt{4mk-\lambda^2}}{2}t + b\sin\frac{\sqrt{4mk-\lambda^2}}{2}t\right)$ (a, b：任意定数)

微分して. $\frac{dx}{dt} = -\frac{\lambda}{2}e^{-\frac{\lambda}{2}}\left(a\cos\frac{\sqrt{4mk-\lambda^2}}{2}t + b\sin\frac{\sqrt{4mk-\lambda^2}}{2}t\right) + e^{-\frac{\lambda}{2}t}\cdot\frac{\sqrt{4mk-\lambda^2}}{2}\cdot\left(-a\sin\frac{\sqrt{4mk-\lambda^2}}{2} + b\cos\frac{\sqrt{4mk-\lambda^2}}{2}\right)$

$= e^{-\frac{\lambda}{2}}\left\{\left(-\frac{\lambda}{2}a + \frac{\sqrt{4mk-\lambda^2}}{2}b\right)\cos\frac{\sqrt{4mk-\lambda^2}}{2}t + \left(-\frac{\lambda}{2}b - \frac{\sqrt{4mk-\lambda^2}}{2}a\right)\sin\frac{\sqrt{4mk-\lambda^2}}{2}t\right\}$

条件. $x(0) = A$, $\frac{dx}{dt}(0) = 0$ より. $a = A$, $b = \frac{2}{\sqrt{4mk-\lambda^2}}\cdot\frac{\lambda}{2}A = \frac{\lambda}{\sqrt{4mk-\lambda^2}}A$. だから.

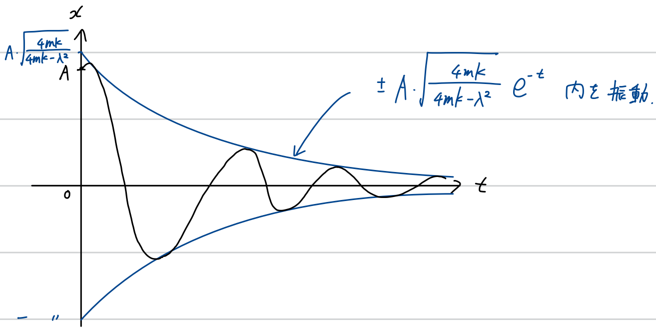
$x = A\cdot e^{-t}\left(\cos\frac{\sqrt{4mk-\lambda^2}}{2}t + \frac{\lambda}{\sqrt{4mk-\lambda^2}}\cdot\sin\frac{\sqrt{4mk-\lambda^2}}{2}t\right)$.



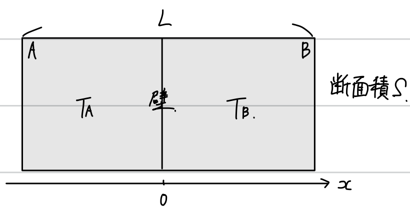
(5)

$$x = A\cdot\sqrt{1+\frac{\lambda^2}{4mk-\lambda^2}}\cdot e^{-t}\cos\left(\frac{\sqrt{4mk-\lambda^2}}{2}t - \tan^{-1}\frac{\lambda}{\sqrt{4mk-\lambda^2}}\right).$$

$$= A\sqrt{\frac{4mk}{4mk-\lambda^2}}\cdot e^{-t}\cos\left(\frac{\sqrt{4mk-\lambda^2}}{2}t-\varphi\right)$$



3.



(1) 気体A, Bの物質量を n_A, n_B とすると.

A, Bの圧力, 体積共に等しいので.

$$(pV=)\ n_A R\cdot T_A = n_B\cdot R\cdot T_B. \quad \therefore \frac{n_A}{n_B}=\frac{T_B}{T_A}$$

モル比は, 質量比に等しいので. $m_A : m_B = T_B : T_A$.

(2) 気体の定積モル熱容量を C_V とおくと.

系は断熱材に覆われているため. 系に熱の出入りがなく, 内部エネルギーは変化しないので.

$$dU_A + dU_B = 0. \rightarrow n_A\cdot C_V(T-T_A) + n_B C_V(T-T_B) = 0.$$

$$n_A T - n_A T_A = -n_B T + n_B T_B.$$

$$(n_A + n_B)T = n_A T_A + n_B T_B.$$

$$T = \frac{n_A T_A + n_B T_B}{n_A + n_B}$$ ここで.(1)の $n_A = \frac{T_B}{T_A} n_B$ を用いて.

$$= \frac{2 n_B T_B}{\frac{T_A + T_B}{T_A} n_B} = \frac{2 n_B T_A T_B}{(T_A + T_B) n_B} = \frac{2 T_A \cdot T_B}{T_A + T_B}$$

(3) $pV_A = n_A R T$ 　状態方程式

$$\underline{+) \; pV_B = n_B R T}$$

$$p \cdot LS = (n_A + n_B) R \cdot \frac{2 T_A \cdot T_B}{T_A + T_B} = \frac{2}{T_A + T_B}\left(\underline{n_A R T_A} \cdot T_B + \underline{n_B R T_B} \cdot T_A\right) = \frac{2}{T_A + T_B}\left(P_0 \cdot \frac{LS}{2} T_B + P_0 \cdot \frac{LS}{2} \cdot T_A\right)$$

$$= P_0 \cdot LS.$$

$$\therefore \; p = P_0.$$

(4) $V_A = S \times \left(\frac{L}{2} + x\right).$

状態方程式. $P_0\left(\frac{SL}{2} + Sx\right) = n_A R T$

$$\underline{-) \; P_0 \cdot \frac{SL}{2} = n_A R T_A.}$$

$$P_0 S \cdot x = n_A R \cdot (T - T_A) = n_A R \cdot \frac{2 T_A T_B - T_A^2 - T_A T_B}{T_A + T_B} = n_A R \cdot \frac{T_A (T_B - T_A)}{T_A + T_B} = n_A R T_A \cdot \frac{T_B - T_A}{T_A + T_B}$$

$$= P_0 \cdot \frac{LS}{2} \cdot \frac{T_B - T_A}{T_A + T_B}$$

$$\underline{x = \frac{L(T_B - T_A)}{2(T_A + T_B)}} /\!/$$