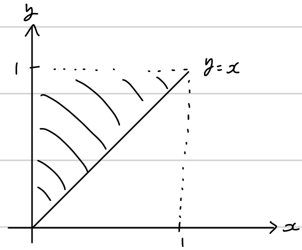
$\boxed{1}$

(1) $\iint_D \sqrt{y^2-x^2}\ dxdy \qquad D=\{(x,y) \mid 0 \leqq x \leqq y \leqq 1\}$



$= \int_0^1 \int_0^y \sqrt{y^2-x^2} \cdot dx \cdot dy$

$\int_0^y \sqrt{y^2-x^2} \cdot dx$ について.

$x = y\sin\theta$ とおくと. $dx = y\cos\theta \cdot d\theta$

$x: 0 \longrightarrow y$

$\theta: 0 \rightarrow \frac{\pi}{2}$ なので. $\int_0^{\frac{\pi}{2}} y^2\cos^2\theta \cdot d\theta = y^2\left[\frac{\theta}{2}+\frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}y^2$

$\int_0^1 \frac{\pi}{4}y^2 \cdot dy = \frac{\pi}{4}\cdot\left[\frac{1}{3}y^3\right] = \frac{\pi}{12}$

(2) $x+2y+z+e^{2z}-1 = 0$

(2-1)

x で偏微分して. $1+\frac{\partial z}{\partial x}+2\cdot\frac{\partial z}{\partial x}e^{2z}=0.$

$(1+2e^{2z})\frac{\partial z}{\partial x} = -1 \Rightarrow \frac{\partial z}{\partial x} = -\frac{1}{1+2e^{2z}}$

y で偏微分して. $\frac{\partial z}{\partial y} = -\frac{2}{1+2e^{2z}}$

$\frac{\partial^2 z}{\partial x^2} = \frac{1}{(1+2e^{2z})^2}\cdot 2\cdot 2e^{2z}\cdot\frac{\partial z}{\partial x}$

$= -\frac{4e^{2z}}{(1+2e^{2z})^3}$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{(1+2e^{2z})^2}\cdot 2\cdot 2e^{2z}\cdot\frac{\partial z}{\partial y}$

$= -\frac{8e^{2z}}{(1+2e^{2z})^3}$

$\frac{\partial^2 z}{\partial y^2} = \frac{2}{(1+2e^{2z})^2}\cdot 2\cdot 2e^{2z}\cdot\frac{\partial z}{\partial y}$

$= -\frac{16e^{2z}}{(1+2e^{2z})^3}$

(2-2)

法線ベクトルは $\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial z}{\partial z}\right) = \left(-\frac{1}{1+2e^{2z}}, -\frac{2}{1+2e^{2z}}, 1\right)$

$(x, y, z) = (-2, 1, 0)$ において $\left(-\frac{1}{3}, -\frac{2}{3}, 1\right)$ だから

求める平面は $-\frac{1}{3}(x+2) - \frac{2}{3}(y-1) + z = 0$

$$-x-2-2y+2+3z=0$$

$$\underline{x+2y-3z=0}$$

1変数

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$

(2-3)

2変数のテイラー展開は $(x, y) = (a, b)$ 周りで

$$f(x,y) = \sum_{i=0}^{n} \frac{1}{i!}\left\{(x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right\}^i f(a,b)$$ である。

これに従えば

$$f(x,y) = f(0,0) + \frac{\partial f(0,0)}{\partial x}x + \frac{\partial f(0,0)}{\partial y}y + \frac{1}{2!}\frac{\partial^2 f(0,0)}{\partial x^2}x^2 + \frac{1}{2!}\frac{\partial^2 f(0,0)}{\partial x \partial y}xy + \frac{1}{2!}\frac{\partial^2 f(0,0)}{\partial y^2}y^2$$

$f(0,0)$ を求める

増減表

$(0,0)$ で $z + e^{2z} = 1$ となる z を求めると、$z=0$ が唯一満たすので、$f(0,0) = 0$.

従って、$\frac{\partial f(0,0)}{\partial x} = -\frac{1}{3}$, $\frac{\partial f(0,0)}{\partial y} = -\frac{2}{3}$, $\frac{\partial^2 f(0,0)}{\partial x^2} = -\frac{4}{27}$

$\frac{\partial^2 f(0,0)}{\partial x \partial y} = -\frac{8}{27}$, $\frac{\partial^2 f(0,0)}{\partial y^2} = -\frac{16}{27}$

従って、$f(x,y) = -\frac{1}{3}x - \frac{2}{3}y - \frac{4}{27}x^2 - \frac{8}{27}xy - \frac{16}{27}y^2$