数学.　①, ②, ③, ④ : 1:43.

① 18分.

(1) $\vec{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = a\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + b\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + c\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ とおく.

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & x \\ 1 & 2 & 1 & y \\ 2 & 3 & 2 & z \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & x \\ 0 & 3 & 1 & -x+y \\ 0 & 5 & 2 & -2x+z \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & x \\ 0 & 15 & 5 & -5x+5y \\ 0 & 15 & 6 & -6x+3z \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & x \\ 0 & 3 & 1 & -x+y \\ 0 & 0 & 1 & -x-5y+3z \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & x \\ 0 & 1 & 0 & 2y-z \\ 0 & 0 & 1 & -x+5y+3z \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 0 & x+2y-z \\ 0 & 1 & 0 & 2y-z \\ 0 & 0 & 1 & -x+5y+3z \end{array}\right).$$

従って. $\underline{\vec{u} = (x+2y-z)\vec{v_1} + (2y-z)\vec{v_2} + (-x+5y+3z)\vec{v_3}}$ //

(2) (1)より, $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \vec{v_1} - \vec{v_3}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2\vec{v_1} + 2\vec{v_2} + 5\vec{v_3}$, $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -\vec{v_1} - \vec{v_2} + 3\vec{v_3}$ だから,

$f\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$, $f\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + 5\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 9 \end{pmatrix}$, $f\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -6 \\ 1 \end{pmatrix}$ である.

fの表現行列をAとおくと. $\left(f\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\ f\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\ f\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) = A$ であるから.

Aの各列の一次関係を調べる.

$$A = \begin{pmatrix} 0 & 9 & 1 \\ -1 & 1 & -6 \\ 0 & 9 & 1 \end{pmatrix} \to \begin{pmatrix} 0 & 9 & 1 \\ -1 & 1 & -6 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 9 & -9 & 54 \\ 0 & 9 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 9 & 0 & 55 \\ 0 & 9 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

行基本変形で各列の一次関係は変化しないから. $f\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $f\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ が線型独立.

従って. $\dim \mathrm{Im} f = 2$ で $\left\{\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 9 \\ 1 \\ 9 \end{pmatrix}\right\}$ を基底にとれる.

(3) $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ を満たす $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ を求める.

$A \to \begin{pmatrix} 9 & 0 & 55 \\ 0 & 9 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ より. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = a\begin{pmatrix} -55 \\ -1 \\ 9 \end{pmatrix}$ (aは任意).

従って. $\dim \mathrm{Ker} f = 1$ で $\left\{\begin{pmatrix} -55 \\ -1 \\ 9 \end{pmatrix}\right\}$ を基底にとれる.

$\boxed{2}$ 24分

(1) $|A-\lambda E| = \begin{vmatrix} 1-\lambda & 2 & 1 \\ -1 & 4-\lambda & 1 \\ 2 & -4 & -\lambda \end{vmatrix} = \begin{vmatrix} 2-\lambda & 2-\lambda & 2-\lambda \\ -1 & 4-\lambda & 1 \\ 2 & -4 & -\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ -1 & 4-\lambda & 1 \\ 2 & -4 & -\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ -1 & 5-\lambda & 2 \\ 2 & -6 & -2-\lambda \end{vmatrix} = (2-\lambda)\{(\lambda-5)(\lambda+2)+12\}$

$= (2-\lambda)(\lambda^2-3\lambda+2) = (2-\lambda)(\lambda-2)(\lambda-1)$ 　　 $\underline{\lambda = 2\ (重解),\ 1}$

(2) (i) $\lambda = 1$ について.

$A-E = \begin{pmatrix} 0 & 2 & 1 \\ -1 & 3 & 1 \\ 2 & -4 & -1 \end{pmatrix} \to \begin{pmatrix} 0 & 2 & 1 \\ -1 & 3 & 1 \\ 0 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -3 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

従って. $A\vec{x} = \vec{x}$ を満たす $\vec{x}$ は. $a\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ (aは任意) 　　 従って $\left\{\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\right\}$ を基底にとる.

(ii) $\lambda = 2$ について

$A-2E = \begin{pmatrix} -1 & 2 & 1 \\ -1 & 2 & 1 \\ 2 & -4 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

従って $A\vec{x} = 2\vec{x}$ を満たす $\vec{x}$ は $a\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ (a,bは任意) 　　 従って. $\left\{\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right\}$ を基底にとる.

(3) $\left(f(\vec{p_1})\ \ f(\vec{p_2})\ \ f(\vec{p_3})\right) = \left(\vec{p_1}\ \ \vec{p_2}\ \ \vec{p_3}\right) \cdot M$

$AB = BM$ 　　 $\vec{p_1}, \vec{p_2}, \vec{p_3}$ は線型独立だから. B^{-1} が存在する.

$B^{-1}AB = M.$

$B = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$ ととると. $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$

③　8分

$f(x,y) = xy \cdot e^{-x^2-y^2}$

(1) $f_x = y \cdot e^{-x^2-y^2} - 2x^2y \cdot e^{-x^2-y^2}$　,　$f_y = x \cdot e^{-x^2-y^2} - 2xy^2 \cdot e^{-x^2-y^2}$

(2) $f_x = 0$ より. $y(1-2x^2) = 0$. 従って. $y = 0$ または. $x = \pm\frac{1}{\sqrt{2}}$

$f_y = 0$ より. $x(1-2y^2) = 0$. 従って. $x = 0$ または. $y = \pm\frac{1}{\sqrt{2}}$.

よって. $(a,b) = (0,0), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(3) $f_{xx} = -2xy \cdot e^{-x^2-y^2} - 4xy \cdot e^{-x^2-y^2} + 4x^3y\, e^{-x^2-y^2} = (-6xy + 4x^3y)\, e^{-x^2-y^2}$

$f_{yy} = (-6xy + 4xy^3)\, e^{-x^2-y^2}$

$f_{xy} = (1-2x^2)\left(e^{-x^2-y^2} - 2y^2 e^{-x^2-y^2}\right) = (1-2x^2)(1-2y^2)\, e^{-x^2-y^2}$

ヘッシアン $H(x,y) = 4x^2y^2(2x^2-3)(2y^2-3)\, e^{-2(x^2+y^2)} - (1-2x^2)^2(1-2y^2)^2\, e^{-2(x^2+y^2)}$

$= \left\{4x^2y^2(2x^2-3)(2y^2-3) - (1-2x^2)^2(1-2y^2)\right\} e^{-2(x^2+y^2)}$

$H(0,0) = (0-1) \cdot 1 = -1 < 0$

$H\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left((-2)(-2) - 0\right) e^{-2} = 4e^{-2} > 0$,　$f_{xx}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = (-3+1)\, e^{-1} = -2e^{-1} < 0$

$H\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 4e^{-2} > 0$　,　$f_{xx}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = (3-1)\, e^{-1} = 2e^{-1} > 0$.

$H\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 4e^{-2} > 0$　,　$f_{xx}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = (3-1)\, e^{-1} = 2e^{-1} > 0$.

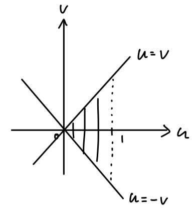
$H\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 4e^{-2} > 0$,　$f_{xx}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = (-3+1)\, e^{-1} = -2e^{-1} < 0$.　より.

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ で 極大値. $\frac{1}{2e}$

$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ で 極小値 $-\frac{1}{2e}$ をとる.

$\boxed{4}$ 53分.

(1) $\iint_D \frac{x+y}{1+(x-y)}\,dxdy$.　　D: $x \geqq 0,\ y \geqq 0,\ x+y \leqq 1$



$x+y=u,\ x-y=v$ とおくと.

$x=\frac{u+v}{2}$,　$y=\frac{u-v}{2}$ より.　$u+v \geqq 0$,　$u-v \geqq 0$,　$u \leqq 1$.
$v \geqq -u$　　$v \leqq u$

従って. Dは $E=\{(u,v) \mid 0 \leqq u \leqq 1,\ -u \leqq v \leqq u\}$ に移る.

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{2}$ より. $\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{2}$ だから.

$$\iint_D \frac{x+y}{1+(x-y)}\,dxdy = \frac{1}{2}\int_0^1\left(\int_{-u}^{u}\frac{u}{1+v}\,dv\right)du = \frac{1}{2}\int_0^1 u\left[\log|1+v|\right]_{v=-u}^{v=u}du$$

$$= \frac{1}{2}\int_0^1 u\cdot\log\frac{1+u}{1-u}\,du$$

$$\int u\cdot\log\frac{1+u}{1-u}\,du = \frac{1}{2}u^2\log\frac{1+u}{1-u} - \frac{1}{2}\int u^2\cdot\frac{1-u}{1+u}\cdot\frac{1-u+1+u}{(1-u)^2}\,du = \frac{1}{2}u^2\log\frac{1+u}{1-u} - \frac{1}{2}\int u^2\cdot\frac{2}{1-u^2}\,du$$

$$= \frac{1}{2}u^2\log\frac{1+u}{1-u} + \int\left(1+\frac{1}{u^2-1}\right)du = \frac{1}{2}u^2\log\frac{1+u}{1-u} + u + \frac{1}{2}\int\left(\frac{1}{u-1}-\frac{1}{u+1}\right)du$$

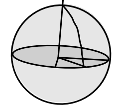
$$= \frac{1}{2}u^2\log\frac{1+u}{1-u} + u + \frac{1}{2}\log\frac{1-u}{1+u}$$

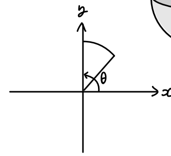
$$= u + \frac{1}{2}(u^2-1)\log\left(\frac{1+u}{1-u}\right).$$

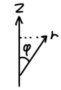
従って. $\frac{1}{2}\int_0^1 u\cdot\log\frac{1+u}{1-u}\,du = \frac{1}{2}\left[u+\frac{1}{2}(u^2-1)\log\frac{1+u}{1-u}\right]_0^1$

$$\lim_{u\to1}\frac{\log\frac{1+u}{1-u}}{\frac{1}{u^2-1}} = \lim_{u\to1}\frac{\frac{1-u}{1+u}\cdot\frac{-1-u-1+u}{(1-u)^2}}{-\frac{2u}{(u^2-1)^2}} = \lim_{u\to1}\frac{\frac{-2}{1-u^2}}{-\frac{2u}{(u^2-1)^2}} = \lim_{u\to1}\frac{-2\cdot(1-u^2)}{-2u} = 0.$$ だから.

$$\frac{1}{2}\left[u+\frac{1}{2}(u^2-1)\log\frac{1+u}{1-u}\right]_0^1 = \frac{1}{2}(1+0-0-0) = \underline{\frac{1}{2}}_{\,/\!/}$$







(2) $\displaystyle\iiint_E xyz\ dxdydz$　　$E:\ y \geqq x \geqq 0,\ z \geqq 0,\ x^2+y^2+z^2 \leqq 1$

$x = r\sin\varphi\cdot\cos\theta,\quad y = r\sin\varphi\cdot\sin\theta,\quad z = r\cos\varphi$　とおくと.

E は. $F = \left\{(r,\theta,\varphi)\ \middle|\ 0 \leqq r \leqq 1,\ \frac{\pi}{4} \leqq \theta \leqq \frac{\pi}{2},\ 0 \leqq \varphi \leqq \frac{\pi}{2}\right\}$ に移る.

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = \begin{vmatrix} \sin\varphi\cos\theta & \sin\varphi\sin\theta & \cos\varphi \\ -r\sin\varphi\sin\theta & r\sin\varphi\cos\theta & 0 \\ r\cos\varphi\cos\theta & r\cos\varphi\sin\theta & -r\sin\varphi \end{vmatrix} = r^2\sin\varphi \begin{vmatrix} \sin\varphi\cos\theta & \sin\varphi\sin\theta & \cos\varphi \\ -\sin\theta & \cos\theta & 0 \\ \cos\varphi\cos\theta & \cos\varphi\sin\theta & -\sin\varphi \end{vmatrix}$$

$$= r^2\sin\varphi\left(-\sin^2\varphi\underline{\cos^2\theta} - \sin^2\varphi\underline{\sin^2\theta} - \cos^2\varphi\underline{\sin^2\theta} - \cos^2\varphi\underline{\cos^2\theta}\right)$$

$$= r^2\sin\varphi(-1) = -r^2\sin\varphi. \qquad \therefore \left|\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}\right| = r^2\sin\varphi.$$

従って. $\displaystyle\iiint_E xyz\cdot dxdydz = \int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^1 r^3\sin^2\varphi\cos\varphi\cdot\cos\theta\cdot\sin\theta \times r^2\sin\varphi\ dr\,d\theta\,d\varphi.$

$$= \int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^1 r^5\sin^3\varphi\cdot\cos\varphi\cdot\cos\theta\cdot\sin\theta\ dr\,d\theta\,d\varphi$$

$$= \int_0^{\frac{\pi}{2}}\sin^3\varphi\cdot\cos\varphi\cdot d\varphi \times \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\sin\theta\cdot\cos\theta\cdot d\theta \times \int_0^1 r^5\cdot dr$$

$$= \left[\frac{1}{4}\sin^4\varphi\right]_0^{\frac{\pi}{2}} \times \left[\frac{1}{2}\sin^2\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \times \frac{1}{6}$$

$$= \frac{1}{4}\times\frac{1}{2}\left(1-\frac{1}{2}\right)\times\frac{1}{6}$$

$4\cdot2\cdot2\cdot6$

$$= \underline{\frac{1}{96}}\ /\!/$$

5　　15分.

(1) $z^4+1=0$

$z=re^{i\theta}$ とおくと.

$r=1$, $4\theta=\pi+2n\pi$,　　$\theta=\frac{\pi}{4}, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$

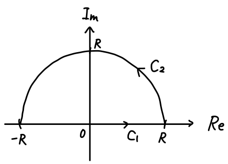
$\therefore z=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$,　$-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$,　$-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}$,　$\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}$.

(2) $\mathrm{Res}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)=\dfrac{1}{\sqrt{2}\cdot 2\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)\cdot\sqrt{2}i}=\dfrac{1}{4\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)}$

同様に. 他の極についても留数をとると.

$\mathrm{Res}(\alpha)=\dfrac{1}{4\alpha}\left(=\dfrac{\overline{\alpha}}{4}\right)$ である.

(3) $I=\int_0^\infty f(x)\,dx=\frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{x^4+1}\cdot dx$.



図のような経路 C_1+C_2 で $R\to\infty$ とした

$\oint_{C_1+C_2} f(z)\cdot dz$ を考える.

C_1+C_2 内に含まれる $f(z)$ の極は. $z=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$, $-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$ なので.

$\mathrm{Res}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)=\dfrac{1}{4\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)}=\dfrac{\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}}{4}$,　　$\mathrm{Res}\left(-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)=\dfrac{-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}}{4}$

従って $\oint_{C_1+C_2} f(z)\,dz=2\pi i\cdot\left(-\dfrac{i\cdot\sqrt{2}}{4}\right)=\dfrac{\pi}{\sqrt{2}}$.

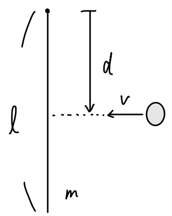
$\int_{C_2} f(z)\cdot dz$ は. $R\to\infty$ としたとき. $\left|z\times\frac{1}{z^4+1}\right|\to 0$ なので. $\int_{C_2} f(z)=0$.

$\oint_{C_1+C_2} f(z)\cdot dz=\int_{C_1} f(z)\cdot dz=\dfrac{\pi}{\sqrt{2}}$ であり.　$\int_{C_1} f(z)\cdot dz=\int_{-\infty}^{\infty} f(x)\cdot dx$ だから.

求める値は. $\underline{\dfrac{\pi}{2\sqrt{2}}}$ //

物理.

1.



(1) $I=\int_0^l \frac{m}{l}\cdot x^2\cdot dx = \frac{m}{l}\left[\frac{1}{3}x^3\right]_0^l = \frac{1}{3}ml^2$

(2) 角運動量が保存されるので.

$mvl = m\cdot l\cdot \omega\cdot l + I\omega$

$I=\frac{1}{3}ml^2$ より. $mvl = ml^2\omega + \frac{1}{3}ml^2\omega. = \frac{4}{3}ml^2\omega$

$\therefore\ \omega = \frac{3v}{4l}$

(3) $K = \frac{1}{2}m\cdot\left(\frac{3v}{4}\right)^2 + \frac{1}{2}I\cdot\left(\frac{3v}{4l}\right)^2 = \frac{9}{32}mv^2 + \frac{1}{6}ml^2\cdot\frac{9v^2}{16l^2} = mv^2\left(\frac{9}{32}+\frac{3}{32}\right) = \frac{3}{8}mv^2$

(4) 角運動量が保存されるので.

$mv\cdot d = m\cdot d\cdot\omega\cdot d + I\omega.$

$I=\frac{1}{3}ml^2$ より. $mvd = md^2\cdot\omega + \frac{1}{3}ml^2\cdot\omega$

$3vd = (l^2+3d^2)\omega.\quad \underline{\omega = \frac{3vd}{l^2+3d^2}}$ //

(5). $K = \frac{1}{2}m\cdot\left(\frac{3vd^2}{l^2+3d^2}\right)^2 + \frac{1}{2}I\cdot\left(\frac{3vd}{l^2+3d^2}\right)^2$

$= \frac{1}{2}m\left(\frac{3vd^2}{l^2+3d^2}\right)^2 + \frac{1}{6}ml^2\left(\frac{3vd}{l^2+3d^2}\right)^2$

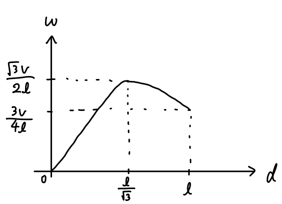
$= \frac{3}{2}mv^2\,\frac{d^2(3d^2+l^2)}{(l^2+3d^2)^2} = \frac{3}{2}mv^2\cdot\frac{d^2}{l^2+3d^2}$

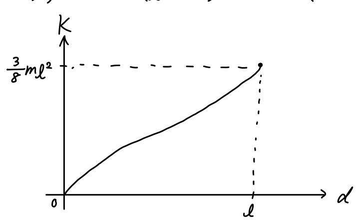
(6) グラフを書くために $\frac{d\omega}{dd}$, $\frac{dK}{dd}$ を求める.

$d=\frac{l}{\sqrt{3}}$ のとき $\omega = \frac{\sqrt{3}v}{2l}$

$\frac{d\omega}{dd} = 3v\cdot\left(\frac{1}{l^2+3d^2} + d\cdot\frac{-6d}{(l^2+3d^2)^2}\right) = 3v\cdot\frac{l^2-3d^2}{(l^2+3d^2)^2}$

$\frac{dK}{dd} = \frac{3}{2}mv^2\left(\frac{2d}{l^2+3d^2} + d^2\cdot\frac{-6d}{(l^2+3d^2)^2}\right) = \frac{3}{2}mv\cdot\frac{2d(l^2+3d^2-3d^2)}{(l^2+3d^2)^2} = 3mv\cdot\frac{dl^2}{(l^2+3d^2)^2}$

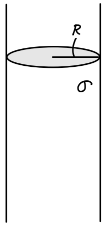




変曲点とかも

時間あれば.

2



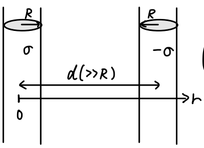
(1) $Q = 2\pi R \times 1 \times \sigma = 2\pi R\sigma$

(2) 長さ1，半径rでAと同軸の円筒表面Sについて．

ガウスの法則を適用して．$E_p(r) \cdot 2\pi r \times 1 = \dfrac{2\pi R\sigma}{\varepsilon_0}$

$$E_p(r) = \frac{R\sigma}{\varepsilon_0 r}$$

(3) $\phi_p = -\displaystyle\int_R^r \frac{R\sigma}{\varepsilon_0 r}dr = -\frac{R\sigma}{\varepsilon_0}\log\frac{r}{R}$



(4) $E_S(r) = E_p(r) + E_p(d-r) = \dfrac{R\sigma}{\varepsilon_0}\left(\dfrac{1}{r} + \dfrac{1}{d-r}\right)$.

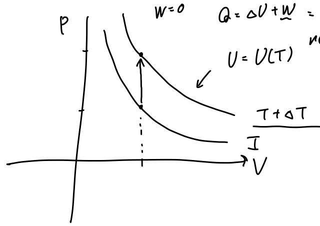
(5) (電位差を求めるのに基準??) → 多分 Aから見たBの電位を聞いてる．

$$\Delta\phi_{AB} = -\int_R^{d-R}\frac{R\sigma}{\varepsilon_0}\left(\frac{1}{r}+\frac{1}{d-r}\right)dr = -\frac{R\sigma}{\varepsilon_0}\left[\log\left|\frac{r}{d-r}\right|\right]_R^{d-R} = -\frac{R\sigma}{\varepsilon_0}\left(\log\frac{d-R}{R} - \log\frac{R}{d-R}\right)$$

$$= -\frac{2R\sigma}{\varepsilon_0}\log\frac{d-R}{R}$$

$d-R \fallingdotseq d$と近似すれば．$\Delta\phi_{AB} = -\dfrac{2R\sigma}{\varepsilon_0}\cdot\log\dfrac{d}{R}$

(6). $C = \dfrac{2\pi R\sigma}{|\Delta\phi_{AB}|} \fallingdotseq \dfrac{\pi\varepsilon_0}{\log\frac{d}{R}}$



$Q = \Delta U + \underline{W} = Q = \Delta U$

$\nu C_V dT = \Delta U$

3.

(1) 気体のエネルギー変化 ΔU は.

$$\Delta U = \int_{T_A}^{T_B} C_V\, dT = \int_{T_A}^{T_B} (C_V^0 + aT)\, dT = \left[C_V^0 T + \frac{1}{2}aT^2\right]_{T_A}^{T_B} = C_V^0(T_B - T_A) + \frac{1}{2}a(T_B^2 - T_A^2)$$

気体が外部にする仕事 W は. 圧力一定より. $W = p_A \cdot V_0$

熱力学第一法則より. 吸収する熱量 Q を用いて. $Q = \Delta U + W$ が成立つので.

$$Q_{AB} = C_V^0(T_B - T_A) + \frac{1}{2}a(T_B^2 - T_A^2) + p_A V_0$$

(2) 断熱変化でポアソンの関係式が成り立つ.

$$p_A \cdot (2V_0)^\gamma = p_C \cdot (3V_0)^\gamma$$

$$T_B(2V_0)^{\gamma-1} = T_C \cdot (3V_0)^{\gamma-1} \qquad \frac{T_B}{T_C} = \left(\frac{3}{2}\right)^{\frac{2}{3}} = 1.3 \qquad 従って. T_B = 1.3\,T_C$$

状態方程式 $p_A \cdot V_0 = RT_A$, $p_A \cdot 2V_0 = RT_B$ より. $T_B = 2T_A$. 従って $T_A < T_C < T_B$.

(3) (1)と同様に

$$\Delta U = \int_{T_C}^{T_D} (C_V^0 + aT)\, dT = C_V^0(T_D - T_C) + \frac{1}{2}a(T_D^2 - T_C^2)$$

$$W = \int p \cdot dV = 0.$$

より. $Q_{CD} = -(\Delta U + W) = -C_V^0(T_D - T_C) - \frac{1}{2}a(T_D^2 - T_C^2)$.

状態Dから状態Aは断熱変化だから. $Q_{DA} = 0$.

(4) $\eta = \dfrac{Q_{AB} - Q_{CD}}{Q_{AB}} = 1 + \dfrac{C_V^0(T_D - T_C) + \frac{1}{2}a(T_D^2 - T_C^2)}{C_V^0(T_B - T_A) + \frac{1}{2}a(T_B^2 - T_A^2) + p_A \cdot V_0}$