数学.　[1], [2], [3], [4] : 1:43.

[1]　18分.

(1) $\vec{u} = \begin{pmatrix}x\\y\\z\end{pmatrix} = a\begin{pmatrix}1\\1\\2\end{pmatrix} + b\begin{pmatrix}-1\\2\\3\end{pmatrix} + c\begin{pmatrix}0\\1\\2\end{pmatrix}$ とおく.

$$\left(\begin{array}{ccc|c}1&-1&0&x\\1&2&1&y\\2&3&2&z\end{array}\right) \to \left(\begin{array}{ccc|c}1&-1&0&x\\0&3&1&-x+y\\0&5&2&-2x+z\end{array}\right) \to \left(\begin{array}{ccc|c}1&-1&0&x\\0&15&5&-5x+5y\\0&15&6&-6x+3z\end{array}\right) \to \left(\begin{array}{ccc|c}1&-1&0&x\\0&3&1&-x+y\\0&0&1&-x-5y+3z\end{array}\right) \to \left(\begin{array}{ccc|c}1&-1&0&x\\0&1&0&2y-z\\0&0&1&-x+5y+3z\end{array}\right)$$

$$\to \left(\begin{array}{ccc|c}1&0&0&x+2y-z\\0&1&0&2y-z\\0&0&1&-x+5y+3z\end{array}\right).$$

従って. $\underline{\vec{u} = (x+2y-z)\vec{v_1} + (2y-z)\vec{v_2} + (-x+5y+3z)\vec{v_3}}$ //

(2)　(1)より, $\begin{pmatrix}1\\0\\0\end{pmatrix} = \vec{v_1} - \vec{v_3}$, $\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\vec{v_1} + 2\vec{v_2} + 5\vec{v_3}$, $\begin{pmatrix}0\\0\\1\end{pmatrix} = -\vec{v_1} - \vec{v_2} + 3\vec{v_3}$ だから,

$f\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix} - \begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}0\\-1\\0\end{pmatrix}$, $f\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\begin{pmatrix}1\\1\\1\end{pmatrix} + 2\begin{pmatrix}1\\2\\1\end{pmatrix} + 5\begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}9\\1\\9\end{pmatrix}$, $f\begin{pmatrix}0\\0\\1\end{pmatrix} = -\begin{pmatrix}1\\1\\1\end{pmatrix} - \begin{pmatrix}1\\2\\1\end{pmatrix} + 3\begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\-6\\1\end{pmatrix}$ である.

fの表現行列をAとおくと. $\left(f\begin{pmatrix}1\\0\\0\end{pmatrix}\ f\begin{pmatrix}0\\1\\0\end{pmatrix}\ f\begin{pmatrix}0\\0\\1\end{pmatrix}\right) = A$ であるから.

Aの各列の一次関係を調べる.

$$A = \begin{pmatrix}0&9&1\\-1&1&-6\\0&9&1\end{pmatrix} \to \begin{pmatrix}0&9&1\\-1&1&-6\\0&0&0\end{pmatrix} \to \begin{pmatrix}9&-9&54\\0&9&1\\0&0&0\end{pmatrix} \to \begin{pmatrix}9&0&55\\0&9&1\\0&0&0\end{pmatrix}$$

行基本変形で 各列の一次関係は変化しないから. $f\begin{pmatrix}1\\0\\0\end{pmatrix}, f\begin{pmatrix}0\\1\\0\end{pmatrix}$ が線型独立.

従って. $\dim \mathrm{Im} f = 2$ で $\left\{\begin{pmatrix}0\\-1\\0\end{pmatrix}, \begin{pmatrix}9\\1\\9\end{pmatrix}\right\}$ を基底にとれる.

(3)　$A\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ を満たす $\begin{pmatrix}x\\y\\z\end{pmatrix}$ を求める.

$A \to \begin{pmatrix}9&0&55\\0&9&1\\0&0&0\end{pmatrix}$ より. $\begin{pmatrix}x\\y\\z\end{pmatrix} = a\begin{pmatrix}-55\\-1\\9\end{pmatrix}$ (aは任意).

従って. $\dim \mathrm{Ker} f = 1$ で $\left\{\begin{pmatrix}-55\\-1\\9\end{pmatrix}\right\}$ を基底にとれる.

$\boxed{2}$ 24分

(1) $|A-\lambda E| = \begin{vmatrix} 1-\lambda & 2 & 1 \\ -1 & 4-\lambda & 1 \\ 2 & -4 & -\lambda \end{vmatrix} = \begin{vmatrix} 2-\lambda & 2-\lambda & 2-\lambda \\ -1 & 4-\lambda & 1 \\ 2 & -4 & -\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ -1 & 4-\lambda & 1 \\ 2 & -4 & -\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ -1 & 5-\lambda & 2 \\ 2 & -6 & -2-\lambda \end{vmatrix} = (2-\lambda)\{(\lambda-5)(\lambda+2)+12\}$

$= (2-\lambda)(\lambda^2-3\lambda+2) = (2-\lambda)(\lambda-2)(\lambda-1)$ $\underline{\lambda = 2\ (重解),\ 1}$ //

(2) (i) $\lambda=1$ について.

$A-E = \begin{pmatrix} 0 & 2 & 1 \\ -1 & 3 & 1 \\ 2 & -4 & -1 \end{pmatrix} \to \begin{pmatrix} 0 & 2 & 1 \\ -1 & 3 & 1 \\ 0 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -3 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

従って. $A\vec{x}=\vec{x}$ を満たす $\vec{x}$ は. $a\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ (aは任意) 従って $\left\{\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\right\}$ を基底にとる.

(ii) $\lambda=2$ について

$A-2E = \begin{pmatrix} -1 & 2 & 1 \\ -1 & 2 & 1 \\ 2 & -4 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

従って $A\vec{x}=2\vec{x}$ を満たす $\vec{x}$ は $a\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}+b\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ (a,bは任意) 従って. $\left\{\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right\}$ を基底にとる.

(3) $\left(f(\vec{p_1})\ f(\vec{p_2})\ f(\vec{p_3})\right) = \left(\vec{p_1}\ \vec{p_2}\ \vec{p_3}\right)\cdot M$

$AB = BM$ $\vec{p_1}, \vec{p_2}, \vec{p_3}$ は線型独立だから. B^{-1} が存在する.

$B^{-1}AB = M.$

$B = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$ ととると. $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

③ 8分

$f(x,y) = xy \cdot e^{-x^2-y^2}$

(1) $f_x = y \cdot e^{-x^2-y^2} - 2x^2y \cdot e^{-x^2-y^2}$, $f_y = x \cdot e^{-x^2-y^2} - 2xy^2 \cdot e^{-x^2-y^2}$

(2) $f_x = 0$ より. $y(1-2x^2) = 0$. 従って. $y=0$ または. $x = \pm\frac{1}{\sqrt{2}}$

$f_y = 0$ より. $x(1-2y^2) = 0$. 従って. $x=0$ または. $y = \pm\frac{1}{\sqrt{2}}$.

よって. $(a,b) = (0,0), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(3) $f_{xx} = -2xy \cdot e^{-x^2-y^2} - 4xy \cdot e^{-x^2-y^2} + 4x^3y\, e^{-x^2-y^2} = (-6xy + 4x^3y)\, e^{-x^2-y^2}$

$f_{yy} = (-6xy + 4xy^3)\, e^{-x^2-y^2}$

$f_{xy} = (1-2x^2)\left(e^{-x^2-y^2} - 2y^2 e^{-x^2-y^2}\right) = (1-2x^2)(1-2y^2)\, e^{-x^2-y^2}$

ヘッシアン $H(x,y) = 4x^2y^2(2x^2-3)(2y^2-3)\, e^{-2(x^2+y^2)} - (1-2x^2)^2(1-2y^2)^2 e^{-2(x^2+y^2)}$

$= \left\{4x^2y^2(2x^2-3)(2y^2-3) - (1-2x^2)^2(1-2y^2)\right\} e^{-2(x^2+y^2)}$

$H(0,0) = (0-1) \cdot 1 = -1 < 0$

$H\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left((-2)(-2) - 0\right) e^{-2} = 4e^{-2} > 0$, $f_{xx}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = (-3+1)\, e^{-1} = -2e^{-1} < 0$

$H\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 4e^{-2} > 0$, $f_{xx}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = (3-1)\, e^{-1} = 2e^{-1} > 0$.

$H\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 4e^{-2} > 0$, $f_{xx}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = (3-1)\, e^{-1} = 2e^{-1} > 0$.

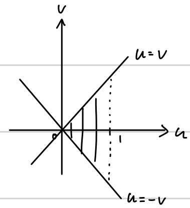
$H\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 4e^{-2} > 0$, $f_{xx}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = (-3+1)\, e^{-1} = -2e^{-1} < 0$. より.

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ で 極大値 $\frac{1}{2e}$

$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ で 極小値 $-\frac{1}{2e}$ をとる.

④ 53分.

(1) $\displaystyle\iint_D \frac{x+y}{1+(x-y)}\,dxdy$.　　D: $x \geqq 0,\ y \geqq 0,\ x+y \leqq 1$



$x+y=u,\ x-y=v$ とおくと.

$x = \dfrac{u+v}{2}$, $y = \dfrac{u-v}{2}$ より.　$u+v \geqq 0$ ($v \geqq -u$), $u-v \geqq 0$ ($v \leqq u$), $u \leqq 1$.

従って. D は $E = \left\{ (u,v) \,\middle|\, 0 \leqq u \leqq 1,\ -u \leqq v \leqq u \right\}$ に移る.

$\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\dfrac{1}{2}$ より. $\left| \dfrac{\partial(x,y)}{\partial(u,v)} \right| = \dfrac{1}{2}$ だから.

$$\iint_D \frac{x+y}{1+(x-y)}\,dxdy = \frac{1}{2}\int_0^1 \left(\int_{-u}^{u} \frac{u}{1+v}\,dv \right) du = \frac{1}{2}\int_0^1 u \Big[\log|1+v| \Big]_{v=-u}^{v=u} du$$

$$= \frac{1}{2}\int_0^1 u \cdot \log\frac{1+u}{1-u}\,du$$

$$\int u \cdot \log\frac{1+u}{1-u}\,du = \frac{1}{2}u^2\log\frac{1+u}{1-u} - \frac{1}{2}\int u^2 \cdot \frac{1-u}{1+u} \cdot \frac{1-u+1+u}{(1-u)^2}\,du = \frac{1}{2}u^2\log\frac{1+u}{1-u} - \frac{1}{2}\int u^2 \cdot \frac{2}{1-u^2}\,du$$

$$= \frac{1}{2}u^2\log\frac{1+u}{1-u} + \int\left(1 + \frac{1}{u^2-1}\right)du = \frac{1}{2}u^2\log\frac{1+u}{1-u} + u + \frac{1}{2}\int\left(\frac{1}{u-1} - \frac{1}{u+1}\right)du$$

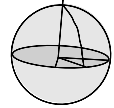
$$= \frac{1}{2}u^2\log\frac{1+u}{1-u} + u + \frac{1}{2}\log\frac{1-u}{1+u}$$

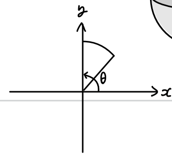
$$= u + \frac{1}{2}(u^2-1)\log\left(\frac{1+u}{1-u}\right).$$

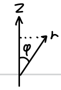
従って、$\displaystyle\frac{1}{2}\int_0^1 u \cdot \log\frac{1+u}{1-u}\,du = \frac{1}{2}\left[u + \frac{1}{2}(u^2-1)\log\frac{1+u}{1-u} \right]_0^1$

$$\lim_{u\to 1} \frac{\log\frac{1+u}{1-u}}{\frac{1}{u^2-1}} = \lim_{u\to 1} \frac{\frac{1-u}{1+u}\cdot\frac{-1-u-1+u}{(1-u)^2}}{-\frac{2u}{(u^2-1)^2}} = \lim_{u\to 1} \frac{\frac{-2}{1-u^2}}{-\frac{2u}{(u^2-1)^2}} = \lim_{u\to 1} \frac{-2\cdot(1-u^2)}{-2u} = 0.$$ だから.

$$\frac{1}{2}\left[u + \frac{1}{2}(u^2-1)\log\frac{1+u}{1-u} \right]_0^1 = \frac{1}{2}(1+0-0-0) = \underline{\frac{1}{2}}_{\,/\!/}$$







(2) $\iiint_E xyz \, dxdydz$　　$E: y \geqq x \geqq 0, \ z \geqq 0, \ x^2+y^2+z^2 \leqq 1$

$x = r\sin\varphi \cdot \cos\theta, \quad y = r\sin\varphi \cdot \sin\theta, \quad z = r\cos\varphi$　　とおくと.

E は. $F = \left\{ (r, \theta, \varphi) \,\middle|\, 0 \leqq r \leqq 1, \ \frac{\pi}{4} \leqq \theta \leqq \frac{\pi}{2}, \ 0 \leqq \varphi \leqq \frac{\pi}{2} \right\}$ に移る.

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = \begin{vmatrix} \sin\varphi\cos\theta & \sin\varphi\sin\theta & \cos\varphi \\ -r\sin\varphi\sin\theta & r\sin\varphi\cos\theta & 0 \\ r\cos\varphi\cos\theta & r\cos\varphi\sin\theta & -r\sin\varphi \end{vmatrix} = r^2\sin\varphi \begin{vmatrix} \sin\varphi\cos\theta & \sin\varphi\sin\theta & \cos\varphi \\ -\sin\theta & \cos\theta & 0 \\ \cos\varphi\cos\theta & \cos\varphi\sin\theta & -\sin\varphi \end{vmatrix}$$

$$= r^2\sin\varphi \left(-\sin^2\varphi\underline{\cos^2\theta} - \sin^2\varphi\,\underline{\sin^2\theta} - \cos^2\varphi\,\underline{\sin^2\theta} - \cos^2\varphi\,\underline{\cos^2\theta} \right)$$

$$= r^2\sin\varphi\,(-1) = -r^2\sin\varphi. \qquad \therefore \left| \frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} \right| = r^2\sin\varphi.$$

従って. $\iiint_E xyz \cdot dxdydz = \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^1 r^3\sin^2\varphi\cos\varphi \cdot \cos\theta \cdot \sin\theta \times r^2\sin\varphi \, dr\,d\theta\,d\varphi.$

$$= \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^1 r^5\sin^3\varphi \cdot \cos\varphi \cdot \cos\theta \cdot \sin\theta \, dr\,d\theta\,d\varphi$$

$$= \int_0^{\frac{\pi}{2}} \sin^3\varphi \cdot \cos\varphi \cdot d\varphi \times \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta \cdot \cos\theta \cdot d\theta \times \int_0^1 r^5 \cdot dr$$

$$= \left[\frac{1}{4}\sin^4\varphi \right]_0^{\frac{\pi}{2}} \times \left[\frac{1}{2}\sin^2\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \times \frac{1}{6}$$

$$= \frac{1}{4} \times \frac{1}{2}\left(1 - \frac{1}{2}\right) \times \frac{1}{6}$$

4·2·2·6

$$= \underline{\frac{1}{96}} \ //$$

$\boxed{5}$　15分.

(1) $z^4+1=0$

$z = re^{i\theta}$ とおくと.

$r=1$, $4\theta = \pi + 2n\pi$,　$\theta = \frac{\pi}{4}, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$

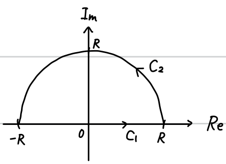
$\therefore z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}},\quad -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}},\quad -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}},\quad \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}.$

(2) $\mathrm{Res}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}\cdot 2\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\cdot\sqrt{2}i} = \frac{1}{4\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)}$

同様に. 他の極についても留数をとると.

$\mathrm{Res}(\alpha) = \frac{1}{4\alpha}\left(= \frac{\overline{\alpha}}{4}\right)$ である.

(3) $I = \int_0^{\infty} f(x)\,dx = \frac{1}{2}\int_{-\infty}^{\infty} \frac{1}{x^4+1}\,dx.$



図のような経路 C_1+C_2 で $R\to\infty$ とした

$\oint_{C_1+C_2} f(z)\cdot dz$ を考える.

C_1+C_2 内に含まれる $f(z)$ の極は. $z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ なので.

$\mathrm{Res}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \frac{1}{4\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)} = \frac{\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}}{4}$,　$\mathrm{Res}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \frac{-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}}{4}$

従って $\oint_{C_1+C_2} f(z)\,dz = 2\pi i\cdot\left(-\frac{i\cdot\sqrt{2}}{4}\right) = \frac{\pi}{\sqrt{2}}.$

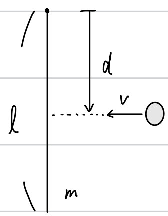
$\int_{C_2} f(z)\cdot dz$ は. $R\to\infty$ としたとき. $\left|z\times\frac{1}{z^4+1}\right| \to 0$ なので. $\int_{C_2} f(z) = 0.$

$\oint_{C_1+C_2} f(z)\cdot dz = \int_{C_1} f(z)\cdot dz = \frac{\pi}{\sqrt{2}}$ であり.　$\int_{C_1} f(z)\cdot dz = \int_{-\infty}^{\infty} f(x)\cdot dx$ だから.

求める値は. $\underline{\frac{\pi}{2\sqrt{2}}}$ //

物理

1.



(1) $I = \int_0^l \frac{m}{l} \cdot x^2 \cdot dx = \frac{m}{l}\left[\frac{1}{3}x^3\right]_0^l = \frac{1}{3}ml^2$

(2) 角運動量が保存されるので.

$mvl = m \cdot l \cdot \omega \cdot l + I\omega$

$I = \frac{1}{3}ml^2$より. $mvl = ml^2\omega + \frac{1}{3}ml^2\omega. = \frac{4}{3}ml^2\omega$

$\therefore \omega = \frac{3v}{4l}$

(3) $K = \frac{1}{2}m \cdot \left(\frac{3v}{4}\right)^2 + \frac{1}{2}I \cdot \left(\frac{3v}{4l}\right)^2 = \frac{9}{32}mv^2 + \frac{1}{6}ml^2 \cdot \frac{9v^2}{16l^2} = mv^2\left(\frac{9}{32} + \frac{3}{32}\right) = \frac{3}{8}mv^2$

(4) 角運動量が保存されるので.

$mv \cdot d = m \cdot d \cdot \omega \cdot d + I\omega.$

$I = \frac{1}{3}ml^2$より. $mvd = md^2 \cdot \omega + \frac{1}{3}ml^2 \cdot \omega$

$3vd = (l^2 + 3d^2)\omega.$ $\quad \underline{\omega = \frac{3vd}{l^2+3d^2}}$ //

(5). $K = \frac{1}{2}m \cdot \left(\frac{3vd^2}{l^2+3d^2}\right)^2 + \frac{1}{2}I \cdot \left(\frac{3vd}{l^2+3d^2}\right)^2$

$= \frac{1}{2}m\left(\frac{3vd^2}{l^2+3d^2}\right)^2 + \frac{1}{6}ml^2\left(\frac{3vd}{l^2+3d^2}\right)^2$

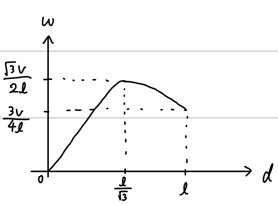
$= \frac{3}{2}mv^2 \frac{d^2(3d^2+l^2)}{(l^2+3d^2)^2} = \frac{3}{2}mv^2 \cdot \frac{d^2}{l^2+3d^2}$

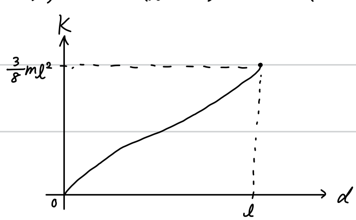
(6) グラフを書くために. $\frac{d\omega}{dd}$, $\frac{dK}{dd}$を求める.

$d = \frac{l}{\sqrt{3}}$のとき. $\omega = \frac{\sqrt{3}v}{2l}$

$\frac{d\omega}{dd} = 3v \cdot \left(\frac{1}{l^2+3d^2} + d \cdot \frac{-6d}{(l^2+3d^2)^2}\right) = 3v \cdot \frac{l^2-3d^2}{(l^2+3d^2)^2}$

$\frac{dK}{dd} = \frac{3}{2}mv^2\left(\frac{2d}{l^2+3d^2} + d^2 \cdot \frac{-6d}{(l^2+3d^2)^2}\right) = \frac{3}{2}mv \cdot \frac{2d(l^2+3d^2-3d^2)}{(l^2+3d^2)^2} = 3mv \cdot \frac{dl^2}{(l^2+3d^2)^2}$

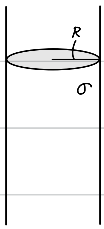




変曲点とかも

時間あれば.

$\boxed{2}$



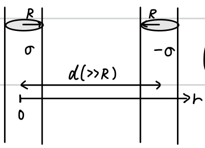
(1) $Q = 2\pi R \times 1 \times \sigma = 2\pi R\sigma$

(2) 長さ1, 半径 r で A と同軸の円筒表面 S について.

ガウスの法則を適用して. $E_p(r)\cdot 2\pi r \times 1 = \dfrac{2\pi R\sigma}{\varepsilon_0}$

$$E_p(r) = \frac{R\sigma}{\varepsilon_0 r}$$

(3) $\phi_p = -\displaystyle\int_R^r \frac{R\sigma}{\varepsilon_0 r}dr = -\frac{R\sigma}{\varepsilon_0}\log\frac{r}{R}$



(4) $E_S(r) = E_p(r) + E_p(d-r) = \dfrac{R\sigma}{\varepsilon_0}\left(\dfrac{1}{r} + \dfrac{1}{d-r}\right)$.

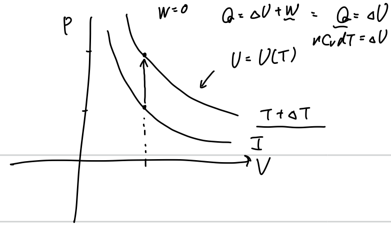
(5) (電位差を求めるのに基準??) → 多分 Aから見たBの電位を聞いてる.

$$\Delta\phi_{AB} = -\int_R^{d-R}\frac{R\sigma}{\varepsilon_0}\left(\frac{1}{r}+\frac{1}{d-r}\right)dr = -\frac{R\sigma}{\varepsilon_0}\left[\log\left|\frac{r}{d-r}\right|\right]_R^{d-R} = -\frac{R\sigma}{\varepsilon_0}\left(\log\frac{d-R}{R} - \log\frac{R}{d-R}\right)$$

$$= -\frac{2R\sigma}{\varepsilon_0}\log\frac{d-R}{R}$$

$d-R \fallingdotseq d$ と近似すれば. $\Delta\phi_{AB} = -\dfrac{2R\sigma}{\varepsilon_0}\cdot\log\dfrac{d}{R}$

(6). $C = \dfrac{2\pi R\sigma}{|\Delta\phi_{AB}|} \fallingdotseq \dfrac{\pi\varepsilon_0}{\log\frac{d}{R}}$



3.

(1) 気体のエネルギー変化 ΔU は.

$$\Delta U=\int_{T_A}^{T_B} C_V\,dT=\int_{T_A}^{T_B}(C_V^0+aT)\,dT=\left[C_V^0T+\frac{1}{2}aT^2\right]_{T_A}^{T_B}=C_V^0(T_B-T_A)+\frac{1}{2}a(T_B^2-T_A^2)$$

気体が外部にする仕事 W は. 圧力一定より. $W=p_A\cdot V_0$

熱力学第一法則より. 吸収する熱量 Q を用いて. $Q=\Delta U+W$ が成立つので.

$$Q_{AB}=C_V^0(T_B-T_A)+\frac{1}{2}a(T_B^2-T_A^2)+p_A V_0$$

(2) 断熱変化でポアソンの関係式が成り立つ.

$$P_A\cdot(2V_0)^\gamma=P_C\cdot(3V_0)^\gamma$$

$$T_B(2V_0)^{\gamma-1}=T_C\cdot(3V_0)^{\gamma-1}\qquad \frac{T_B}{T_C}=\left(\frac{3}{2}\right)^{\frac{2}{3}}=1.3\qquad 従って.\ T_B=1.3T_C$$

状態方程式 $P_A\cdot V_0=RT_A$, $P_A\cdot 2V_0=RT_B$ より. $T_B=2T_A$. 従って $T_A<T_C<T_B$.

(3) (1)と同様に

$$\Delta U=\int_{T_C}^{T_D}(C_V^0+aT)\,dT=C_V^0(T_D-T_C)+\frac{1}{2}a(T_D^2-T_C^2)$$

$$W=\int p\cdot dV=0.$$

より. $Q_{CD}=-(\Delta U+W)=-C_V^0(T_D-T_C)-\frac{1}{2}a(T_D^2-T_C^2).$

状態Dから状態Aは断熱変化だから. $Q_{DA}=0$.

(4) $$\eta=\frac{Q_{AB}-Q_{CD}}{Q_{AB}}=1+\frac{C_V^0(T_D-T_C)+\frac{1}{2}a(T_D^2-T_C^2)}{C_V^0(T_B-T_A)+\frac{1}{2}a(T_B^2-T_A^2)+p_A\cdot V_0}$$