ALTRO:
A Fast Solver For Constrained Trajectory Optimization

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The Big Picture

Trajectory optimization is a versatile tool with many important applications in robotic motion planning. Solution methods for this problem are typically classified as:

Indirect Methods (e.g. iLQR)
- Fast
- Lightweight
- Anytime Dynamic Feasibility
- Limited Constraint Handling
- Difficult to Initialize

Direct Methods (e.g. DIRCOL)
- Numerically Robust
- Arbitrary Constraints
- Infeasible Initialization
- Typically Slow
- Large-scale sparse NLP solvers

ALTRO (Augmented Lagrangian TRajectory Optimizer) combines the best aspects of indirect and direct methods.

Augmented Lagrangian sqrt(iLQR) + State Augmentation + Active-set Projection Method

paper: https://rexlab.stanford.edu/papers/altro-iros.pdf

code: https://github.com/RoboticExplorationLab/TrajectoryOptimization.jl

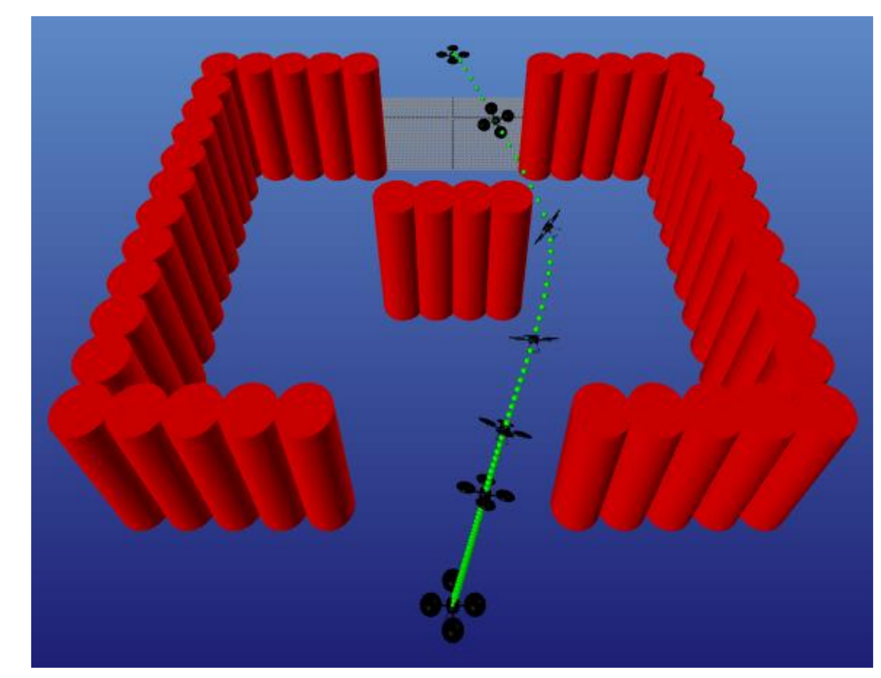
Trajectory Optimization

ALTRO uses Iterative LQR in an Augmented Lagrangian framework to handle constraints and solves optimization problems of the form:

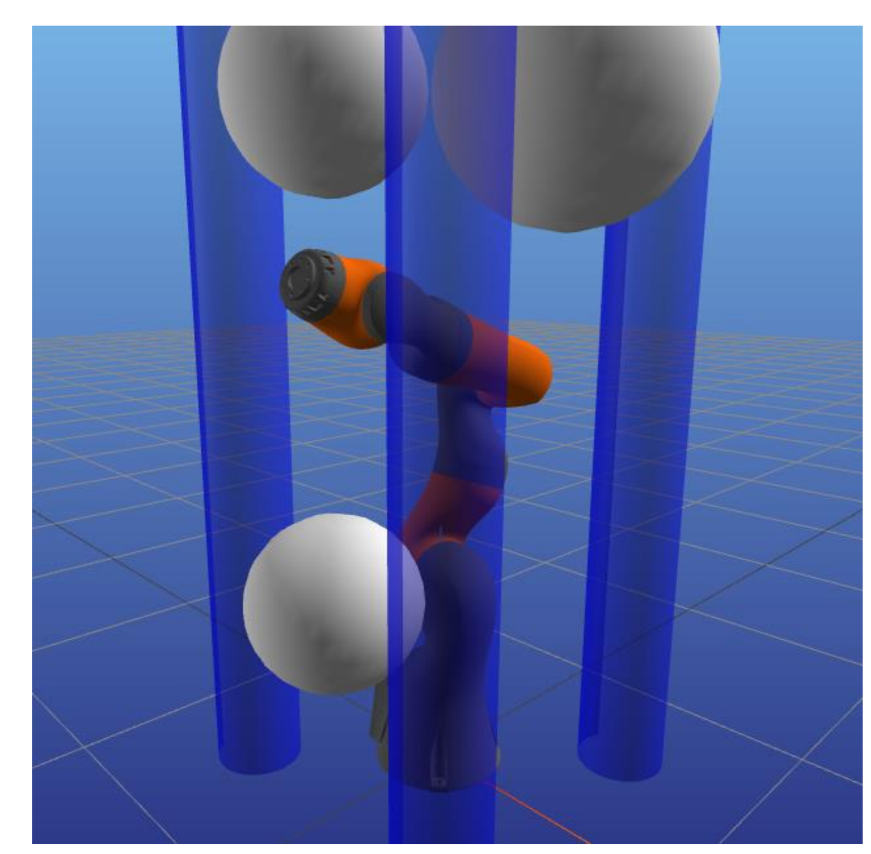
$$\begin{aligned} \underset{x_{0:N},\, u_{0:N-1}}{\text{minimize}} \quad & \ell_N(x_N) + \sum_{k=0}^{N-1} \ell_k(x_k, u_k, \Delta t) \\ \text{subject to} \quad & x_{k+1} = f_k(x_k, u_k, \Delta t), \\ & c_k(x_k, u_k) \, \{\leq, =\} \, 0 \end{aligned}$$

Examples

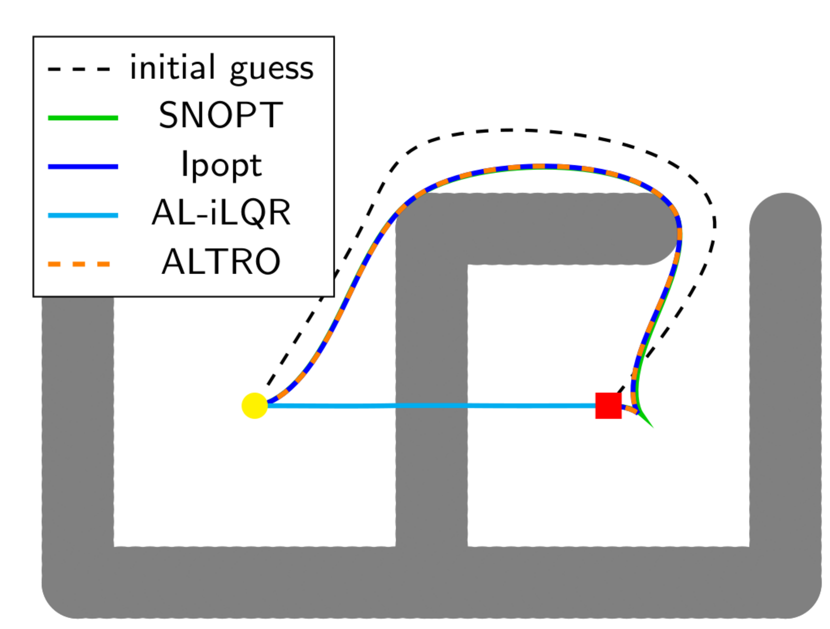
quadrotor maze



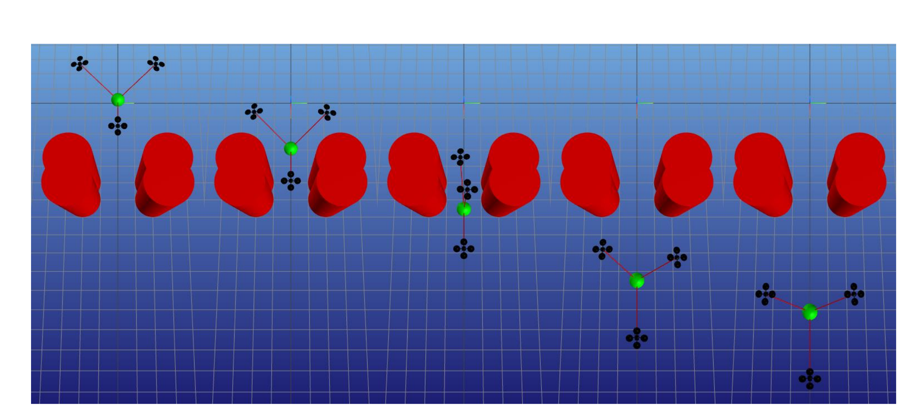
kuka obstacles



car escape

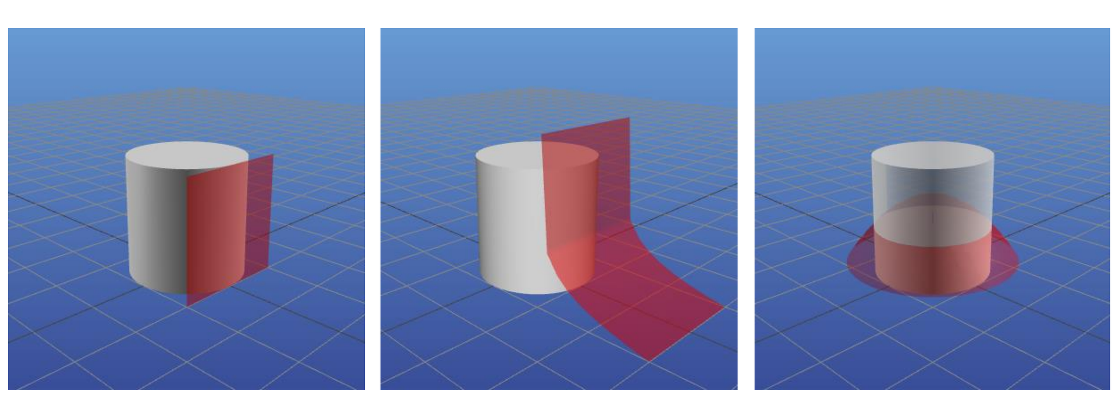


distributed team lift



Augmented Lagrangian

Framework for solving Iterative LQR with constraints.



This approach (right) is arguably better suited for obstacle avoidance than SQP (left) or interior point (center) methods.

Square-root Iterative LQR

This Riccati recursion improves numerical conditioning.

$$S_k \leftarrow \mathbf{QR}\left(\begin{bmatrix} \sqrt{(Q_{xx})_k} + \sqrt{(Q_{xx})_k}^{-T}(Q_{ux})_k^T K_k \\ \mathbf{DD}(\sqrt{(Q_{uu})_k}, \sqrt{(Q_{xx})_k}^{-T}(Q_{ux})_k^T) K_k \end{bmatrix}\right)$$

State Augmentation

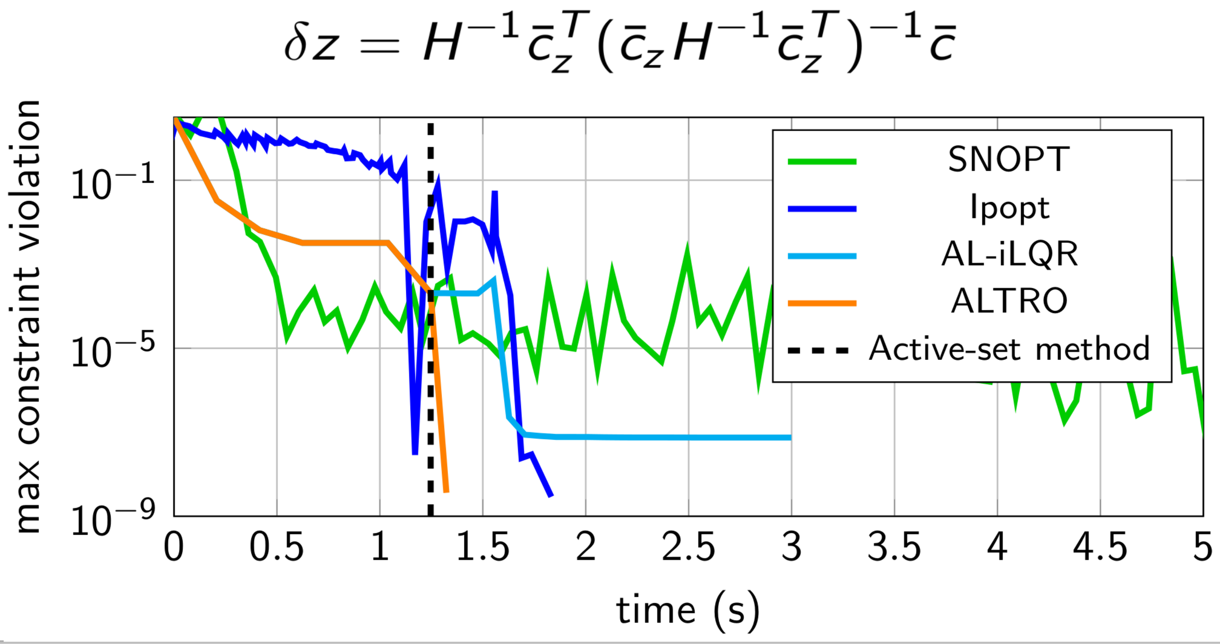
Augmenting the state and controls enables initializing state trajectories and solving minimum time problems.

$$\tilde{x}_{k+1} = \begin{bmatrix} f_k(x_k, u_k, \sqrt{\Delta t_k}) + s_k \\ \sqrt{\Delta t_k} \end{bmatrix} \quad \tilde{u}_k = \begin{bmatrix} u_k \\ s_k \\ \sqrt{\Delta t_k} \end{bmatrix}$$

Active-set Projection Method

A coarse solution from iLQR is used to warm start an active-set method used for solution polishing. The search direction for the primals is iteratively projected onto the constraint manifold to ensure feasibility.

$$\delta z = H^{-1} \bar{c}_z^T (\bar{c}_z H^{-1} \bar{c}_z^T)^{-1} \bar{c}$$



Performance

ALTRO compared to DIRCOL using SNOPT or Ipopt.

system	ALTRO	Ipopt	SNOPT
block move	**12 ms**	26 ms	31 ms
parallel park	**63 ms**	138 ms	385 ms
pendulum	**65 ms**	213 ms	646 ms
car w/ 3 obs.	**136 ms**	935 ms	2.4 s
cartpole	**661 ms**	1.1 s	3.8 s
acrobot	**1.2 s**	11.9 s	6.1 s
car escape	**1.2 s**	9.2 s	37.2 s
quadrotor line	**2.3 s**	4.9 s	7.9 s
kuka w/ obs.	**9.2 s**	-	-
quadrotor maze	**33.0 s**	-	-