## Lab 3 report

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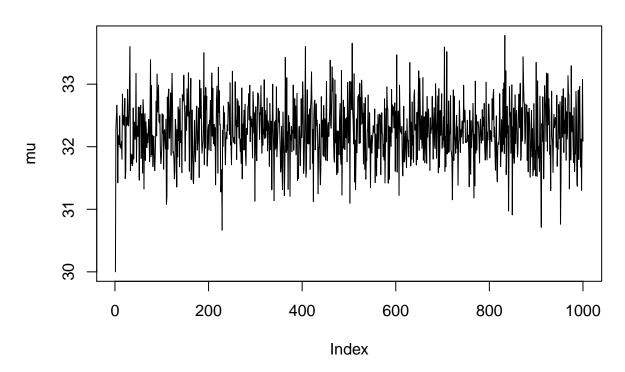
#### Assignment 1

#### 1.a

Assuming that the daily precipitation are independent normally distributed with parameters  $\mu$  and  $\sigma^2$  we use a Gibbs sampler to simulate from the joint posterior  $p(\mu, \sigma^2 | y_1, ..., y_n)$ 

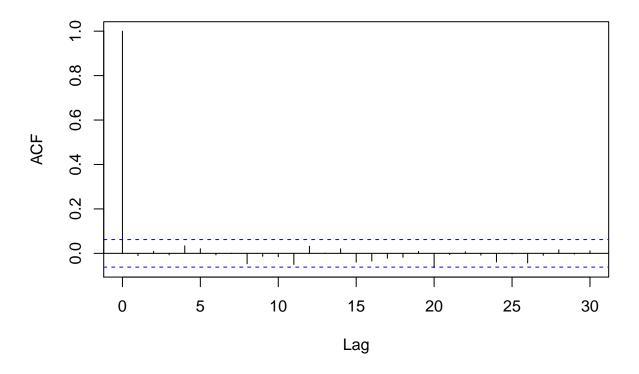
Let see if the sampler converges, starting with the trace plot of  $\mu$ :

#### Traceplot of mu



From the traceplot  $\mu$  seems to converge to a value between 32 and 33. For further analysis we compute the ACF and the inefficiency factor:

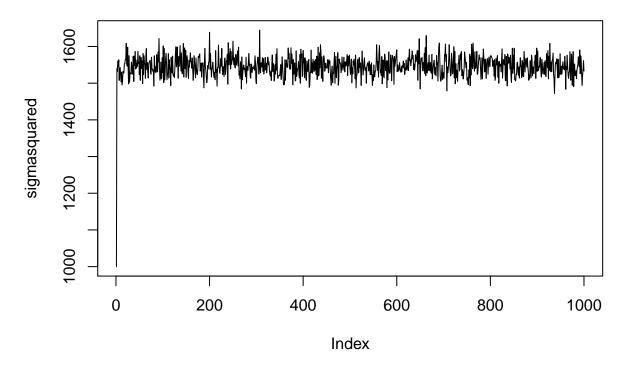
### ACF of traceplot of mu



## [1] 0.4458138

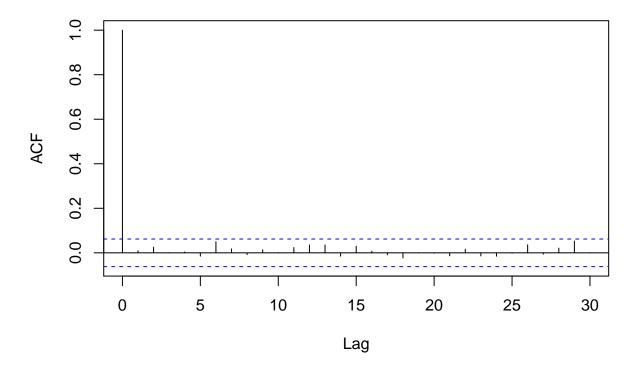
They both shows that the sampler is very efficience since the draws seem to be independent. The traceplot of  $\sigma^2$ :

### Traceplot of sigma squared



Also  $\sigma^2$  seems to converge, to a value around 1550. The ACF and inefficiency factor:

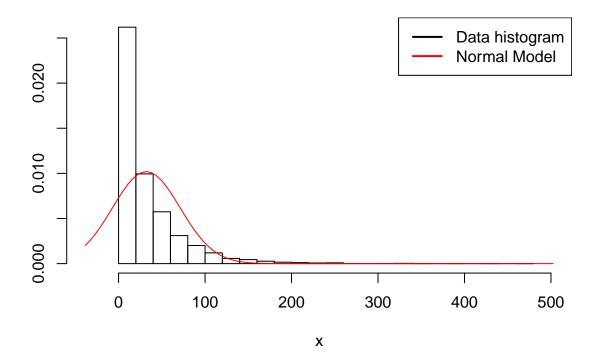
### ACF of traceplot of sigma squared



## [1] 1.51204

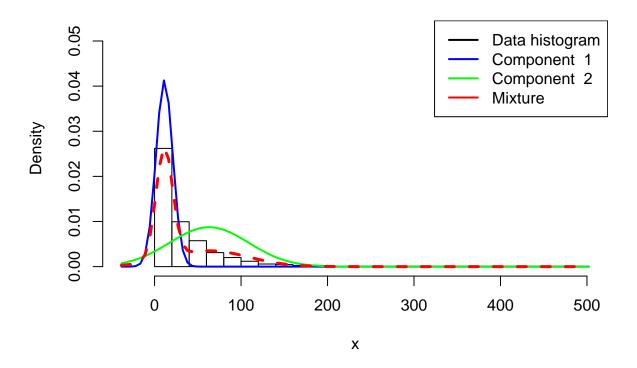
As for  $\mu$ , we can say that the draws seem to be independent. In general the Gibbs sampler is very good. In this final plot we report the histogram of the data together with the normal distribution using the posterior mean values for the two parameters:

### Histogram of Japan daily precipitation



1.b
Using mixture normal model, here we present the plot for the last iteration with the two components.

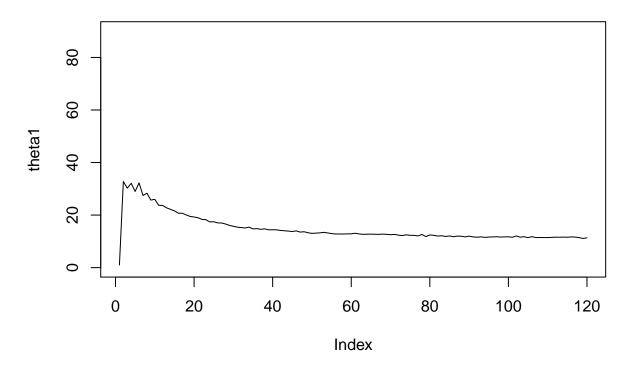
### **Iteration number 120**



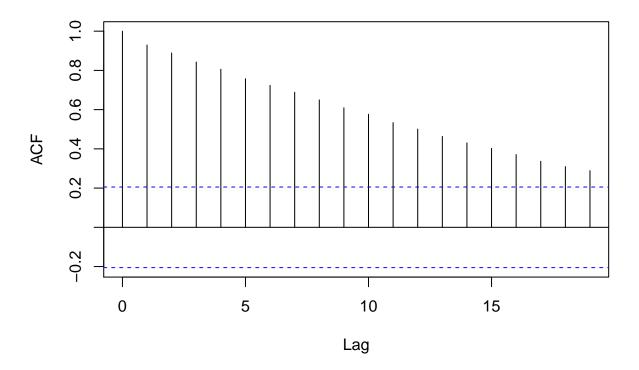
Let check the convergence and the efficiency of all the parameters.

Traceplot, ACF and inefficiency factor of  $\mu_1$ :

# Traceplot of theta1





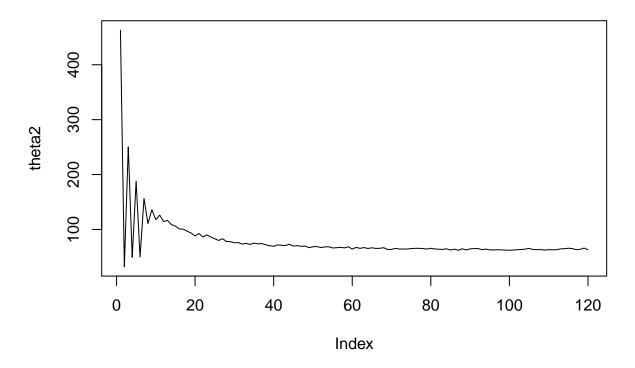


## [1] "IF: 22.6397995588737"

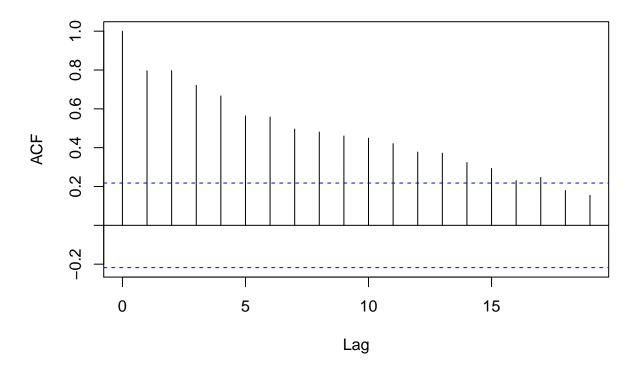
 $\mu_1$  seems to converge to a value around 20 but the draws seem to be highly positively correlated, which leads to a high IF, which means that the number of equivalent independent draws is merely a fraction of the actual draws.

Traceplot, ACF and inefficiency factor of  $\mu_2$ :

# Traceplot of theta2





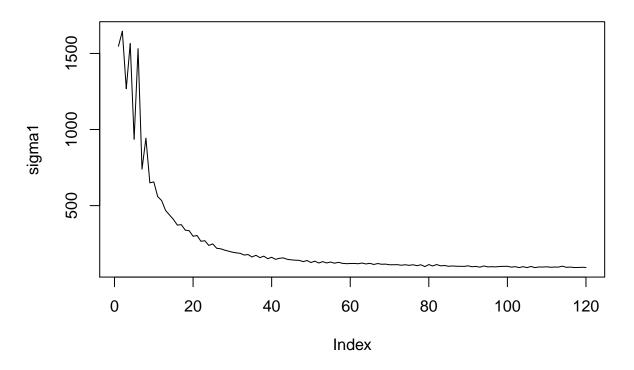


## [1] "IF: 18.1694761144715"

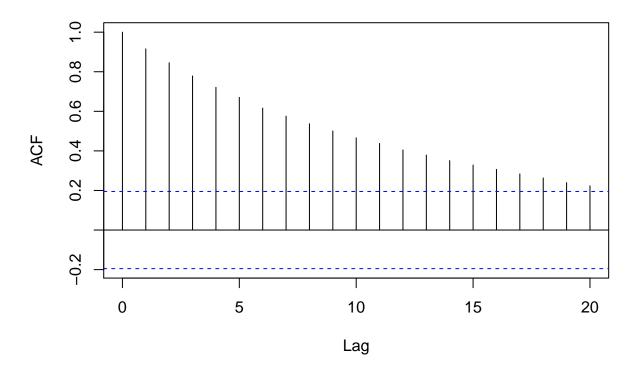
 $\mu_2$  seems to converge to a value around 100 but the draws seem to be positively correlated with draws an even number of lags apart. The IF number is rather large here as well.

Traceplot, ACF and inefficiency factor of  $\sigma_1^2$ :

# Traceplot of sigma1



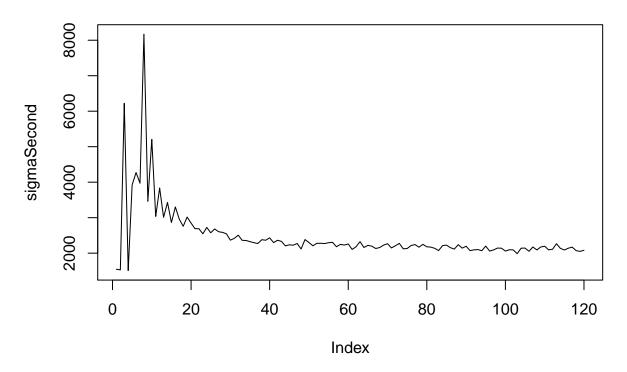




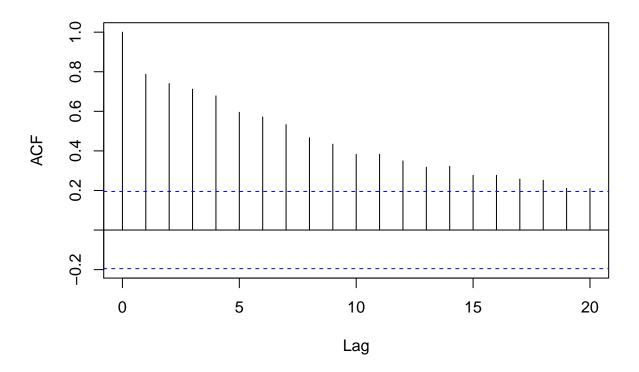
## [1] 20.24201

 $\sigma_1^2$  converges to a value close to 90, the draws seem to be highly positively correlated, which leads to a high IF, which means that the number of equivalent independent draws is merely a fraction of the actual draws. Traceplot, ACF and inefficiency factor of  $\sigma_2^2$ :

# Traceplot of sigma2



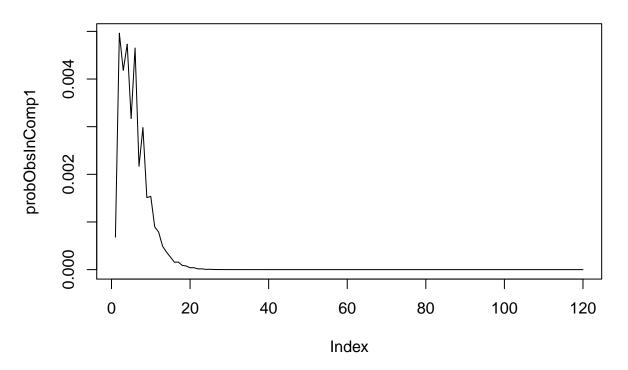




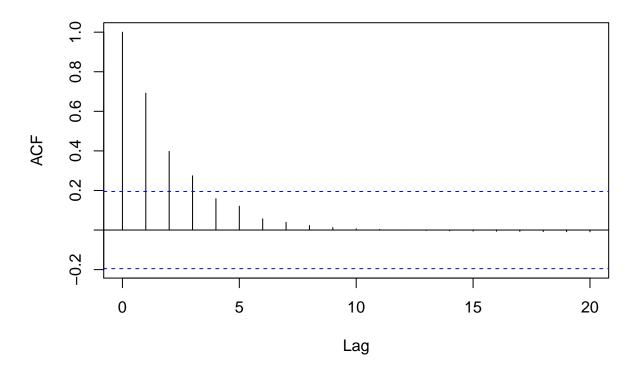
## [1] 18.09024

 $\sigma_2^2$  converges to a value close to 2100, the comment for the ACF and IF of  $\sigma_1^2$  are true also in this case. Traceplot, ACF and inefficiency factor of  $\pi_1$ :

# Traceplot of pi1



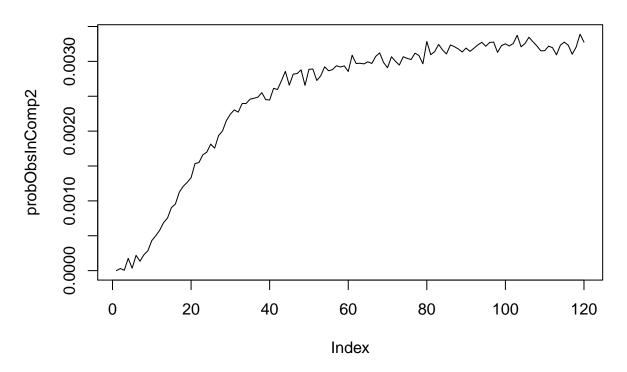




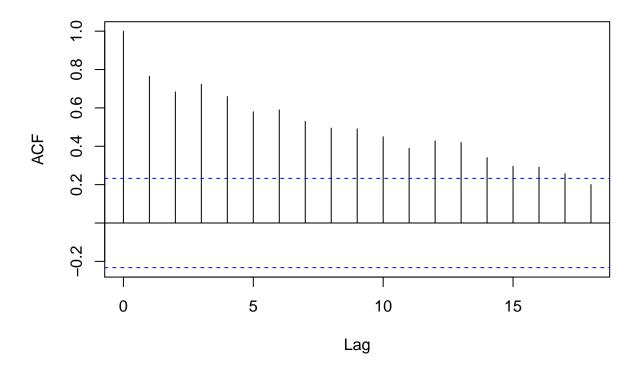
## [1] 4.482566

 $\pi_1$  converges to a value that is virtually zero and apart for the first 4 lags the draws seems to be independent. Traceplot, ACF and inefficiency factor of  $\pi_2$ :

## Traceplot of pi2







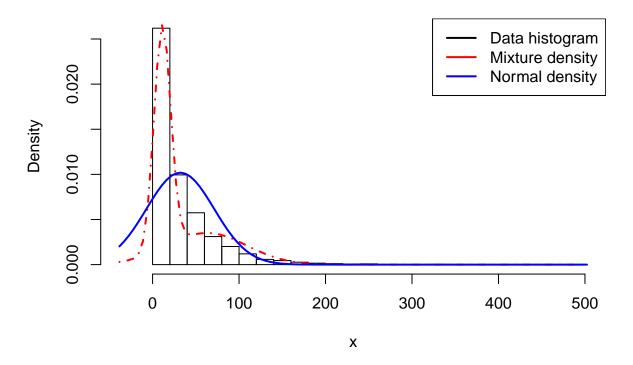
## [1] 17.77187

Also  $\pi_2$  converges to a value close to zero, but in this case from the ACF and IF we can see that the draws seem to be highly positively correlated.

#### 1.c

Histogram of the data with the model in 1.a and 1.b using the posterior mean values for all the parameters:

### Final fitted density

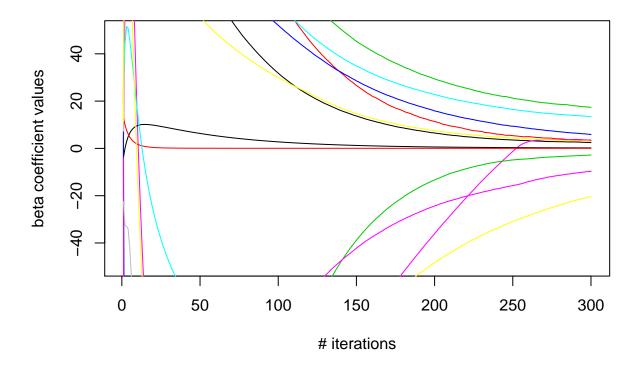


The mixture of normal models seems to be the best in terms of fitting.

#### Assignment 2

We are going to use a gibbs sampler to find the regression coefficients to be used in the probit regression model and then compare the performance of the regression coefficients  $\beta$  to regression coefficients drawn from  $\mathcal{N}(\tilde{\beta}, J^{-1})$ .

#### beta coefficents vs # iterations



## [1] "Classification rate for coefficients drawn from posterior mode normal dist.: 90.5 %"

## [1] "Classification rate for gibbs sampler coefficients: 88.3 %"

We see that the plot shows that the gibbs sampler does make the regression coefficients converge to relatively small magnitudes after initially diverging from zero. We also see that the classification performance of the gibbs sampler coefficients is as good as the one obtained from coefficients found from the posterior mode normal distribution.