Time series analysis Lab3

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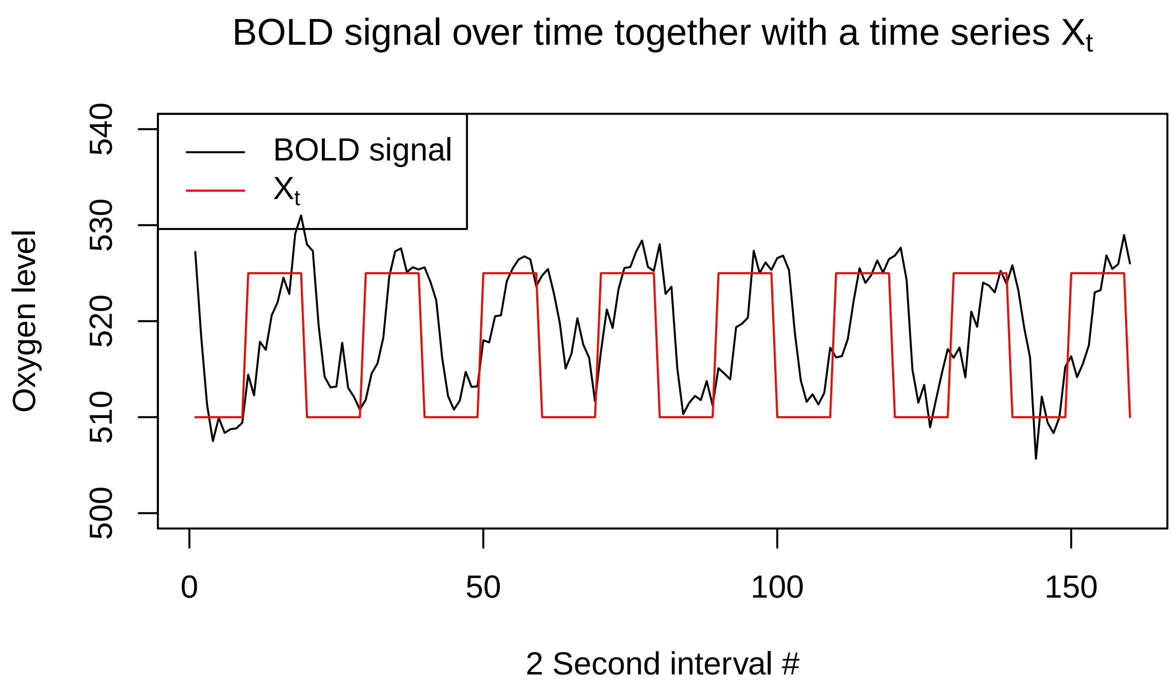
Tuesday, October 06, 2015

Problem 1

1.a.

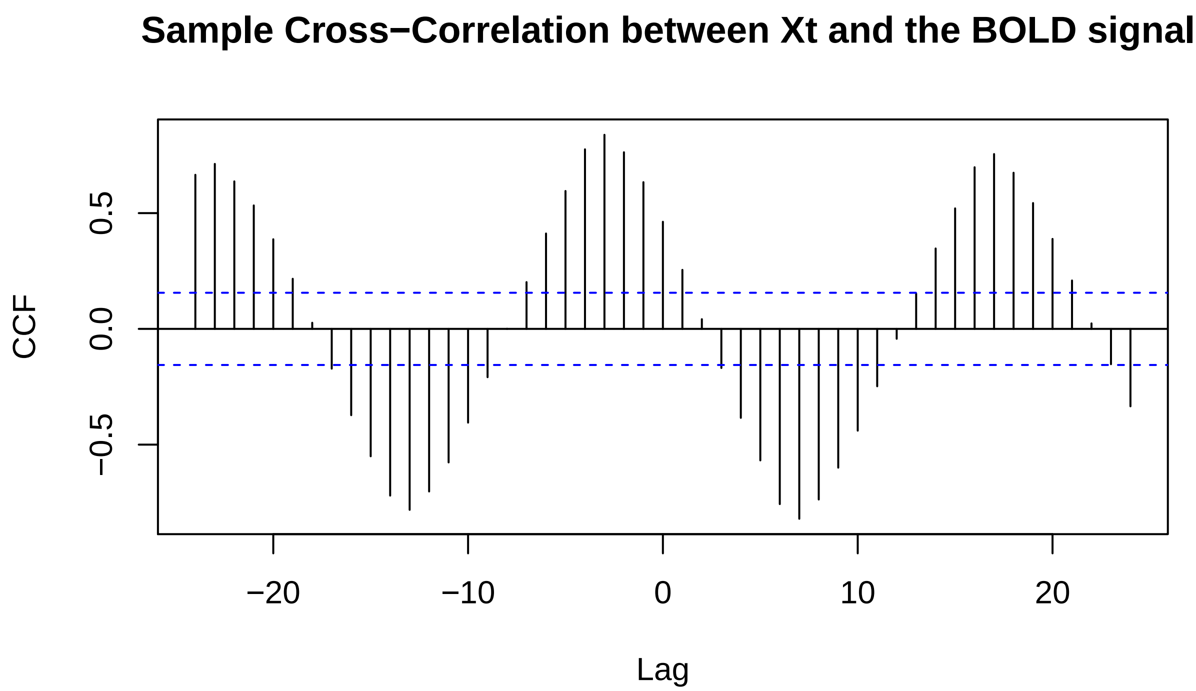
We plot the BOLD signal and the time series X_t. The height scale of X_t has been changed for better visuals.



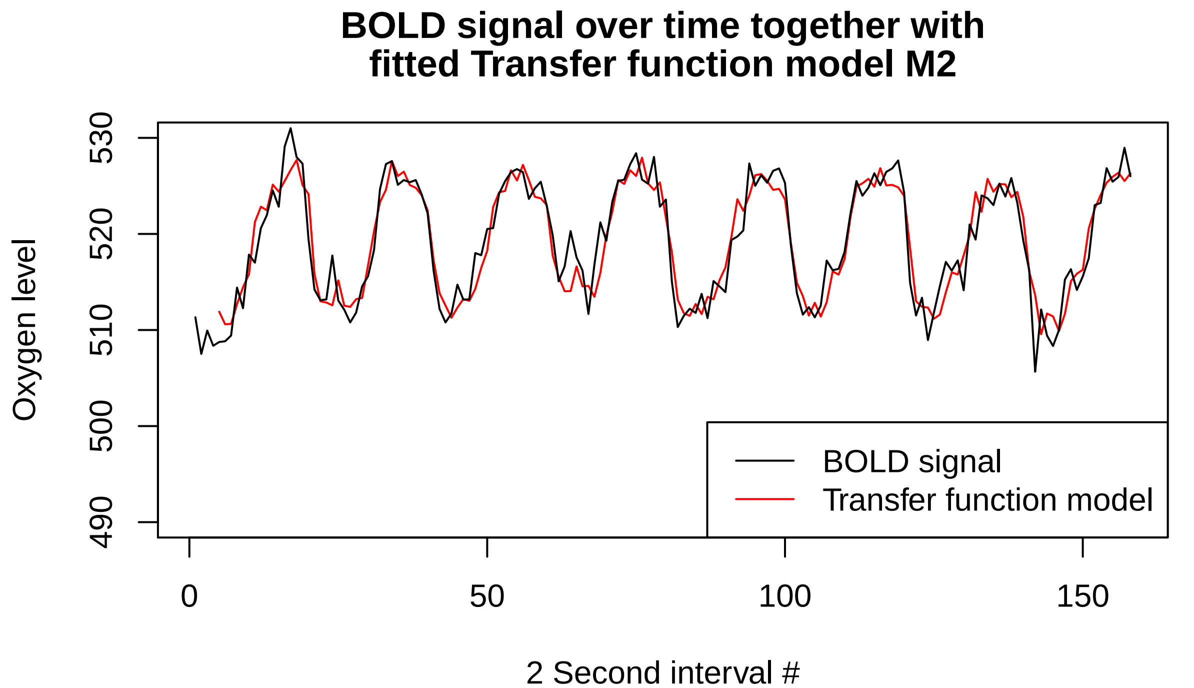
We see that the BOLD signal has a clear lag against X_t throughout the time series. It is also clear that the first two measurements of the BOLD signal are instrumentation artefacts, so we remove them from the time series henceforth. No clear outliers except maybe the first peak, periodicity of X_t is 20 seconds.

1.b.

Now let us take a look at the corss-correlation between BOLD signal and X_t.



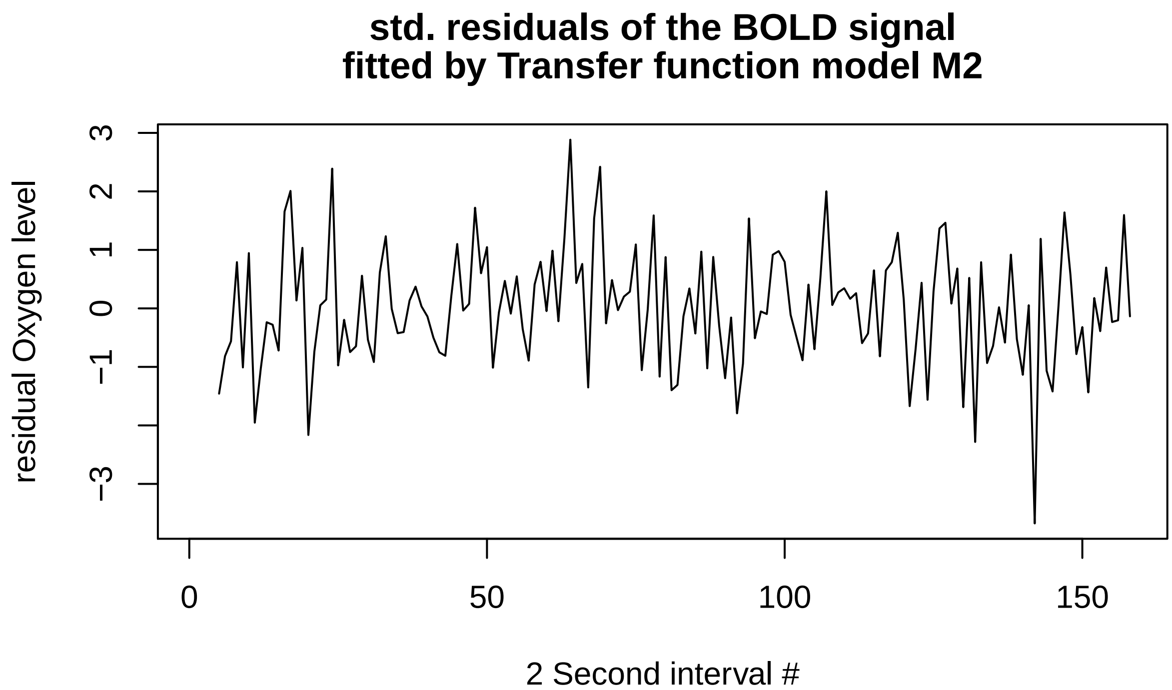
It seems as if the lag is about 3-4 seconds on average between X_t and BOLD signal. That seems to be the physiological lag in neuronal activity in the brain when it comes to observing images. We now try to find a transfer function model for the BOLD signal, taking X_t as its covariate.

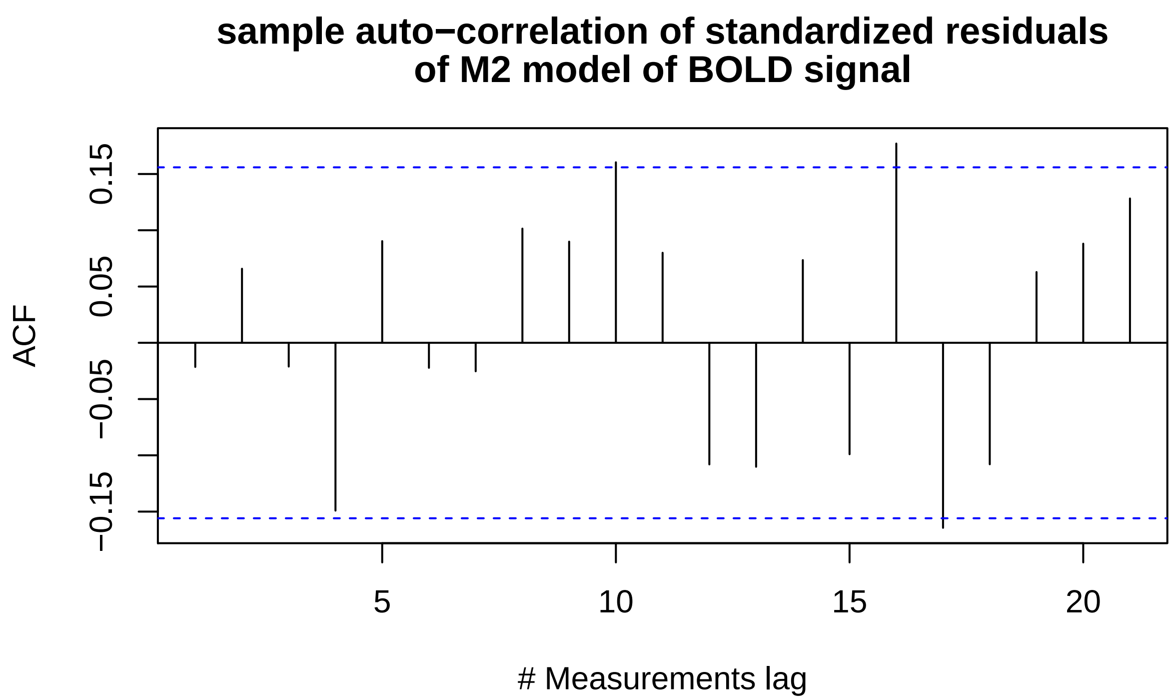


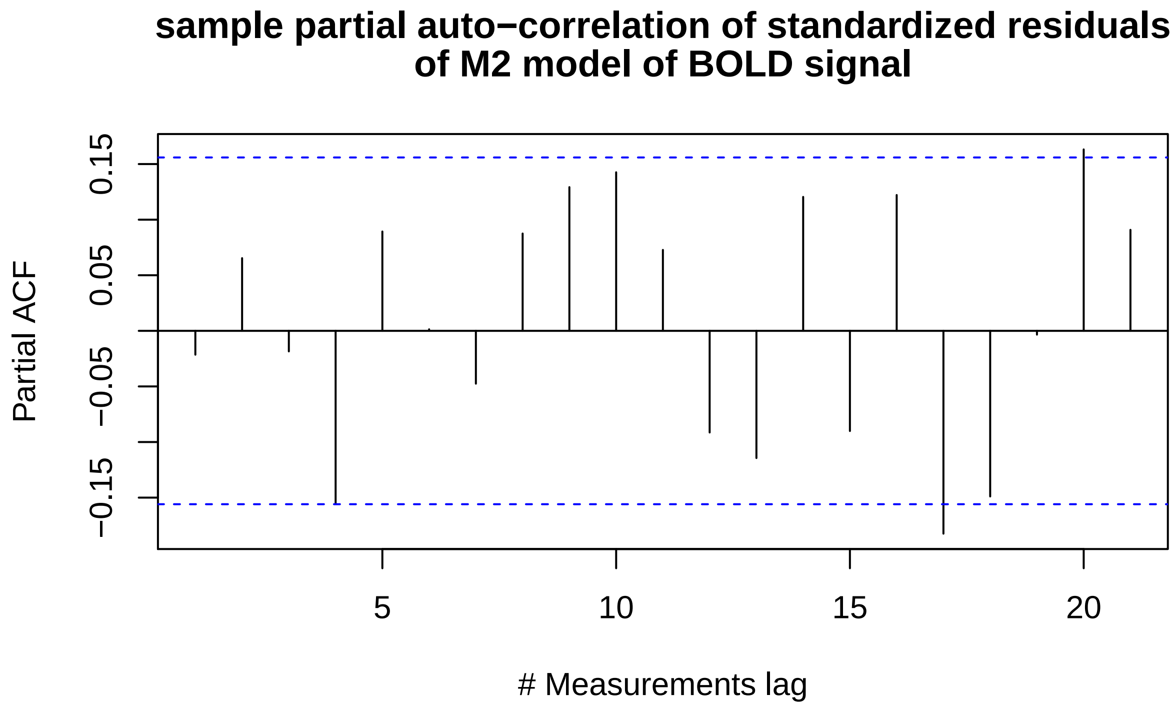
We find that this transfer function with parameters b=0, r=2, s=4, and an AR(1) transfer function for the noise is good. The mathematical expression,where let us say Y_t is the BOLD signal, is

$$Y_t = \beta_0 + \frac{\left(\omega_0 - \omega_1 B - \omega_2 B^2 - \omega_3 B^3 - \omega_4 B^4\right)}{(1 - \delta_1 B - \delta_2 B^2)} X_t + \frac{1}{1 - \phi_1 B} e_t$$

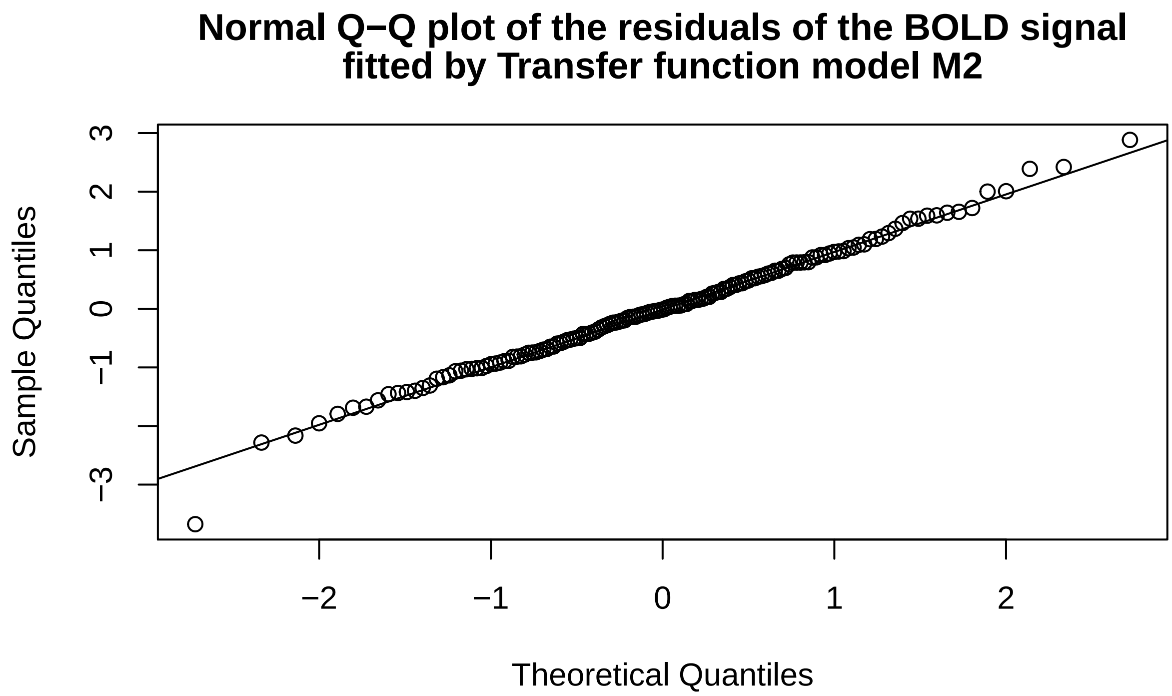
where e_t is standard white noise.How good is this transfer function? Let us take a look at the residuals.


std. residuals of the BOLD signal
fitted by Transfer function model M2
residual Oxygen level
3
2
1
0
−1
−3
0
50
100
150
2 Second interval #


sample auto−correlation of standardized residuals
of M2 model of BOLD signal
ACF
0.15
0.05
−0.05
−0.15
5
10
15
20
Measurements lag



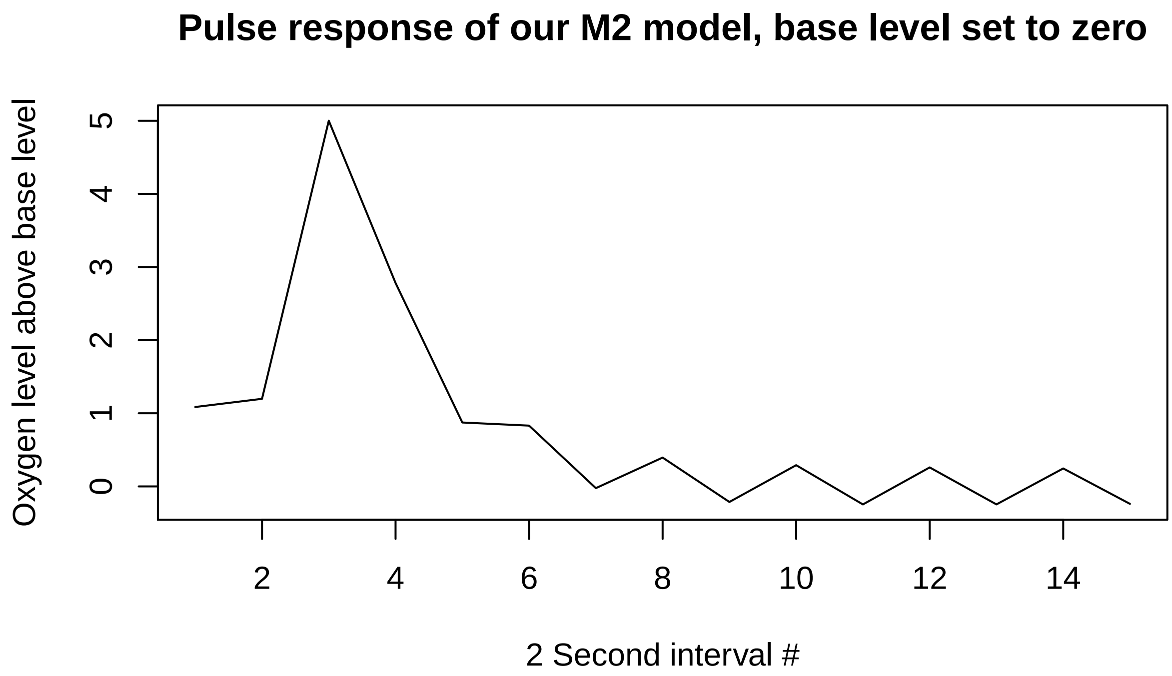

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 o o o o o o o o o o o  o  o  o
## 1 x o o o o o o o o o o  o  o  o
## 2 x x o o o o o o o o o  o  o  o
## 3 o x x o o o o o o o o  o  o  o
## 4 x x o x o o o o o o o  o  o  o
## 5 o x o o x o o o o o o  o  o  o
## 6 o x o o o o o o o o o  o  o  o
## 7 x x x o o o o o o o o  o  o  o
```



It feels like the residuals are reasonably well behaved, and almost gaussian in appearance. No problems, in other words. As regards the lags above 20, we belive that it is not necessary to include parameters in the transfer function for lags greater than 20 because the covariate X_t has periodicity 20. For instance, X_{25} returns the same as X_5 and and any parameter multiplied with X_{25} could just as well be multiplied with X_5. We feel that any parameter beyond lag 20 would just contribute to instability in the transfer function model.

1.c.

Now let us look at the pulse response of the model, with no noise. β_0 is set as the zero level.



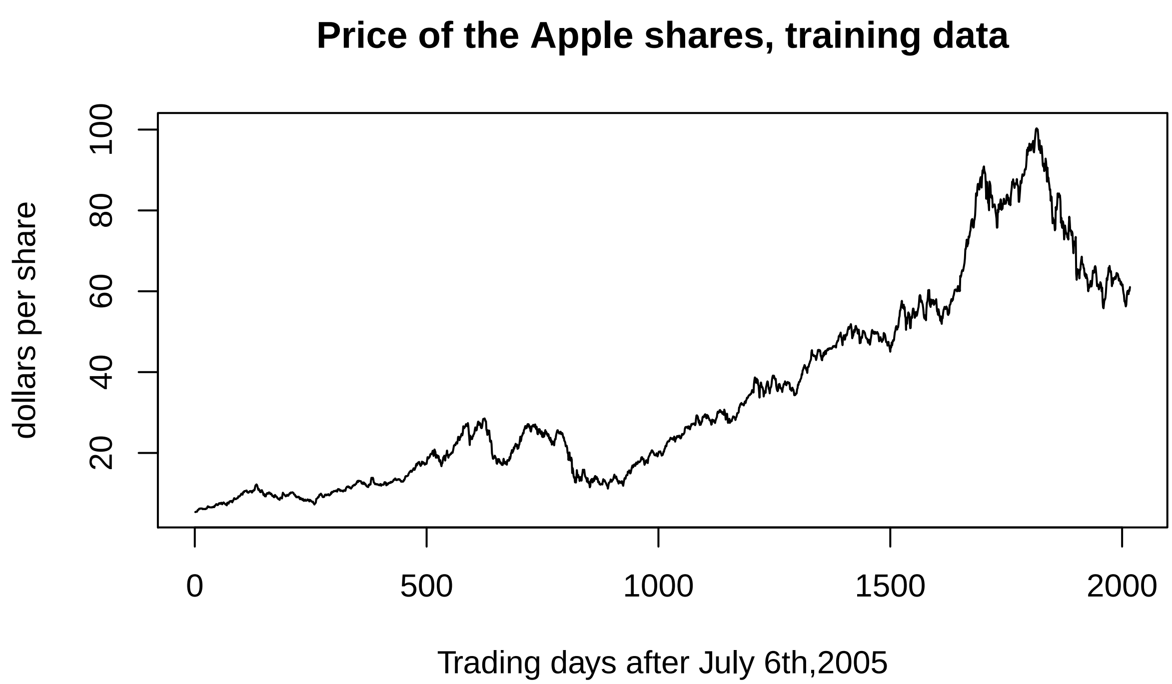
The graph is reminiscent of the canonical HRF in the lab instructions. Coincidence? We think not. Probably this is to be expected. A smaller mesurement interval is to be desired.

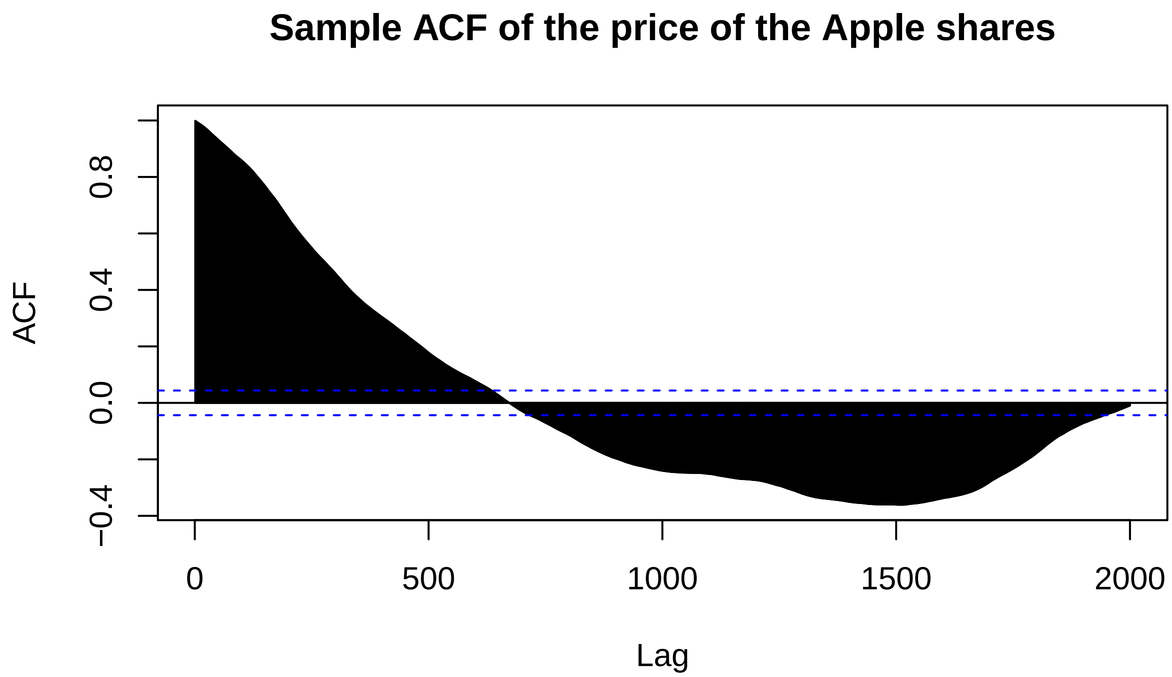
Problem 2

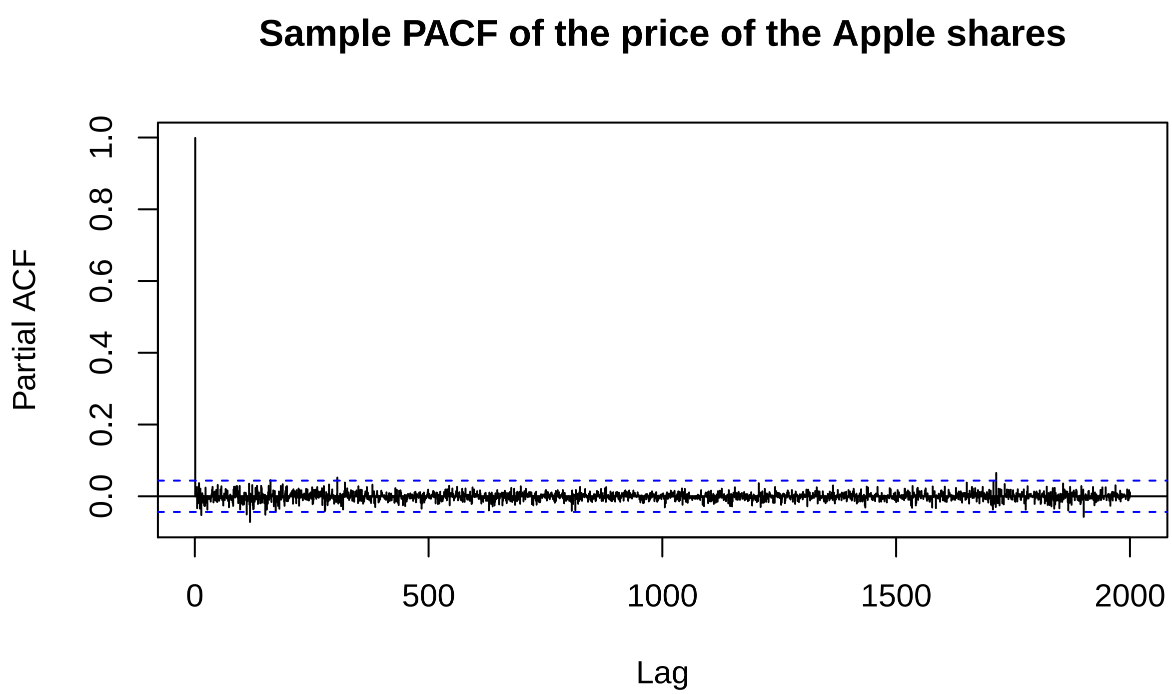
2.a.

Let us take a look at the training data.

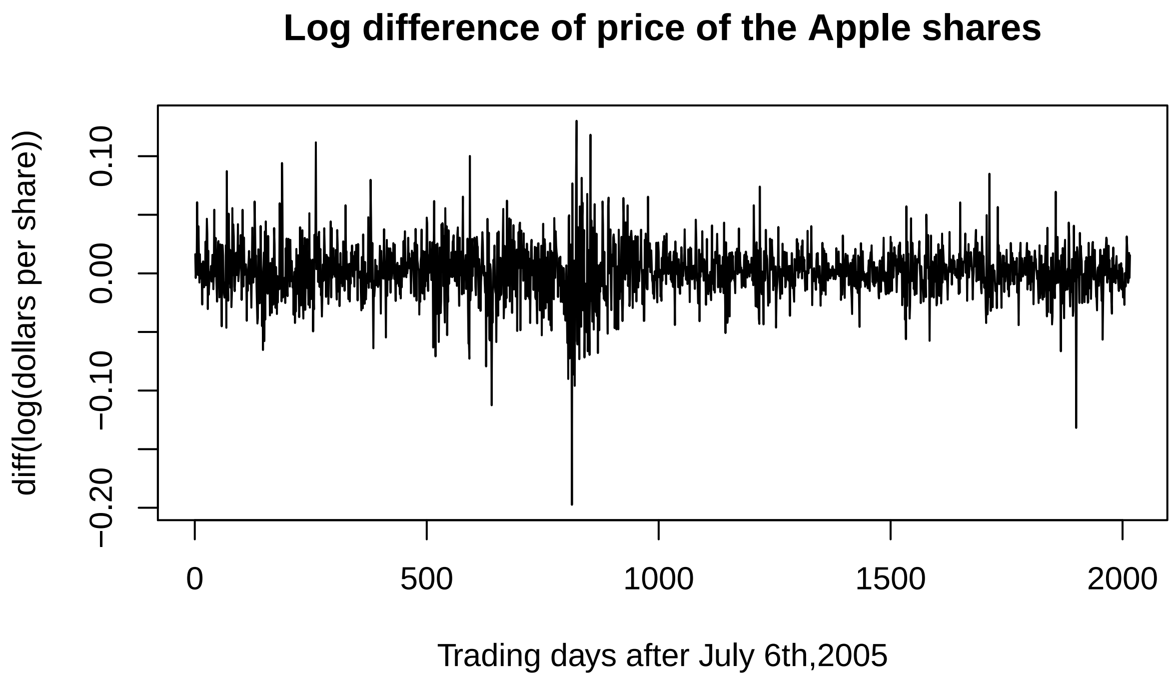

Price of Apple share from 070705 to 070715
dollars per share
20
60
100
0
500
1000
1500
2000
2500
Trading days after July 6th,2005

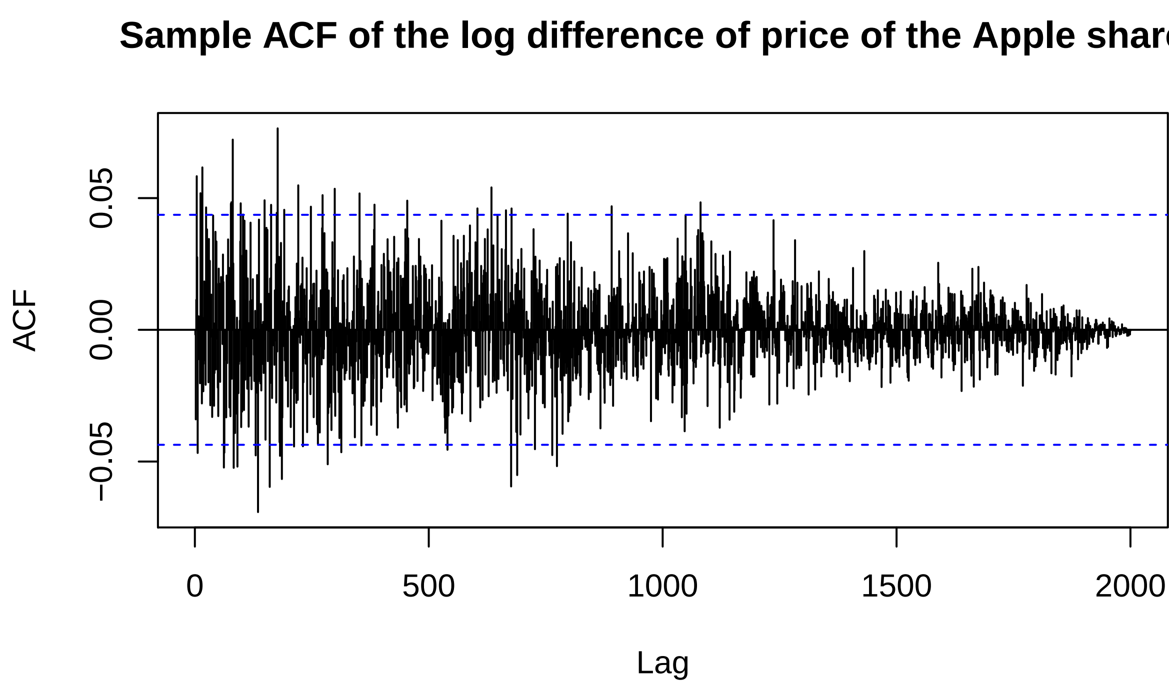

Price of the Apple shares, training data
dollars per share
20
40
60
80
100
0
500
1000
1500
2000
Trading days after July 6th,2005

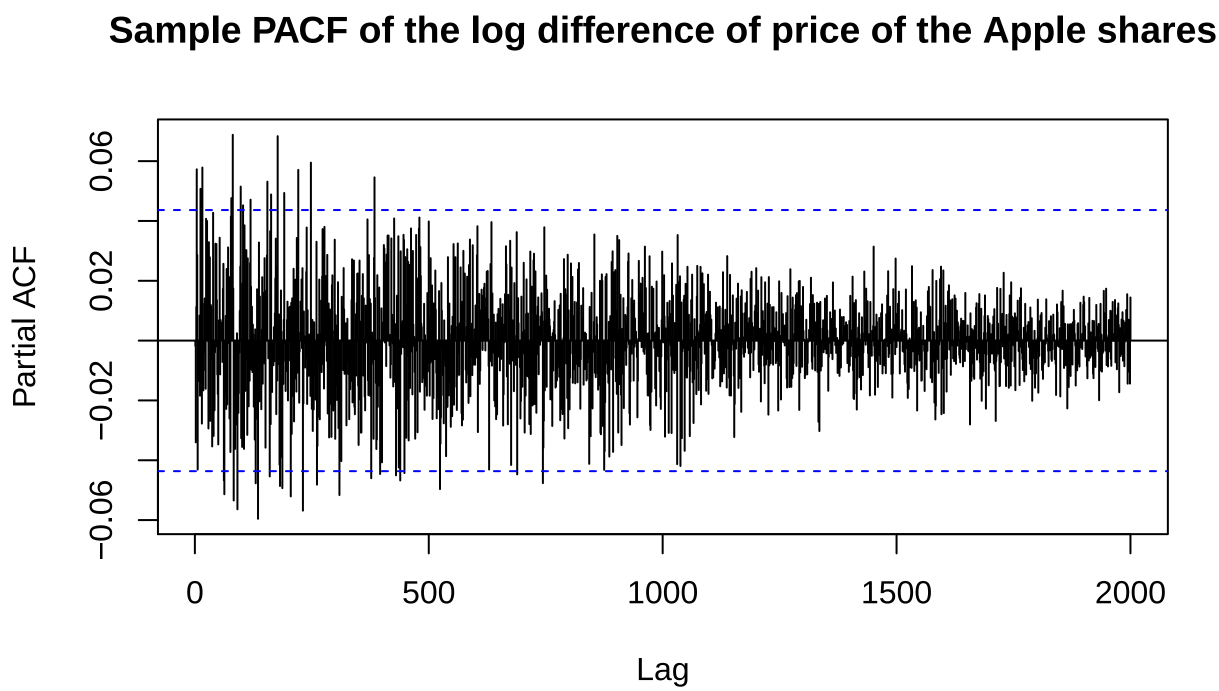


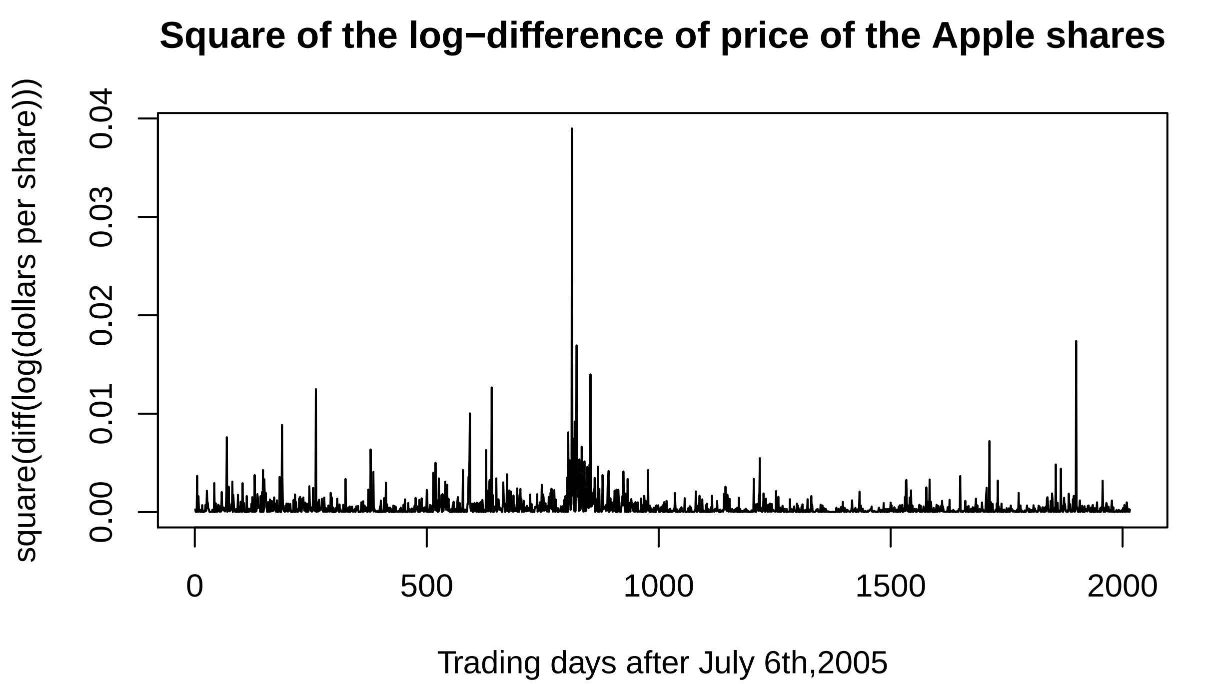


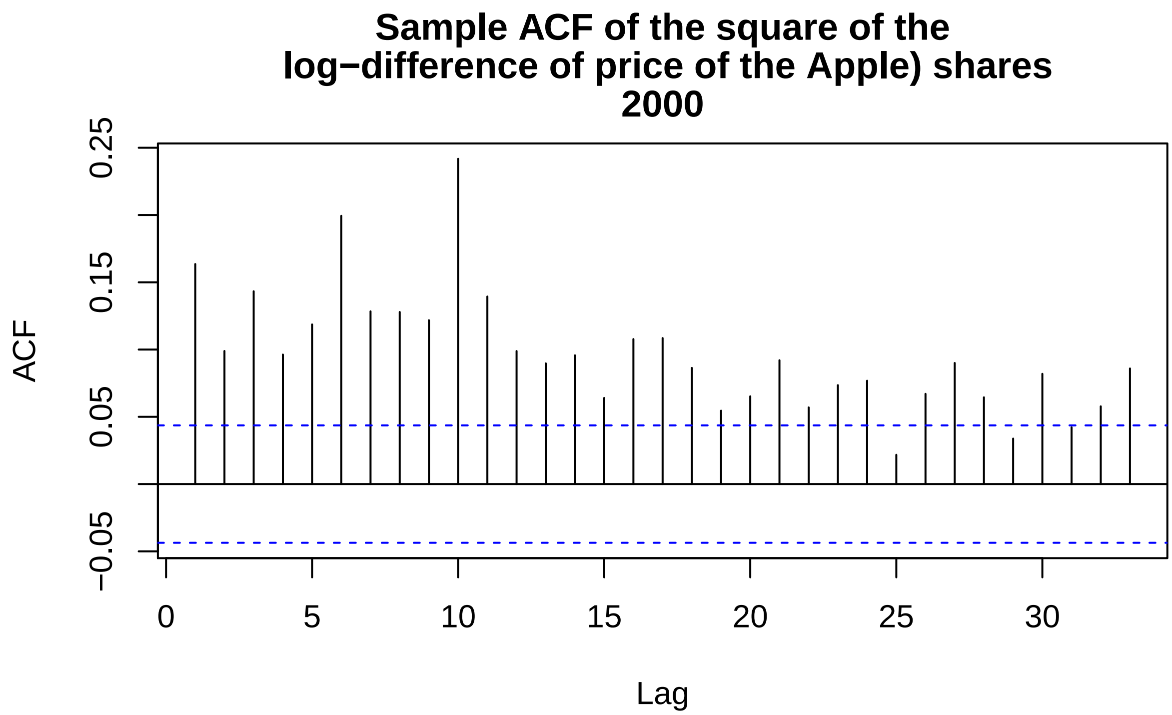
It does not look like the training data is stationary or white. Let us look further at the Log difference of this data.

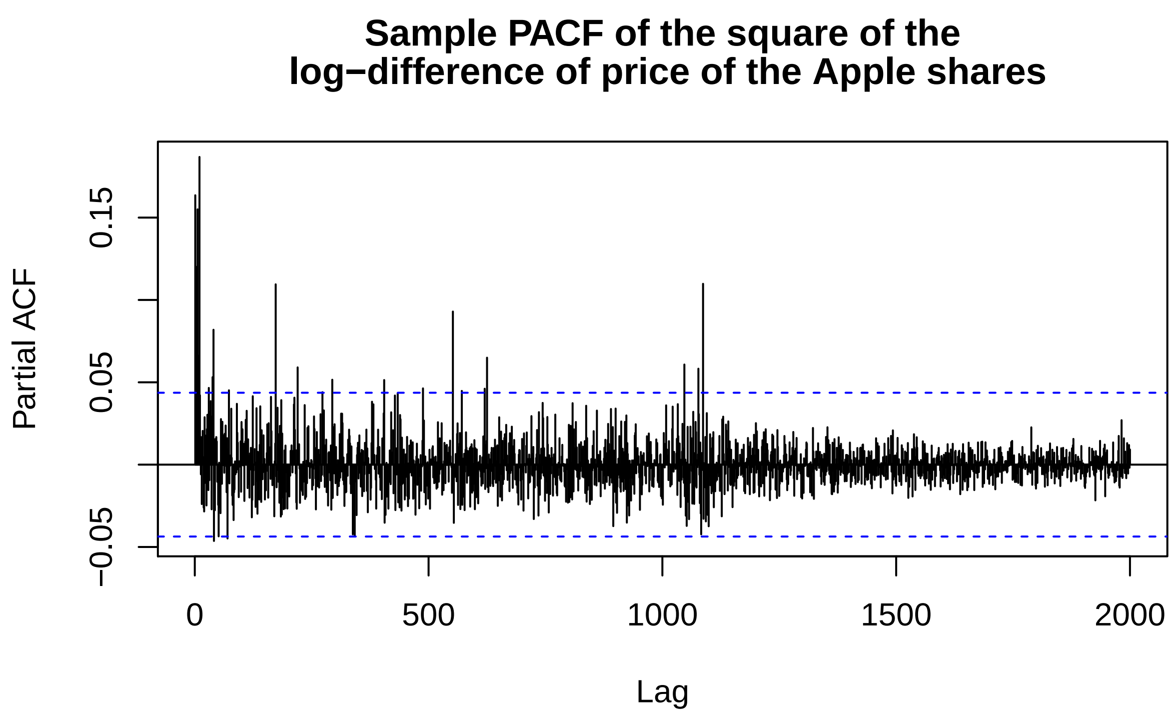

Log difference of price of the Apple shares
diff(log(dollars per share))
0.10
0.00
−0.10
−0.20
0
500
1000
1500
2000
Trading days after July 6th,2005


Sample ACF of the log difference of price of the Apple shares
ACF
0.05
0.00
−0.05
0
500
1000
1500
2000
Lag


Sample PACF of the log difference of price of the Apple shares
Partial ACF
0.06
0.02
−0.02
−0.06
0
500
1000
1500
2000
Lag


Square of the log−difference of price of the Apple shares
square(diff(log(dollars per share)))
0.04
0.03
0.02
0.01
0.00
0
500
1000
1500
2000
Trading days after July 6th,2005





In the log differenced data we see clear signs of volatility clustering, most clearly we see that the ACF has greater magnitude at lower lags, which is a sign of clustering. We can test the data and the square of the data using Box-Ljung on squared data, and EACF on squared data and absolute value of data below.

```r
Box.test(r_app2, type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  r_app2
## X-squared = 54.015, df = 1, p-value = 1.99e-13
```

```r
eacf(r_app2)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x x x  x  x  x
## 1 x x x o o x o o o x o  o  o  o
## 2 x x o o o x o o o x x  o  o  o
## 3 x x o x o x o o o x x  o  o  o
## 4 x o x x o o o o o x x  o  o  o
## 5 x x x x x o o o o x o  x  x  o
## 6 x x x x x x o o o x o  o  o  o
## 7 x x o x x x x o o x o  o  o  o
```

```r
eacf(abs(r_app))
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x x x  x  x  x
## 1 x o o o o o o o o o x  x  o  o
## 2 x x o o o o o o x o x  x  o  o
## 3 x x x o o o o o o o x  o  o  o
## 4 x x x x o o o o o o x  o  o  o
## 5 x x x x x o o o o o x  o  o  o
## 6 x x x o x x o o o o x  o  o  o
## 7 x x x o x o o o o o o  o  o  o
```

We interpret the results from the tests as indicating an GARCH model of order $(p, q) = (3, 1)$ or $(p, q) = (3, 2)$ as suitable for the log differenced data. Model picking shows..

```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     4.063608e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##      4     5.000000e-02     1.000e+00
##      5     5.000000e-02     1.000e+00
##      6     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF     PRELDF     RELDX    STPPAR    D*STEP    NPRELDF
```

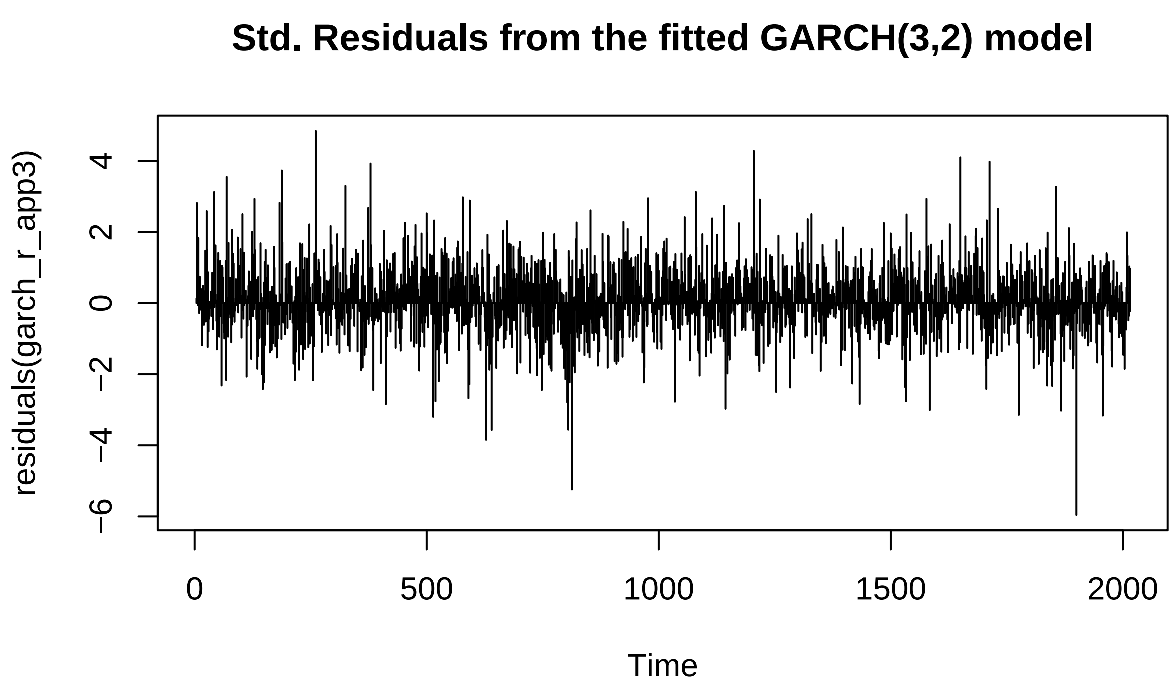
```
##      0    1 -6.617e+03
##      1    7 -6.620e+03  4.37e-04  7.11e-04  2.9e-04  5.7e+09  2.9e-05  2.02e+06
##      2    8 -6.620e+03  1.55e-05  1.74e-05  2.2e-04  2.0e+00  2.9e-05  2.21e+01
##      3   15 -6.653e+03  5.02e-03  7.08e-03  4.1e-01  2.0e+00  8.7e-02  2.19e+01
##      4   17 -6.681e+03  4.11e-03  3.85e-03  2.4e-01  2.0e+00  8.7e-02  7.37e+00
##      5   19 -6.724e+03  6.42e-03  7.95e-03  3.2e-01  2.0e+00  1.7e-01  1.20e+02
##      6   25 -6.724e+03  1.35e-05  2.32e-05  3.6e-06  3.5e+02  1.7e-06  4.47e-02
##      7   33 -6.724e+03  5.77e-05  7.74e-05  3.4e-02  2.0e+00  2.9e-02  2.33e-02
##      8   36 -6.725e+03  1.54e-04  1.63e-04  1.3e-01  1.8e+00  1.1e-01  8.37e-03
##      9   38 -6.726e+03  2.82e-05  2.96e-05  2.7e-02  2.0e+00  2.3e-02  1.65e-02
##     10   40 -6.729e+03  4.72e-04  5.89e-04  1.7e-01  9.2e-01  1.8e-01  1.28e-03
##     11   42 -6.730e+03  1.84e-04  1.87e-04  1.5e-02  1.5e+00  1.8e-02  9.77e-04
##     12   44 -6.736e+03  9.59e-04  8.87e-04  1.0e-01  4.4e-01  1.4e-01  1.02e-03
##     13   46 -6.739e+03  3.07e-04  5.04e-04  2.4e-02  1.1e+00  4.8e-02  1.02e-03
##     14   47 -6.741e+03  3.23e-04  4.81e-04  2.4e-02  1.3e+00  4.8e-02  1.26e-03
##     15   49 -6.741e+03  2.67e-05  3.31e-05  1.9e-03  1.7e+00  4.8e-03  1.33e-04
##     16   51 -6.741e+03  2.64e-05  4.01e-05  7.3e-03  7.7e-01  1.6e-02  6.47e-05
##     17   53 -6.741e+03  4.86e-06  9.11e-06  3.4e-03  7.8e-01  4.5e-03  1.39e-05
##     18   54 -6.741e+03  7.71e-07  9.04e-07  4.9e-03  0.0e+00  7.5e-03  9.04e-07
##     19   55 -6.741e+03  2.08e-07  2.00e-07  2.9e-03  0.0e+00  4.8e-03  2.00e-07
##     20   56 -6.741e+03  8.41e-08  5.16e-08  1.5e-03  0.0e+00  2.7e-03  5.16e-08
##     21   57 -6.741e+03  3.28e-08  1.20e-08  1.3e-03  0.0e+00  2.0e-03  1.20e-08
##     22   58 -6.741e+03  1.67e-09  8.20e-11  1.2e-04  0.0e+00  1.9e-04  8.20e-11
##     23   59 -6.741e+03  8.19e-11  2.11e-12  1.6e-05  0.0e+00  2.7e-05  2.11e-12
##     24   60 -6.741e+03 -1.51e-11  5.11e-15  6.1e-07  0.0e+00  1.0e-06  5.11e-15
##
##  ***** RELATIVE FUNCTION CONVERGENCE *****
##
##  FUNCTION    -6.741111e+03   RELDX        6.061e-07
##  FUNC. EVALS      60         GRAD. EVALS      24
##  PRELDF       5.109e-15      NPRELDF      5.109e-15
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    1.931311e-05     1.000e+00    -1.365e+00
##      2    7.823113e-02     1.000e+00    -1.825e-04
##      3    7.633125e-02     1.000e+00    -2.631e-04
##      4    1.801731e-01     1.000e+00    -2.749e-04
##      5    6.195782e-03     1.000e+00    -2.415e-04
##      6    6.249228e-01     1.000e+00    -3.116e-04
```

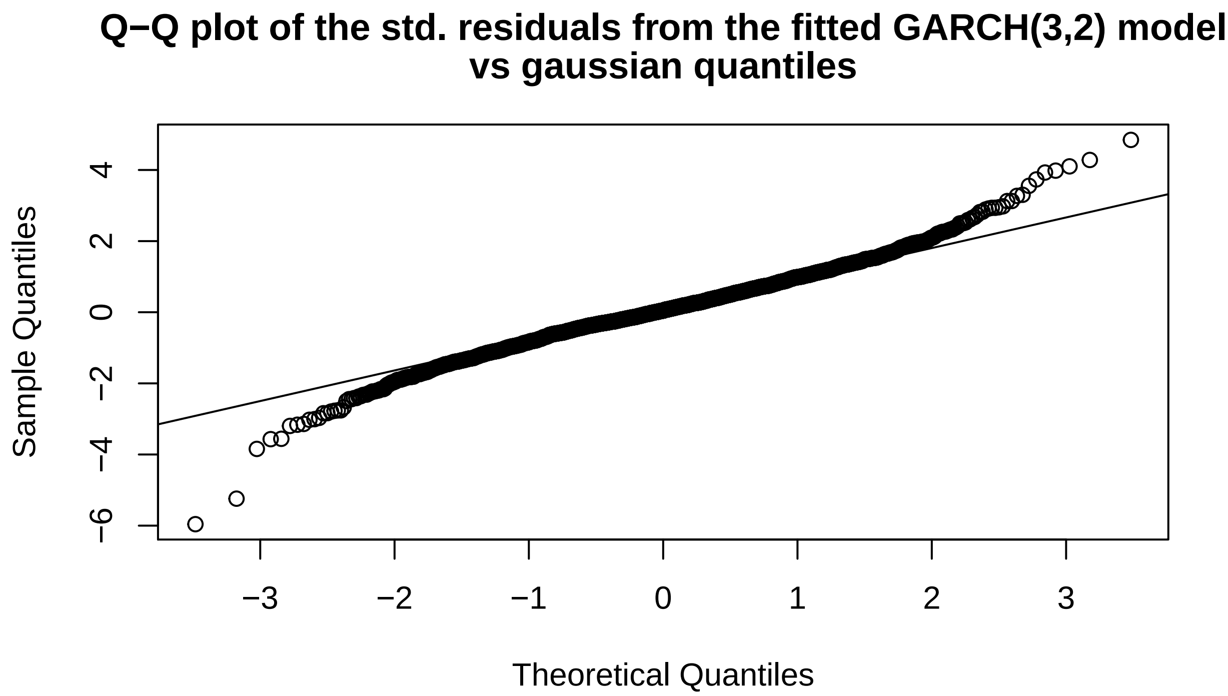
```r
summary(garch_r_app3)
```

```
##
## Call:
## garch(x = r_app, order = c(3, 2))
##
## Model:
## GARCH(3,2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -5.95884 -0.49647  0.04783  0.66403  4.84566
##
```

```
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0 1.931e-05   4.995e-06    3.866  0.00011 ***
## a1 7.823e-02   1.810e-02    4.322 1.54e-05 ***
## a2 7.633e-02   2.450e-02    3.115  0.00184 **
## b1 1.802e-01   1.507e-01    1.195  0.23201
## b2 6.196e-03   1.513e-01    0.041  0.96734
## b3 6.249e-01   7.845e-02    7.966 1.55e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
## X-squared = 397.3135, df = 2, p-value < 2.2e-16
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 0.0033, df = 1, p-value = 0.9541
```

.. That we are going to go with GARCH(3,2), it shows relative function convergence and it has non-significant Box-Ljung test of squared residuals. This means that the squared residuals magnitudes are white noise-like and pass a chi-squared test for significance at the established p-value. From the summary we see that coefficient b1 is maybe equal to zero and b2 is most probably equal to zero since they are not very significant.





We see that the residuals probably are not gaussian. None of the GARCH models we investigated had guassian residuals or non-significant Jarque-Bera tests.

2.c.

The one ahead predictor of conditional variance, $\hat{\sigma}^2_{t+1|t}(1)$ is easily computed by applying the definition of prediction to our knowledge position at time t.

$$\begin{aligned}
\hat{\sigma}^2_{t+1|t}(1) &= \mathbf{E}\left[\sigma^2_{t+1|t}|\sigma^2_{t|t-1}, \epsilon_t, r_t, \sigma^2_{t-1|t-2}, \dots\right] \\
&= \mathbf{E}\left[\omega + \beta_1\sigma^2_{t|t-1} + \beta_3\sigma^2_{t-2|t-3} + \alpha_1 r_t^2 + \alpha_2 r^2_{t-1}|\sigma^2_{t|t-1}, \epsilon_t, r_t, \sigma^2_{t-1|t-2}, \dots\right] \\
&= \omega + \beta_1\sigma^2_{t|t-1} + \beta_3\sigma^2_{t-2|t-3} + \alpha_1\mathbf{E}\left[r_t^2|\sigma^2_{t|t-1}, \epsilon_t, r_t, \sigma^2_{t-1|t-2}, \dots\right] + \alpha_2\mathbf{E}\left[r^2_{t-1}|\sigma^2_{t|t-1}, \epsilon_t, r_t, \sigma^2_{t-1|t-2}, \dots\right] \\
&= \omega + \beta_1\sigma^2_{t|t-1} + \beta_3\sigma^2_{t-2|t-3} + \alpha_1 r_t^2 + \alpha_2 r^2_{t-1}
\end{aligned}$$

We can also recursively compute the several ahead predictor of conditional variance $\hat{\sigma}^2_{t+h|t}(h)$ according to the technique in course book (12.2.12) if we wish to (and we do).