# From Regular Expressions To Deterministic Automata

**Paper**

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**Implementation & Presentation**

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# Paper Overview

- A new algorithm to go from regular expressions to DFAs
- Refines Brzozowski's elegant algorithm into McNaughton and Yamada's faster algorithm

# Paper Overview

**Brozowski's construction of a DFA**
Input: Regular Expression *E*
Output: DFA *D*

1. The states of D are the distinct derivatives w^{-1}E, for all strings *w*
2. Construct a transition under symbol *a* from state *p* to state *q* if and only if *p* is for derivative w^{-1}E, for some *w*, and *q* is for [(*wa*)^(-1)]E
3. The state for *E* is the start state.
4. A state is an accepting state if and only if it is for a derivative w^{-1}E, for some *w*, and delta([w^(-1)]E) = 1; that is the empty string is in [L(*wa*)^(-1)]E.

**McNaughton and Yamada's construction of a DFA**
Input: Marked Regular Expression *E'*
Output: DFA *M'*

1. *M'* has a state for the continuation of each marked symbol in *E'*
2. Construct a transition under symbol *a* from state *p* to state *q* for the continuation of *a* if and only if *p* is for some continuation *C* and *C* can generate a string with a leading *a*
3. The state for the entire expression *E'* is the start state.
4. A state is an accepting state if and only if it is for a continuation *C* and delta(*C*)=1

# Paper Overview

**Brozowski's construction of a DFA**
Input: Regular Expression *E*
Output: DFA *D*

- Introduces the derivative and delta of a regular expression

**McNaughton and Yamada's construction of a DFA**
Input: Marked Regular Expression *E'*
Output: DFA *M'*

- Introduces marked regular expressions

**New Algorithm**
Input: Marked Regular Expression *E'*
Output: DFA *M'*

- Constructs transitions using *first(E')* and *follow(b)*, where *b* is a state
- Accepting states are found by testing if a special end character is in *follow(b)*

# Take away from Brzozowski

Delta Notation

- $\delta(E)$ stands for *1* if $L(E)$ contains the empty string
- $\delta(E)$ is computed recursively on $E$
  - $\delta(0) = 0$
  - $\delta(1) = 1$ (where *1* is taken to mean $\varepsilon$)
  - $\delta(E_1 \cup E_2) = \delta(E_1) + \delta(E_2)$
  - $\delta(E_1E_2) = \delta(E_1) \cdot \delta(E_2)$
  - $\delta(E^*) = 1$
- It is clear then that $\delta(E)$ represents the number of possibilities the expression has of containing the empty string $\varepsilon$.

# Take aways from McNaughton and Yamada

## Marked Expressions

- Mark all input symbols in a regular expression
  - *(ab ∪ b)* ba* becomes $(a_1b_2 \cup b_3)^* b_4a_5$
- These subscripted symbols are distinct and will become the states of an automaton

## First

- $first(E) = \{a : av \in L(E)\}$
- The set of possible first symbols in the language of a regular expression *E*

## Follow

- $follow_E(a) = \{b : uabv \in L(E)\}$
- The set of possible symbols that come immediately after the symbol *a* in the language of *E*
- For a marked symbol $a_1$, *follow*($a_1$) will contain the set of marked symbols corresponding to the next possible states in the automaton

# New Algorithm

Input: Marked Regular Expression *E'*
Output: DFA *M'*

1. *M'* has a start state plus a state for each marked symbol *a* in *E'*
2. Construct a transition from the start state to the state for *a* if and only if *a* is in *first(E')*
3. Construct a transition from the state *b* to the state *a* if and only if *a* is in *follows(b)*
4. The start state is an accepting state if and only if *delta(E')*=1
5. The state for *a* is an accepting state if and only if ! is in *follows(a)*

# General Idea of Using New Algorithm

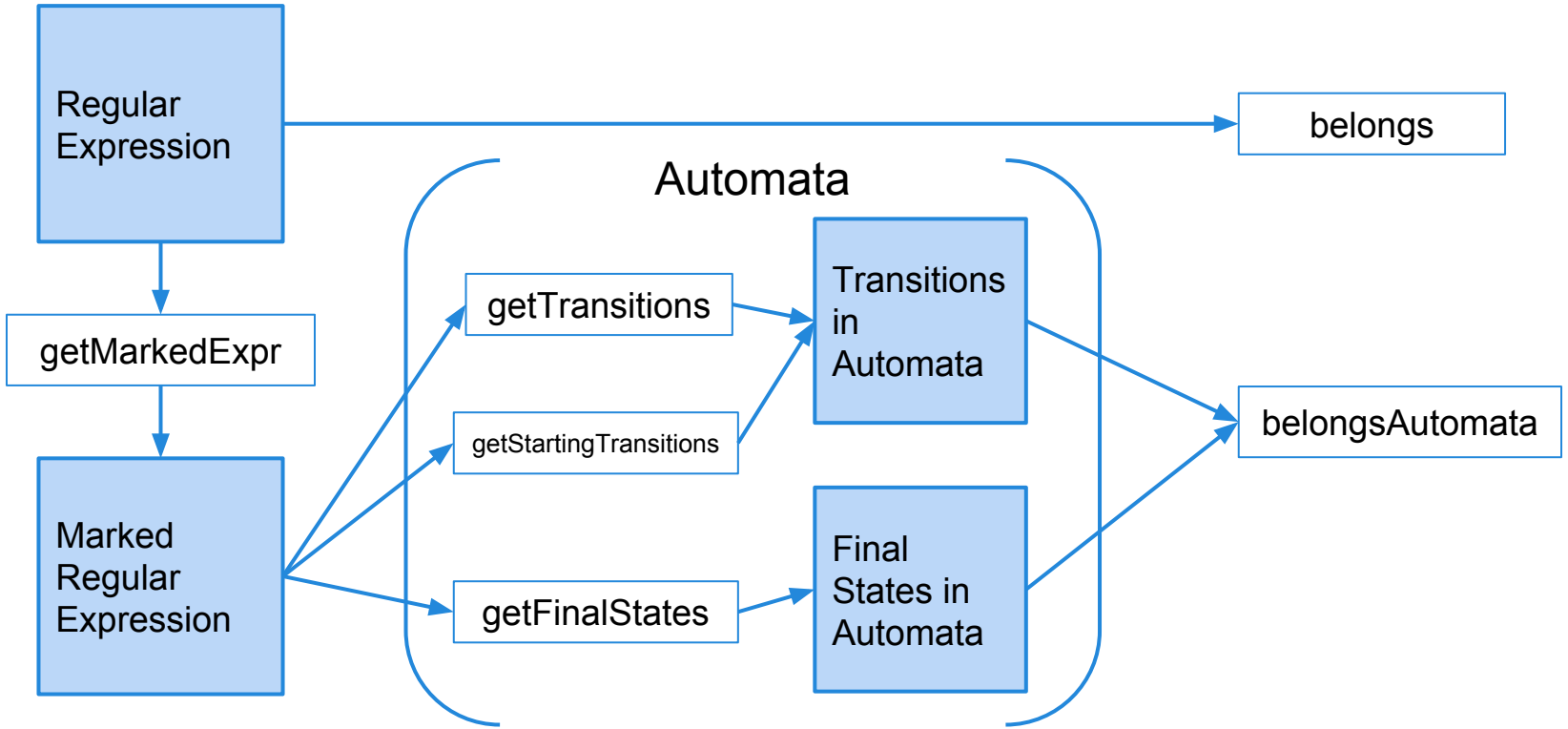
1. Construct an automata from a marked expression
2. The marks on the transitions are then erased, resulting in an NFA for the original unmarked expression

- Note: This approach works for the usual operations of union, concatenation, and iteration, but does not work with intersection and complement. This is because marking and unmarking do not preserve the language generated by  regular expressions with these operations

# Project Implementation Overview

- We originally intended to implement all 3 algorithms, however we only implemented the new algorithm

# What we did implement



# Implementation

- We used OCaml's algebraic types extensively.
  - Regex, consisting of a char, a Union, a Concatenation, a Star, or Epsilon.
  - These are all custom datatypes that use Cartesian product with regex.

```
type regex =
        | Eps
        | Lit of char
        | Union of regex * regex
        | Concat of regex * regex
        | Star of regex;;
```

```
type markedRegex =
        | MEps
        | MLit of char * int
        | MUnion of markedRegex * markedRegex
        | MConcat of markedRegex * markedRegex
        | MStar of markedRegex;;
```

# Implementation

## Regular Expression Functions

(marked & unmarked)

- belongs
  - In: string and expr
  - Out: is the string accepted by expr
- delta
  - In: expr
  - Out: *delta(expr)* --> the potential for a string in the expr to include epsilon
- follow
  - In: expr and symbol
  - Out:The set of possible symbols that come immediately after the symbol *a* in the language of *E*
- first
  - In: expr
  - Out: a list of starting symbols in the expression
- last
  - In: expr
  - Out: a list of ending symbols in the expression

# Implementation

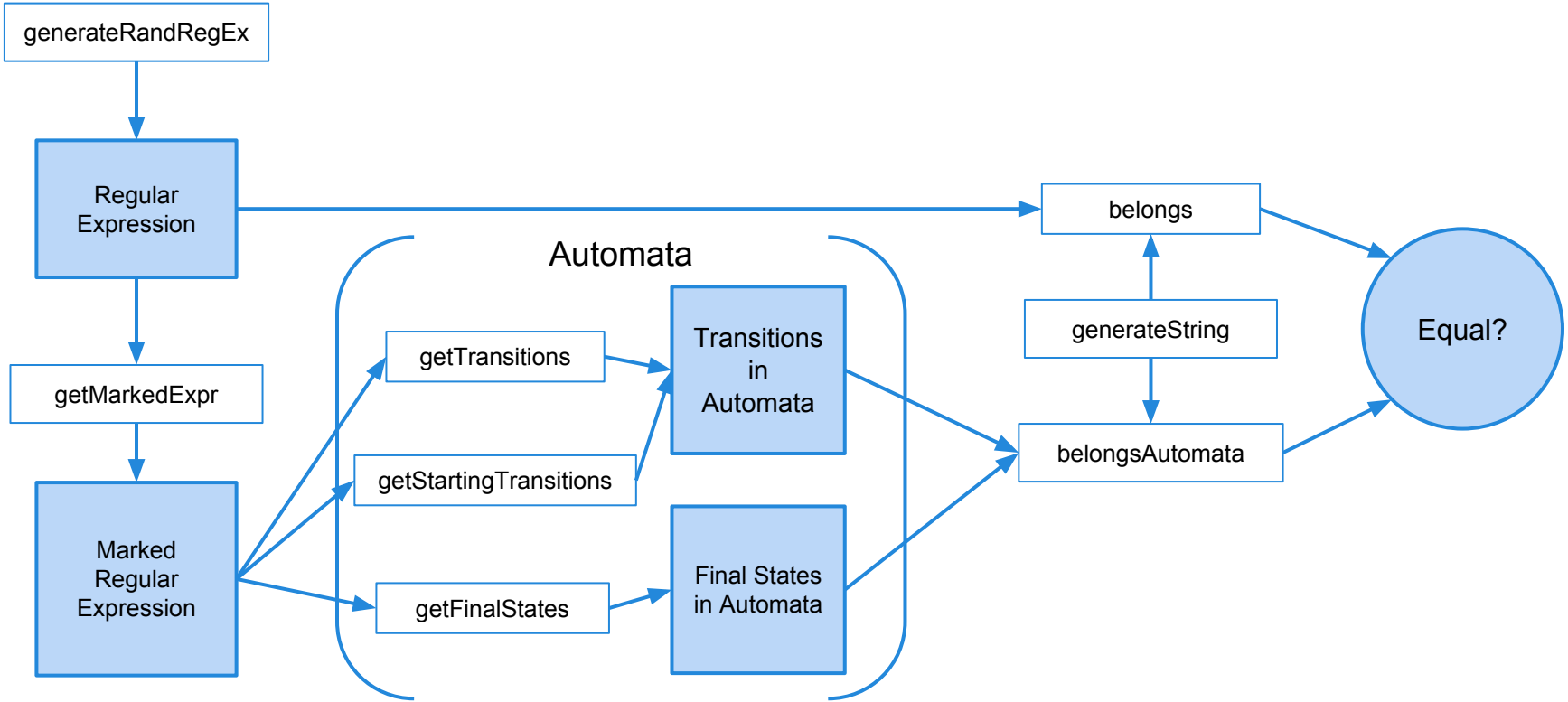
## Automata

- belongs:
  - In: string, transitionlist, finalstates and currentstate
  - Out: is string accepted by automata
- getStates:
  - In: expr
  - Out: states
- getTransitions:
  - In: expr
  - calls two helper functions getTransitionsFromStates expr getStates
  - Out: prepares a list of transitions
- getfinalsates:
  - In: expr and states
  - Out: accepting states
- getNextStates:
  - In: transitionLists and currentstate
  - Out: possible next states
- getStartingTransitions:
  - In: expr and startingStates
  - Out: transitions from first to next state

# Testing the Implementation

- Created a function test
  - make an alphabet
  - randomly generates
    - string
    - regex
  - makes automata from regex
  - calls belong on regex and automata
  - compares the two

# Testing the Implementation



# Script

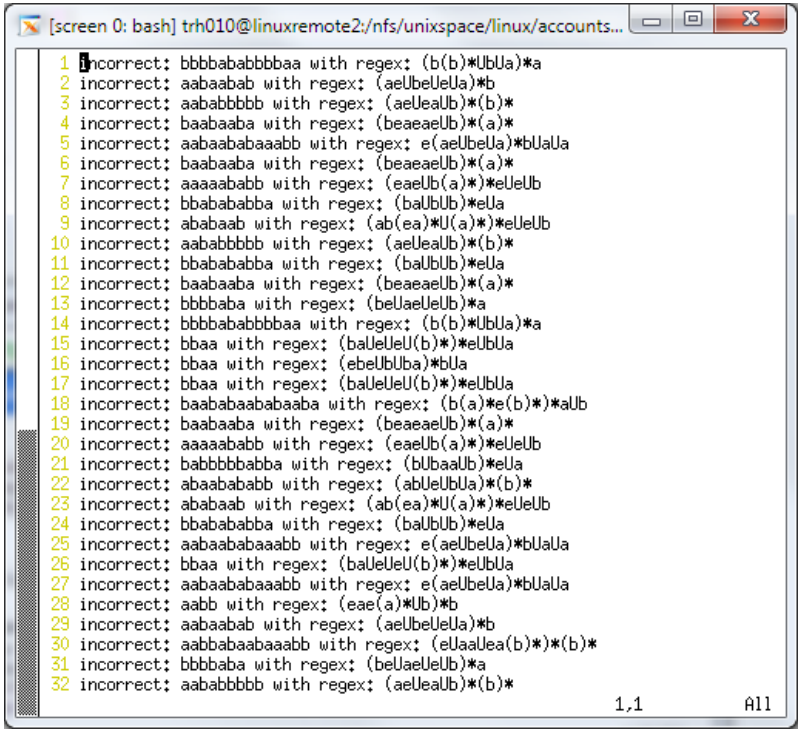
- Script runs endlessly concatenating output to a file in which we grep errors.

```
#!/bin/bash
echo running script
while true
do
        ./proj_phase3.mlo >> output.txt 2>> errors.txt
done
exit
```

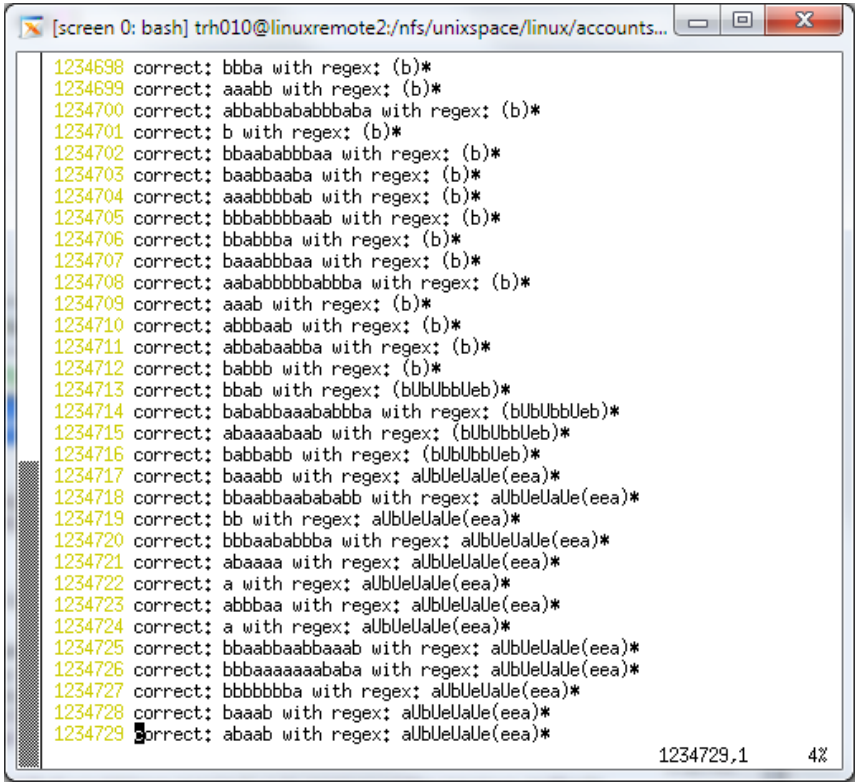
```
#!/bin/bash
cat output.txt | grep "incorrect"
exit
```

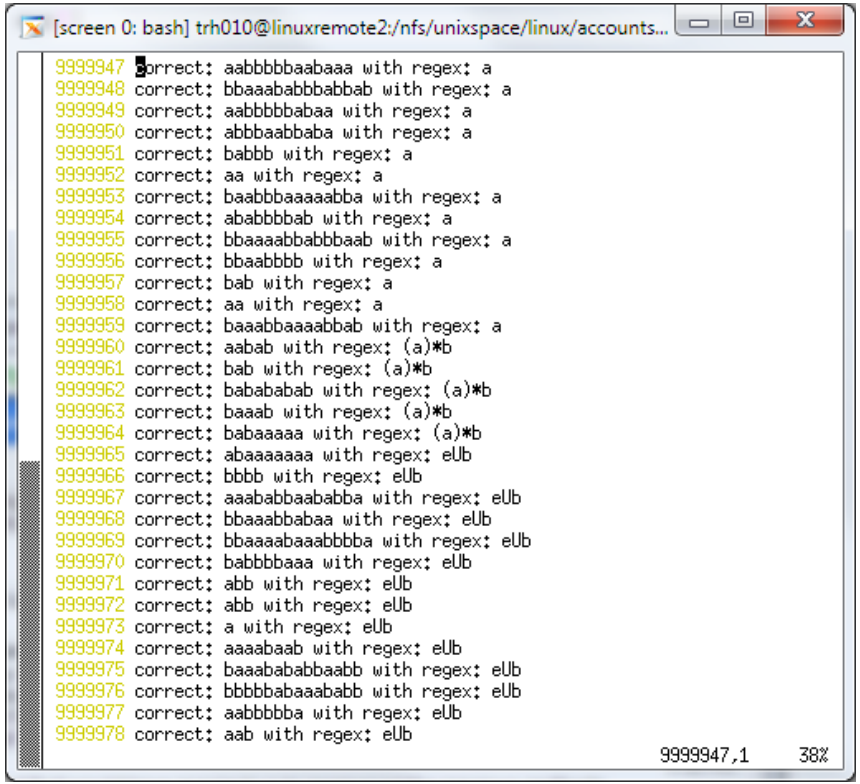
# Results

- Test cases: 25,107,109
  (actually still running…)
  - generated regular expressions are not minimized
- Incorrect cases: 32
- Accuracy: 99.999872546%
- So we made a small mistake somewhere...

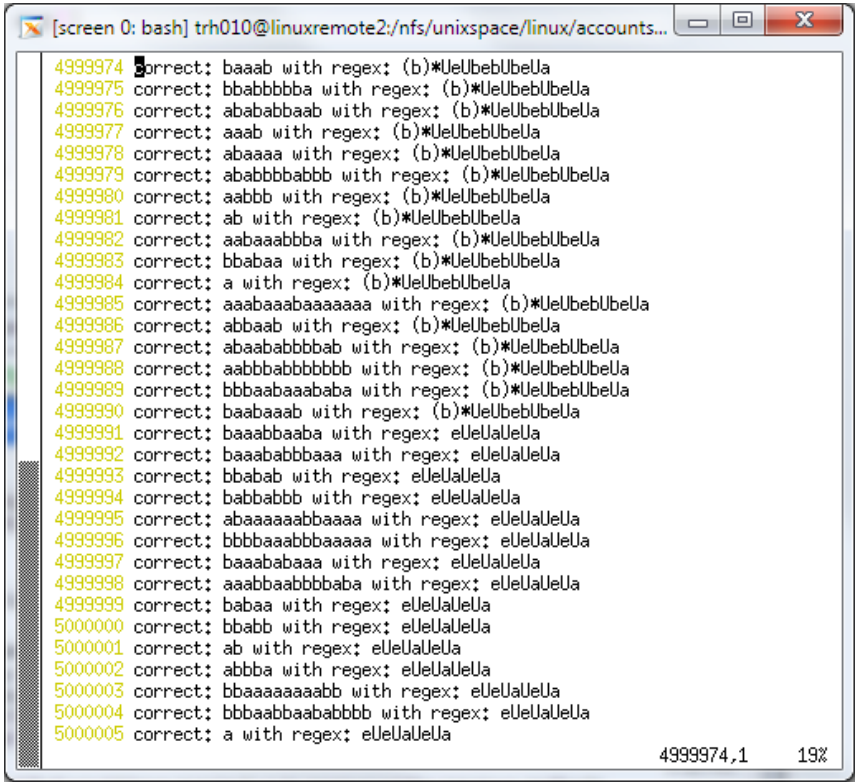


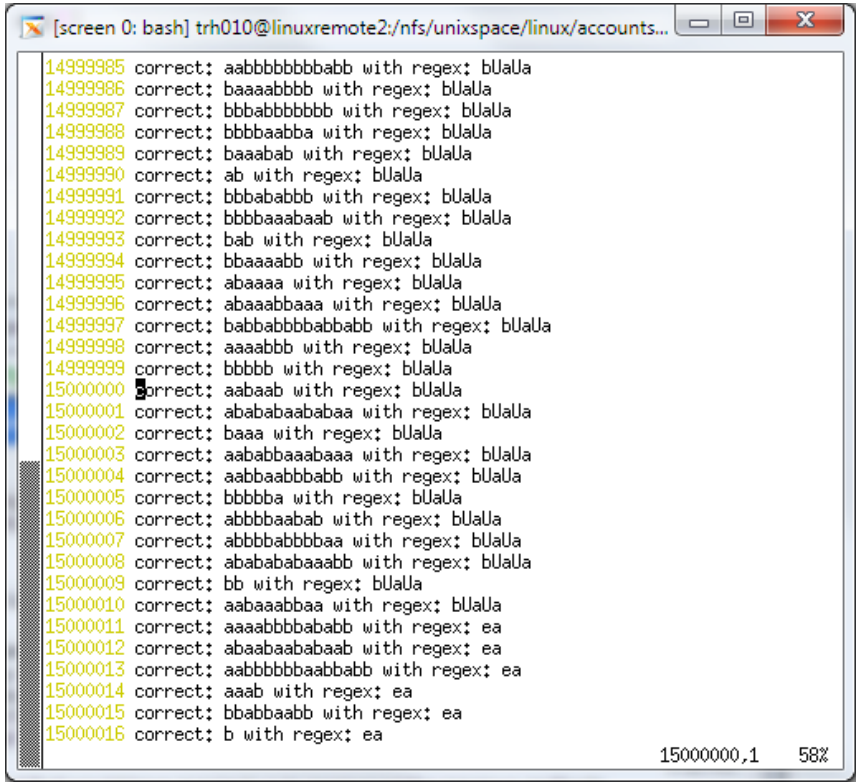
# We have a lot correct though...



# and more...


[screen 0: bash] trh010@linuxremote2:/nfs/unixspace/linux/accounts...
4999974 correct: baaab with regex: (b)*UeUbebUbeUa
4999975 correct: bbabbbbba with regex: (b)*UeUbebUbeUa
4999976 correct: ababbbaab with regex: (b)*UeUbebUbeUa
4999977 correct: aaab with regex: (b)*UeUbebUbeUa
4999978 correct: abaaaa with regex: (b)*UeUbebUbeUa
4999979 correct: ababbbbabbb with regex: (b)*UeUbebUbeUa
4999980 correct: aabbb with regex: (b)*UeUbebUbeUa
4999981 correct: ab with regex: (b)*UeUbebUbeUa
4999982 correct: aabaaabbba with regex: (b)*UeUbebUbeUa
4999983 correct: bbabaa with regex: (b)*UeUbebUbeUa
4999984 correct: a with regex: (b)*UeUbebUbeUa
4999985 correct: aaabaaabaaaaaaa with regex: (b)*UeUbebUbeUa
4999986 correct: abbaab with regex: (b)*UeUbebUbeUa
4999987 correct: abaabaabbbbab with regex: (b)*UeUbebUbeUa
4999988 correct: aabbbabbbbbbb with regex: (b)*UeUbebUbeUa
4999989 correct: bbbaabaaababa with regex: (b)*UeUbebUbeUa
4999990 correct: baabaaab with regex: (b)*UeUbebUbeUa
4999991 correct: baaabbaaba with regex: eUeUaUeUa
4999992 correct: baaababbbaaa with regex: eUeUaUeUa
4999993 correct: bbabab with regex: eUeUaUeUa
4999994 correct: babbabbb with regex: eUeUaUeUa
4999995 correct: abaaaaaabbaaaa with regex: eUeUaUeUa
4999996 correct: bbbbaaabbbaaaaa with regex: eUeUaUeUa
4999997 correct: baaababaaa with regex: eUeUaUeUa
4999998 correct: aaabbaabbbbaba with regex: eUeUaUeUa
4999999 correct: babaa with regex: eUeUaUeUa
5000000 correct: bbabb with regex: eUeUaUeUa
5000001 correct: ab with regex: eUeUaUeUa
5000002 correct: abbba with regex: eUeUaUeUa
5000003 correct: bbaaaaaaaabb with regex: eUeUaUeUa
5000004 correct: bbbaabbaabaabbbb with regex: eUeUaUeUa
5000005 correct: a with regex: eUeUaUeUa
4999974,1 19%


[screen 0: bash] trh010@linuxremote2:/nfs/unixspace/linux/accounts...
14999985 correct: aabbbbbbbbabb with regex: bUaUa
14999986 correct: baaaabbbb with regex: bUaUa
14999987 correct: bbbabbbbbbb with regex: bUaUa
14999988 correct: bbbbaabba with regex: bUaUa
14999989 correct: baaabab with regex: bUaUa
14999990 correct: ab with regex: bUaUa
14999991 correct: bbbabab bb with regex: bUaUa
14999992 correct: bbbbaaabaab with regex: bUaUa
14999993 correct: bab with regex: bUaUa
14999994 correct: bbaaaabb with regex: bUaUa
14999995 correct: abaaaa with regex: bUaUa
14999996 correct: abaaabbaaa with regex: bUaUa
14999997 correct: babbabbbbabbabb with regex: bUaUa
14999998 correct: aaaabbb with regex: bUaUa
14999999 correct: bbbbb with regex: bUaUa
15000000 correct: aabaab with regex: bUaUa
15000001 correct: abababaababaa with regex: bUaUa
15000002 correct: baaa with regex: bUaUa
15000003 correct: aababbaaabaaa with regex: bUaUa
15000004 correct: aabbaabbbabb with regex: bUaUa
15000005 correct: bbbbba with regex: bUaUa
15000006 correct: abbbbaabab with regex: bUaUa
15000007 correct: abbbbabbbbaa with regex: bUaUa
15000008 correct: abababababaabb with regex: bUaUa
15000009 correct: bb with regex: bUaUa
15000010 correct: aabaaabbaa with regex: bUaUa
15000011 correct: aaaabbbbabab with regex: ea
15000012 correct: abaabaabababaab with regex: ea
15000013 correct: aabbbbbbaabbabb with regex: ea
15000014 correct: aaab with regex: ea
15000015 correct: bbabbaabb with regex: ea
15000016 correct: b with regex: ea
15000000,1 58%

# Questions?