<u>Def</u> : Let Ω be some abstract space.
If $\mathcal{C}_1 \subset \mathcal{C}_2$ & let μ_i be a measure defined on $\mathcal{C}_i$, $i = 1, 2$ such that

$$\mu_1(C_1) = \mu_2(C_1) \quad \forall\, C_1 \in \mathcal{C}_1$$

i.e. μ_1 & μ_2 agrees on all sets in $\mathcal{C}_1$. Then μ_1 is known as a restriction of μ_2 on $\mathcal{C}_1$ and μ_2 is known as an extension of μ_1 on $\mathcal{C}_2$.

<u>Example</u> : Let $\Omega = R$.
Define $\mathcal{C}_1 = \{\phi, \Omega\}$, $\mathcal{C}_2 = \{\phi, \Omega, A, A'\}$,
$\mathcal{C}_3 = \{\phi, \Omega, B, B'\}$

$\mu_1 : \mathcal{C}_1 \to R$ s.t. $\mu_1(\phi) = 0$ & $\mu_1(\Omega) = 1$.

$\mu_2 : \mathcal{C}_2 \to R$ s.t. $\mu_2(\phi) = 0$, $\mu_2(\Omega) = 1$,
$\mu_2(A) = \mu_2(A') = \frac{1}{2}$

$\mu_3 : \mathcal{C}_3 \to R$ s.t. $\mu_3(\phi) = 0$, $\mu_3(\Omega) = 1$
$\mu_3(B) = 1/3$, $\mu_3(B') = 2/3$.

Note that $\mathcal{C}_1 \subset \mathcal{C}_2$, $\mathcal{C}_1 \subset \mathcal{C}_3$

μ_1 agrees with μ_2 on $\mathcal{C}_1$
μ_1 '' '' μ_3 on $\mathcal{C}_2$

$\Rightarrow$ μ_1 is restriction of μ_2 on $\mathcal{C}_1$
& μ_1 is '' '' μ_3 on $\mathcal{C}_1$

OR

μ_2 is extension of μ_1 on $\mathcal{C}_2$.
μ_3 '' '' '' '' '' $\mathcal{C}_3$.

Note that there can be more than one extensions of a measure from $\mathcal{C}$ to $\mathcal{C}^*$, where $\mathcal{C} \subset \mathcal{C}^*$.

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