Exam Seat No. ______________

38.a/45

THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA

Date:30.11.2024 **FS MSC I Examination** **Time:**

Day: Saturday **STA2101C01 (Measure Theory)** **11.30 AM to 2.30 PM**

(4 credits) **Total marks:70**

***Instructions*: (i) Write all the answers in the answer book.**

(ii) Numbers on the right side indicates marks of the respective questions.

1 (a) In each of the following questions, there are two statements α and β. Read the statements and choose the correct alternative from the following.

(A) only α is true (B) only β is true

(C) both α and β are true (D) Neither α nor β is true. (10x2=20)

(i) (α) If $A_n \uparrow$, then $\lim A_n = \cup_{n=1}^{\infty} A_n$
C
(β) If $A_n \downarrow$, then $\lim A_n = \cap_{n=1}^{\infty} A_n$

(ii) (α) A monotone field is a σ –field.
A (β) A monotone class is a σ –field.

(iii) (α) Intersection of two σ –fields is a σ –field.
A (β) Union of two σ –fields is a σ –field.

(iv) (α) A finite set function is always σ – finite.
B
(β) A σ – finite set function is always finite.

(v) (α) A Baire function of a measurable function is also measurable.
C (β) A Borel function of a measurable function is also measurable.

(vi) (α) if $|X|$ is measurable, then X is also measurable.
A (β) If X is measurable then $|X|$ is also measurable.

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38.b / 45

(vii) (α) A measurable function f is said to be integrable if $\int f\, d\mu$ exists.
(β) A measurable function f is said to be integrable if anyone from
$\int f^{+} d\mu$ or $\int f^{-} d\mu$ is finite.
should be and.

(viii) (α) If $A_n \uparrow A$, then $\mu(A_n) \uparrow \mu(A)$.
A (β) If $A_n \downarrow A$, then $\mu(A_n) \downarrow \mu(A)$.

(ix) (α) Sections or measurable sets are measurable.
C (β) Sections or measurable functions are measurable.

(x) (α) A finitely additive set function is always countably additive.
C (β) A countably additive set function ψ is finitely additive if $\psi(\phi) = 0$.

(b) Choose the correct alternative. (4x2=8)

(xi) According to Fatou's lemma, which of the following is correct?
(A) $\int \liminf f_n\, d\mu \;\leq\; \lim inf \int f_n\, d\mu$
(B) $\int \liminf f_n\, d\mu \;\geq\; \lim inf \int f_n\, d\mu$
(C) $\int \limsup f_n\, d\mu \;\leq\; \lim sup \int f_n\, d\mu$ (D) None of these.

(xii) Which of the following measure/s is/are finite?
(α) Lebesgue measure (β) Lebesgue -Stiltjes measure (δ) counting Measure
(A) Only α (B) only β (C) only δ (D) all the three.

(xiii) " Indefinite integrals are σ- additive". This result is application of which of the following ?
(A) Lebesgue's decomposition theorem
(B) Monotone convergence theorem
(C) Lebesgue's dominated convergence theorem
(D) Fatou's lemma.

(xiv) " Taking derivative inside the integral sign" is application of which of the following ?
(A) Lebesgue's decomposition theorem
(B) Monotone convergence theorem
(C) Lebesgue's dominated convergence theorem
(D) Fatou's lemma.

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2. (a) Prove that $\lim\inf A_n \subset \lim\sup A_n$. (4)

(b) Prove that a monotone field is a σ –field. (2)

(c) Let $A_n = [-1, \frac{1}{n}), n \geq 1$. Check whether $\lim A_n$ exists or not. (3)

OR

(c) Explain how to generate Borel σ –field? (3)

3. (a) Prove that $\lim\inf \mu(A_n) \geq \mu(\lim\inf A_n)$. (5)

(b) Let $F(x) = \begin{cases} 0 & \text{if } x < -3 \\ \frac{1}{4} & \text{if } -3 \leq x < 0 \\ \frac{x+1}{4} & \text{if } 0 \leq x < \frac{1}{2} \\ 1 & \text{if } x \geq \frac{1}{2}. \end{cases}$

Previous Paper

Find L-S measure μ_F of the following sets.

(i) $A = \{0, \frac{1}{2}\}$, (ii) $B = (0, \frac{1}{2})$ (iii) $C = [-3, 0]$ (iv) $(-3, \frac{1}{2}]$ (4)

2/3 1/3 1/4 5/6

(c) Define finitely additive set function and countably additive set function (4)

OR

(c) Define finite set function and σ – finite set function. (4)

4. (a) Define(i) Convergence in Measure (ii) Convergence a.e. (4)

(b) State the three linearity properties and three order preserving properties associated with integration. (6)

OR

(b) State and prove Monotone convergence theorem. (6)

Exam Seat No. 09

Department of Statistics

Date:23.09.2024 | Mid Semester Examination(M.Sc. Sem 1) | Time:

Day: Monday | STA2101C01 (Measure Theory) | 3.00 to 4.00 PM

Instructions: (i) Write all the answers in the answer book. (ii) Max marks: 30

(ii) Numbers on the right side indicates marks of the respective questions.

1. In each of the following questions, there are two statements α and β.

Read the statements and choose the correct alternative from the following.

(A) only α is true (B) only β is true

(C) both α and β are true (D) Neither α nor β is true. (6x2=12)

(i) (α) Every subset of $\mathcal{R}$ (real line) is a Borel set.
(β) Every Borel set is a subset of $\mathcal{R}$.

(ii) (α) Every finite field is a σ –field.
(β) Every σ –field is a field.

(iii) (α) A finitely additive set function is always countably additive.
(β) A countably additive set function ψ is finitely additive if $\psi(\phi) = 0$.

(iv) (α) Almost everywhere (a.e.) convergence imply convergence in measure provided the measure is finite.
(β) If the measure is not finite, then a.e. convergence may or may not imply convergence in measure.

(v) (α) A finite set function is always σ – finite.
(β) A σ – finite set function is always finite.

(vi) (α) If $A_n \uparrow A$, then $\mu(A_n) \uparrow \mu(A)$
(β) If $A_n \downarrow A$, then $\mu(A_n) \downarrow \mu(A)$.

2. Check whether $\lim\limits_{n\to\infty} A_n$ exists or not for the sequence $A_n = (\frac{-1}{2n+1}, \; 1 - \frac{1}{2n}]$. (4)

3. Prove that a field is a σ- field if and only if it is a monotone class. (4)

4. Prove that $\mu(\limsup A_n) \geq \limsup \mu(A_n)$. (4)

5. State the constructive definition of a measurable function.(2)

6. Define convergence almost everywhere(a.e.). Derive the criterion for a.e. convergence. (4)

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