Exam Seat No. ____________________

Department of Statistics

Date:23.09.2024 Mid Semester Examination(M.Sc. Sem 1) Time:

Day: Monday STA2101C01 (Measure Theory) 3.00 to 4.00 PM

Instructions: (i) Write all the answers in the answer book. (ii) Max marks: 30

(ii) Numbers on the right side indicates marks of the respective questions.

1. In each of the following questions, there are two statements α and β.

   Read the statements and choose the correct alternative from the following.

   (A) only α is true (B) only β is true

   (C) both α and β are true (D) Neither α nor β is true. (6x2=12)

   (i) (α) Every subset of $\mathcal{R}$ (real line) is a Borel set.
   (β) Every Borel set is a subset of $\mathcal{R}$. c

   (ii) (α) Every finite field is a σ –field.
   (β) Every σ –field is a field. C

   (iii) (α) A finitely additive set function is always countably additive.
   (β) A countably additive set function ψ is finitely additive if $\psi(\phi) = 0$. B

   (iv) (α) Almost everywhere (a.e.) convergence imply convergence in measure provided the measure is finite. C
   (β) If the measure is not finite, then a.e. convergence may or may not imply convergence in measure.

   (v) (α) A finite set function is always σ – finite.
   (β) A σ – finite set function is always finite. A

   (vi) (α) If $A_n \uparrow A$, then $\mu(A_n) \uparrow \mu(A)$.
   (β) If $A_n \downarrow A$, then $\mu(A_n) \downarrow \mu(A)$. c

2. Check whether $\lim_{n\to\infty} A_n$ exists or not for the sequence $A_n = (\frac{-1}{2n+1},\ 1 - \frac{1}{2n}]$. (4)

3. Prove that a field is a σ- field if and only if it is a monotone class. (4)

4. Prove that $\mu(\limsup A_n) \geq \limsup \mu(A_n)$ (4)

5. State the constructive definition of a measurable function.(2)

6. Define convergence almost everywhere(a.e.). Derive the criterion for a.e. convergence. (4)

****

Date:24-9-2024	Department of statistics	Time:3.00PM-4.00PM
Day: Tuesday	M.Sc. Sem I (Mid Sem sept 2024) STA2102C02 -Linear Models(4 credits)	Marks:30

Q-I Answer briefly following questions[2 x 6] [12]

1. State Gauss Markov linear model with assumptions
2. Define parametric function.
3. In the usual notations state the necessary and sufficient condition for the linear parametric function $l'\beta$ of the parameters to be linearly estimable. $\underline{C}$ $l'\beta$
4. Define estimation space.
5. For what purpose Scheffe's test is used?
6. State the test statistics for Tukey's test.

Q-II Do as Directed [18]

1. Explain in how many broad areas Gauss Markov linear model may be classified. Explain each of them. [4]
2. Explain the method of estimating parameters when the parameters are subject to a set of consistent linear restrictions, also state the model. [5]

OR

2. In the usual notations prove that $R_0^2 \sim \sigma^2 \chi^2_{(n-r)}$ where r is the rank of matrix X. $\lambda = \underline{\beta}' A \underline{\beta}$ [5]
3. Consider three independent random variables, Y_1, Y_2 and Y_3 having common variance σ^2 and expectations $E(Y_1) = \mu_1 + \mu_3$, $E(Y_2) = \mu_1 + \mu_2$, and $E(Y_3) = \mu_1 + \mu_3$ Determine the condition of estimability of the parametric function $l'\mu = l_1\mu_1 + l_2\mu_2 + l_3\mu_3$ [3]
4. Explain the use of OC curves in linear models also explain for what purpose power of F test is used? [6]

Exam Seat No. ____________

Mid Semester Examination
Faculty of Science
Department of Statistics
M.Sc. Semester–I Examination
STA2111C11 : Sampling Theory

Date : 27-09-2024
Day: Friday
Time : 3.00 to 4.00 pm

Q1. Choose the correct alternative (12x1=12)

1. The Horvitz-Thompson estimator is defined by
   (a) $\sum_i^n \frac{Y_i}{\pi_i}$ (b) $\frac{Y_i}{\pi_i}$ (c) $Y_i\pi_i$ (d) $\frac{\pi_i}{Y_i}$
2. Sampling frame is a term used for
   (a) a list of random numbers (b) a list of voters
   (c) a list of sampling units of population (d) none of the above
3. An unbiased estimator of the gain due to ppswr sampling as compared to srswr is
   (a) $\frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y_i^2}{P_i}\right)\left(N-\frac{1}{P_i}\right)$ (b) $\frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y_i^2}{P_i}\right)\left(n-\frac{1}{P_i}\right)$
   (c) $\frac{1}{n^2}\sum_{i=1}^{n}\left(\frac{Y_i^2}{P_i}\right)\left(N-\frac{1}{P_i}\right)$ (d) none of the above.
4. Random Group Method" is better than "probability Proportional to Size with replacement" whenever the groups are of
   (a) different size (b) proportional size
   (c) same size (d) none of the above
5. For effective stratification, strata should be formed in such a way that units within each strata are homogeneous w.r.t. the variable
   (a) X (b) $\frac{Y_{ij}}{P_{ij}}$ (c) Y (d) none of the above
6. If $Y_i \propto X_i$ in PPS Sampling then
   (a) $V_{pps}(\hat{Y}_{unst}) < 2\,V_{prop}(\hat{Y}_{st})$ (b) $V_{pps}(\hat{Y}_{unst}) = V_{prop}(\hat{Y}_{st})$
   (c) $V_{pps}(\hat{Y}_{unst}) \le V_{prop}(\hat{Y}_{st})$ (d) $V_{pps}(\hat{Y}_{unst}) \ge V_{prop}(\hat{Y}_{st})$
7. In random group method
   (a) $V_1E_2[\hat{Y}_{RHC}|G_1, \dots, G_n] > 0$ (b) $V_1E_2[\hat{Y}_{RHC}|G_1, \dots, G_n] < 0$
   (c) $V_1E_2[\hat{Y}_{RHC}|G_1, \dots, G_n] \neq 0$ (d) $V_1E_2[\hat{Y}_{RHC}|G_1, \dots, G_n] = 0$
8. A larger sample size is required when:
   (a) The population of interest for a study is less diverse
   (b) A low level of precision is required
   (c) The population of interest is easily recruited to the study
   (d) A high level of precision is required
9. If larger units have greater probability of their inclusion in the sample, it is known as
   (a) selection with replacement (b) selection with probability proportional to size
   (c) selection with equal probability (d) judgement sampling
10. Under SRS, the product estimator is less precise than the expansion estimator, when the variables X and Y have
    (a) high negative correlation (b) low positive correlation
    (c) high positive correlation (d) none of the above
11. Let $e_0 = \frac{\hat{Y}-Y}{Y}$ then e_0 satisfied
    (a) $E(e_0) = 0$ (b) $E(e_0^2) = V(\hat{Y})/Y^2$
    (c) both the above are correct. (d) both the above are wrong.

12. Which of the following statement is true
(a) less the standard error, better it is (b) less the variance, better it is
(c) both (a) and (b) are correct (d) standard error is always unity

Q2 (a.) Define Ratio estimator. Derive bias and mean square error for ratio estimator. [06]

(b.) Define product estimator. State the bias and mean square error for product estimator. [03]

Q3 (a.) Define PPS systematic sampling. Give Hartley and Rao estimator for population total and variance of the estimator. [06]

OR

(a.) Define PPS stratified sampling. Give estimator for population total and its Variance under pps stratified sampling. [06]

(b.) Define Murthy's unordered estimator. [03]

------X------

$P) + \frac{(1+P)(n-1)}{(N-n)}$

$M_{OR} = (1+P)($

$M_{OR}.$

$(1+P)\frac{Y_1}{P_1} + (1-P)\frac{Y_2}{P_2}$

(1, 2)

(2, 1)

a
c
c
c
b
c
d
d

Date: 26/09/2024

The Maharaja Sayajirao University of Baroda
Faculty of Science, Department of Statistics
M.Sc. Semester – I (Mid-Semester Examination)

Time: 3:00 to 4:00 p.m.
Marks: 30

STA2104C04 – Decision Theory

Q.1 Do as directed: (2 mark each)

(1) Define: non randomized and randomized decision rules.

(2) Define: A statistical decision problem.

(3) For a decision problem with $\mathcal{X} = \{x_1, x_2\}$, whose loss table is given as below, construct the risk table:

	a_1	a_2
θ_1	2	-1
θ_2	-2	3

$P_{\theta_1}(x_1) = 1/2$

$P_{\theta_2}(x_1) = 1/3$

(4) State the various criterions that can be used in order to obtain the optimal decision rules.

(5) Define: convex set, convex hull.

(6) Define: A decision tree. State the importance of decision tree in decision theory.

Q.2 (6 marks each):

(a) Prove that, a decision rule δ_0 is minimax if and only if

$$R(\theta', \delta_0) \leq \sup_{\theta \in \Theta} R(\theta, \delta) \ \forall \ \theta' \in \Theta \ \& \ \delta \in D^*$$

OR

Define: behavioral decision rule, Bayes risk, Bayes decision rule, ϵ – Bayes rule, Risk set, comparison between decision problem and game theory problem.

(b) Define: admissible decision rule, equivalent decision rules, essentially complete class, complete class, minimal complete class, minimal essentially complete class.

(c) Define: unbiased decision rule, least favorable prior distribution. State and prove the necessary and sufficient condition under which a prior distribution will be least favorable.

OR

Let $\Theta = (0, \infty)$, $\mathcal{a} = \mathcal{R}$ and loss function is squared error. Suppose $(X \mid \theta) \sim Poisson\ (\theta)$ and the prior distribution of θ is

$$g(\theta) = \frac{e^{-\theta/\beta}\, \theta^{\alpha-1}}{\Gamma\alpha\, \beta^{\alpha}} \ ; \ \theta > 0 \quad \text{where } \alpha, \beta > 0$$

(i) Show that the posterior distribution of θ given $X = x$ is $G\left(\alpha + x, \frac{\beta}{\beta+1}\right)$.

(ii) show that the Bayes rule is $d_{\alpha,\beta}(x) = \frac{\beta\,(\alpha+x)}{(\beta+1)}$.

p. distn.

******

THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA

M.Sc. (SEM I) MID- SEM ADDITIONAL EXAMINATION

DAY: TUESDAY DATE: 24.09.2024

TIME: 3:00PM-4:00PM

SUBJECT/PAPER: STATISTICS/MULTIVARIATE ANALYSIS-I

PAPER CODE: STA-2103 C03 TOTAL MARKS: 30

PART-1(1X12=12) MARKS

1. Let X be an observation matrix of the order 8 x6 then there are

(A) 8 observations on 6 individuals (B) 6 observations on 8 individual (C) 8 observations on 8 individuals(D) 6 observations on 6 individuals.

2. Consider a random vector X of the order 7X1 then its associated variance –covariance matrix $\sum$ will have

(A) 7 distinct elements (B) 14 distinct elements (C) 28 distinct elements

(D) 21 distinct elements.

3. If the expression of the exponent of a bivariate normal distribution is

$-1/102\ [\ (x-2)^2\ -\ 2.8(x+2)(y-1)\ +4(y-1)^2]$ then the mean of X and Y are

(A) 2, 1 (B) -2, 1 (C) -2,-1 (D) -2, 1 respectively

4. The Multivariate Normal Distribution (MVND) can be defined

(A) if the associated VCM is non-singular (B) if the associated VCM is singular (C) if the associated VCM is non-singular as well singular (D) it has nothing to do with the nature VCM.

5. For a STANDARD MVND the characteristic function is given by

(A) $e^{it'-1/2\, t't}$ (B) $e^{it'\mu-1/2\, t't}$ (C) $e^{it'\mu-1/2\, t'\Sigma t}$ (D) $e^{-1/2\, t't}$

6. Let X'= (2, 1) μ'= (4,3) and the variance-covariance matrix be an identity matrix then the value of the Q.F. $(X-\mu)' \Sigma^{-1} (X-\mu)$ will be

(A) 12 (B) 14 (C) 16 (D) 32

7. The random vector X_{8x1} was partitioned into two sub-sets (X^1 and X^2) one having 4 components and other the remaining ones, the corresponding partitioning of the mean vector μ and the variance-covariance matrix ∑ are $\mu' = (\mu_1, \mu_2)'$ and ∑ =

Σ_{11}	Σ_{12}
Σ_{21}	Σ_{22}

then the Marginal distribution of X^1 will be

(A) $N_4(\mu, \Sigma)$ (B) $N_4(\mu_1, \Sigma_{11})$ (C) $N_4(\mu_1, \Sigma_{12})$ (D) $N_4(\mu_2, \Sigma_{22})$

8. Considering the partitioning of Q.7 the correct value of the partial variance –covariance matrix would be

(A) $\Sigma_{11.2} = \Sigma_{22} - \Sigma_{12}\Sigma^{-1}{}_{22}\Sigma_{21}$ (B) $\Sigma_{11.2} = \Sigma_{22} - \Sigma_{12}\Sigma^{-1}{}_{22}\Sigma_{21}$

(C) $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12}\Sigma^{-1}{}_{11}\Sigma_{21}$ (D) $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12}\Sigma^{-1}{}_{22}\Sigma_{21}$

9. Let X follows $N_p(\mu, \Sigma)$ and Y =CX is a singular transformation ,C is a matrix of order tXp with rank 't' then Y will

(A) have no distribution (B) be Np (μ, ∑) (C) be Nt (Cμ, C∑C') (D) be Nt (Cμ, C∑C') with no proper p.d.f.

10. Let $X_1, X_2 \ldots\ldots\ldots X_N$ be a random sample of N vectors from $N_p(\mu, \Sigma)$ then the MLE of Σ when μ is known will be

(A) the sample variance –covariance matrix S (B) depending upon the sample mean vector (C) depending upon the sample mean vector and μ (D) depending upon μ.

11. Consider the partitioning of Q.7 then the equation of the regression plane of X^1 on X^2 can be obtained with the help of

(A) the marginal distribution of X^1 (B) the marginal distribution of X^2

(C) the conditional distribution of $X^1 \mid X^2$ (D) the conditional distribution of $X^2 \mid X^1$

12. Let the random vector **X** follows $N_p(\mu_1, \Sigma_1)$ and another random vector Y follows $N_p(\mu_2, \Sigma_2)$ consider a random vector **Z= 3X + 4Y** then Y will be distributed as

(A) $N_p(\mu_1 + \mu_2, \Sigma_1 + \Sigma_2)$ (B) $N_p(3\mu_1 + 4\mu_2, \Sigma_1 + \Sigma_2)$

(C) $N_p(3\mu_1 + 4\mu_2, 9\Sigma_1 + 16\Sigma_2)$ (D) $N_p(\mu_1 + \mu_2, 9\Sigma_1 + 16\Sigma_2)$.

PART-2 **18 MARKS**

1. ATTEMPT ALL THE QUESTIONS

2. FIGURES WITHIN PARANTHESIS INDICATE THE MARKS.

1.(A) Given a random vector **X** $_{px1}$ assuming that the components of X are independent but not identically Normally distributed, then derive the joint probability density function of the components of **X** $_{px1}$ (4)

(B) The random vector X_{px1} was partitioned into two sub-sets (X^1 and X^2) and accordingly the mean vector μ, and Σ are portioned then derive NASC for the independence of X^1 with X^2. (3)

(C)Show that the variance-covariance matrix Σ associated with a random vector is at least positive semi -definite. Why this condition is necessary? (2)

OR

(C)What is the density free approach for a Multivariate normal distribution? (2)

2.(A) Let $X_1, X_2 \ldots\ldots\ldots X_N$ be a random sample of N vectors from $N_p(\mu, \Sigma)$ then obtain the MLE^s of μ and Σ when both are unknown. Mention the properties of these estimators. (6)

2.(B) Let the random vector X be 2x1 accordingly μ is 2x1 and Σ is 2x2 matrix. Derive the pdf of X. (3)

OR

2.(B) Show that a random vector X follows Np (μ, Σ) iff the components of X are distributed as univariate normal. (3)

$(x-\mu)$

$c^{-1}y = x$

****

$(x-\mu)'\Sigma^{-1}(x-\mu)$

$cx = y$