80a/45

Exam Seat No. 1800610

Date:4-12-2024 | The Maharaja Sayajirao University of Baroda | Time:11.30AM-2.30PM

FSMSC-I (October 2024)

Subject-Statistics

Day: Wednesday | Multivariate Analysis-I STA2103C03 | Marks:70

N.B. All the questions must be answered in the answer book only

Q-I **Choose the correct alternative and write only answer in the answer book** **[14]**

1. Fisher's linear discriminant function is obtained under the assumption of
(A) equality of two covariance matrices
(B) inequality of two covariance matrices
(C) equality of two covariance matrices and also these are known
(D) equality of two covariance matrices and also these are unknown.

2. For a multivariate data, the variance –covariance matrix is of order pxp the number of canonical correlations which one can find will depend on
(A) the order of the matrix
(B) the number of roots of VCM
(C) the number of positive roots of VCM
(D) the number of non-zero roots of VCM.

3. The best regions of classification can be obtained if
(A) The p.d.f. of the populations are known
(B) the p.d.f. of the populations are known, costs of misclassification are known
(C) the p.d.f. of the populations are known, costs of misclassification are known and the a priori probabilities are also known
(D) Nothing is known.

4. The variance covariance matrix of a random vector X is
(A) square, symmetric and singular
(B) square, symmetric and non- singular
(C) square, asymmetric and non- singular
(D) square, skew symmetric and non- singular.

5. In the problem of classification the average cost is defined as
(A) Probability of classification X cost of classification
(B) Probability of classification X cost of mis-classification
(C) Probability of mis-classification X cost of classification
(D) Probability of mis-classification X mis- cost of classification.

6. The general formula for the canonical correlation in usual notations is where α and γ are the vectors of appropriate order
(A) $\alpha' \sum_{12} \gamma$
(B) $\gamma' \sum_{12} \alpha$
(C) $\alpha' \sum_{11} \gamma$
(D) $\alpha' \sum_{22} \gamma$.

806/45

7. Diagnosing the disease based on a blood test report is an example of

(A) classification problem (B) discrimination problem

(C) Hypothesis testing problem (D) Estimation problem

(b) Do as Directed [14]

1. In the usual notations state the pdf for $N_2(\mu,\sum)$
2. In the usual notations state the reproductive property for a Multi-variate Normal distribution.
3. In the usual notations state the reproductive property for a Wishart distribution.
4. State the difference between classification and discrimination
5. Define canonical correlation
6. For a random vector X if each of its components are independent. Write down the $\sum$ matrix and its inverse.
7. In the general theory of the problem of classification in usual notations state the best region of classification

Q-II **Attempt any TWO** [14]

1. Discuss the properties of MLE of μ and $\sum$ for a multivariate normal distribution.
2. Let the random vector X follows $N_p(\mu,\sum)$, consider the transformation Y=CX, where 'C' is a non-singular matrix of order 'p' obtain the distribution of Y.
3. Let the random vector X follows $N_p(\mu,\sum)$, obtain the characteristic function of **X.**

Q-III **Attempt any TWO** [14]

1. Derive the Chi-square distribution using the Wishart distribution.
2. Obtain the characteristic function of a Wishart distribution.
3. Prove the reproductive property for a Multi-variate Normal distribution.

Q-IV **Do as Directed(Any TWO)** [14]

1. Discuss the problem of classification of a random vector **X** in to one of the two multivariate normal populations $N_p(\mu^1,\sum)$ **and** $N_p(\mu^2,\sum)$ obtain the best region of classification.
2. Write down the estimating equations for the Canonical correlation provide the structure of solution of first canonical variables and first canonical correlations.
3. Explain Fisher's approach for solving a discrimination problem. Use this approach to obtain a linear discriminant function to discriminate the multivariate observations coming from two multivariate populations.

****

Exam Seat No._____

Date:1-04-2025 | The Maharaja Sayajirao University of Baroda FSMSC-I (April 2025) | Time:11.30AM-2.30PM

Day: Tuesday | Multivariate Analysis-I STA2103C03 | Marks:70

N.B. All the questions must be answered in the answer book only

Q-1 **Choose the correct alternative and write only answer in the answer book**

1. Let $X \sim N_p(\mu, \Sigma)$ and $Y = AX + C$ where A is a (q x p) matrix and C is any q-vector, then Y is a_____
   (A) p-variate normal distribution
   (B) q-variate normal distribution
   (C) (q x p) variate normal distribution
   (D) (p x q) variate normal distribution

2. If $X \sim N_p(\underline{\mu}, \Sigma)$, then $(X-\underline{\mu})' \Sigma^{-1} (X-\underline{\mu})$ is distributed as
   (A) Chi-square
   (B) F
   (C) T
   (D) None of these

3. In the usual notations if $A \sim W_p(\Sigma, n)$ and B is a q x p matrix, then $B'AB \sim$ _____
   (A) $W_p(B\Sigma, n)$
   (B) $W_p(B\Sigma B', n)$
   (C) $W_p(B\Sigma, n-1)$
   (D) $W_p(B\Sigma B', n-1)$

4. If $A \sim W_p(\Sigma, n)$ then the diagonal submatrices of A has
   (A) Normal distribution
   (B) Chi-Square distribution
   (C) Wishart distribution
   (D) Cauchy distribution

5. If $A \sim W_p(I, m)$ and $B \sim W_p(I, n)$ and are independent, $m \geq p$, then $|A|/|A + B|$ has
   (A) Wilk's lambda distribution with parameters p, m, n
   (B) Wilk's lambda distribution with parameters m, p, n
   (C) Wilk's lambda distribution with parameters p, n, m
   (D) Wilk's lambda distribution with parameters m, n, p

6. In the usual notation characteristic function of Wishart distribution is given by
   (A) $1/|I - iK\Sigma|^{n/2}$
   (B) $1/|I-iK\Sigma|^{n}$

(C) $1/|I-iK\Sigma|$
(D) None of these

7. Fisher's linear discriminant function is obtained under the assumption of
(A) equality of two covariance matrices
(B) inequality of two covariance matrices
(C) equality of two covariance matrices and also these are known
(D) equality of two covariance matrices and also these are unknown.

8. For a multivariate data, the variance –covariance matrix is of order pxp the number of canonical correlations which one can find will depend on
(A) the order of the matrix
(B) the number of roots of VCM
(C) the number of positive roots of VCM
(D) the number of non-zero roots of VCM.

9. The best regions of classification can be obtained if
(A) The p.d.f. of the populations are known
(B) the p.d.f. of the populations are known, costs of misclassification are known
(C) the p.d.f. of the populations are known, costs of misclassification are known and the a priori probabilities are also known
(D) Nothing is known.

10. The general formula for the canonical correlation in usual notations is where α and γ are the vectors of appropriate order
(A) $\alpha'\sum_{12}\gamma$
(B) $\gamma'\sum_{12}\alpha$
(C) $\alpha'\sum_{11}\gamma$
(D) $\alpha'\sum_{22}\gamma$.

11. Diagnosing the disease based on a blood test report is an example of

(A) classification problem (B) discrimination problem

(C) Hypothesis testing problem (D) Estimation problem

12. If X is $N_p(0,I)$ and a be a non -zero p-vector, $a'X/\sqrt{a'a}$ is_____

(A) p variate normal
(B) univariate normal

(C) chi-square
(D) none of these

13. Let x_1, x_2, x_3, x_4, x_5 and x_6 be a random sample of size of size 6 from $N(0, \sigma^2)$, then in the usual notation mgf of $\Sigma x_i^2/\sigma^2$ is _____

(A) $(1-2t)^{-4}$
(B) $(1-2t)^{-5}$
(C) $(1-2t)^{-6}$
(D) $(1-2t)^{-3}$

14. Let $X \sim N_p(\mu, \Sigma)$ then AX and BX are independent if and only if _____where A(nxp) and B(q x p) are matrices of constants

(A) $A\Sigma B=1$
(B) $A\Sigma B' = 0$
(C) $A\Sigma B = 0$
(D) $A\Sigma B' = 1$

Q-II **Attempt any TWO questions** **[14]**

1. Define p-variate normal distribution. Derive its characteristic function.
2. State the properties of p-variate normal distribution and prove any one of the property.
3. Explain Spherical multivariate normal distribution.

Q-III **Attempt any TWO questions** **[14]**

1. Define Wishart distribution and hence state its properties.
2. In the usual notation if A_i (i=1,2,------r) is Wishart with n_i d.f., then derive the characteristic function of $A= A_1 + A_2 +$ -------- $+ A_r$
3. Derive the Chi-square distribution using the Wishart distribution.

Q-IV **Do as Directed** **[14]**

1. Explain by giving examples canonical correlation analysis
2. Explain Fisher's linear Discriminant function.

****