Exam Seat No. ____________

Mid Semester Examination
Faculty of Science
Department of Statistics
M.Sc. Semester–I Examination
STA2111C11 : Sampling Theory

Date : 16-09-2023
Day: Saturday
Time : 3.00 to 4.00 pm

Q1. Choose the correct alternative (12x1=12)

1. Under Simple Random Sampling in Ratio method
   (a) $B(\hat{Y}_R) = \left[\frac{N^2(N-n)}{Nn}\right] Y \left\{\left[\frac{S_x^2}{X^2}\right] - \left[\frac{S_{xy}}{XY}\right]\right\}$
   (b) $\hat{Y}_R = \frac{\sum Y_i}{\sum X_i} \cdot X$
   (c) $MSE(\hat{Y}_R) = \left[\frac{N^2(N-n)}{Nn}\right] Y^2 \left\{\left[\frac{S_y^2}{Y^2}\right] + \left[\frac{S_x^2}{X^2}\right] - 2\left[\frac{S_{xy}}{XY}\right]\right\}$
   (d) all three
2. In random group method population is randomly divided in to groups which are
   (a) mutually exclusive but not exhaustive.
   (b) not mutually but exhaustive
   (c) mutually exclusive and exhaustive.
   (d) only exhaustive.
3. In random group method
   (a) $V_1E_2\,[\hat{Y}_{RHC}|G_1, \dots, G_n] > 0$
   (b) $V_1E_2\,[\hat{Y}_{RHC}|G_1, \dots, G_n] < 0$
   (c) $V_1E_2\,[\hat{Y}_{RHC}|G_1, \dots, G_n] \neq 0$
   (d) $V_1E_2\,[\hat{Y}_{RHC}|G_1, \dots, G_n] = 0$
4. A larger sample size is required when:
   (a) The population of interest for a study is less diverse
   (b) A low level of precision is required
   (c) The population of interest is easily recruited to the study
   (d) A high level of precision is required
5. In Lahiri's method unit is selected finally if
   (a) $R = X_i$
   (b) $R \leq X_i$
   (c) $R \geq X_i$
   (d) none of the above
6. if $Y_i \propto X_i$ in PPS Sampling then
   (a) $V_{pps}(\hat{Y}_{unst}) < 2\,V_{prop}(\hat{Y}_{st})$
   (b) $V_{pps}(\hat{Y}_{unst}) = V_{prop}(\hat{Y}_{st})$
   (c) $V_{pps}(\hat{Y}_{unst}) \leq V_{prop}(\hat{Y}_{st})$
   (d) $V_{pps}(\hat{Y}_{unst}) \geq V_{prop}(\hat{Y}_{st})$
7. A sample consists of
   (a) all units of the population
   (b) 50 percent of the population
   (c) any fraction of the population
   (d) 5 percent unit of the population
8. If larger units have greater probability of their inclusion in the sample, it is known as
   (a) selection with replacement
   (b) selection with probability proportional to size
   (c) selection with equal probability
   (d) judgement sampling
9. Under SRS, the product estimator is more precise than the expansion estimator, when the variables X and Y have
   (a) high negative correlation
   (b) low positive correlation
   (c) high positive correlation
   (d) none of the above
10. Let $e_0 = \frac{\hat{Y} - Y}{Y}$ then e_0 satisfied
    (a) $E(e_0) = 0$
    (b) $E(e_0^2) = V(\hat{Y})/Y^2$
    (c) both the above are correct.
    (d) both the above are wrong.
11. Which of the following statement is true
    (a) less the standard error, better it is
    (b) less the variance, better it is
    (c) both (a) and (b) are correct
    (d) standard error is always unity
12. Whenever the groups are of same size in Random group method
    (a) $V(\hat{Y}_{RHC}) > V(\hat{Y}_{PPS})$
    (b) $V(\hat{Y}_{RHC}) = V(\hat{Y}_{PPS})$
    (c) $V(\hat{Y}_{RHC}) < V(\hat{Y}_{PPS})$
    (d) none of the above

Q2.(a.) What is random group Method? (03)

(b.) Prove that under ppswor, $\hat{Y}_{DR}$ is unbiased for the population total and unbiased estimator of (06)

$$V(\hat{Y}_{DR}) \text{ is } \frac{1}{n(n-1)}\sum_{i=1}^{n} \left(t_i - \underline{t}\right)^2$$

Q3(a.) Explain Lahiri's method with suitable example. In.which situation Lahiri method is more suitable than cumulative method? (03)

(b.) In pps sampling with out replacement, $\hat{Y}_{HT}$ is unbiased and its sampling variance is given by $V_{HT}(\hat{Y}_{HT}) = \sum_{i=1}^{N} \frac{(1-\pi_i)Y_i^2}{\pi_i} + \sum_{i}^{N} \sum_{i\neq j}^{N} \frac{(\pi_{ij}-\pi_i\pi_j)}{\pi_i\pi_j} Y_i Y_j$ where π_{ij} is the probability of inclusion of both the ith and jth unit in the sample.Prove it. (06)

OR

(b.) Derive bound for bias of ratio estimator. (06)

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Exam Seat No. ___________

THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA

Date: 20-12-23 FS MSC(P) Examination Time: 11.30 a.m to 2:30 p.m

Day : Wednesday **Sampling Theory (STA2111C11)**

Total Marks: 70

Q: 1. Choose the correct option and write in the answer sheet provided. [28]

1. Random group method is better than probability proportional to size with replacement whenever
   (a) the groups are of the different size. (b) the groups are of the same size.
   (c) the sizes are in increasing order (d) none of the above
2. Sampling frame is a term used for
   (a) a list of random numbers (b) a list of voters
   (c) a list of sampling units of population (d) none of the above
3. In Lahiri's method of pps sampling, the probability that no unit is selected at any draw is
   (a) $1-\frac{\overline{X}}{M}$ (b) $1-\frac{\overline{X}}{N}$
   (c) $\frac{\overline{X}}{M}$ (d) None of the above
4. An unbiased estimator of the gain due to ppswr sampling as compared to srswr is
   (a) $\frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y_i^2}{P_i}\right)\left(N-\frac{1}{P_i}\right)$ (b) $\frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y_i^2}{P_i}\right)\left(n-\frac{1}{P_i}\right)$
   (c) $\frac{1}{n^2}\sum_{i=1}^{n}\left(\frac{Y_i^2}{P_i}\right)\left(N-\frac{1}{P_i}\right)$ (d) none of the above.
5. For effective stratification strata should be formed in such a way that units with in each stratum are
   (a) heterogeneous w.r to the variable $\frac{Y_{ij}}{P_{ij}}$ (b) homogeneous w.r to the variable $\frac{Y_{ij}}{P_{ij}}$
   (c) homogeneous w.r to the variable Y (d) none of the above.
6. An unbiased estimator of the population total Y, derived by Hartley and Rao is given by
   (a) $\frac{X}{n}\sum_{i=1}^{n}\frac{Y_i}{X_i}$ (b) $\frac{1}{n}\sum_{i=1}^{n}\frac{Y_i}{P_i}$
   (c) both the above are correct. (d) both the above are wrong.
7. Let $e_o = \frac{\hat{Y}-Y}{Y}$ then e_o satisfied
   (a) $E(e_o) = 0$ (b) $E(e_o^2) = V(\hat{Y})/Y^2$
   (c) both the above are correct. (d) both the above are wrong.
8. The exact bias of the product estimator are given by
   (a) $B(\hat{Y}_p) = \frac{Cov(\hat{X},\hat{Y})}{Y}$ (b) $B(\hat{Y}_p) = \frac{Cov(\hat{X},\hat{Y})}{X}$
   (c) $B(\hat{Y}_p) = \frac{Cov(\hat{X},\hat{Y})}{XY}$ (d) none of the above.
9. Product estimator $\hat{Y}_p$, is more efficient than $\hat{Y}$ if
   (a) $\rho_{xy} < \frac{1}{2}\frac{C(X)}{C(Y)}$ (b) $\rho_{xy} < -\frac{1}{2}\frac{C(X)}{C(Y)}$
   (c) $\rho_{xy} > -\frac{1}{2}\frac{C(X)}{C(Y)}$ (d) none of the above.

10. An unbiased estimator of the population total Y, derived by Hartley and Rao is given by

~~(a)~~ $\frac{X}{n}\sum_{i=1}^{n} \frac{Y_i}{X_i}$ (b) $\frac{1}{n}\sum_{i=1}^{n} \frac{Y_i}{P_i}$

(c) both the above are correct. (d) both the above are wrong.

11. When the samples are drawn independently in the two phases of sampling, the approximate bias of the ratio estimator is

~~(a)~~ $B(\hat{Y}_{RD}) = Y\left[\frac{V(\hat{X})}{X^2} - \frac{Cov(\hat{X},\hat{Y})}{XY}\right]$ (b) $B(\hat{Y}_{RD}) = Y\left[\frac{V(\hat{X})}{Y^2} - \frac{Cov(\hat{X},\hat{Y})}{XY}\right]$

(c) $B(\hat{Y}_{RD}) = Y\left[\frac{V(\hat{X})}{X^2} + \frac{Cov(\hat{X},\hat{Y})}{XY}\right]$ (d) none of the above.

12. The combined ratio estimator in stratified sampling is given by

(a) $\hat{Y}_{RC} = \sum_{h=1}^{L} \left[\frac{\hat{Y}_h}{\hat{X}_h}\right] X_h$ (b) $\hat{Y}_{RC} = \sum_{h=1}^{L} \left[\frac{\hat{X}_h}{\hat{Y}_h}\right] X_h$

(c) $\hat{Y}_{RC} = \left[\frac{\sum_{h=1}^{L} \hat{Y}_h}{\sum_{h=1}^{L} \hat{X}_h}\right] Y$ ~~(d)~~ $\hat{Y}_{RC} = \left[\frac{\sum_{h=1}^{L} \hat{Y}_h}{\sum_{h=1}^{L} \hat{X}_h}\right] X$

13. Under srs in regression estimation

(a) $V(\hat{Y}_{SRS}) > MSE(\hat{Y}_{LR})$ (b) $MSE(\hat{Y}_R) > MSE(\hat{Y}_{LR})$

~~(c)~~ both (a) and (b) are correct. (d) both (a) and (b) are wrong.

14. The bias of the estimator $\hat{Y}_{RO}$ is

(a) (N-1)S_{zx} ~~(b) -(N-1)~~ S_{zx}

(c) $\frac{(1-N)}{N} Szx$ (d) none of the above

Q2 (a) Explain Cumulative Total method with suitable example. In which situation cumulative total method is not preferred than Laheri's method. (04)

(b) The probability of selecting the i[th] unit in the first effective draw is $\frac{X_i}{X}$ in Lahiri's method of pps sampling. Prove it. (05)

(c) Under ppswor, $\hat{Y}_{DR}$ is unbiased for the population total and unbiased estimator of $\hat{Y}_{DR}$ is $\frac{1}{n(n-1)}\sum_{i=1}^{n} (t_i - \bar{t})^2$ (05)

OR

(c) Show that in stratified pps sampling, $V_{pps}(\hat{Y}_{unst}) \geq V_{prop}(\hat{Y}_{st})$ (05)

Q3(a) Explain Ratio estimator, Ratio type estimator & Almost unbiased ratio estimator. (04)

(b) The ratio estimator $\hat{Y}_R$ is more efficient than the expansion estimator $\hat{Y}$,

if $\rho > \frac{1}{2}\left[\frac{C_x}{C_y}\right]$ Where, $C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$

and ρ is the coefficient of correlation. (05)

(c) The approximate bias and mean square error of the ratio estimator are

B($\hat{Y}_R$)=$Y\left\{\left[\frac{V(\hat{X})}{X^2}\right] - \left[\frac{Cov(\hat{X},\hat{Y})}{XY}\right]\right\}$ and MSE[$\hat{Y}_R$]=$Y^2\left\{\left[\frac{V(\hat{Y})}{Y^2}\right] + \left[\frac{V(\hat{X})}{X^2}\right] - 2\left[\frac{Cov(\hat{X},\hat{Y})}{XY}\right]\right\}$.

Prove it. (05)

OR

(c) Prove that the bias of the estimator $\hat{Y}_{R_0}$ is B[$\hat{Y}_{R_0}$]= $-[N-1]S_{zx}$

Where $S_{zx} = \frac{1}{N-1}\sum_i [Z_i - \bar{Z}][X_i - \bar{X}]$, $Z_i = \frac{Y_i}{X_i}$ (05)

Q4. (a) (a) Explain Sampling and non sampling error. What are the sources of non sampling error? (04)

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(b) When SRS is used in both phases of sampling and samples are drawn independently

$B(\hat{Y}_{RD}) = \frac{N^2(N-n)}{N\,n} Y[C_{XX} - C_{XY}]$ and (05)

$MSE(\hat{Y}_{RD}) = \frac{N^2(N-n)}{N\,n} Y^2[C_{YY} + C_{XX} - 2C_{XY}] + \frac{N^2(N-n_1)}{N\,n_1} Y^2 C_{XX}$

Where $C_{XX} = \frac{S_X{}^2}{X^2}, C_{YY} = \frac{S_Y{}^2}{Y^2}, C_{XY} = \frac{S_{XY}}{XY}$

(c) When the samples are drawn independently in two phases of sampling with the help of SRS the variance of the difference estimator is

$V(\hat{Y}_{DD}) = N^2[fS_Y{}^2 + \lambda^2(f + f')S_x{}^2 - 2\lambda f S_{XY}]$ Where $f = \frac{N-n}{Nn}$ $and\ f' = \frac{N-n\prime}{Nn\prime}$

Where n' and n are sample sizes corresponding to the first and second phases of sampling. (05)

OR

(c) When the second phase sample is a subsample of the first phase sample, the approximate mean square error is

$MSE(\hat{Y}_{RD}) = Y^2\left[\frac{V(\hat{Y})}{Y^2} + \frac{V(\hat{X})}{X^2} + \frac{V(\hat{X}_d)}{X^2} - 2\,\frac{Cov\,(\hat{X},\hat{Y})}{X\,Y} - 2\,\frac{Cov\,(\hat{X},\hat{X}_d)}{X^2} + 2\,\frac{Cov\,(\hat{Y},\hat{X}_d)}{Y\,X}\right]$

(05)

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THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA

15/12/2023 FSM.Sc. Previous Examination(Nov 2023) Time: 11.30 am

Friday STA2102C02-Linear Models(4 credits) to 2.30 pm

Total Marks: 70

Instructions: ***(i)All the questions have to be answered in the answerbook only.***

Q-I (a) Fill in the blanks [14]

1. In the usual notations Variance inflation factor is given by_________

2. In the usual notations Press residuals are represented as__________________

3. Euclidean norm of matrix A = $\begin{bmatrix} 5 & -4 & 2 \\ -1 & 2 & 3 \\ -2 & 1 & 0 \end{bmatrix}$ is_______________ .

4. Full form of MINQUE is________________.

5. In the usual notation if the value of standardized residual d_i is ________, it indicates outlier.

6. In the usual notation Ridge estimator is given by________

7. The vector of residuals **e** in terms of hat matrix H can be represented as___________

Q-I (b) Choose the correct alternative [8]

1. In the usual notation a linear function l'y is said to belong to error if ________ irrespective of the values of $\Theta_1, \Theta_2 \text{-----} \Theta_m$.
   (A) E(l'y) > 0
   (B) E(l'y) < 0
   (C) E(l'y) = 0
   (D) None of these

2. What is the meaning of the term "hetroscedasticity"?
   (A)The variance of the errors is not constant
   (B) The variance of the dependent variable is not constant

(C) The errors are not linearly independent of each other
(D)The errors have non zero mean

3. In the plot of residuals against the fitted values outward opening funnel pattern indicates that the
   (A) variance is a decreasing function of y.
   (B) variance is an increasing function of y.
   (C) variance is constant.
   (D) Nothing can be said.

4. If there is strong multi-collinearity between x_j and any subset of the other (p-1) regressors, then the value of R_j^2 will be.
   (A) close to unity
   (B) close to zero
   (C) close to -1
   (D) can be anything.

Q-I (c) Do as Directed [6]

1. State the properties of residuals
2. State one way random effect model with assumptions
3. State the form of C_p statistics

Q-II Answer briefly [11]

1. If $l'\beta$ is any estimable linear function of the parameters β_1, β_2------ ------- β_p, then in the usual notation prove that [7]
(i) there exists a unique linear function $c'Y$of the random variables Y_1, Y_2 ---------Y_n such that $c \in V(A')$ and $E(c'Y) = l'\beta$.
(ii) $Var(c'Y)$is less than the the variance of any other linear unbiased estimator of $l'\beta$.

2. Explain the Gauss Markoff set up for uncorrelated variables. [4]

OR

2. Define (i) error space (ii) estimation space [4]

Q-III Do as Directed [11]

1. State and prove the second fundamental theorem of least square theory. [6]

OR

1. Assuming the Gauss Markov linear set up for the observations and assuming that the observations are normally and independently distributed, derive the ANOVA test for the hypothesis that k independent and consistent conditions have assigned values: $H'\beta = \Theta_0$. [6]

2. Explain briefly Scheffe's test [5]

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Q-IV **Attempt any two** [10]

1. Explain briefly MINQUE theory for estimation of parameters in linear models
2. Explain normal probability plots and its uses
3. Explain Studentized residuals

Q-V **Attempt any two** [10]

1. Write a note on Mallow's C_p statistic
2. Explain the ill effects of multicollinearity
3. Explain principal component regression

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THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA

M.Sc. (SEM I) END SEM EXAMINATION

DAY: SATURDAY DATE: 16.12.2023

YEAR: 2023 TIME: 11.30AM-2.30 PM

SUBJECT/PAPER: STATISTICS/MULTIVARIATE ANALYSIS

PAPER CODE: STA-2103 C03 TOTAL MARKS: 70

QUESTION-1 (28 MARKS)

MCQ (1X28=28)

1. Let there be 10 observations on 20 individuals then the total number of observations would be

(A) 10 (B) 20 (C) 200 (D) 100

2. Let **X** be the random vector and Σ be the associated Variance – covariance matrix (VCM) then the condition $X' \Sigma \mathbf{X} \geq 0$ is necessary for

(A) The probability density function of a multivariate distribution to be defined (B) The probability density function of a multivariate distribution to be bounded (C) neither (A) nor (B) (D) both (A) and (B).

3. Let **X** has $\mathbf{N_7}$ **(μ, Σ)** distribution then the total number of distinct elements of Σ matrix would be

(A) 7 (B) 14 (C) 21 (D) 28

4. Let X follows $N_p(\mu,\Sigma)$ and Y =CX is a non-singular transformation then the value of the jackobian of transformation would be

(A) $|J| = |C|$ (B) $|J| = |C^{-1}|$ (C) $|J| = |C'|$ (D) $|J| = |CC'|$

5. Let X follows $N_p(\mu, \Sigma)$ consider the transformation $(X-\mu) = CY$ where C is a non-singular matrix then Y will be distributed as

(A) $N_p(0,\Sigma)$ (B) $N_p(0,I_p)$ (C) $N_p(\mu,I_p)$ (D) $N_p(1, \Sigma)$

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6. Let X follows $N_p(\mu,\Sigma)$, and X be partitioned into two sub-sets X^1 and X^2 having q and (p-q) components and accordingly μ and Σ are partitioned. Then in usual notation the conditional mean will be

(A) $\mu^1 + \Sigma_{12}\Sigma^{-1}_{22}$ ~~(B)~~ $\mu^1 + \Sigma_{12}\Sigma^{-1}_{22}(X^2 - \mu^2)$ (C) $\mu^1 + \Sigma_{11}\Sigma^{-1}_{22}(X^2 - \mu^2)$
(D) $\mu^1 - \Sigma_{12}\Sigma^{-1}_{22}(X^2 - \mu^2)$.

7. For a singular distribution the density function g(X) for a multivariate normal distribution can be written with the help of

(A) Eigen values of Σ (B) Eigen vectors of Σ (C) diagonal elements of Σ (D) g-inverse elements of Σ.

8. Let X follows $N_p(\mu,\Sigma)$, consider the transformation Z= D X where D is a singular matrix of the order 5X7 then Z will be distributed as

(A) $N_7(D\mu, D\Sigma D')$ (B) $N_5(D\mu, D\Sigma D'$ (C) $N(D\mu, D\Sigma D')$ order with the rank of D (D) $N_p(D\mu, D\Sigma D')$

9. Let X follows $N_p(\mu,\Sigma)$ then the MLE of Σ is obtained by using the

(A) Unbiasedness property of MLE (B) Sufficiency property of MLE

(C) Consistency property of MLE (D) Invariance property of MLE.

10. Let the sample variance –covariance matrix A be distributed as

$W_p(n, \Sigma)$ then its characteristic function Φ is given by

(A) $E(e^{iUA})$ ~~(B)~~ $E(e^{tr.\, iUA})$ (C) $E(e^{tr.\, iU})$ (D) $E(e^{tr.\, iA})$

11. Let the sample variance –covariance matrix A be distributed as

$W_p(n, \Sigma)$ then the Marginal distribution of any subset of A will also be wishart with

(A) different degrees of freedom (B) different degrees of freedom and same Σ (C) same degrees of freedom and same Σ ~~(D)~~ same degrees of freedom and subsequent partition of Σ.

12. Let the sample variance –covariance matrix A be distributed as

$W_p(n, \Sigma)$ then from the $W_p(n, \Sigma)$ distribution a chi-square distribution with 'n' degrees of freedom can be obtained by

(A) premultiplying A^{-1} by an arbitrary non-null vector L (B) premultiplying A by an arbitrary non-null vector L (C) Post multiplying A by an arbitrary non-null vector L (D) premultiplying and post multiplying A by an arbitrary non-null vector L.

13. In the problem of classification

(A) we do not know the number of different categories into which observations are classified (B) we know the number of different categories into which observations are classified (C) categories can be arbitrary (D) the number of different categories is decided by the researcher.

14. While classifying a random vector **X** of observations in to two regions R_1 and R_2 the regions should be

(A) Mutually exclusive (B) Exhaustive (C) mutually exclusive and Exhaustive (D) independent

15. The average cost of misclassification is based on

(A) Prior probabilities, cost of misclassification and the probability of misclassification (B) cost of misclassification and the probability of misclassification (C) prior probabilities and cost of misclassification

(D) Prior probabilities cost of classification and the probability of misclassification.

16. The two regions R_1 and R_2 the regions of classification can be completely specified if we know

(A) the probability distribution functions of the two regions(B) the two costs of misclassifications (C) apriori probabilities of classifications.

(D) all of above.

17. In the problem of classification for two multivariate normal populations $N_p(\mu^1, \Sigma)$ and $N_p(\mu^2, \Sigma)$ the best region of classification R_1 will be given by

(A) R_1: $X'\Sigma(\mu^1 - \mu^2) \geq k$ (some constant) (B) R_1: $X'\Sigma^{-1}(\mu^1 - \mu^2) \geq k$

(C) R_1: $X'\Sigma^{-1}(\mu^1 - \mu^2) \leq k$ (D) R_1: $X'\Sigma(\mu^1 - \mu^2) \geq k$

18. In the problem of classification for two multivariate normal populations $N_p(\mu^1, \Sigma)$ and $N_p(\mu^2, \Sigma)$ in order to calculate the probabilities of mis-classification we must find the distribution of a linear combination of **X** the random vector using

(A) the two mean vectors μ^1 and μ^2 (B) the variance covariance matrix Σ

(C) the two mean vectors μ^1 and μ^2 and the variance covariance matrix Σ (D) any one of the mean vectors μ^1 and μ^2 and Σ.

19. As per the Fisher's criterion in order to discriminate a random vector **X** between two multivariate normal populations $N_p(\mu^1, \Sigma)$ and $N_p(\mu^2, \Sigma)$ for the linear compound **l' X** the vector **l** is to be choosen in such a way that

(A) $[E_1(\mathbf{l}'\mathbf{X}) - E_2(\mathbf{l}'\mathbf{X})]^2$ should be minimum under certain condition

(B) $[E_1(\mathbf{l}'\mathbf{X}) - E_2(\mathbf{l}'\mathbf{X})]^2$ should be maximum under certain condition

(C) $[E_1(\mathbf{l}'\mathbf{X}) - E_2(\mathbf{l}'\mathbf{X})]$ should be minimum under certain condition

(D) $[E_1(\mathbf{l}'\mathbf{X}) - E_2(\mathbf{l}'\mathbf{X})]$ should be maximum under certain condition.

20. In order to calculate the correlation between height and age, weight, father's height and mother's height of a group of individuals, one must calculate

(A) Canonical correlation (B) Partial correlation (C) Multiple correlation (D) pair wise correlation.

21. Canonical correlation analysis(CCA) is

(A) A dimension reduction technique (B) an interdependence analysis technique (C) just another way of finding the correlation (D) an alternative to multiple correlation technique

22. Before performing CCA one must test (in usual notations)

(A) $H_0 : \Sigma_{11} = 0$ (B) $H_0 : \Sigma_{22} = 0$ (C) $H_0 : \Sigma_{12} = 0$ (D) $H_0 : \Sigma_{11} = 0$ and $H_0 : \Sigma_{22} = 0$ simultaneously.

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22-e
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23. If the VCM $\sum$ associated to a random vector **X** is of the order pxp and **X** is partitioned into two subsets having p_1 and p_2 $(p_1 < p_2)$ components respectively then the canonical correlations ρ will correspond to

(A) p_1 non-zero roots of $\sum$ (B) p_2 non-zero roots of $\sum$ (C) 2 p_1 non-zero roots of $\sum$ (D) 2 p_2 non-zero roots of $\sum$.

24. Let the random vector **X** is partitioned into two subsets X_1 and X_2 having p_1 and p_2 $(p_1 < p_2)$ components and the associated VCM $\sum$ is partitioned accordingly and let $U=\alpha' X_1$ and $V=\gamma' X_2$ then maximizing the canonical correlation is equivalent to the maximization of

(A) $\alpha' \sum_{11} \gamma$ subjected to some condition (B) $\alpha' \sum_{12} \gamma$ subjected to some condition (C) $\alpha' \sum_{22} \gamma$ subjected to some condition (D) $\alpha \sum_{12} \gamma$ subjected to some condition.

25. In the discriminant analysis for discriminating a random vector **X** between two multivariate normal populations the linear discriminant function is obtained when

(A) The two associated VCM^s are different (B) the two associated VCM^s are same (C) their mean vectors are different (D) their mean vectors are same.

26. For a random vector **X** partitioned into two subsets in such a way that one subset has only one component and the remaining ones are in the other subset then for the best regression equation one should take which of the following values of β (in usual notations)

(A) $\sigma_{12} \sum_{22}^{-1}$ (B) $-\sigma_{12} \sum_{22}^{-1}$ (C) $\sigma_{12} \sum_{11}^{-1}$ (D) $-\sigma_{12} \sum_{11}^{-1}$

27. By the property of the solutions of canonical correlations ,if A and Γ denote the matrices of canonical vectors ,then which of the following results(in usual notations) hold?

(A) $\Gamma' \sum_{11} \Gamma$ = Null matrix (B)) $\Gamma' \sum_{11} \Gamma$ =identity matrix of order p_2

(C) $\Gamma' \sum_{22} \Gamma$ =identity matrix of order p_2 (D) $\Gamma' \sum_{22} \Gamma$ =identity matrix of any order.

28. Let the random vector **X** has $N_p(0, I_p)$ distribution then the distribution of **X** is known as

(A) An elliptical multivariate normal distribution (B) spherical multivariate normal distribution(C) symmetrical multivariate normal distribution (D) Circular multivariate normal distribution.

PART-2 **MAX.MARKS:42**

EACH QUESTION CARRIES EQUAL MARKS.

FIGURES WITHIN PARANTHESIS INDICATE MARKS.

1. (A) Let the random vector **X** follows $N_p(\mu, \Sigma)$, consider the partitioning of X into two subsets X^1 **and** X^2 having 'q' and (p-q) components respectively. Also μ and Σ partitioned accordingly. Obtain the Marginal distribution of X^1. Also, mention the application of the distribution. (7).

1. (B) Let the random vector **X** follows $N_p(\mu, \Sigma)$, obtain the MLE^s of μ and Σ and discuss their properties. (7).

OR

1. (B) Define a multivariate normal distribution.

Let the random vector **X** follows $N_p(\mu, \Sigma)$, consider the partitioning of X into two subsets X^1 **and** X^2 having 'q' and (p-q) components respectively. Also μ and Σ partitioned accordingly. Derive the necessary and sufficient condition for the two subsets being independent. (7)

2. (A) Define the sample moment matrix. Derive its distribution. Which distribution can be derived from it as a special case? (7)

2. (B) Obtain the marginal distribution of a Wishart distribution. State and prove the reproductive property of this distribution. Obtain chi-square distribution from it without using its probability density function. (7)

OR

2. (B) Obtain the joint distribution of sample mean vector and the sample variance-covariance matrix, mention the application of this result. (7)

SU 3. (A) Discuss the problem of classification and discrimination with example. Derive the best region of classification foe classifying a random vector X into two multivariate normal populations with different mean vectors but same VCM. (7)

3. (B) why do we require the canonical correlation analysis (CCA)?

Derive the first canonical correlation for a given random vector **X**. (7)

OR

3. (B) Define Fisher's linear discriminant function. Obtain the two regions of discrimination in case of discrimination between two multivariate normal populations with same VCM. Provide the estimates of these regions. What are drawbacks of the problem of classification? (7)

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M.Sc. (SEM I) MID- SEM EXAMINATION

DAY: WEDNESDAY

DATE: 27.09. 2023

TIME: 1:15-2:15 PM

SUBJECT/PAPER: STATISTICS/MULTIVARIATE ANALYSIS

PAPER CODE: STA-2103 C 03

TOTAL MARKS: 30

PART-1 (12 MARKS)

1. Suppose that the data is collected on 10 individuals for their 6 characteristics then the data matrix **X** will be of the order

(A) 6X6 (B) 6X10 (C) 10X6 (D) 10X10

2. Consider a random vector **X** of the order 6X1 then its associated variance – covariance matrix $\sum$ will have

(A) 12 distinct elements (B) 36 distinct elements (C) 21 distinct elements

(D) 18 distinct elements.

3. If the expression of the exponent of a bivariate normal distribution is

$-1/102\ [\ (x+2)^2 - 2.8(x+2)(y-1) + 4(y-1)^2]$ then the mean of X and Y are

(A) 2, 1 (B) -2, 1 (C) -2,-1 (D) 2,-1 respectively

4. In a general if the density function of a multivariate normal density is written as

$f(\mathbf{X}) = k\, e^{-1/2}\ (\mathbf{X}-\mathbf{B})'\ A\ (\mathbf{X}-\mathbf{B})$ where **X** and **B** are the vectors of order 'p' and A is positive definite matrix then the probability density function of **X** is defined when

(A) $(\mathbf{X}-\mathbf{B})'\ A\ (\mathbf{X}-\mathbf{B}) > 0$ (B) $(\mathbf{X}-\mathbf{B})'\ A\ (\mathbf{X}-\mathbf{B}) < 0$ (C) $(\mathbf{X}-\mathbf{B})'\ A\ (\mathbf{X}-\mathbf{B}) \geq 0$

(D) $(\mathbf{X}-\mathbf{B})'\ A\ (\mathbf{X}-\mathbf{B}) \leq 0$

5. Let the random vector **X** of order px1 be distributed as $N_p(0, \Sigma)$ then its characteristic function will be given by

(A) $\exp(-1/2\, t' \Sigma t)$ (B) $\exp(-1/2\, it')$ (C) $\exp(-1/2\, it'\mu)$

(D) $\exp(-1/2\, it'\mu - t' \Sigma t.)$

6. Let $X' = (1, 2)$ $\mu' = (3,4)$ and the variance-covariance matrix be an identity matrix then the value of the Q.F. $(X-\mu)' \Sigma^{-1} (X-\mu)$ will be

(A) 12 (B) 14 (C) 16 (D) 32

7. Consider the partitioning of the mean vector μ and the variance-covariance matrix Σ as $\mu' = (\mu_1 , \mu_2)'$ and $\Sigma = \begin{array}{|c|c|}\hline \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \\ \hline \end{array}$

As the random vector $X_{10\times1}$ was partitioned into two sub-sets (X^1 and X^2) one having 4 components and other the remaining ones, then the Marginal distribution of X^1 will be

(A) $N_4(\mu,\Sigma)$ (B) $N_6(\mu_1, \Sigma_{11})$ (C) $N_4(\mu_1,\Sigma_{11})$ (D) $N_6(\mu_2, \Sigma_{22})$

8. Considering the partitioning of Q.7 the correct value of the partial variance – covariance matrix would be

(A) $\Sigma_{11.2} = \Sigma_{22} - \Sigma_{12}\Sigma^{-1}_{22}\Sigma_{21}$ (B) $\Sigma_{11.2} = \Sigma_{22} - \Sigma_{12}\Sigma^{-1}_{pp}\Sigma_{21}$

(C) $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12}\Sigma^{-1}_{11}\Sigma_{21}$ (D) $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12}\Sigma^{-1}_{22}\Sigma_{21}$

9.Let $X_1, X_2, \ldots\ldots\ldots X_N$ be a random sample of N vectors from $N_p(\mu, \Sigma)$ then the MLE of Σ when μ is known will be

(A) the sample variance –covariance matrix S (B) depending upon the sample mean vector (C) depending upon the sample mean vector and μ (D) depending upon μ.

10. In usual notations which of the following is the correct expression for the characteristic function of a Wishart distribution?

(A) $1/|I - 2iU\Sigma|$ (B) $1/|I - 2iU\Sigma|^{(N-1)}$ (C) $1/|I - 2iU\Sigma|^{(N-1)/2}$

(D) $1/|I - 2iU\Sigma|^{-(N-1)/2}$

11.In the proposed density of Wishart distribution the value of the constant 'C' For p=1 is equal to

(A) $1/2^{n/2}$ (B) $1/2^{n/2}\ \Gamma n/2$ (C) $1/\ \Gamma n/2$ (D) $2^{n/2}\ \Gamma n/2$

12. Let the sample variance-covariance matrix 'A' be distributed as $W_p(n,\Sigma)$ then we can obtain chi-square distribution from it by

(A) pre multiplying by a vector L (say) of order px1 (B) post multiplying by a vector L (say) of order px1 (C) pre and post multiplying by a vector L (say) of order px1 (D) none of the above.

PART-2 (18 marks)

1. Attempt ALL questions

2. Figures within parenthesis indicate marks.

1. (A) In a general if the density function of a multivariate normal density is written as

$f(\mathbf{X}) = k\, e^{-1/2}\ (\mathbf{X}\text{-}B)'\ A\ (\mathbf{X}\text{-}B)$ where **X** and **B** are the vectors of order 'p' and A is positive definite matrix then obtain the value of 'k' so that it represents the pdf of a Multivariate normal density. (3)

1. (B) obtain the characteristic function of a $N_p(\mu, \Sigma)$ distribution. (3)

1. (C) Consider the partitioning of Q.7 then state and prove the necessary condition for the independence of two sub-sets of the vector **X**. (3)

OR

1. (C) Write down the expression of conditional distribution of X^1 given X^2 form this write the expression of the conditional mean and the regression of X^1 on X^2 (3)

2. (A) Let $\mathbf{X}_1,\mathbf{X}_2 \ldots\ldots\ldots \mathbf{X}_N$ be a random sample of N vectors from $N_p(\mu, \Sigma)$ write down the expression for the likelihood function. What are the MLE^s of μ and Σ when both are unknown.(Do NOT derive) (3)

2. (B) Show that for p=1 the chi-square distribution can be derived from the $W_p(n,\Sigma)$ distribution. (3)

2.(C) State and prove the reproductive property of Wishart distribution. (3)

OR

2.(C) Let the matrix A be distributed as $W_p(n,\Sigma)$ distribution. Then obtain the marginal distribution of any partition of the matrix 'A'. (3)

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Department of Statistics
MSc. Previous Sem I
End Sem Practical Examination 2023-2024
STA2105C05: Multivariate Analysis – I Lab

Date: 22/12/23

Time:2 Hours
Marks:40

Note: Attempt all questions

Ques: 1 Linear Combination of the Cholesterol Measurements: Measurements were taken on n heart-attack patients on their cholesterol levels. For each patient, measurements were taken 0, 2, and 4 days following the attack. Treatment was given to reduce cholesterol level. Find the Mean and variance of Y, where **Y = X1 – X2**. The sample mean vector and Variance-Covariance matrix is:

Variable	Mean
X1 = 0-Day	273.48
X2 = 2-Day	220.18
X3 = 4-Day	249.99

		X1	X2	X3
		0-Day	**2-Day**	**4-Day**
X1	**0-Day**	6722	8015	813
X2	**2-Day**	8015	6022	1349
X3	**4-Day**	813	1349	5681

Ques: 2 Compute Multivariate Normal distribution and also plot the probability graph for the following data:

Sr. no	X1	X2	X3	X4
1	30.6	810	745	1698
2	17.1	860	920	1714
3	31.8	780	730	1623
4	23.9	705	710	1527
5	26.8	770	720	1685
6	34.6	695	700	1750
7	32.1	780	800	1678
8	23.5	740	740	1718
9	30.7	825	825	1699
10	21.3	790	790	1770
11	25.8	780	730	1682
12	31.4	920	830	1402
13	29.8	850	780	1724
14	35.7	755	740	1654
15	29.7	845	805	1651

Ques: 3 A soft drink bottler is analysing the vending machine service routes in his distribution system. He is interested in predicting the amount of time required by the route driver to service the vending machines in an outlet. This service activity includes stocking the machine with beverage products and minor maintenance or housekeeping. The industrial engineer responsible for the study has suggested that the two most important variables affecting the delivery time(y) are the no. of cases of product stocked (x_1) and the distance walked by the route driver (x_2). The engineer has collected 25 observations on delivery time which are shown below.

Observation number	Delivery time(minutes)	Number of cases x_1	Distance (feet) x_2
1	16.68	17	560
2	11.50	13	220
3	12.03	3	340
4	14.88	14	80
5	13.75	6	150
6	18.11	7	330
7	8.00	12	110
8	17.83	7	210
9	79.24	30	1460
10	21.50	5	605
11	40.33	16	688
12	21.00	10	215
13	13.50	4	255
14	19.75	6	462
15	24.00	19	448
16	29.00	10	776
17	15.35	6	200
18	19.00	7	132
19	9.50	3	36
20	35.10	17	770

i. Draw the scatter plot of y and x1 and y & x_2 and give your comments.

ii. Obtain correlation matrix and give your conclusions.

iii. Fit a multiple linear regression model relating delivery time, no. of cases stocked and distance travelled.

iv. Construct the ANOVA table and test for its significance of regression.

v. Calculate t statistics for testing the hypothesis $H_0:\beta_1=0$ and $H_0: \beta_2=0$. Also given 90% confidence interval and give your interpretation.

vi. Obtain point estimate of delivery time when $x_{h1}=8$ & $x_{h2}=275$.

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Exam Seat No. ____________

THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA

Date:14.12.2023 **FS MSC I Examination** **Time:**

Day: Thursday **STA2101C01 (Measure Theory)** **11.30 AM to 2.30 PM**

(4 credits)

***Instructions*: (i) Write all the answers in the answer book.**

(ii) Numbers on the right side indicates marks of the respective questions.

1 (a) Match the following (5x1=5)

(i) lim inf A_n	$(\alpha)\ \cap_{n=1}^{\infty} A_n$
(ii) lim sup A_n	$(\beta)\phi$ (null set)
(iii) lim A_n, where $A_n \uparrow$	$(\gamma)\ \{\omega \in \Omega \mid \omega \in A_n$ infinitely often $\}$
(iv) lim A_n, where $A_n \downarrow$	$(\delta)\{\ \omega \in \Omega \mid \omega \in all\ A_n$, except possibly A finite no. of them}
(v) lim A_n, where A_n's are disjoint	$(\lambda)\ \cup_{n=1}^{\infty} A_n$

(b) In each of the following questions, there are two statements α and β. Read the statements and choose the correct alternative from the following.

(A) only α is true (B) only β is true

(C) both α and β are true (D) Neither α nor β is true. (8x2=16)

(i) (α) Every subset of $\mathcal{R}$ (real line) is a Borel set.
(β) Every Borel set is a subset of $\mathcal{R}$.

(ii) (α) Every finite field is a σ –field.
(β) Every σ –field is a field.

(iii) (α) Intersection of two σ –fields is a σ –field.
(β) Union of two σ –fields is a σ –field.

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(iv) (α) A finitely additive set function is always countably additive.

(β) A countably additive set function ψ is finitely additive if $\psi(\phi) = 0$.

(v) (α) Almost everywhere (a.e.) convergence imply convergence in measure provided the measure is finite.

(β) If the measure is not finite, then a.e. convergence may or may not imply convergence in measure.

(vi) (α) A measurable function f is said to be integrable if $\int f\, d\mu$ exists.

(β) A measurable function f is said to be integrable if anyone from $\int f^{+} d\mu$ or $\int f^{-} d\mu$ is finite.

(vii) (α) A finite set function is always $\sigma -$ finite.

(β) A $\sigma -$ finite set function is always finite.

(viii) (α) If $A_n \uparrow A$, then $\mu(A_n) \uparrow \mu(A)$.

(β) If $A_n \downarrow A$, then $\mu(A_n) \downarrow \mu(A)$.

(c) Fill in the blanks (7x1=7)

(i) The smallest $\sigma -$field is ____________________.

(ii) Lebesgue measure is ________________ but Lebesgue Stiltjes(L-S) measure is ______________(choose between finite, not finite).

(iii) The Lebesgue measure of set of rationals between [2, 10] is __________.

(iv) The only functions measurable with respect to trivial $\sigma -$field are ________________________.

(v) A set function ψ is said to be absolutely continuous with respect to a measure μ, ($\psi << \mu$), if for a measurable set A, ________________.

(vi) All discrete distributions are absolutely continuous with respect to the ____________________.

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2. (a) Prove that $\liminf A_n \subset \limsup A_n$. (4)

(b) Prove that a monotone field is a σ –field. (2)

(c) Let $A_n = \left(0,\ 1 - \frac{1}{n}\right)$, if n is even and $A_n = (0,\ 1 + \frac{1}{n}]$ if n is odd. Check whether $\lim A_n$ exists or not. (4)

OR

(c) Explain how to generate Borel σ –field? (4)

3. (a) Prove that $\liminf \mu(A_n) \geq \mu(\liminf A_n)$. (5)

(b) Define Lebesgue measure (5)

OR

(b) Define Lebesgue -Stiltjes Measure. (5)

4. (a) Define (i) Convergence in Measure (ii) Convergence a.e. (4)

(b) State the three linearity properties and three order preserving properties associated with integration. (3)

(c) State and prove Monotone convergence theorem. (6)

OR

(c) (i) State Fatou's lemma. (2)

(ii) State Dominated convergence theorem. (2)

(iii) Evaluate $\lim_{n\to\infty} \int_1^2 \frac{n}{1+nx^3}\, dx$. (2)

5. (a) State (i) Lebesgue's decomposition theorem. (2)

(ii) Radon Nykodym theorem. (2)

OR

(a) State (i) Caratheodory extension theorem.(2)

(ii) Fubini's theorem. (2)

(b) Explain the concept of product measure space. (5)

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[illegible] Semester Examination

M.Sc. Statistics Semester I

Measure Theory (STA 2101C01)

Date:12.9.2023
Day: Tuesday

Time:
3.00 to 4.00 PM

Note: (i) Write all the answers in the answer book.

(ii) Numbers on the right indicates marks.

1. In each of the following questions there are two statements S_1 and S_2. Read the Statements and write the correct alternative from the following. **(6x2=12)**

(A) Both S_1 and S_2 are correct. (B) Only S_1 is correct

(C) Only S_2 is correct (D) Neither S_1 nor S_2 is correct.

(i) S_1 : Every measure is always continuous from below.

S_2 : Every measure is always continuous from above.

(ii) S_1: Every subset of real line is a Borel set.

S_2: Every Borel set is a subset of real line.

(iii) S_1: A monotone field is a σ-field.

S_2 : A σ-field is always a monotone class.

(iv) S_1: $\lim\inf \mu(A_n) \leq \mu(\lim\inf A_n)$

S_2 : $\lim\sup \mu(A_n) \geq \mu(\lim\sup A_n)$, when μ is finite.

(v) S_1: A finitely additive set function is always countably additive.

S_2: A countably additive set function ψ is finitely additive if $\psi(\phi)$=0.

(VI) S_1: A simple function is always a measurable function.

S_2: A limit of a sequence of simple functions is always a measurable function.

Na[redacted]

2. Answer the following.

(i) Let $F(x) = \begin{cases} 0 & \text{if } x < 3 \\ \frac{x}{6} - \frac{1}{8} & \text{if } 3 \leq x \leq 4 \\ 1 & \text{if } x \geq 4. \end{cases}$

Find μ_F for the following sets

(1) A={3, 4} (2) [3, 4] (3) (3, 4] (4) [3,4). (4)

(ii) Give an example of sequence of open intervals whose limit is a closed Interval. (3)

(iii) Define a σ-finite set function.(2)

(iv) State the two descriptive definitions of a measurable function. Prove their equivalence. (5)

(v) Explain in detail Lebesgue measure. (4)

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$\frac{1}{2}$

$\frac{9}{6} - \frac{1}{8}$

$\frac{1}{2}$

Date: 15/09/2023

The Maharaja Sayajirao University of Baroda
Faculty of Science, Department of Statistics
M.Sc. Semester – I (Mid-Semester Examination)

Time: 3:00 to 4:00 p.m.
Marks: 30

STA2104C04 – Decision Theory

Note: Answer to all the questions, including Q.1, must be written in the answer book provided ONLY.

Q.1 (A) Choose the correct option and write in the answer book: (1 mark each)

(1) A decision rule is mapping from

(a) Sample space to parameter space
(b) Parameter space to action space
(c) Sample space to action space
(d) None of these

(2) A randomized decision for a decision problem $(\Theta, \mathcal{a}, L)$, with sample space $\mathfrak{X}$ and decision set $\mathcal{D}$, is associated with a probability distribution over

(a) Θ (b) $\mathcal{D}$ (c) $\mathfrak{X}$ (d) none of these

(3) Which of the following can be used as a strategy for reducing the number of decision rules out of all possible decision rules?

(a) Principle of unbiasedness
(b) The Bayes Principle
(c) Principle of invariance
(d) Either (a) or (c)

(4) A decision theory problem can be referred to as a problem of point estimation when

(a) $\mathcal{a} = (-\infty, \infty)$
(b) $\mathcal{a}$ is countably infinite space
(c) $\mathcal{a}$ is finite
(d) Any of the above

(5) Let P_1 and P_2 be two randomized actions. Then which of the following will also be a randomized action?

(a) $\frac{1}{2}(P_1 + P_2)$
(b) $0.4\, P_1 + 0.6\, P_2$
(c) Both (a) and (b)
(d) $0.4\, P_1 + 0.4\, P_2$

(6) The Bayes decision rule $d(x)$ for a quadratic loss function can be obtained as

(a) $d(x) = E(X|\theta)$
(b) $d(x) = E(\theta|X = x)$
(c) $d(x) = E(X|a)$
(d) $d(x) = E(\theta|a)$

Q.1 (B) Do as directed: (2 marks each)

(1) A farmer must decide whether or not to plant his crop early. If he plants early and the monsoon occurs timely, then he will gain Rs. 50000 in extra harvest but if he plants early and the monsoon occurs late, then he will lose Rs. 20000 as the cost of reseeding. If he doesn't plant early, his gain will be Rs. 0. Describe the parameter space, action space and loss table based on this information.

(2) Explain the basic difference between randomized decision rule and behavioral decision rule.

(3) Discuss the decision theory problem as a problem of testing of hypothesis.

Q.2 Attempt Any TWO: (6 marks each)

(a) Define: statistical decision problem, randomized action.

Show that risk function $R(\theta, \delta)$ is a convex set.

(b) Define: prior distribution, Bayes risk, Bayes decision rule, minimax decision rule.

(c) Define: unbiased decision rule, least favorable prior distribution. State and prove the necessary and sufficient condition under which a prior distribution will be least favorable.

Q.3 Answer Any [illegible] marks)

(a) Let $\Theta = [illegible]$ and loss function is squared error. Suppose $(X \mid \theta) \sim$ Poisson [illegible] and the prior distribution of θ is

$$[illegible](\theta) = \frac{e^{-\theta/\beta}\, \theta^{\alpha-1}}{\Gamma\alpha\, \beta^{\alpha}} \;;\; \theta > 0 \quad \text{where } \alpha, \beta > 0$$

(i) Show that the posterior distribution of θ given $X = x$ is $G\left(\alpha + x, \frac{\beta}{\beta+1}\right)$.

(ii) Show that the Bayes rule is $d_{\alpha,\beta}(x) = \frac{\beta\,(\alpha+x)}{(\beta+1)}$.

(b) An insurance company is faced with taking one of the following 2 actions: decrease sales force by [illegible] work force. Depending upon the economy is good, mediocre or bad, [illegible] lose following amount of money:

	Decrease [illegible] force by [illegible]	Ma[illegible] force
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Suppose [illegible] variable that can be used to make a prediction about the state of economy [illegible] will predict the state of economy correctly is 0.6, whereas [illegible] indicator variable will predict the state of economy wrongly is 0.2 for each [illegible]. Prepare the risk table for this decision problem.

18/12/2023
Monday

The Maharaja Sayajirao University of Baroda

M.Sc. Statistics – Semester I
(End-Semester Examination)

Time: 11:30 to 2:30 p.m.
Marks: 70

STA2104C04 – Decision Theory

Note: All answers are to be written in the answer sheet provided.

Q.1: Choose the correct option and write it (only a/b/c/d) in the answer book [2x14 = 28]

(1) Let $'a'$ be the correct action with respect to the parameter value θ, then $L(\theta, a) =$

(a) Minimum (b) zero (c) $-\infty$ (d) 1

(2) Let P_1 and P_2 be two randomized actions. Then which of the following is true?

(a) Any linear function of P_1 and P_2 will also be a randomized action
(b) Any function of P_1 and P_2 will also be a randomized action
(c) Any convex combination of P_1 and P_2 will also be a randomized action
(d) All of these

(3) The behavioral decision rule guides the statistician with respect to

(a) How to randomize the actions at each move, after observing the outcome of the experiment
(b) How to choose the actions randomly at each move, before observing the outcome of the experiment
(c) Deciding the actions to be taken based on outcome of the experiment (fixed course of actions, without randomization)
(d) Deciding the actions to be taken, irrespective of the outcome of the experiment (fixed course of actions, without randomization)

(4) Which of the following method/s can help in ordering the decision rules, in context of obtaining the optimal decision rule?

(a) Bayes principle
(b) Minimax principle
(c) Both (a) and (b)
(d) Neither (a) nor (b)

(5) The Bayes decision rule $d(x)$ for a loss function proportional to an absolute error can be obtained as

(a) $d(x) = E(X|\theta)$
(b) $d(x) = E(\theta|X = x)$
(c) $d(x) =$ median of the distribution of $(X|\theta)$
(d) $d(x) =$ median of the distribution of $(\theta|X = x)$

(6) A decision rule δ_1 is said to be admissible if

(a) There does not exist any rule δ for which $R(\theta, \delta) \leq R(\theta, \delta_1)$ for all $\theta \in \Theta$
(b) There does not exist any rule δ for which $R(\theta, \delta) < R(\theta, \delta_1)$ for atleast one $\theta \in \Theta$
(c) There does not exist any rule better than δ_1
(d) All of the above

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US

... following statements is incorrect ?

(a) Minimal complete class of decision rules always exist
(b) If a minimal complete class exists, it will consist of only admissible rules
(c) A class of admissible rules will be a subset of complete class of decision rules
(d) No proper subset of minimal complete class of rules will be complete.

(8) In usual notations, $\underline{V} =$

(a) $\sup_{\delta \in D^*} \inf_{\tau \in \Theta^*} r(\tau, \delta)$
(b) $\sup_{\tau \in \Theta^*} \inf_{\delta \in D^*} r(\tau, \delta)$
(c) $\inf_{\tau \in \Theta^*} \sup_{\delta \in D^*} r(\tau, \delta)$
(d) $\inf_{\tau \in \Theta^*} \sup_{\delta \in D^*} r(\tau, \delta)$

(9) A minimax decision rule δ will be Bayes with respect to prior distribution τ if

(a) τ is least favorable prior distribution
(b) Minimax theorem holds
(c) δ is Bayes rule with respect to τ
(d) Both (a) and (b) hold

(10) Which of the following should hold in order that a Bayes rule with respect to prior distribution $(p_1, p_2, \ldots, p_k)$ exist ?

(a) $p_j > 0 \; for \; all \; j$
(b) Risk set is bounded from below
(c) Parameter space is closed from below
(d) All of these

(11) Which of the following statement is correct?

(a) If a statistic T is boundedly complete, then it is also complete
(b) For a boundedly complete sufficient statistic T, for every real-valued bounded function g, $E_\theta[g(T)] = 0 \Rightarrow g(t) = 0 \; for \; all \; t$
(c) Both (a) and (b)
(d) Neither (a) nor (b)

(12) For a random sample $X_1, X_2, \ldots, X_n$ from $U(0, \theta)$, which of the following is a complete sufficient statistic ?

(a) $\sum X_i$ (b) $Min\, X_i$ (c) $Max\, X_i$ (d) both (b) and (c)

(13) For a decision problem, the principle of invariance involves transformation of

(a) Sample space
(b) Parameter space
(c) Action space
(d) All of these

(14) Let S denote the risk set and S_0 denote the set of all risk points corresponding to nonrandomized actions. Then which of the following statements is incorrect ?

(a) S is a convex subset of Euclidian k-space, where k is the number of parameters
(b) S_0 is a convex hull of S
(c) S is the intersection of all convex sets containing S_0
(d) All of the above are correct

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Q.2 Attempt Any FOUR: (16 marks)

(a) (i) Define a decision problem and compare it with a game theory problem.

(ii) Compare between a nonrandomized and randomized decision rules.

(b) Define behavioral decision rule, Bayes rule, minimax rule, prior distribution.

(c) Prove that, a decision rule δ_0 is minimax if and only if

$$R(\theta', \delta_0) \leq \sup_{\theta \in \Theta} R(\theta, \delta) \ \forall \ \theta' \in \Theta \ \& \ \delta \in D^*$$

(d) Consider the following loss table for some decision problem:

θ \ a	a_1	a_2	a_3	a_4
θ_1	3	2	5	4
θ_2	2	-2	3	-1

Construct a convex polyhedron corresponding to each pure action. Obtain the minimax action and the corresponding value of expected loss, graphically.

(e) (i) Prove that, the risk set is a convex set.

(ii) Define: limit of Bayes rule, generalized Bayes rule.

Q.3 Attempt Any FOUR: (16 marks)

(a) Define: equivalent decision rules, essentially complete class, complete class, minimal complete class.

(b) Prove that, if $\mathbb{C}$ is complete and contains no proper essentially complete subset, then $\mathbb{C}$ is minimal complete and minimal essentially complete.

(c) Define support of a distribution. If δ_0 is Bayes rule with respect to a probability distribution τ on R for which $r(\tau, \delta_0)$ is finite, and if support of τ is R, then prove that δ_0 is admissible.

(d) Prove that, for a decision problem with finite parameter space, if the risk set is bounded from below then maximin value and minimax value will coincide.

(e) Suppose the probability of occurrence of an event A is θ, for a problem in which $\Theta = \left\{\frac{1}{4}, \frac{3}{4}\right\}, a = R, \ L(\theta, a) = (\theta - a)^2$.Obtain a minimax decision rule.

Q.4 Attempt Any TWO: (10 marks)

(a) Define complete sufficient statistic, minimal complete sufficient statistic. Obtain complete sufficient for θ where $X_j \sim B(m, \theta); j = 1, 2, \ldots, n$.

(b) Define: Invariant prior distributions, Invariance of a statistical decision problem, Invariance of a randomized decision rule, Invariance of a behavioral decision rule.

(c) Define invariance of a family of distributions. Suppose $X \sim N(\theta, 1), \Theta = a = R, L(\theta, a) = (\theta - a)^2$. Show that $N(\theta, 1)$ is invariant under the change of origin.

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