~~(b) Should investor purchase economic forecast report for $5000?~~

(a) Which investment option should be chosen without buying the forecast.

Sol^n Given :-

Investor has 3 options

(i) High risk stock portfolio

(II) Government funds

(III) Real estate

options	profit (good market)	loss (Bad market)
High risk Portfolio	$ 40,000	- $10,000
Gov. funds	$6,000	$6,000
Real estate	$ 20,000	+ $ 5,000

(i) High risk stock portfolio :-

P(0.6) Good Market → 40% return → $ 40,000

P(0.4) Bad Market → 10% loss → - $ 10,000

(II) Gov. Bonds → 6% return fixed

Good Market → $ 6000

Bad Market → $ 6000

(III) Real estate :-
Good Market :- 20% return → $+20,000 → P(0.7)
Bad Market :- 5% return → + $ 5,000 → P(0.3)

(i) EMV of high-risk stock

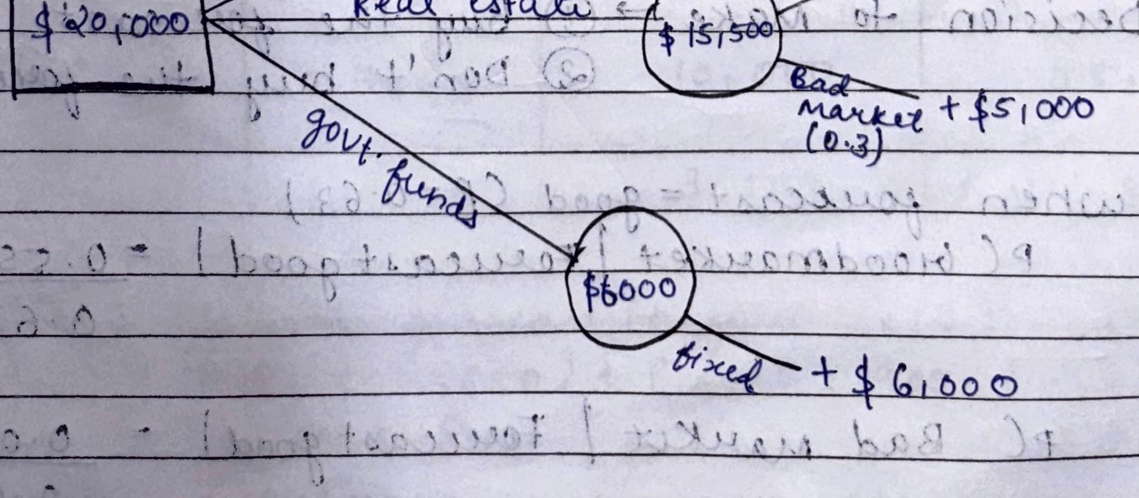
⇒ $(0.6)(40,000) + (0.4)(-10,000)$
$= \$20,000$

(II) EMV of real estate

⇒ $(0.7)(20,000) + (0.3)(5,000)$
⇒ $15,500

(III) EMV of govt. funds
⇒ $ 6000 (fixed)

Decision to be made :- ① High risk stock ② govt. fund
③ Real estate.



Conclusion :-

Based on expected Monetary values (EMV)
High risk stock portfolio → \$ 20,000
Govt. fund → \$ 6000
real estate fund → \$ 15,500

The investor should choose the high risk stock portfolio, as it has the highest EMV without purchasing the forecast.

(b) Should the investor purchase the forecast before deciding?

→ stock portfolio
- good market = +40,000
- Bad Market = - 10,000

→ Real estate
- good market = +20,000
- Bad Market = +5,000

Decision to make → ① Buy the forecast
② Don't buy the forecast

① when forecast = good ($P = 0.63$)

$P(\text{Good market} \mid \text{Forecast good}) = \dfrac{0.55}{0.63} = 0$

$P(\text{Bad Market} \mid \text{Forecast good}) = \dfrac{0.08}{0.63}$

(2) When forecast = Bad (P = 0.37)

P(Good market | Bad forecast) $= \frac{0.10}{0.37} = 0.270$

P(Bad market | Bad forecast) $= \frac{0.27}{0.37} = 0.730$

Pay-off table (forecast good)

Market	probability	Stock	Real estate
Good	0.873	+40,000	+20,000
Bad	0.127	−10,000	+5,000
		+33650	+18095.

Pay-off table [forecast Bad]

Market	prob.	Stock	Real estate
Good	0.270	+40,000	+20,000
Bad	0.730	−10,000	+5,000
		3,500	$9050

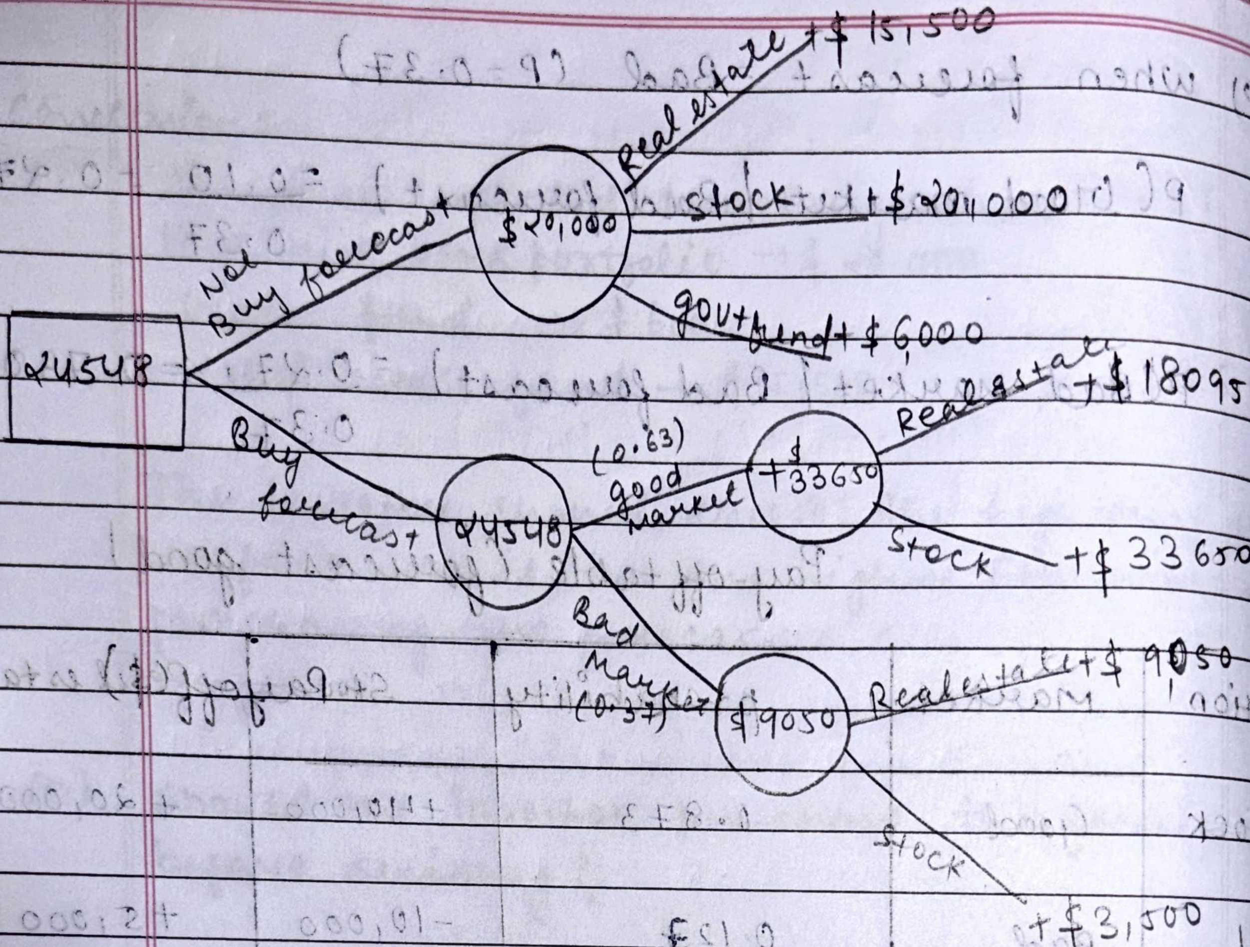
∴ Final EV with forecast (before cost)

$= (0.63 \times 33650) + (0.37 \times 9050)$

$= 24{,}548$

→ After substracting forecast cost,

Net = \$ 24,548 − \$ 5000 = \$ 19,548.



Conclusion:-
The investor should not purchase the economic forecast because it reduces their expected Monetary value from \$ 20,000 to \$ 19,548 even after using the forecast effectively.

(C) What is the Expected value of forecast?

⇒ EV(with forecast before cost) − EV without forecast

⇒ 24,548 − 20,000
⇒ \$ 4,548

Conclusion:- The Expected Value of the forecast is \$ 4,548.

This means the forecast adds informational value but not enough to justify its cost of \$ 5,000.

19/7/2025.

Practical - 2

Two-person Game

Q1. An company named Fresh diary Pvt. Ltd. produces pacakets of fresh milkshakes that are distributed to supermarkets every morning. The milkshake pacakets are perishable and cannot be stored for the next day. The demand fluctuates daily and can be 50, 100, 150 or 200 pacakets per day. The company earns a profit of RS. 12 per packet. (due to spoilage). Additionally, if the company fails to meet demand, there is a penalty of Rs. 5 per unfulfilled pacaket (loss of customer trust and emergency procurement). The production manager must decide how many pacakets to produce daily. Prepare a loss table for the given problem.

Soln:- Here, producer (i.e. company) is statistician.
∴ Demand is Nature.

Parameter space → $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$

$= \{50, 100, 150, 200\} \in \Theta$

θ_1 = 50 pacakets are demanded per day.
θ_2 = 100 packets are demanded per day.
θ_3 = 150 " " " "
θ_4 = 200 " " " "

Action space $= \mathcal{A} = \{a_1, a_2, a_3, a_4\}$

$= \{50, 100, 150, 200\}$

$a_1 = 50$ packets are produced per day.
$a_2 = 100$ " " " "
$a_3 = 150$ " " " "
$a_4 = 200$ " " " "

profit → ₹20 per packet
loss → ₹12 (packet spoiled)
penalty → ₹5 per unfulfilled packet.

LOSS TABLE

$\Theta \backslash \mathcal{A}$	$a_1 = 50$	$a_2 = 100$	$a_3 = 150$	$a_4 = 200$
$\theta_1 = 50$	$50 \times 20 = -1000$	$(-50 \times 20 + 50 \times 12)$	$(-50 \times 20 + 100 \times 12)$	$(-50 \times 20 + 150 \times 12)$
$\theta_2 = 100$	$(50 \times 20 + 50 \times 5) = -750$	$-(100 \times 20)$	$(-100 \times 20) + 50 \times 12$	$(-100 \times 20) + (-100 \times 12)$
$\theta_3 = 150$	$(50 \times 20) - (100 \times 5) = -500$	$-(100 \times 20 + 50 \times 5)$	(-150×20)	$(-150 \times 20) + (50 \times 12)$
$\theta_4 = 200$	$(50 \times 20) - (150 \times 5) = -250$	$-(100 \times 20 + 100 \times 5)$	$(-150 \times 20 + 50 \times 5)$	(-200×20)

$\boxed{\text{LOSS TABLE}}$

$\Theta \backslash \mathcal{A}$	$a_1 = 150$	$a_2 = 100$	$a_3 = 150$	$a_4 = 200$
$\theta_1 = 50$	-1000	-400	200	800
$\theta_2 = 100$	-750	-2000	-1400	-800
$\theta_3 = 150$	-500	-1750	-3000	-2400
$\theta_4 = 200$	-250	-1500	-2750	-4000

Q2 A company smart cool Pvt. Ltd. Manufactures air conditioners (ACs) for residential buildings and has categorized its customers based on their peak cooling requirements. Silver Residency generally requires 1.5 Ton ACs and green view Apartment generally requires 2 Ton ACs. However, customers often do not report their Actual requirement truthfully in surveys. A study revealed that in silver residency, 60% of customers, report honestly, while 40% of overstate their need by 0.5 ton. In green view apartment, 50% of customers report honestly, while the remaining 50% understate their requirement by 0.5 ton. The company has to decide which AC capacity to install based on the reported requirements to minimise losses arising from either installing over capacity ACs (leading to higher investment cost) or under capacity - ACs (leading to customer complaints and emergency replacement). The loss is calculated as follows: if the company install a higher capacity than required, it incurs a loss of ₹5000 per 0.5 ton Extra installed (over-capacity cost) whereas installing a lower capacity result in a loss of ₹2,000 per 0.5 ton short (due to customer complaints and emergency replacements). There is no loss if the correct capacity is installed.

(i) write down action space and parameter space.

classmate

Date

Page

sol^n :- Action space = $\mathcal{a} = \{a_1, a_2\}$

Parameter space = Θ = $\{\theta_1, \theta_2\}$

$\theta_1 = 1.5$ ton AC is to be installed

$\theta_2 = 2$ ton AC " " "

$a_1 = 1.5$ ton AC is required to install

$a_2 = 2.0$ ton AC " " " .

Loss → loss of ₹ 5,000 per 0.5 ton extra installed

→ loss of ₹ 2,000 per 0.5 ton short

→ In silver residency

truth → 60% = 0.6

False → 40% = 0.4.

→ In Green View Apartments

truth :- 50% = 0.5

false :- 50% = 0.5.

Action space :- Installations.

Parameter space - demanded installments

	$\theta_1^{(S.R)}$	$\theta_2^{(G.V)}$
(S.R) 1.5	0.6	0.5
(G.V) 2	0.4	0.5

LOSS TABLE.

	$a_1 = 1.5$	$a_2 = 2$
$\theta_1 = 1.5$	0	5000
$\theta_2 = 2$	2000	0

Non-randomised decision table.

x	d_1	d_2	d_3	d_4
x_1	a_1	a_1	a_2	a_2
x_2	a_1	a_2	a_1	a_2

Loss $L(\theta, d)$

	d_1		d_2		d_3		d_4		$P_\theta(x)$	
x	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2
x_1	0	2000	0	2000	5000	0	5000	0	0.6	0.5
x_2	0	2000	5000	0	0	2000	5000	0	0.4	0.5
	0	2000	2000	1000	3000	1000	5000	0		

RISK TABLE

$\Theta \backslash D$	d_1	d_2	d_3	d_4
$\theta_1 = 1.5$	0	2000	3000	5000
$\theta_2 = 2$	2000	1000	1000	0

overall expected loss under.

$\boxed{E(d_1) = 1.5 \times 0 + 2 \times 2000 = 4000}$ → Minimum.

$E(d_2) = 2000 \times 1.5 + 1000 \times 2 = 5000$

$E(d_3) = 1.5 \times 3000 + 2 \times 1000 = 6500$

$E(d_4) = 1.5 \times 5000 + 0 \times 2 = 7500$

Therefore, d_1 is best decision to take.

Q3: Given that a decision problem with parameter state $\Theta = \{3/4, 1/4\}$ and loss $L(\theta_1, a_1) = L(\theta_2, a_2) = 0$, $L(\theta_1, a_2) = L(\theta_2, a_1) = 1$
Suppose X is an exponential distⁿ with mean θ and consider sample space $= \{(0,1), (1,\infty)\}$.

(i) Find best of all non randomised decision rules of the statistician.

(ii) Prepare a risk table.

Given :-
Parameter space $\Theta = \left\{\frac{3}{4}, \frac{1}{4}\right\}$

Loss function :-

$$L(\theta, a_1) = \begin{cases} 0 & \text{if } \theta = 3/4 \\ 1 & \text{if } \theta = 1/4 \end{cases}$$

$$L(\theta, a_2) = \begin{cases} 1 & \text{if } \theta = 3/4 \\ 0 & \text{if } \theta = 1/4. \end{cases}$$

Sample space $X = \{0, 1\}$
$X \sim$ Exponential (θ)

(i) Since, sample space is $\{0,1\}$, each non randomised decision rule maps 0 and 1 to an action $\{a_1, a_2\}$.

Hence, we have 4 possible non-randomized decision rules:

x	d_1	d_2	d_3	d_4
x_1	a_1	a_1	a_2	a_2
x_2	a_1	a_2	a_1	a_2

We calculate Risk function as

$$R(\theta, d) = \sum_x L(\theta, d) \cdot P(x)$$

where,

when $\theta = \frac{3}{4}$

$$P(0) = \int_0 \frac{3}{4} e^{-\frac{3}{4}x} dx = 1 - e^{-3/4} = 0.5276$$

$$P(1) = e^{-3/4} = 0.4724$$

when $\theta = \frac{1}{4}$

$$P(0) = \int_0 \frac{1}{4} e^{-\frac{x}{4}} dx = 1 - e^{-1/4} = 0.2212$$

$$P(1) = e^{-1/4} = 0.7788$$

Now, calculate risk values for each decision rule.

$$R\left(\frac{3}{4}, d_1\right) = 0 \times 0.5276 + 0 \times 0.4724 = 0$$

$$R\left(\frac{1}{4}, d_1\right) = 1 \times 0.2212 + 1 \times 0.7788 = 1$$

$R\left(\frac{3}{4}, d_2\right) = 0 \times 0.5276 + 1 \times 0.4724$
$= 0.4724$

$R\left(\frac{1}{4}, d_2\right) = 1 \times 0.2212 + 0 \times 0.7788$
$= 0.2212$

$R\left(\frac{3}{4}, d_3\right) = 0 \times 0.4724 + 1 \times 0.5276$
$= 0.5276$

$R\left(\frac{1}{4}, d_3\right) = 0 \times 0.2212 + 1 \times 0.7788$
$= 0.7788$

$R\left(\frac{3}{4}, d_4\right) = 1 \times 0.5276 + 1 \times 0.4724$
$= 1$

$R\left(\frac{1}{4}, d_4\right) = 0 \times 0.2212 + 0 \times 0.7788$
$= 0$

(ii) $\boxed{\text{RISK TABLE}}$

$\Theta \backslash D$	d_1	d_2	d_3	d_4
$\theta_1 = 3/4$	0	0.4724	0.5276	1
$\theta_2 = 1/4$	1	0.2212	0.7788	0

Best non-randomised decision rule :-

Max risk of $d_1 = 1$

" " " $d_2 = 0.4724$

" " " $d_3 = 0.7788$

" " " $d_4 = 1$

Thus, best rule is d_2.

Q4. The management of Royal Stay hotels, a luxury Hotel chain, needs to decide how many rooms to keep ready every weekend, considering fluctuating customer demand. preparing a room in advance involves cleaning, decorating and staff allocation which incurs a fixed cost. If a prepared room remains unoccupied, it leads to loss of ₹ 1,500 per room due to wasted preparation on the other hand if the hotel fails to prepare enough rooms and additional guests arrive, it results in a penalty of ₹ 1,000 per unprepared room due to emergency arrangements and loss of reputation. The possible weekend demand for rooms can be 20, 40, or 60 rooms. The Hotel earns a profit of ₹ 3,000 per room occupied The Manager has three alternative actions to choose from prepare 20, 40 rooms or 60 rooms. According to local guide, the probabilities of demand are 0.8 for 20 room 0.5 for 40 room and 0.3 for 60 rooms.

(i) prepare loss table for given problem.
(ii) prepare risk table considering only five decision rules.

Sol^n :- $\mathfrak{a}$ → action space = No. of rooms to prepare
= {20, 40, 60}.

(H) → parameter space ⇒ Demand of rooms
= { 20, 40, 60 }.

→ profit for occupied room = ₹ 3000

→ Loss if prepared room remains unoccupied = ₹1500

→ loss if guest arrives and room is not ready (emergency) = ₹2000

→ probabilities

P(Demand = 20) = 0.2

P(Demand = 40) = 0.5

P(Demand = 60) = 0.3

(i)

60,000 - 30,000 = 30,000

LOSS TABLE

H \ a	prepare 20	prepare 40	prepare 60
demand 20	0	₹ 40,000	₹ 80,000
demand 40	3000 × 20 - (20 × 1500) = ₹ 30,000	0	₹ 40,000
demand 60	= 3000 × 20 - (40 × 1500) ₹ 60,000	₹ 30,000	0

loss table

H \ a	prepare 20	prepare 40	prepare 60
demand 20	0	(20 × 3000) - (20 × 1500)	(20 × 3000) - (40 × 1500)
40	(20 × 3000) - (20 × 2000)	0	(40 × 3000) - (20 × 1500)
60	(20 × 3000) - (40 × 2000)	(40 × 3000) - (20 × 2000)	0

H \ a	20	40	60
20	0	₹ 30,000	₹ 60,000
40	₹ 40,000	0	₹ 30,000
60	₹ 80,000	₹ 40,000	0

(II)

Risk table

Risk for prepare 20 rooms = (0.2 × 0) + (0.5 × 40,000) + (0.3 × 80,000) = ₹44,000.

Risk for prepare 40 rooms

⇒ (0.2 × 30,000) + (0.5 × 0) + (0.3 × 40,000)

= ₹18,000

Risk for prepare 60 rooms

= (0.2 × 60,000) + (0.5 × 30,000) + (0 × 0.3)

= ₹ 27,000

2/08/2025.

Practical - 3.

Q1.

Soln :-

Loss table

H \ a	a_1	a_2
θ_1	1	3
θ_2	1	0

$P(\theta_1 | x_1) = 1/5$ $\quad$ $P(\theta_1 | x_2) = 4/5$

$P(\theta_2 | x_1) = 1/4$ $\quad$ $P(\theta_2 | x_2) = 3/4$

Step 1 :- No. of deterministic rule = $a^{x} = 2^2 = 4$.

Step 2 :- Decision rule table

H \ D	d_1	d_2	d_3	d_4
x_1	a_1	a_1	a_2	a_2
x_2	a_1	a_2	a_1	a_2

Step 3 :- Loss table $L(\theta, d)$

$x \backslash D$	$P_\theta(x)$			d_1		d_2		d_3		d_4	
	θ_1	θ_2		θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2
x_1	1/5	1/4		1	1	1	1	3	0	3	0
x_2	4/5	3/4		1	1	3	0	1	1	3	0
				1	1	3/5	1/4	7/5	3/4	3	0

Step 4 :- RISK table $R(\theta, d)$

$\Theta \backslash D$	d_1	d_2	d_3	d_4
θ_1	1	13/5	7/5	3
θ_2	1	1/4	3/4	0

let $S_1 = [1/4, 3/4, 0, 0]$

$$R(\theta_1, \delta) = \frac{1}{4}(1) + \frac{3}{4}\left(\frac{13}{5}\right) + 0 + 0$$

$$= \frac{1}{4} + \frac{39}{20} = 2.2$$

$$R(\theta_2, \delta) = \frac{1}{4}(1) + \frac{3}{4}\left(\frac{1}{4}\right) + 0 + 0$$

$$= 0.4375$$

Q 2

(i) No. of deterministic rule $= a^x = 4^3 = 64.$

(ii)

sol^n $Dim(D^*) = m^n - 1$
$= 64 - 1 = 63.$

(iii) $Dim(D) = n(m-1) = 3(4-1)$
$= 9.$

(IV)

sol^n D^* is higher dimensional.

Q3.

sol^n:- $P(\theta_1|x_1) = 0.6$ $\quad P(\theta_2|x_1) = 0.5$
$P(\theta_1|x_2) = 0.4$ $\quad P(\theta_2|x_1) = 0.5$

LOSS TABLE

Θ \ a	a_1	a_2
θ_1	0	5000
θ_2	2000	0

decision table

x \ D	d_1	d_2	d_3	d_4
x_1	a_1	a_1	a_2	a_2
x_2	a_1	a_2	a_1	a_2

Loss $L(\theta, d)$

x \ D	$P_\theta(x)$		d_1		d_2		d_3		d_4	
	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2
x_1	0.6	0.5	0	2000	0	2000	5000	0	5000	0
x_2	0.4	0.5	0	2000	5000	0	0	2000	5000	0

RISK table

Θ \ D	d_1	d_2	d_3	d_4
θ_1	0	2000	3000	5000
θ_2	2000	1000	1000	0

$\delta = \{1/3, 1/6, 1/4, 1/4\}$

$R(\theta_1|\delta) = 1/3(0) + 2000(1/6) + 3000(1/4) + 5000(1/4)$
$= 2333.33$

$R(\theta_2|\delta) = 2000(1/3) + 1000(1/6) + 1000(1/4) + 1/4(0)$
$= 1083.33$

Q4.

soln:

$\Theta = \{\theta_1, \theta_2\}$ $\quad$ $x = \{x_1, x_2\}$

$A = \{a_1, a_2\}$

$P(\theta_1 | x_1) = 0.23$ $\quad$ $P(\theta_1 | x_2) = 0.77$

$P(\theta_2 | x_1) = 0.7$ $\quad$ $P(\theta_2 | x_2) = 0.3$

x \ D	$P_\theta(x)$ θ_1	$P_\theta(x)$ θ_2	d_1 θ_1	d_1 θ_2	d_2 θ_1	d_2 θ_2	d_3 θ_1	d_3 θ_2	d_4 θ_1	d_4 θ_2
x_1	0.23	0.7	0	3	0	3	2	1	2	1
x_2	0.77	0.3	0	3	2	1	0	3	2	1
			0	3	1.54	2.4	0.46	1.6	2	1

H \ D	d_1	d_2	d_3	d_4
θ_1	0	1.54	0.46	2
θ_2	3	2.4	1.6	1

$S_1 = \{0.2, 0.7, 0.1, 0\}$

$R(\theta_1, S_1) = 0(0.2) + 1.54(0.7) + 0.46(0.1) + 0(2)$
$= 1.124$

$R(\theta_2, S_1) = 3(0.2) + 2.4(0.7) + 1.6(0.1) + 1(0)$
$= 2.44.$

$S_2 = \{0.3, 0.5, 0.2, 0\}$

$R(\theta_1, S_2) = 0.862.$ $\quad$ $R(\theta_2, S_2) = 2.42.$

$S_3 = \{0.5, 0.5, 0, 0\}$

$R(\theta_1, S_3) = 0.77$ $\quad$ $R(\theta_2, S_3) = 2.7$

$\delta_4 = \{0.6, 0.4, 0, 0\}$

$R(\theta_1, \delta_4) = 0.616$

$R(\theta_2, \delta_4) = 2.76$

$\delta_5 = \{0.2, 0.8, 0, 0\}$

$R(\theta_1, \delta_5) = 1.232$ $\quad$ $R(\theta_2, \delta_5) = 2.52$